F-Theory GUTs and Discrete Symmetry

PLANCK 2016 - Valencia

Andrew K Meadowcroft

University of Southampton

May 26th 2016

F-Theory GUTs and Discrete Symmetry

- **Phenomenological implications of a minimal F-theory GUT with discrete symmetry**
  JHEP 1510 (2015) 041-*Karozas, King, Leontaris, AKM*

- **MSSM from F-theory SU**(5) **with Klein Monodromy**
  Geometric R-parity and application to achieve an SU(5) MSSM
  [arXiv:1512.09148]-*M.Crispin-Romão, Karozas, King, Leontaris, AKM*

- **Diphoton excess from** E_6 **in F-theory GUTs**
  Offers explanation for the 750 GeV bump using an E_6 inspired model from earlier work Phys. Lett. B **757** (2016) 73-*Karozas, King, Leontaris, AKM*

What is F-theory?

F-Theory and Symmetries

F-Theory is a 12 Dimensional formulation of Type IIB string theory:

F-Theory and Symmetries

F-Theory is a 12 Dimensional formulation of Type IIB string theory:

- 4 Large spacetime dimensions
- 6 Compactified internal dimensions

F-Theory and Symmetries

F-Theory is a 12 Dimensional formulation of Type IIB string theory:

- 4 Large spacetime dimensions
- 6 Compactified internal dimensions
- 2 Dimensions due to elliptic fibration

F-Theory and Symmetries

F-Theory is a 12 Dimensional formulation of Type IIB string theory:

- 4 Large spacetime dimensions
- 6 Compactified internal dimensions
- 2 Dimensions due to elliptic fibration

The elliptic fibration of the space codes the Gauge Group supported by 7-branes in the internal manifold - Kodaira.

The maximum symmetry enhancement is E_8, which acts as a parent symmetry for any GUT group . . .

F-Theory and Symmetries

F-Theory is a 12 Dimensional formulation of Type IIB string theory:

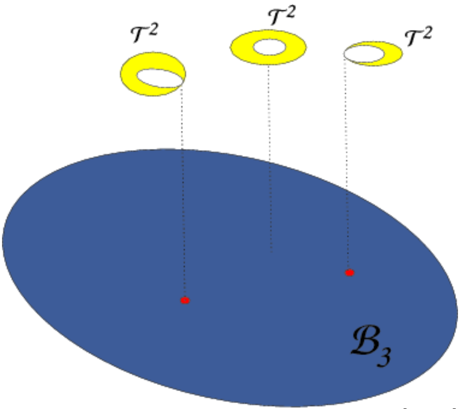
- 4 Large spacetime dimensions
- 6 Compactified internal dimensions
- 2 Dimensions due to elliptic fibration

The elliptic fibration of the space codes the Gauge Group supported by 7-branes in the internal manifold - Kodaira.

The maximum symmetry enhancement is E_8, which acts as a parent symmetry for any GUT group ...

$$E_8 \supset E_6 \times SU(3)_\perp$$
$$E_8 \supset SO(10) \times SU(4)_\perp$$
$$E_8 \supset SU(5) \times SU(5)_\perp$$

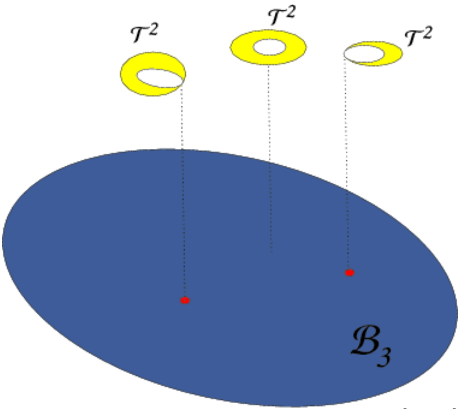
F-Theory and Symmetries



arXiv:1203.6277 [hep-th]

When the elliptic fibre becomes singular, A D7 brane is present and intersecting the GUT-surface. This indicates a symmetry enhancement - a matter curve.

F-Theory and Symmetries



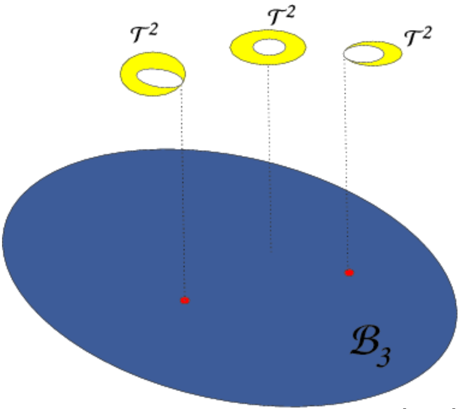
arXiv:1203.6277 [hep-th]

When the elliptic fibre becomes singular, A D7 brane is present and intersecting the GUT-surface. This indicates a symmetry enhancement - a matter curve.

For a given GUT surface, the parent E_8 descends to give a perpendicular group as well as the GUT group. For example:

$$E_8 \rightarrow SU(5)_{GUT} \times SU(5)_{\perp}$$

F-Theory and Symmetries



arXiv:1203.6277 [hep-th]

When the elliptic fibre becomes singular, A D7 brane is present and intersecting the GUT-surface. This indicates a symmetry enhancement - a matter curve.

For a given GUT surface, the parent E_8 descends to give a perpendicular group as well as the GUT group. For example:

$$E_8 \to SU(5)_{GUT} \times SU(5)_{\perp}$$

Depending on how the weights of the **perpendicular group** identify under **"monodromy action"** we can have a **family symmetry** structure accompanying our matter.

