Flavor Structure of Grand Unified Theories and Related Errors in Proton Lifetime Estimates

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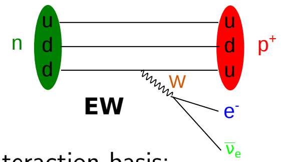
Joint work with Michal Malinský (IPNP)

Introduction: flavour structure of gauge interactions



- Interaction basis:

$$\mathcal{O}_\beta = \frac{g^2}{8{m_W}^2}\bar{u}_a\gamma_\mu d_a \bar{e}_b\gamma^\mu\nu_b$$

general Yukawa matrices Y_u, Y_d

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$$Y_u^{\text{diag}} = U_C^T Y_u U, \quad Y_d^{\text{diag}} = D_C^T Y_d D$$

$$\left|\left(U^\dagger D\right)_{11}\right| = \left|V_{11}^{CKM}\right| \approx \cos\theta_C$$

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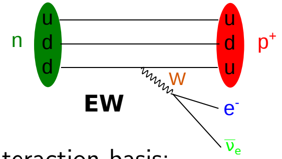

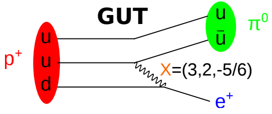

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general Y_u, Y_d, Y_l

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$$Y_l^{\text{diag}} = E_C^T Y_l E$$

$$\Rightarrow \left(U_C^\dagger U\right)_{11} \left(E_C^\dagger D\right)_{11} \text{ factor}$$

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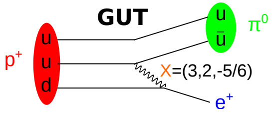

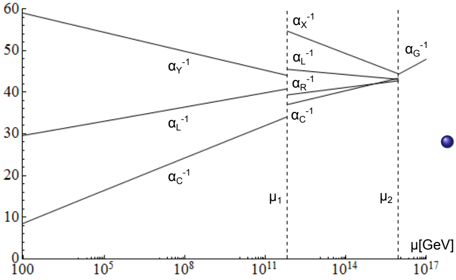

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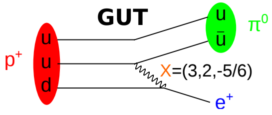
Outline:

1. Could the proton decay be hidden due to the flavour structure?
2. In case the Yukawa sector can be fitted, are such fits robust if Planck suppressed operators considered?



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Proton decay rates induced by gauge d=6 operators

[Nath, Pérez:Phys.Rept.(2007)], [Dorsner, Pérez: Phys.Lett.B625(2005)]

(dominant in non-SUSY GUTs, present in SUSY GUTs)

$$\Gamma(p \to \pi^0 e_\alpha^+) \propto \left\{ \left| c(e_\alpha, d^C) \right|^2 + \left| c(e_\alpha^C, d) \right|^2 \right\},$$

$$\Gamma(p \to K^0 e_\alpha^+) \propto \left\{ \left| c(e_\alpha, s^C) \right|^2 + \left| c(e_\alpha^C, s) \right|^2 \right\},$$

$$\Gamma(p \to \pi^+ \overline{\nu}) \propto \sum_{l=1}^{3} \left| c(\nu_l, d, d^C) \right|^2,$$

$$\Gamma(p \to K^+ \overline{\nu}) \propto \sum_{l=1}^{3} \left| B_1 c(\nu_l, d, s^C) + B_2 c(\nu_l, s, d^C) \right|^2$$

$$c(e_\alpha, d_\beta^C) = k_1{}^2 \left(U_C^\dagger U \right)_{11} \left(D_C^\dagger E \right)_{\beta\alpha} + k_2{}^2 \left(D_C^\dagger U \right)_{\beta 1} \left(U_C^\dagger E \right)_{1\alpha},$$

$$c(e_\alpha^C, d_\beta) = k_1{}^2 \left[\left(U_C^\dagger U \right)_{11} \left(E_C^\dagger D \right)_{\alpha\beta} + \left(U_C^\dagger D \right)_{1\beta} \left(E_C^\dagger U \right)_{\alpha 1} \right],$$

