Adventures with Stringy Axions

Wieland Staessens
based on 16xx.xxxxx (1503.01015, 1503.02965 [hep-th])
with G. Shiu

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CSIC
Consejo Superior de Investigaciones Científicas
UAM
Universidad Autónoma de Madrid

Axions and Inflation

- Axions = CP-odd $\mathbb{R}$ scalars with $a \to a + \varepsilon$
  - ⋆ pert. shift symmetry $\to$ ~~dim 6 operators~~
  - ⋆ Nonpert. effects $\to$ $V_{inf} \sim \Lambda^4 \left[1 - \cos \frac{a}{f_a}\right]$

$\Rightarrow$ natural inflation Freese-Frieman-Olinto (1990)

- strong UV sensitivity of Large Field Inflation
  $\to$ need for UV complete theory with quantum gravity $\sim$ String Theory
  - ⋆ But $\exists$ no-go theorems and arguments forbidding $f > M_{Pl}$ (1 axion)

Banks-Dine-Fox-Gorbatov ('03), Svrček-Witten ('06), Arkhani-Hamed-Motl-Nicolis-Vafa ('06), Conlon-Krippendorf ('16)

$\rightsquigarrow$ multiple axions: N-flation (2005), aligned natural inflation (2004), axion monodromy (2008, 2014), kinetic mixing (2015)

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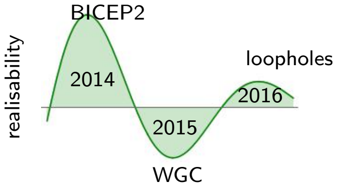

(generalised) WGC: kinematic constraints on f_a for multiple axions??

See Uranga's talk

The hidden treasures of EFT

- axion monodromy remains untouched by WGC + loopholes to WGC
  Kappl-Nilles-Winkler ('15), Choi-Kim('15)
- Ambiguous definition of $f_{\rm eff}$ for N axions & $M(\geq N)$ instantons
  $\rightsquigarrow$ consider the diameter of axion moduli space
  Bachlechner-Long-McAllister (2014/15), Junghans (2015)
- Presence of other (heavy) fields
  $\rightsquigarrow$ affects inflation process
  $\Rightarrow 0.8 M_{\rm Pl} < f_{\rm eff} < M_{\rm Pl}$ consistent with data
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+ explore inflationary models from strong dynamics

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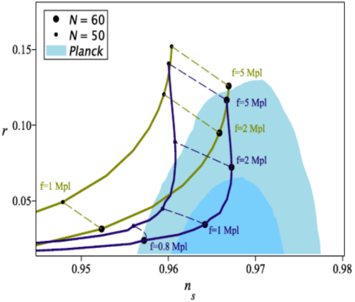

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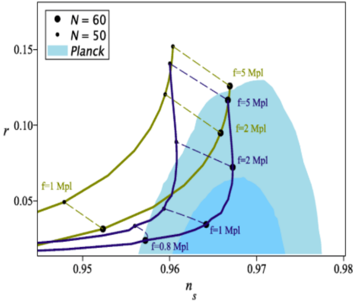

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Stringy Axions and Stückelberg-mechanism

reviews: Baumann (2009), Baumann-McAllister (2009,2014), Westphal (2014), Grimm-Louis (2005), Grimm-Lopes (2011), Kerstan-Weigand (2011), Grimm-Louis (2004), Jockers-Louis (2005), Haack-Krefl-Lust-Van Proeyen-Zagermann (2006)

- Closed string axions a^i from dim. red. of p-forms $C_{(p)}$ on $\mathcal{M}_{1,3} \times \mathcal{X}_6/\Omega\mathcal{R}$ ($C_{(p)} \in$ RR-forms + NS 2-form in Type II)

$$a^i \equiv (2\pi)^{-1} \int_{\Sigma^i} C_{(p)}, \qquad p-\text{cycle } \Sigma^i \subset \mathcal{X}_6, \qquad i \in \{1, \ldots, \begin{matrix} h_{11} \\ h_{21}+1 \end{matrix} \}$$