F-Theory and Symmetries

The Weierstrass equation for elliptically fibred spaces:

$$y^2 = x^3 + f(z)x + g(z)$$

F-Theory and Symmetries

The Weierstrass equation for elliptically fibred spaces:

$$y^2 = x^3 + f(z)x + g(z)$$

SU(5) spectral cover equation:

$$\mathcal{C}_5 : \; b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_1 s^4 + b_0 s^5$$

F-Theory and Symmetries

The Weierstrass equation for elliptically fibred spaces:

$$y^2 = x^3 + f(z)x + g(z)$$

SU(5) spectral cover equation:

$$\mathcal{C}_5 :\; b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_1 s^4 + b_0 s^5$$

The roots of this equation can be identified with the weights of the fundamental representation of the perpendicular group, which are paired with the antisymmetric representation of the GUT SU(5) - the 10s.

The SU(5) $5/\overline{5}$s can be described by a similar, degree ten polynomial

$$248 \to (24, 1) + (1, 24) + (10, 5) + (\overline{10}, \overline{5}) + (5, \overline{10}) + (\overline{5}, 10)$$

$$\mathcal{C}_5 = b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_1 s^4 + b_0 s^5 = \sum_k b_k s^{5-k}$$

In general, the spectral cover equation can be factorised. Depending on how the roots are related, there may be monodromy actions relating the roots. For example, if $\mathcal{C}_5$ factorises:

$$\mathcal{C}_5 \to (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s)(a_6 + a_7 s)(a_8 + a_9 s) = 0$$

F-Theory and Symmetries

$$\mathcal{C}_5 = b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_1 s^4 + b_0 s^5 = \sum_k b_k s^{5-k}$$

In general, the spectral cover equation can be factorised. Depending on how the roots are related, there may be monodromy actions relating the roots. For example, if $\mathcal{C}_5$ factorises:

$$\mathcal{C}_5 \rightarrow (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s)(a_6 + a_7 s)(a_8 + a_9 s) = 0$$

Assuming the quadratic part cannot be factorised in the same field as the original b_k coefficients, the two roots can be shown to be:

$$s_{\pm} = \frac{-a_2 \pm \sqrt{w}}{2a_3}$$
$$w = e^{i\theta}|w|$$
$$\sqrt{w} = e^{i\theta/2}\sqrt{|w|}$$

Under $\theta \rightarrow \theta + 2\pi$, the roots interchange: they are related by the action.

F-Theory and Symmetries

The spectral cover equation has an additional symetry property that we can also exploit:

$$\sigma : s \rightarrow s e^{i\phi}$$
$$b_k \rightarrow b_k e^{i(\chi+(k-6)\phi)}$$
$$\sum_k b_k s^{5-k} \rightarrow e^{i(\chi-\phi)} \sum_k b_k s^{5-k}$$

F-Theory and Symmetries

The spectral cover equation has an additional symetry property that we can also exploit:

$$\sigma : s \to s e^{i\phi}$$

$$b_k \to b_k e^{i(\chi+(k-6)\phi)}$$

$$\sum_k b_k s^{5-k} \to e^{i(\chi-\phi)} \sum_k b_k s^{5-k}$$

When we factorise the spectral cover in any way, the resulting parts must be consistent with this symmetry. Consider:

$$C_5 \to C_4 \times C_1$$

$$a_n \to e^{i(3-n)\phi} a_n$$

For a given choice of $\phi = \frac{2\pi}{N}$, the coefficients of the split spectral cover will transform differently. This can be used to give a matter parity in non-*ad hoc* way.

Consider $N = 2$:

$$a_n \to e^{i(3-n)\pi} a_n$$

This will give a parity that alternates:

$$a_1, a_3, a_5, a_7 \to -$$
$$a_2, a_4, a_6 \to +$$

While the defining equation of the 10s of the GUT group is:

$$b_5 = a_1 a_6$$

So the curves naturally have different parities.

a_n	$N = 2$	$N = 3$	$N = 4$	$N = 5$
a_1	$-$	α^2	β^2	γ^2
a_2	$+$	α	β	γ
a_3	$-$	1	1	1
a_4	$+$	α^2	β^3	γ^4
a_5	$-$	α	β^2	γ^3
a_6	$+$	1	β	γ^2
a_7	$-$	α^2	1	γ

We are of course free to experiment with other choices for N.

In each case, we also have some freedom to use the phases we discarded for this example. This will shift $a_{1,\ldots,5}$ or $a_{6,7}$ depending on phase choice.

A Model with a D_4 Monodromy

The $\mathcal{C}_4$ Spectral Cover

The most interesting classes of Family symmetry groups are S_4 and a number of its subgroups. This corresponds to a splitting of the spectral cover:

$$\mathcal{C}_5 \to \mathcal{C}_4 \times \mathcal{C}_1$$
$$\left(a_1 + a_2 s + a_3 s^2 + a_4 s^3 + a_5 s^4\right) \times \left(a_6 + a_7 s\right)$$

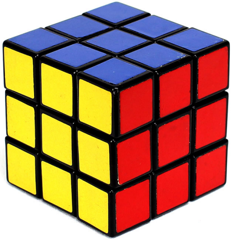
The quartic part encompasses a monodromy action on four weights of $SU(5)_\perp$. The most general action on four elements is S_4

The $\mathcal{C}_4$ Spectral Cover

The most interesting classes of Family symmetry groups are S_4 and a number of its subgroups. This corresponds to a splitting of the spectral cover:

$$\mathcal{C}_5 \to \mathcal{C}_4 \times \mathcal{C}_1$$
$$\left(a_1 + a_2 s + a_3 s^2 + a_4 s^3 + a_5 s^4\right) \times \left(a_6 + a_7 s\right)$$

The quartic part encompasses a monodromy action on four weights of $SU(5)_\perp$. The most general action on four elements is S_4



retromash.com

D_4 from F-theory

The equation defining the properties of the matter curves for the 10s of SU(5) is the s^0 term of the spectral cover equation, while the equation for the 5s arises due to consistency conditions:

$$b_5 = a_1 a_6$$

$$R = \left(a_2^2 a_7 + a_2 a_3 a_6 \mp a_0 a_1 a_6^2\right)\left(a_3 a_6^2 + (a_2 a_6 + a_1 a_7) a_7\right)$$

