Inflation from the most general $f(R)$ theory

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- GR - super successful, consistent with the observational data, gravitational waves detection etc. Could we just leave GR as it is? Sure! But modifications of GR can be responsible for dark energy and inflation.

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- Can we still obtain successful inflation with all possible higher order terms?

$f(R)$ as a Brans-Dicke theory

The action of $f(R)$ theory

$$\frac{1}{2}\int d^4x\sqrt{-g}\ R \ \rightarrow \ \frac{1}{2}\int d^4x\sqrt{-g}\ f(R)\,. \tag{1}$$

The action of $f(R)$ can be rewritten as

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\varphi R - U(\varphi) + \mathcal{L}_{\rm m}\right]\ , \tag{2}$$

where

$$\varphi = F(R)\ , \qquad U(\varphi) = \frac{1}{2}(RF - f)$$

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On the other hand the action of Brans-Dicke theory is

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\varphi R-\frac{\omega_{\rm BD}}{2\varphi}\left(\nabla\varphi\right)^2-U(\varphi)+\mathcal{L}_{\rm m}\right] \tag{3}$$

$f(R)$ is a Brans-Dick theory with $\omega_{\rm BD}=0$

From Brans-Dicke to Einstein frame

The gravitational part of the action may be canonical after transformation to Einstein frame

$$\tilde{g}_{\mu\nu} = \varphi g_{\mu\nu} \tag{4}$$

which gives the action of the form of

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2}\tilde{R} - \frac{3}{4}\left(\frac{\tilde{\nabla}\varphi}{\varphi}\right)^2 - \frac{U(\varphi)}{\varphi^2} \right] , \tag{5}$$

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where $V = U/\varphi^2 \, (\varphi = \varphi(\phi))$. BD = GR + scalar field

The Starobinsky model and saddle-point inflation

The oldest and one of the most successful inflationary models

$$f(R) = R + \frac{R^2}{6M^2} \quad \Rightarrow \quad V(\phi) = \frac{3}{4}M^2\left(1 - e^{-\sqrt{2/3}\phi}\right)^2 \tag{6}$$

The model is great because of small r and non-gaussianities, which are pefectly consistent with the data. Nevertheless higher order corrections of the form

$$\sum_{n=3}^{\infty} \alpha_n \frac{R^n}{M^{2(n-1)}} \tag{7}$$

may spoil the plateau. How to get inflation without the R^2 domination? We need a small, but very flat part of the Einstein frame potential, i.e. we need the saddle-point inflation. Problem: Saddle point inflation doesn't fit the data!

Saddle-point inflation with vanishing k derivatives

In general one can define the saddle point with first k derivatives vanishing. In that case $1 - n_s \simeq \frac{2k}{N_\star(k-1)}$ when freeze-out of primordial inhomogeneities happens close to the saddle point. Thus, for sufficiently big k one can fit the Planck data!

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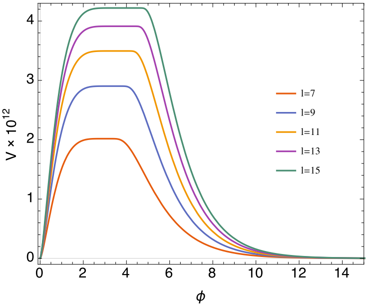
$$f(R) = R + \alpha_2 \frac{R^2}{M^2} + \sum_{n=3}^{l} \alpha_n \frac{R^n}{M^{2(n-1)}} \,, \qquad (8)$$

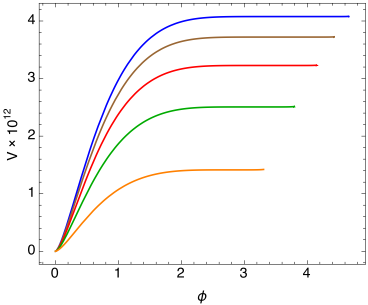
We want first $l-2$ derivatives to vanish, which gives

$$R_s = \sqrt{p}\, M^2 \,, \qquad \alpha_n = (-1)^{n-1} \frac{2(l-3)!}{(l-n)!(n-1)!} p^{\frac{3-n}{2}} \qquad (9)$$

where $p := \sqrt{(l-1)\,(l/2-1)}$. You can sum it up and obtain the analytical form of $f(R)$.