$$c(\nu_l, d_\alpha, d_\beta^C) = k_1{}^2 (U_C^\dagger D)_{1\alpha} (D_C^\dagger N)_{\beta l} + k_2{}^2 (D_C^\dagger D)_{\beta\alpha} (U_C^\dagger N)_{1l}$$

$$k_{1/2} = \frac{g_G}{\sqrt{2} M_{X/X'}}, \quad X = \left(3, 2, -\frac{5}{6}\right), \; X' = \left(3, 2, \frac{1}{6}\right) \quad \alpha, \beta \in \{1, 2\}$$

Proton decay rates: Constraints

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1) In any GUT:

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2) Gauge-group-dependent constraints:

flipped SU(5) [≡ SU(5)×U(1)]	SU(5)	SO(10)
only X' vector boson present ⇒ $k_1 = 0$	only X ⇒ $k_2 = 0$	both X and X'

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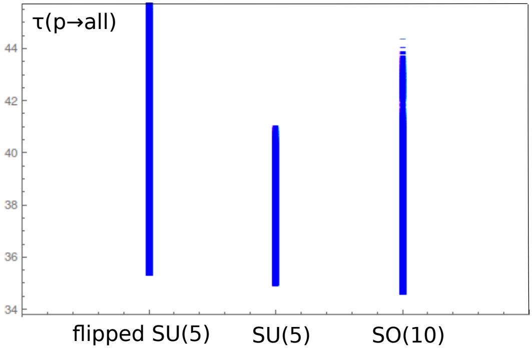
3) Constraints due to the specific choice of Yukawa sector (later)

May the p-decay be hidden due to the flavour structure?

[Dorsner, Pérez: Phys. Lett. B606(2005)], [Nath, Pérez:Phys.Rept.(2007)]

flipped SU(5)	SU(5)	SO(10)
Yes!	constrained by $\lvert V_{13}^{CKM} \rvert \approx 0.003 \neq 0$ [LHCb, A. Oyanguren talk]	No, but...



- $M_X = M_{X'} = 5 \times 10^{15}$ GeV fixed
- U, D, E, N satisfying the constraints by V^{CKM} and V^{PMNS}

Model dependent constraints on Yukawa sector

Example: SO(10) with 10_H, 126_H and type I seesaw

$$\mathcal{L}_Y = Y_{10} 16\,16\,10_H + Y_{126} 16\,16\,\overline{126}_H, \qquad Y_{10} = Y_{10}^T,\ Y_{126} = Y_{126}^T$$

$$\begin{aligned} M_u &= v_{10} Y_{10} + v_{126} Y_{126}, \\ M_d &= v_{10} Y_{10} + v_{126} Y_{126}, \\ M_{\nu_D} &= v_{10} Y_{10} - 3 v_{126} Y_{126}, \\ M_l &= v_{10} Y_{10} - 3 v_{126} Y_{126}, \\ M_{\nu_R} &= v_{126} Y_{126}, \\ M_{\nu_L} &= v_{126} Y_{126} \end{aligned}$$

⇒ fits for mass matrices [K.S.Babu talk][Dueck, Rodejohann: JHEP(2013)]

⇒ U, U_C, D, D_C etc. calculable

But what if one adds higher dimensional operators?

$$\mathcal{L}_Y \ni \frac{\tilde{Y}_{10}}{M_{pl}} 16\,16\,10_H\,45_H + \frac{\tilde{Y}_{126}}{M_{pl}} 16\,16\,\overline{126}_H\,45_H \ldots$$

⇒ corrections of the order of $\frac{M_G}{M_{Pl}} \sim 10^{-2}$ to Yukawa matrices!

Model independent analysis: analytic understanding

- both Y and $Y + \Delta Y$ reproduce the low energy data:

$$Y^{\text{diag}} = U_C^T Y U = \tilde{U}_C^T (Y + \Delta Y) \tilde{U}$$

What are the constraints on $\tilde{U}, \tilde{U}_C$ (and consequently τ_p) if $|\Delta Y| < 10^{-2}$ required???