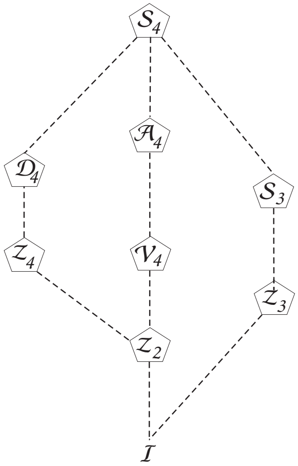
D_4 from F-theory

The equation defining the properties of the matter curves for the 10s of SU(5) is the s^0 term of the spectral cover equation, while the equation for the 5s arises due to consistency conditions:

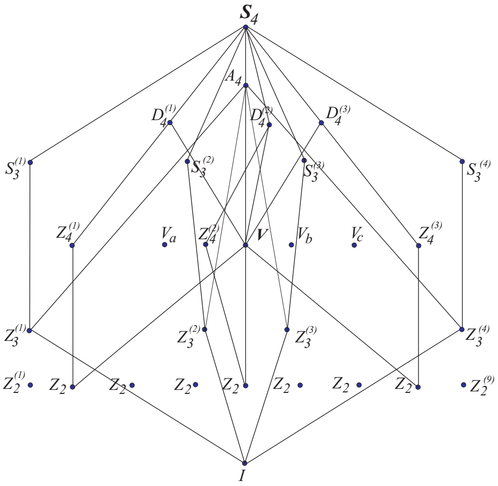
$$b_5 = a_1 a_6$$
$$R = \left(a_2^2 a_7 + a_2 a_3 a_6 \mp a_0 a_1 a_6^2\right)\left(a_3 a_6^2 + (a_2 a_6 + a_1 a_7) a_7\right)$$

Curve	Equation	Homology	N	$SU(5)_\perp$ weight
10_a	a_1	$\eta - 5c_1 - \chi$	$-N$	t_1
10_b	a_6	χ	$+N$	t_5
5_c	$a_2^2 a_7 + a_2 a_3 a_6 \mp a_0 a_1 a_6^2$	$2\eta - 7c_1 - \chi$	$-N$	$2t_1$
5_d	$a_3 a_6^2 + (a_2 a_6 + a_1 a_7) a_7$	$\eta - 3c_1 + \chi$	$+N$	$t_1 + t_5$

D_4 from F-theory



arXiv:1308.1581 [hep-th]



arXiv:1512.09148 [hep-th]

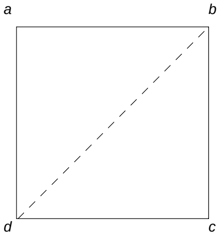
D_4 from F-theory

The generators of D_4 satisfy three relations:

- $A^4 = I$
- $B^2 = I$
- $BAB = A^{-1}$ or $ABA = B$

Geometrically speaking, these correspond to the symmetries of a square: a rotation of $\frac{\pi}{4}$ about the centre point (A) and a flip (B).

A quadruplet is not an irreducible representation of D_4



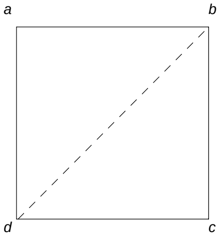
D_4 from F-theory

The generators of D_4 satisfy three relations:

- $A^4 = I$
- $B^2 = I$
- $BAB = A^{-1}$ or $ABA = B$

Geometrically speaking, these correspond to the symmetries of a square: a rotation of $\frac{\pi}{4}$ about the centre point (A) and a flip (B).

A quadruplet is not an irreducible representation of D_4



$$\begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} \rightarrow \frac{1}{2} \begin{pmatrix} t_1 + t_2 + t_3 + t_4 \\ t_1 - t_2 + t_3 - t_4 \\ \sqrt{2}(t_1 - t_3) \\ \sqrt{2}(t_2 - t_4) \end{pmatrix}$$

$$\rightarrow 1_{++} + 1_{+-} + 2$$

Must assume some form of matter curve multifurcation to reconcile.

Low Energy Spectrum	D_4 rep	$U(1)_{t_5}$	Z_2
Q_3, u_3^c, e_3^c	1_{+-}	0	$-$
u_2^c	1_{++}	1	$+$
u_1^c	1_{++}	0	$+$
$Q_{1,2}, e_{1,2}^c$	2	0	$-$
L_i, d_i^c	1_{+-}	0	$-$
ν_3^c	1_{+-}	0	$-$
$\nu_{1,2}^c$	2	0	$-$
H_u	1_{++}	0	$+$
H_d	1_{++}	-1	$+$

- It is possible to construct a model with low energy MSSM content

D_4 from F-theory

Low Energy Spectrum	D_4 rep	$U(1)_{t_5}$	Z_2
Q_3, u_3^c, e_3^c	1_{+-}	0	$-$
u_2^c	1_{++}	1	$+$
u_1^c	1_{++}	0	$+$
$Q_{1,2}, e_{1,2}^c$	2	0	$-$
L_i, d_i^c	1_{+-}	0	$-$
ν_3^c	1_{+-}	0	$-$
$\nu_{1,2}^c$	2	0	$-$
H_u	1_{++}	0	$+$
H_d	1_{++}	-1	$+$

- It is possible to construct a model with low energy MSSM content
- The geometric parity gives some R-parity violating operators at low energies

D_4 from F-theory

Low Energy Spectrum	D_4 rep	$U(1)_{t_5}$	Z_2
Q_3, u_3^c, e_3^c	1_{+-}	0	$-$
u_2^c	1_{++}	1	$+$
u_1^c	1_{++}	0	$+$
$Q_{1,2}, e_{1,2}^c$	2	0	$-$
L_i, d_i^c	1_{+-}	0	$-$
ν_3^c	1_{+-}	0	$-$
$\nu_{1,2}^c$	2	0	$-$
H_u	1_{++}	0	$+$
H_d	1_{++}	-1	$+$

- It is possible to construct a model with low energy MSSM content
- The geometric parity gives some R-parity violating operators at low energies
- The only RPV operator type is $10 \cdot \overline{5} \cdot \overline{5} \rightarrow u^c d^c \tilde{d}^c$ - no proton decay, but Neutron-antineutron oscillations.