Odd and even l





Odd (even) l gives a plateau with a local maximum (saddle point). The scale of inflation and length of the plateau grows with l.

The $l \to \infty$ limit

For $l \to \infty$ one finds

$$f(R) = R\left(e^{-\frac{\sqrt{2}R}{M^2}} + \frac{\sqrt{2}+\alpha_2}{M^2}R\right). \qquad (10)$$

The α_2 tell us about the contribution of R^2 to $f(R)$. Even for $\alpha_2 = 0$ this guy fits the data perfectly well. What is the problem?

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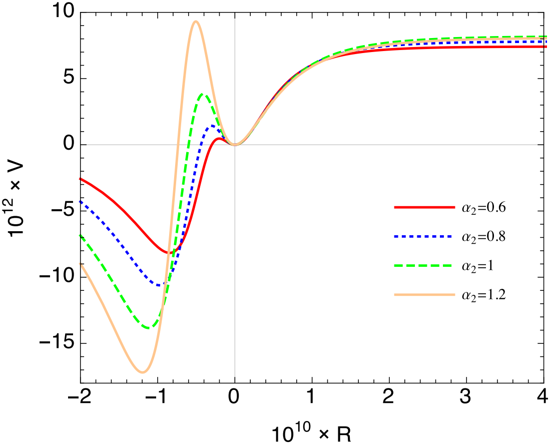
- The saddle point moves to infinity
- Some contribution of the R^2 term is needed ($\alpha_2 > 0$) in order to obtain a GR vacuum.

Einstein frame potential



The GR minimum at $R = 0$ is (surprisingly) stable on both, classical and quantum level!

The stability of the GR vacuum

For any finite α_2 one obtains two vacua - in $R = 0$ (GR) and in some $R < 0$ (anti de-Sitter - the true vacuum). In theory one could reach the true vacuum by

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- Classical evolution of the field - can we pass the local maximum? The EOM of R is following

$$\ddot{R} + 3\frac{\dot{a}}{a}\dot{R} = \frac{1}{3F'}(2f - RF - F''\dot{R}^2)\,. \qquad (11)$$

At the local maximum one finds $F' = 0$ and therefore one requires $\dot{R}^2 = (2f - RF)/F''$. LHS is always positive, but RHS is negative! The only solution, which can pass the local maximum is imaginary and therefore unphysical. this can be avoided for $\alpha_2 \gtrsim 0.7$

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- Quantum tunnelling from GR to anti de-Sitter - two trajectories exist, but none of them gives tunnelling.

The same idea for scalar fields

One could look for similar conditions for the flatness of a general potential in the minimally coupled scalar theory. Let's assume that the universe if filled with a homogeneous scalar field φ with the potential

$$V(\varphi) = V(f(\varphi))\,, \tag{12}$$

where

$$f(\varphi) = \xi \sum_{k=1}^{n} \lambda_k \varphi^k\,. \tag{13}$$

The requirement of vanishing of the first $n-1$ derivatives of V is equivalent to vanishing derivatives of f, which leads to $f(\varphi)$ of the form

$$f(\varphi) = \frac{\xi}{n} (n\,\lambda)^{\frac{-1}{n-1}} \left(1 + \left((n\,\lambda)^{\frac{1}{n-1}}\,\varphi - 1\right)^n\right)\,, \tag{14}$$

where $\lambda := \lambda_n$.

Flat potentials for finite n

Such $f(\varphi)$ has a stationary point at $\varphi_s = (n\,\lambda)^{\frac{-1}{n-1}}$, which is also a stationary point of V. Practically any V as a function of f will give you an inflationary potential!