  Kinetic terms for p-forms $C_{(p)} \in \rightsquigarrow$ kinetic terms for a^i

- Stack of N D-branes on $\mathcal{M}_{1,3} \times \Sigma^i \rightsquigarrow U(N)$ gauge theory in $4+p$ dim
  Reduction of D-brane CS-action $\rightsquigarrow$ couplings for a^i
  - ⋆ $C_3 \wedge \text{Tr}(G \wedge G) \rightarrow$ anomal. coupling $a^i\, Tr(G \wedge G)$
  - ⋆ $C_5 \wedge F \rightarrow$ Stückelberg-coupling $(da^i - k^i A)^2$ under U(1)
- Subset of axions are eaten by anomalous $U(1)$'s $\rightsquigarrow$ survive as global symmetries

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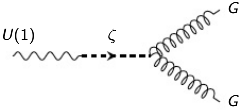
The road to NJL Models

Shiu-WS-Ye('15), Shiu-WS (work in progress)

- $U(1)$ gauge invariance: $A \to A + \mathrm{d}\chi$, $a \to a + k\chi$

$$\mathcal{S}_{sub} = \int \left[-\frac{M_{st}^2}{2} |\mathrm{d}a - kA|^2 - \frac{1}{4g_{U(1)}^2} |F|^2 - \underbrace{\frac{1}{8\pi^2} a \,\mathrm{Tr}(\, G \wedge G)}_{\text{not } U(1) \text{ invariant}} \right]$$



- @ intersection of two D-brane stacks with $U(N) \times U(1)$
  $\rightsquigarrow$ chiral matter in bifund. rep.

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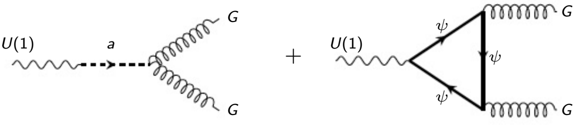

"reversed" GS mechanism

Antoniadis-Kiritsis-Rizos ('02)

Aldazabel-Franco-Ibáñez-Rábadan-Uranga ('01)

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- Using Fierz-identities $\rightsquigarrow$ NJL-type models

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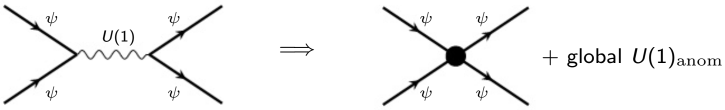

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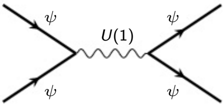
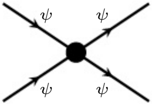


$\Longrightarrow$



$+$ global $U(1)_{\rm anom}$

- Using Fierz-identities $\rightsquigarrow$ NJL-type models

$$\mathcal{L}_{4\psi} = \frac{q_L q_R}{2M_{st}^2}\sum_{i,j=1}^{N_f}\left[\underbrace{(\overline{\psi}^i\psi^j)(\overline{\psi}^j\psi^i) - (\overline{\psi}^i\gamma^5\psi^j)(\overline{\psi}^j\gamma^5\psi^i)}_{\text{typical NJL interactions}}\right] + \frac{1}{2M_{st}^2}\sum\left[q_L^2 V_L^\mu V_{L,\mu} + q_R^2 V_R^\mu V_{R,\mu}\right]$$

Nambu-Jona-Lasinio ('61)

$$V_*^\mu = \overline{\psi}_*\gamma^\mu\psi_*$$

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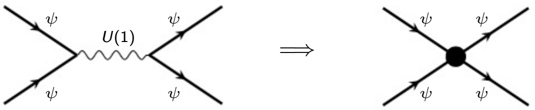