D_4 from F-theory

There are three trilinear R-Parity violating couplings:

$$10 \cdot \overline{5} \cdot \overline{5} \to QL\tilde{d}^c + u^c d^c \tilde{d}^c + LL\tilde{e}^c$$

No single coupling facilitates proton decay.

D_4 from F-theory

There are three trilinear R-Parity violating couplings:

$$10 \cdot \overline{5} \cdot \overline{5} \rightarrow QL\tilde{d}^c + u^c d^c \tilde{d}^c + LL\tilde{e}^c$$

No single coupling facilitates proton decay.

- proton decay (QLD+UDD)

D_4 from F-theory

There are three trilinear R-Parity violating couplings:

$$10 \cdot \overline{5} \cdot \overline{5} \to QL\tilde{d^c} + u^c d^c \tilde{d^c} + LL\tilde{e^c}$$

No single coupling facilitates proton decay.

- proton decay (QLD+UDD)
- Rare meson decays (QLD or QLD+LLE)

D_4 from F-theory

There are three trilinear R-Parity violating couplings:

$$10 \cdot \overline{5} \cdot \overline{5} \rightarrow QL\tilde{d}^c + u^c d^c \tilde{d}^c + LL\tilde{e}^c$$

No single coupling facilitates proton decay.

- proton decay (QLD+UDD)
- Rare meson decays (QLD or QLD+LLE)
- $\tau/\mu \rightarrow e\nu_e\nu_{\tau/\mu}$ processes (LLE)

D_4 from F-theory

There are three trilinear R-Parity violating couplings:

$$10 \cdot \overline{5} \cdot \overline{5} \rightarrow QL\tilde{d}^c + u^c d^c \tilde{d}^c + LL\tilde{e}^c$$

No single coupling facilitates proton decay.

- proton decay (QLD+UDD)
- Rare meson decays (QLD or QLD+LLE)
- $\tau/\mu \rightarrow e\nu_e\nu_{\tau/\mu}$ processes (LLE)
- Atomic parity violation (QLD)

D_4 from F-theory

There are three trilinear R-Parity violating couplings:

$$10 \cdot \overline{5} \cdot \overline{5} \rightarrow QL\tilde{d}^c + u^c d^c \tilde{d}^c + LL\tilde{e}^c$$

No single coupling facilitates proton decay.

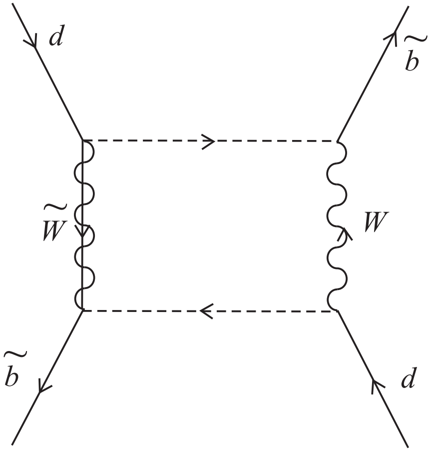
- proton decay (QLD+UDD)
- Rare meson decays (QLD or QLD+LLE)
- $\tau/\mu \rightarrow e\nu_e\nu_{\tau/\mu}$ processes (LLE)
- Atomic parity violation (QLD)
- neutron-antineutron oscillations (UDD)

D_4 from F-theory

Neutron-antineutron oscillations are a seldom considered for BSM physics.

According to Goity and Sher - **[hep-ph/9412208]** - the dominant process is mediated by a boxgraph with W boson and gaugino exchange.

Can set on the coupling λ_{dbu}, coupling to the third generation - large contribution due to factors of m_b^2/m_W^2 in the decay rate.



The decay rate for the box process is **[hep-ph/9412208]**:

$$\Gamma = -\frac{3g^4\lambda^2_{dbu}M^2_{\tilde{b}_{LR}}m_{\tilde{w}}}{8\pi^2 M^4_{\tilde{b}_L}M^4_{\tilde{b}_R}}|\psi(0)|^2\sum_{j,j'}^{u,c,t}\xi_{jj'}J(M^2_{\tilde{w}},M^2_W,M^2_{u_j},M^2_{\tilde{u}_{j'}})$$

$$J(m_1,m_2,m_3,m_4) = \sum_{i=1}^{4}\frac{m_i^4\ln(m_i^2)}{\prod_{k\neq i}(m_i^2-m_k^2)}$$

The experimental bounds on the oscillation time are: $\tau = 1/\Gamma \gtrsim 10^8$

Using the data, along with other known inputs, it is possible to calculate the limits on the coupling. We take $M_{\tilde{b}_L} = M_{\tilde{b}_R} = 500 GeV$, scanning over the parameter space of the stop mass.

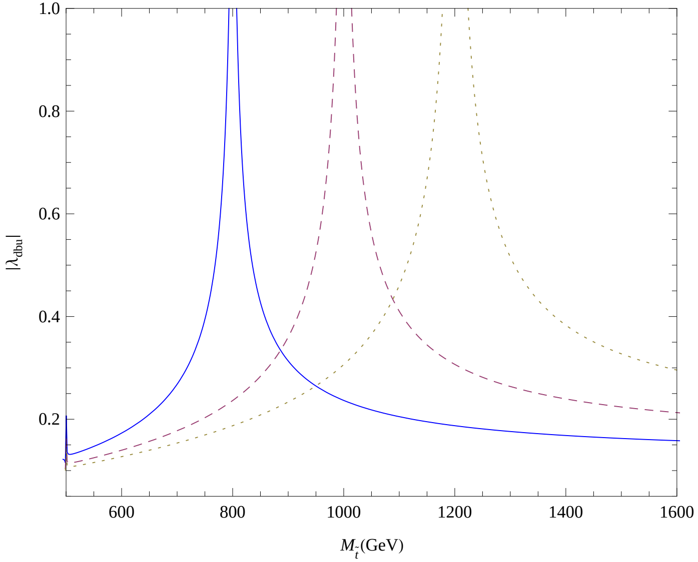


Figure: Bounds on λ_{dbu} using the latest experimental limits. Blue: $M_{\tilde{u}} = M_{\tilde{c}} = 800 GeV$, Dashed: $M_{\tilde{u}} = M_{\tilde{c}} = 1000 GeV$, Dotted: $M_{\tilde{u}} = M_{\tilde{c}} = 1200 GeV$.