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The $f(\varphi)$ simplifies in two cases

$$f(\varphi) = \frac{\xi}{n}\left(1-(1-\varphi)^n\right) \qquad \text{for} \qquad \lambda = \lambda_1\,, \tag{15}$$

$$f(\varphi) = \left(1-\left(1-\frac{\xi}{n}\varphi\right)^n\right) \qquad \text{for} \qquad \lambda = \lambda_2\,, \tag{16}$$

where

$$\lambda_1 := 1/n \quad \text{and} \quad \lambda_2 := \frac{1}{\xi}(\xi/n)^n \tag{17}$$

The λ_1 seems to be troublesome from the point of view of perturbativity, but do we really know what does it mean in this case?

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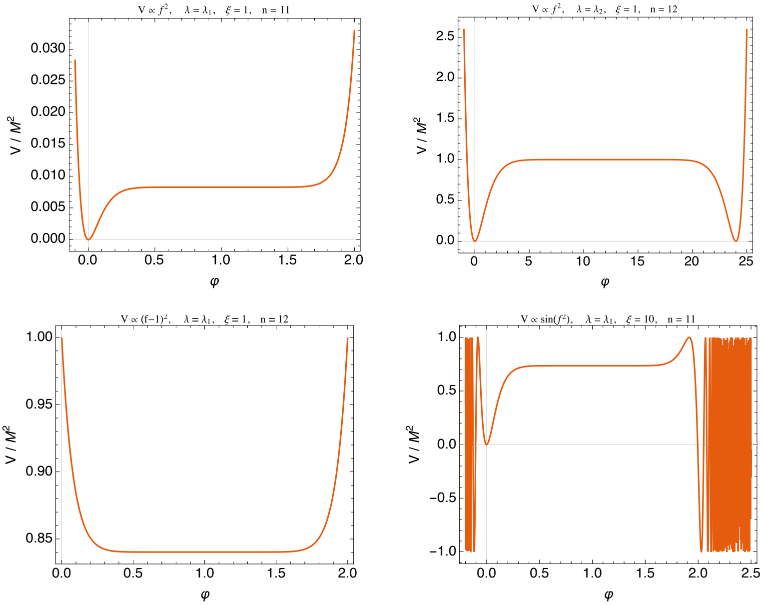

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$$f(\varphi) = 1 - e^{-\xi\varphi} \tag{18}$$

$V \propto f^2$ gives the Brans-Dicke generalisation of the Starobinsky inflation!

$$\omega_{\text{BD}} = \frac{1}{\xi^2} - \frac{3}{2} \tag{19}$$

This means that the Starobinsky potential is objectively one of the very flattest that one can get. This is confirmed by analysing both $f(R)$ and regular, minimally coupled scalar theory.

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Another application of $f(\varphi)$ is induced inflation, which enable multi-phase inflation, arbitrarily low scale and solving the problem of initial conditions.

Flat potentials and α-attractors

α-attractors - theory with kinetic term with a pole

$$\frac{(\partial\psi)^2}{\left(1-\frac{\psi^2}{6\alpha^2}\right)^2}\,. \tag{20}$$

If you'd re-define the field to obtain a canonical kinetic term it would appear, that any potential $V(\psi)$ is stretched around the pole, just like for $V(f(\varphi))$. In fact you can use $f(\varphi)$ as a scalar field and find a direct relation between f and ψ.

$$\psi(f)=\frac{\sqrt{6\alpha}\left((6\alpha(1-f)\xi)^{\sqrt{\frac{2}{3\alpha}}\xi}-1\right)}{(6\alpha(1-f)\xi)^{\sqrt{\frac{2}{3\alpha}}\xi}+1}\,. \tag{21}$$

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- How to get inflationary Einstein frame potential? Just one stationary point needed!
- Consistency with the PLANCK data, no problems with stability
- The idea works for the minimally coupled scalar field as well! The Starobinsky model is one of the examples!
- Total equivalence between the scalar field picture and α-attractors