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- Using Fierz-identities $\rightsquigarrow$ NJL-type models with $N_f = 1$

$$\mathcal{L}_{4\psi} = \frac{q_L q_R}{2M_{st}^2}\left[(\overline{\psi}\psi)(\overline{\psi}\psi) - (\overline{\psi}\gamma^5\psi)(\overline{\psi}\gamma^5\psi)\right] + \ldots$$

NJL & Dynamical Mass Generation

Nambu-Jona-Lasinio ('61), review: Vogl-Weise ('91)

- NJL $= U(1)_{\rm chiral}$ invariant 4ψ-interactions $\quad \psi \to e^{i\alpha\gamma^5}\psi$

$$\bar\psi\psi \to \bar\psi\psi\cos(2\alpha) + i\psi\gamma^5\psi\sin(2\alpha)$$
$$\bar\psi\gamma^5\psi \to \psi\gamma^5\psi\cos(2\alpha) + i\bar\psi\psi\sin(2\alpha)$$

$$\mathcal{L}_{\rm NJL} = \bar\psi i\partial\!\!\!/\psi + \frac{q_L q_R}{2M_{st}^2}\left[(\bar\psi\psi)(\bar\psi\psi) - (\bar\psi\gamma^5\psi)(\bar\psi\gamma^5\psi)\right]$$

- two phases:
  - ⋆ Wigner phase: $\langle\bar\psi\psi\rangle = 0 \rightsquigarrow U(1)_{\rm chiral}$ unbroken and ψ massless
  - ⋆ Nambu-Goldstone-phase : $\langle\bar\psi\psi\rangle \neq 0 \rightsquigarrow \exists\ m_\psi = -\frac{q_L q_R}{M_{st}^2}\langle\bar\psi\psi\rangle$ and $\cancel{U(1)}_{\rm chiral}$

  NG-phase requires satisfied self-consistency condition:

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  and $\exists\, m_\psi \neq 0$ at strong coupling: $\alpha_{U(1)}(\Lambda) > \frac{\pi}{N}$ for $\Lambda < M_{st}$

NJL & Dynamical Mass Generation

Nambu-Jona-Lasinio ('61), review: Vogl-Weise ('91)

$\bar\psi\psi \to \bar\psi\psi\cos(2\alpha) + i\psi\gamma^5\psi\sin(2\alpha)$

$\bar\psi\gamma^5\psi \to \psi\gamma^5\psi\cos(2\alpha) + i\bar\psi\psi\sin(2\alpha)$

- NJL $= U(1)_{\rm chiral}$ invariant 4ψ-interactions $\quad \psi \to e^{i\alpha\gamma^5}\psi$

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NJL & Dynamical Mass Generation (II)

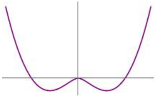
- Verify minimum in NG-phase in large N limit Gross-Neveu ('74)
  - ⋆ auxiliary fields σ, π: $\begin{pmatrix} \sigma \\ \pi \end{pmatrix} \to \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$

$$\mathcal{L}_\sigma = \overline{\psi} i \partial\!\!\!/ \psi - \frac{1}{2g^2}(\sigma^2 + \pi^2) + \left(\sigma \overline{\psi}\psi + i\pi\overline{\psi}\gamma^5\psi\right), \qquad g^2 = \frac{q_L q_R}{M_{st}^2}$$

  - ⋆ effective potential $V_{\rm eff}(\sigma) = \frac{1}{2g^2}\sigma^2 - 2N \int \frac{d^4p}{(2\pi)^4} \ln\left(1 + \frac{\sigma^2}{p^2}\right)$

    $\rightsquigarrow$ minimum: $\frac{dV_{\rm eff}}{d\sigma}\Big|_{\sigma=\sigma_{\rm min}} = 0 \to$ GAP eq

$$V_{\rm eff}(\sigma_{\rm min}) < 0$$



- Bound (scalar) states: (Salpeter-Bethe ('51) or poles of $G^{(4)}_{4\psi}$)
  - ⋆ 0^+ state $\sigma = g\overline{\psi}\psi \quad \rightsquigarrow m^2_{0^+} = 4m^2_\psi$
  - ⋆ 0^- state $\pi = g\overline{\psi}i\gamma^5\psi \rightsquigarrow m^2_{0^-} = 0$