D_4 from F-theory

- Based on our calculation, for a stop mass between 500 and 1600GeV, λ_{dbu} **lies between 0.1 and** $\sim$ **0.5.**

D_4 from F-theory

- Based on our calculation, for a stop mass between 500 and 1600GeV, λ_{dbu} **lies between 0.1 and $\sim$ 0.5.**
- In F-theory it is possible to calculate Yukawa couplings directly, by calculating the integral of overlapping wavefunctions

D_4 from F-theory

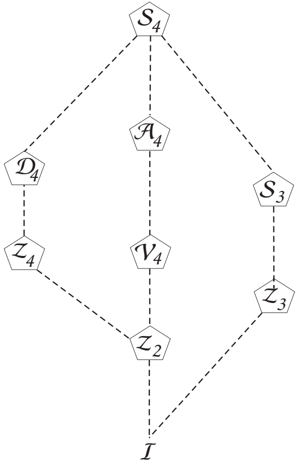
- Based on our calculation, for a stop mass between 500 and 1600GeV, λ_{dbu} **lies between 0.1 and $\sim$ 0.5.**
- In F-theory it is possible to calculate Yukawa couplings directly, by calculating the integral of overlapping wavefunctions
- Taking into account mixing effects, this particular coupling is estimated to be of the order $\lambda_{dbu} \leq 10^{-1}$ - which is compatible with the experimental value.

D_4 from F-theory

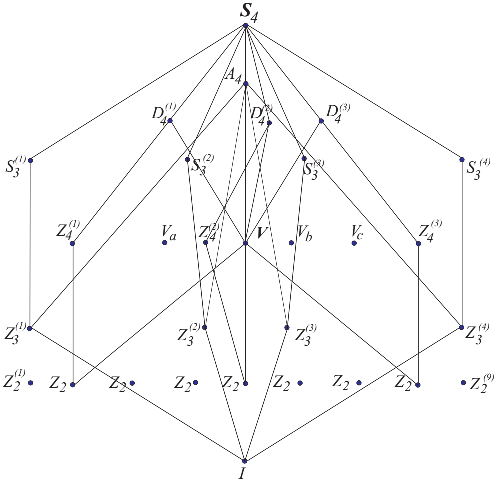
- Based on our calculation, for a stop mass between 500 and 1600GeV, λ_{dbu} **lies between 0.1 and $\sim$ 0.5.**
- In F-theory it is possible to calculate Yukawa couplings directly, by calculating the integral of overlapping wavefunctions
- Taking into account mixing effects, this particular coupling is estimated to be of the order $\lambda_{dbu} \leq 10^{-1}$ - which is compatible with the experimental value.
- The geometric parity implemented in this model gives rise to unexpected R-parity violating effects, which may provide testable predictions of new physics.

Klein Groups and Geometric Parity

Klein Groups and Geometric Parity



arXiv:1308.1581 [hep-th]



arXiv:1512.09148 [hep-th]

Klein Groups and Geometric Parity

	S_4 cycles	Trans. A_4	Trans. V_4
4-cycles	(1234), (1243), (1324), (1342), (1423), (1432)	No	No
3-cycles	(123), (124), (132), (134), (142), (143), (234), (243)	Yes	No
2+2-cycles	(12)(34), (13)(24), (14)(23)	Yes	Yes
2-cycles	(12), (13), (14), (23), (24), (34)	No	No
1-cycles	e	Yes	Yes

Klein Groups and Geometric Parity

	S_4 cycles	Trans. A_4	Trans. V_4
4-cycles	(1234), (1243), (1324), (1342), (1423), (1432)	No	No
3-cycles	(123), (124), (132), (134), (142), (143), (234), (243)	Yes	No
2+2-cycles	(12)(34), (13)(24), (14)(23)	Yes	Yes
2-cycles	(12), (13), (14), (23), (24), (34)	No	No
1-cycles	e	Yes	Yes

- Transitive Klein group: $\{(1),(12)(34),(13)(24),(14)(23)\}$

$$\mathcal{C}_5 = \mathcal{C}_4 \times \mathcal{C}_1$$

Klein Groups and Geometric Parity

	S_4 cycles	Trans. A_4	Trans. V_4
4-cycles	(1234), (1243), (1324), (1342), (1423), (1432)	No	No
3-cycles	(123), (124), (132), (134), (142), (143), (234), (243)	Yes	No
2+2-cycles	(12)(34), (13)(24), (14)(23)	Yes	Yes
2-cycles	(12), (13), (14), (23), (24), (34)	No	No
1-cycles	e	Yes	Yes

- Transitive Klein group: $\{(1),(12)(34),(13)(24),(14)(23)\}$

$$\mathcal{C}_5 = \mathcal{C}_4 \times \mathcal{C}_1$$

- Non-transitive Klein group: $\{(1),(12),(34),(12)(34)\}$

$$\mathcal{C}_5 = \mathcal{C}_2 \times \mathcal{C}_2 \times \mathcal{C}_1$$

Klein Groups and Geometric Parity

Spectral cover equation for a **non-transitive** Klein monodromy:

$$\mathcal{C}_5 \to \mathcal{C}_2 \times \mathcal{C}_2 \times \mathcal{C}_1 : (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s + a_6 s^2)(a_7 + a_8 s)$$

Klein Groups and Geometric Parity

Spectral cover equation for a **non-transitive** Klein monodromy:

$$\mathcal{C}_5 \to \mathcal{C}_2 \times \mathcal{C}_2 \times \mathcal{C}_1 : (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s + a_6 s^2)(a_7 + a_8 s)$$

The defining equations for the SU(5) matter are then:

$$\begin{aligned} P_{10} =& a_1 a_4 a_7 \\ P_5 =& a_5(a_6 a_7 + a_5 a_8)(a_6 a_7^2 + a_8(a_5 a_7 + a_4 a_8))(a_1 - a_5 a_7 c) \\ & (a_1^2 - a_1(a_5 a_7 + 2 a_4 a_8)c + a_4(a_6 a_7^2 + a_8(a_5 a_7 + a_4 a_8))c^2) \end{aligned}$$

This has one extra factor for P_{10} versus the transitive case.