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- Yukawa matrices typically hierarchical (illustration):

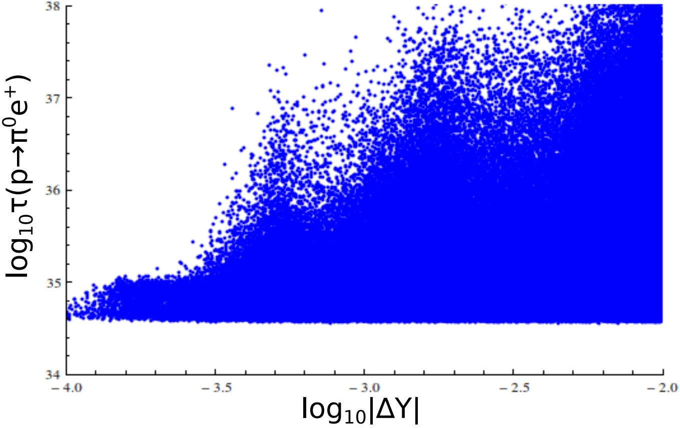
$$Y \sim \begin{pmatrix} < \mathcal{O}(10^{-2}) & < \mathcal{O}(10^{-2}) & \\ < \mathcal{O}(10^{-2}) & < \mathcal{O}(10^{-2}) & \\ & & \mathcal{O}(1) \end{pmatrix}$$

$\Delta Y \sim \mathcal{O}(10^{-2}) \Rightarrow U, \tilde{U}$ **may differ by arbitrary 1-2 rotation!**

Numerics

Individual decay channels

"Golden channel" $p \to \pi^0 e^+$ in SO(10)



$(M_X = 4 \times 10^{15}$ GeV fixed, $\Delta Y < 10^{-2}$ varied)

... total anarchy! (similarly for other gauge groups and decay channels)

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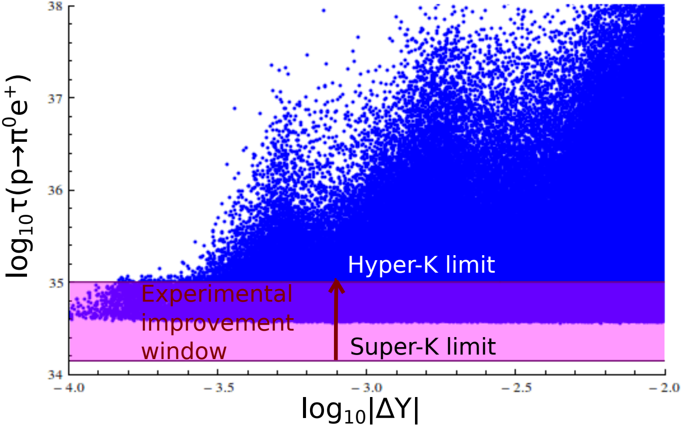

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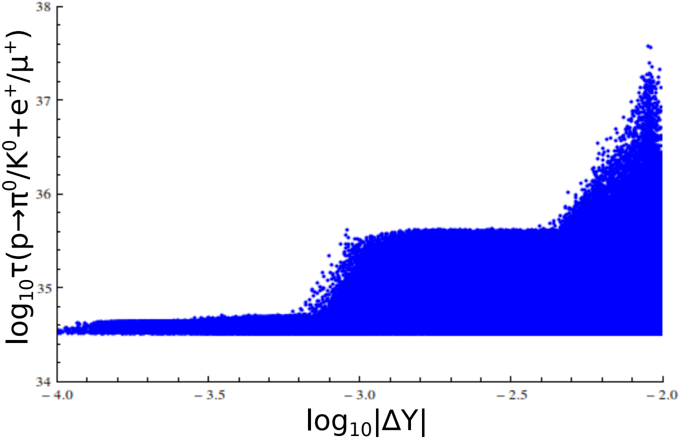
Channels $p \to$ neutral meson + charged lepton summed, SO(10)