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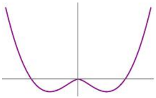

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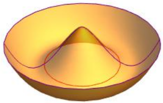

First Inflationary Steps

- Gradient expansion $\rightsquigarrow$ EFT for (σ, π) with inflaton$= \sigma$

$$\mathcal{L}_{(\sigma,\pi)} = \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}(\partial\pi)^2 - \frac{1}{2}4m_\psi^2\sigma^2 - \frac{\lambda}{4}\sigma^4 + \ldots$$

- RGE-analysis with conformal coupling to gravity $\rightsquigarrow$ R^2-type inflation
Hill-Salopek ('92), Inagaki-Odintsov-Sakamoto ('15)

$$V^{\mathrm{RGE}}_{(\sigma,\pi)} = \frac{1}{1 + \frac{D}{6}(\sigma^2)^{1/(1+\frac{\alpha}{\alpha_c})}} \left(\frac{B}{2}\sigma^2 + \frac{C}{4}(\sigma^4)^{1/(1+\frac{\alpha}{\alpha_c})} \right)$$

$\rightsquigarrow$ compatible with Planck 2015 data:
$n_s = 0.961$, $r = 0.0083$ @ weak coupling $\alpha \ll 1$

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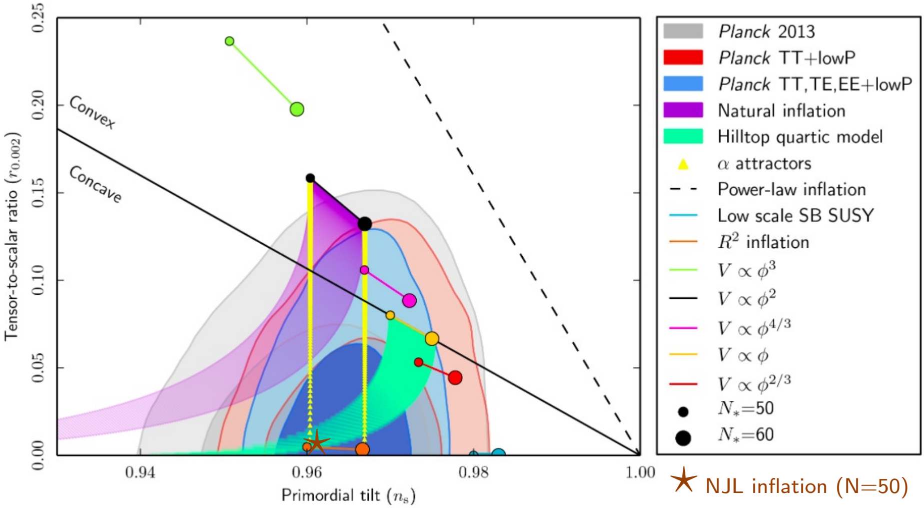

★ NJL inflation (N=50)

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- Strong $SU(N)$ dynamics can spoil (σ, π) set-up: 't Hooft (1976)
  - ⋆ $SU(N)$ gauge instantons produce $\not{U}(1)$ coupling $\kappa\, \det(\overline{\psi}(1 + \gamma^5)\psi) + h.c.$
  - ⋆ $m_\pi^2 \sim |\kappa|^2 < m_\sigma^2 = 4m_\psi^2 + m_\pi^2 \rightsquigarrow \pi = \text{inflaton ?}$
- $\exists$ remaining string axions coupling to $SU(N)$ can play rôle of inflaton?

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  - ⋆ $m_\pi^2 \sim |\kappa|^2$ ... π = inflaton ?
- ∃ remaining stri ... J(N) can play rôle of inflaton?