Klein Groups and Geometric Parity

Spectral cover equation for a **non-transitive** Klein monodromy:

$$\mathcal{C}_5 \to \mathcal{C}_2 \times \mathcal{C}_2 \times \mathcal{C}_1 : (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s + a_6 s^2)(a_7 + a_8 s)$$

The defining equations for the SU(5) matter are then:

$$\begin{aligned} P_{10} =& a_1 a_4 a_7 \\ P_5 =& a_5(a_6 a_7 + a_5 a_8)(a_6 a_7^2 + a_8(a_5 a_7 + a_4 a_8))(a_1 - a_5 a_7 c) \\ & (a_1^2 - a_1(a_5 a_7 + 2a_4 a_8)c + a_4(a_6 a_7^2 + a_8(a_5 a_7 + a_4 a_8))c^2) \end{aligned}$$

This has one extra factor for P_{10} versus the transitive case.

Implement a model with geometric parity based on existing example: Dudas & Palti **[arXiv:1005.5728]**.

Klein Groups and Geometric Parity

Curve	Charge	Spectrum	**D&P**	All possible assignments							
10_1	t_1	$3Q+3u^c+3e^c$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
10_3	t_3	$Q+2e^c$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
10_5	t_5	$-Q-2e^c$	$-$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$
5_1	$-2t_1$	D_u+H_u	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
5_{13}	$-t_1-t_3$	$-3\overline{d^c}-3\overline{L}$	$-$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
5_{15}	$-t_1-t_5$	0		$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
5_{35}	$-t_3-t_5$	$-\overline{H}_d$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
5_3	$-2t_3$	$-\overline{D}_d$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$

- Not possible to implement R-parity in the same way as an *ad hoc* approach

Klein Groups and Geometric Parity

Curve	Charge	Spectrum	**D&P**	All possible assignments							
10_1	t_1	$3Q + 3u^c + 3e^c$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
10_3	t_3	$Q + 2e^c$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
10_5	t_5	$-Q - 2e^c$	$-$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$
5_1	$-2t_1$	$D_u + H_u$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
5_{13}	$-t_1 - t_3$	$-3\overline{d^c} - 3\overline{L}$	$-$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
5_{15}	$-t_1 - t_5$	0		$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
5_{35}	$-t_3 - t_5$	$-\overline{H}_d$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
5_3	$-2t_3$	$-\overline{D}_d$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$

- Not possible to implement R-parity in the same way as an *ad hoc* approach
- Large parameter space to work in due to flux & parity parameters

Klein Groups and Geometric Parity

Curve	Charge	Spectrum	**D&P**	All possible assignments							
10_1	t_1	$3Q + 3u^c + 3e^c$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
10_3	t_3	$Q + 2e^c$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
10_5	t_5	$-Q - 2e^c$	$-$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$
5_1	$-2t_1$	$D_u + H_u$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
5_{13}	$-t_1 - t_3$	$-3\overline{d^c} - 3\overline{L}$	$-$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
5_{15}	$-t_1 - t_5$	0		$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
5_{35}	$-t_3 - t_5$	$-\overline{H}_d$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
5_3	$-2t_3$	$-\overline{D}_d$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$

- Not possible to implement R-parity in the same way as an *ad hoc* approach
- Large parameter space to work in due to flux & parity parameters
- Lots of models are not realistic $\rightarrow$ easy to rule out...

Klein Groups and Geometric Parity

Curve	Charge	Parity	Spectrum
10_1	t_1	i	$M_{10_1} Q + (M_{10_1} - N_1) u^c + (M_{10_1} + N_1) e^c$
10_3	t_3	j	$M_{10_3} Q + (M_{10_3} - N_2) u^c + (M_{10_3} + N_2) e^c$
10_5	t_5	k	$M_{10_5} Q + (M_{10_5} + N_1 + N_2) u^c + (M_{10_5} - N_1 - N_2) e^c$
5_1	$-2t_1$	jk	$M_{5_1} \overline{d^c} + (M_{5_1} - N_1) \overline{L}$
5_{13}	$-t_1 - t_3$	$+$	$M_{5_{13}} \overline{d^c} + (M_{5_{13}} + 2N_1) \overline{L}$
5_{15}	$-t_1 - t_5$	i	$M_{5_{15}} \overline{d^c} + (M_{5_{15}} + N_1) \overline{L}$
5_{35}	$-t_3 - t_5$	j	$M_{5_{35}} \overline{d^c} + (M_{5_{35}} - 2N_1 - N_2) \overline{L}$
5_3	$-2t_3$	$-j$	$M_{5_3} \overline{d^c} + (M_{5_3} + N_2) \overline{L}$

Try to implement an MSSM type model with no exotics or BRPV:

Klein Groups and Geometric Parity

Curve	Charge	Parity	Spectrum
10_1	t_1	i	$M_{10_1} Q + (M_{10_1} - N_1) u^c + (M_{10_1} + N_1) e^c$
10_3	t_3	j	$M_{10_3} Q + (M_{10_3} - N_2) u^c + (M_{10_3} + N_2) e^c$
10_5	t_5	k	$M_{10_5} Q + (M_{10_5} + N_1 + N_2) u^c + (M_{10_5} - N_1 - N_2) e^c$
5_1	$-2t_1$	jk	$M_{5_1} \overline{d^c} + (M_{5_1} - N_1) \overline{L}$
5_{13}	$-t_1 - t_3$	$+$	$M_{5_{13}} \overline{d^c} + (M_{5_{13}} + 2N_1) \overline{L}$
5_{15}	$-t_1 - t_5$	i	$M_{5_{15}} \overline{d^c} + (M_{5_{15}} + N_1) \overline{L}$
5_{35}	$-t_3 - t_5$	j	$M_{5_{35}} \overline{d^c} + (M_{5_{35}} - 2N_1 - N_2) \overline{L}$
5_3	$-2t_3$	$-j$	$M_{5_3} \overline{d^c} + (M_{5_3} + N_2) \overline{L}$

Try to implement an MSSM type model with no exotics or BRPV:

Large parameter space of models to scan, with many possibilities for interesting physics, but difficult to pin down viable models.

MSSM-like option:

$$M_{10_1} = -M_{5_{13}} = 2$$
$$N_1 = M_{10_5} = -M_{5_3} = 1$$
$$N_2 = M_{10_3} = M_{5_1} =$$
$$M_{5_{13}} = M_{5_{35}} = 0$$
$$i = -j = k = -$$

Klein Groups and Geometric Parity

Curve	Charge	Matter Parity	Spectrum
10_1	t_1	$-$	$Q_3 + Q_2 + u_3^c + 3e^c$
10_3	t_3	$+$	$-$
10_5	t_5	$-$	$Q_1 + u_2^c + u_1^c$
5_1	$-2t_1$	$-$	$-\overline{L}_1$
5_{13}	$-t_1 - t_3$	$+$	$2H_u$
5_{15}	$-t_1 - t_5$	$-$	$-\overline{d}_2^c - \overline{d}_1^c - \overline{L}_2$
5_{35}	$-t_3 - t_5$	$+$	$-2\overline{H}_d$
5_3	$-2t_3$	$-$	$-\overline{d}_3^c - \overline{L}_3$
$1_{15} = \theta_7$	$t_1 - t_5$	$-$	N_R^a
$1_{51} = \overline{\theta}_7$	$t_5 - t_1$	$-$	N_R^b

Table: Matter content for a model with the standard matter parity arising from a geometric parity assignment.

Klein Groups and Geometric Parity

- Yukawa couplings for all generations
- Right-handed neutrinos from singlets
- No RPV operators in this model
- No $D/\overline{D}$ - proton decay safe

Curve	Charge	M.Parity	Spectrum
10_1	t_1	$-$	$Q_3 + Q_2 + u^c_3 + 3e^c$
10_3	t_3	$+$	$-$
10_5	t_5	$-$	$Q_1 + u^c_2 + u^c_1$
5_1	$-2t_1$	$-$	$-\overline{L}_1$
5_{13}	$-t_1 - t_3$	$+$	$2H_u$
5_{15}	$-t_1 - t_5$	$-$	$-\overline{d}^c_2 - \overline{d}^c_1 - \overline{L}_2$
5_{35}	$-t_3 - t_5$	$+$	$-2\overline{H}_d$
5_3	$-2t_3$	$-$	$-\overline{d}^c_3 - \overline{L}_3$
$1_{15} = \theta_7$	$t_1 - t_5$	$-$	N^a_R
$1_{51} = \overline{\theta}_7$	$t_5 - t_1$	$-$	N^b_R

- Interesting feature: two copies of each Higgs are required to have a safe spectrum.

Summary

- F-Theory provides a platform for implementing models with SU(5), $SO(10)$, or E_6 GUT groups.

Summary

- F-Theory provides a platform for implementing models with SU(5), $SO(10)$, or E_6 GUT groups.
- Possible to generate interesting family structure by exploiting monodromy actions

Summary

- F-Theory provides a platform for implementing models with SU(5), $SO(10)$, or E_6 GUT groups.
- Possible to generate interesting family structure by exploiting monodromy actions
- Geometric parity assignments are a promising way to generate Matter parity in a non-*ad hoc* way

Summary

- F-Theory provides a platform for implementing models with SU(5), $SO(10)$, or E_6 GUT groups.
- Possible to generate interesting family structure by exploiting monodromy actions
- Geometric parity assignments are a promising way to generate Matter parity in a non-*ad hoc* way
- It is possible to make models that replicate the MSSM with no RPV...

Summary

- F-Theory provides a platform for implementing models with SU(5), $SO(10)$, or E_6 GUT groups.
- Possible to generate interesting family structure by exploiting monodromy actions
- Geometric parity assignments are a promising way to generate Matter parity in a non-*ad hoc* way
- It is possible to make models that replicate the MSSM with no RPV...
- Or models with interesting signatures:
  neutron-antineutron oscillations without proton decay

F-Theory and Symmetries

- **Phenomenological implications of a minimal F-theory GUT with discrete symmetry**
  JHEP 1510 (2015) 041-*Karozas, King, Leontaris, AKM*

- **MSSM from F-theory SU**(5) **with Klein Monodromy**
  Geometric R-parity and application to achieve an SU(5) MSSM
  [arXiv:1512.09148]-*M.Crispin-Romão, Karozas, King, Leontaris, AKM*

- **Diphoton excess from** E_6 **in F-theory GUTs**
  Offers explanation for the 750 GeV bump using an E_6 inspired model from earlier work Phys. Lett. B **757** (2016) 73-*Karozas, King, Leontaris, AKM*

Klein Groups and Geometric Parity

Each coupling must be invariant under the $SU(5)_\perp$ charges, t_i. Consider the coupling:

$$10_1 \cdot 10_1 \cdot 5_{13}$$

The charges for this operator:

$$\begin{aligned} 10_1 &: \; t_1, \\ 5_{13} &: \; -t_1 - t_3, \\ 10_1 \cdot 10_1 \cdot 5_{13} &: \; t_1 - t_3 \end{aligned}$$