… better, but still may be "hidden" (similarly for other gauge groups)

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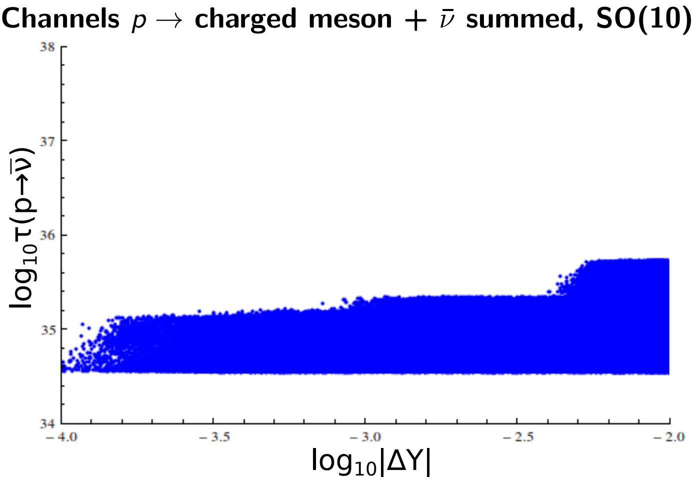

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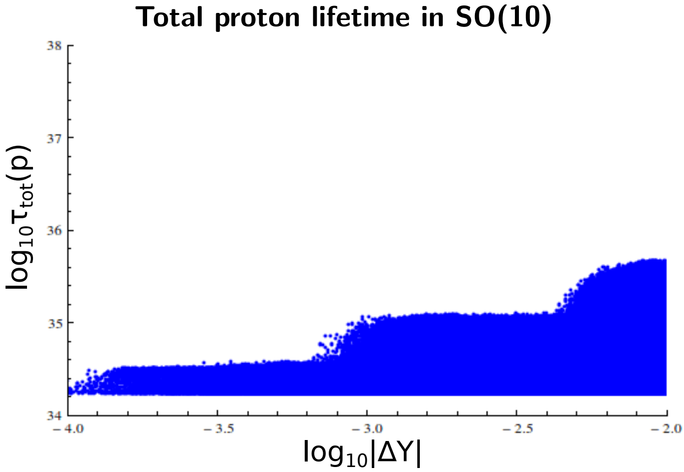

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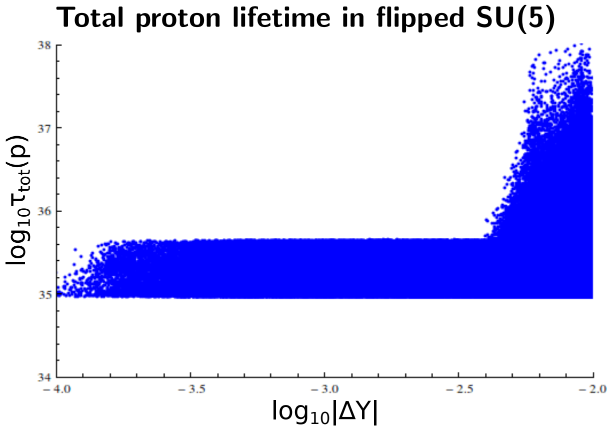

Simplest renormalizable flipped SU(5): tight constraints on $\tau(p)$

$\times$

Non-renormalizable operators $\Rightarrow$ p-decay may be "hidden"!

Conclusions

Range for $\tau_{tot}(p)$ (# of orders of magnitude)

	flipped SU(5)	SU(5)	SO(10)
no assumptions on Yukawa sector	∞	~ 6	$\gtrsim 6$
prediction from renormalizable theory + unknown contribution from M_{pl}	∞	~ 1 (constraints from renormalizable theory rather loose)	~ 1 (tight constraints, Yukawa sector may be fitted)

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- Neutrino channels more robust

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