Need a singlet to balance this:

$$\overline{\theta}_{1,8} : \quad t_3 - t_1$$

So the overall charge can be canceled out for the operator:

$$10_1 \cdot 10_1 \cdot 5_{13} \cdot (\overline{\theta}_1 + \overline{\theta}_8)$$

Klein Groups and Geometric Parity

This model has Yukawa couplings for all the generations of quarks and leptons. Consider for example the up-type quarks, which have SU(5) couplings of type $10 \cdot 10 \cdot 5$.

$$
\begin{aligned}
10_1 \cdot 10_1 \cdot 5_{13} \cdot (\overline{\theta}_1 + \overline{\theta}_8) &\to (Q_3 + Q_2) u_3 H_u (\overline{\theta}_1 + \overline{\theta}_8) \\
10_1 \cdot 10_5 \cdot 5_{13} \cdot \theta_5 &\to ((Q_3 + Q_2)(u_1 + u_2) + Q_1 u_3) H_u \theta_5 \\
10_5 \cdot 10_5 \cdot 5_{13} \cdot \theta_2 \cdot \theta_5 &\to Q_1 (u_1 + u_2) H_u \theta_2 \theta_5
\end{aligned}
$$

Singlets must be used to cancel the $SU(5)_\perp$ charges, so we have a series of non-renormalisable Yukawas

$$
M_{u,c,t} \sim v_u \begin{pmatrix} \epsilon \theta_2 \theta_5 & \theta_2 \theta_5 & \theta_5 \\ \epsilon^2 \theta_5 & \epsilon \theta_5 & \epsilon(\overline{\theta}_1 + \overline{\theta}_8) \\ \epsilon \theta_5 & \theta_5 & \overline{\theta}_1 + \overline{\theta}_8 \end{pmatrix}
$$

The mass matrix is rank 3, with suppressions, ϵ due to the so-called rank theorem, helping to give a hierarchy.

Klein Groups and Geometric Parity

The spectrum contains two singlets that do not have vacuum expectation values, which protects the model from dangerous operators. These singlets, $\theta_7 = N_R^a$ and $\overline{\theta}_7 = N_R^b$, also serve as candidates for right-handed neutrinos. For $\theta_7 = N_R^a$:

$$\overline{5}_3 \cdot 5_{13} \cdot \theta_7 \cdot \overline{\theta}_5 \to L_3 N_R^a H_u \overline{\theta}_5$$
$$\overline{5}_{15} \cdot 5_{13} \cdot \theta_7 \cdot (\overline{\theta}_1 + \overline{\theta}_8) \to L_2 N_R^a H_u (\overline{\theta}_1 + \overline{\theta}_8)$$
$$\overline{5}_1 \cdot 5_{13} \cdot \theta_7 \cdot (\overline{\theta}_1 + \overline{\theta}_8) \cdot \theta_2 \to L_1 N_R^a H_u (\overline{\theta}_1 + \overline{\theta}_8) \theta_2$$

And we also have the operators arising from the N_R^b singlet:

$$\overline{5}_3 \cdot 5_{13} \cdot \overline{\theta}_7 \cdot (\overline{\theta}_1 + \overline{\theta}_8) \cdot \theta_2 \to L_3 N_R^b H_u (\overline{\theta}_1 + \overline{\theta}_8) \theta_2$$
$$\overline{5}_{15} \cdot 5_{13} \cdot \overline{\theta}_7 \cdot \theta_2 \cdot \theta_5 \to L_2 N_R^b H_u \theta_2 \theta_5$$
$$\overline{5}_1 \cdot 5_{13} \cdot \overline{\theta}_7 \cdot \theta_5 \to L_1 N_R^b H_u \theta_5$$

The combination of these operators should reduce the hierarchy and increase mixing for neutrinos.

The right-handed neutrinos also get a Majorana mass:

$$\frac{\langle\theta_2\rangle^2}{\Lambda}\overline{\theta}_7^2 \;+\; \frac{\langle\overline{\theta}_2\rangle^2}{\Lambda}\theta_7^2 \;+\; M\theta_7\overline{\theta}_7$$

This will allow the Seesaw mechanism to be implemented, giving a light effective neutrino mass.

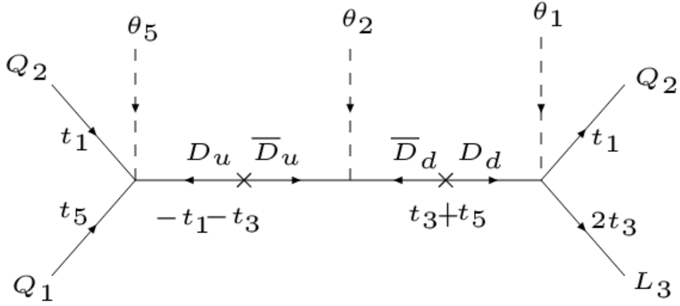
The model has the interesting feature of requiring two copies of the Higgs

$$5_{13}\cdot\overline{5}_{35}\cdot\theta_2 \to M_{ij}H_u^iH_d^j \to M\begin{pmatrix}\epsilon_h^2 & \epsilon_h\\ \epsilon_h & 1\end{pmatrix}\begin{pmatrix}H_u^1\\ H_u^2\end{pmatrix}\begin{pmatrix}H_d^1 & H_d^2\end{pmatrix}$$

The rank theorem tells us that because they are on the same matter curve, only one gets a mass, while the others must have suppressed mass terms: one Higgs will be light, with another having a mass close to the GUT scale.

Klein Groups and Geometric Parity

There are no parity violating operators in the spectrum, however we should still consider dimension six proton decay.



There are no D_u/D_d in the low energy spectrum, however they could appear at the string scale. The process should be highly suppresed due to this, and the non-renormalisability of the internal components of the process.

$n\overline{n}$ oscillations

$$u_R d_R + d_L \to \tilde{b}^*_L + d_L \to \tilde{b}_L + \overline{d}_L \to \overline{u}_R \overline{d}_R + \overline{d}_L$$