Conclusions and Outlook

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- Stringy embedding of Nambu-Jona-Lasinio models through Stückelberg mechanism for closed string axions
- σ-excitation in NG-phase $\rightsquigarrow \sigma =$ small field inflaton candidate

Open issues

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☞ Full String Theory construction with strongly coupled $U(1)$ and moduli stabilisation

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Moltes gràcies
Muchas gracias

Effective Potential

Gross-Neveu ('74) + textbooks

- effective action Γ is Legendre-transform of generating functional $W[J]$ for connected Green's functions:

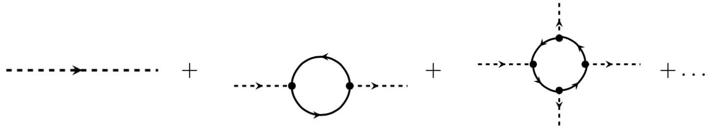
$$\Gamma[\sigma] = \int d^4x\, \sigma(x) J(x) - W[J]$$

- $V_{\rm eff}(\sigma)$ is effective potential for classical field $\sigma = \langle vac|\hat{\sigma}|vac\rangle$ arising from effective action $\Gamma[\sigma]$
- $\Gamma[\sigma]$ is generating functional for 1PI n-point functions

☞

$$V_{\rm eff}(\sigma) = \sum_{n=2}^{\infty} \frac{1}{n!} \sigma^n \underbrace{\tilde{\Gamma}^{(n)}(0,0,\ldots,0)}_{\text{amputated 1 PI}}$$

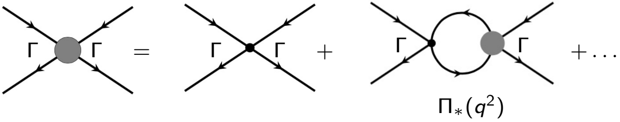
$$= V_{\rm cl}(\sigma) + V_{1-\rm loop}(\sigma) = \frac{1}{2g^2}\sigma^2 - iN \sum_{n=1}^{\infty} \int \frac{1}{2n}\left(\frac{\sigma^2}{p^2}\right)^n$$

$V_{\rm eff} =$ 

Bethe-Salpeter bound states

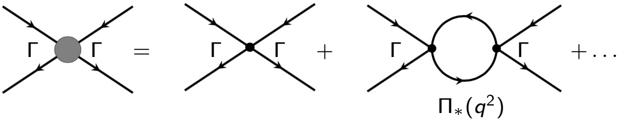
Buballa ('04) + textbooks

- Bethe-Salpeter: presence of bound states assuming mass gap as poles in the Green function



- Modern day formulation: search for poles in T-matrix for 4ψ interactions:

$$T_{4\psi} = \frac{g^2}{1 - g^2 \Pi_{4\psi}(q^2)}$$



R^2-type Inflation

Baumann-McAllister ('15)

- Non-minimally coupled inflation:

$$\mathcal{S} = \int dvol_4 \left[\frac{M_{Pl}^2}{2} \left(1 + \xi \frac{\varphi^2}{M_{Pl}} \right) R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi) \right]$$

perform a conformal rescaling of $g_{\mu\nu}$ & for $\xi \ll 1$

$\rightsquigarrow$ canonically normalised inflaton $\frac{\phi}{M_{Pl}} = \sqrt{\frac{3}{2}} \ln(1 + \frac{\xi\varphi^2}{M_{Pl}^2})$

- $V(\varphi) = \frac{\lambda}{4}\varphi^4 \rightsquigarrow$ Starobinsky's R^2-inflation
- Gauged NJL models Inagaki-Odintsov-Sakamoto ('15)
  - ⋆ Composite scalar σ couples conformally to gravity: Hill-Salopek ('92)

$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2 s^2 - \frac{\lambda}{4}\sigma^4 - \xi\sigma^2 R + \ldots, \qquad \xi = \frac{1}{6} (\text{IR fixed point})$$

  - ⋆ @ weak coupling for $SU(N) \to \phi \sim \sinh^{-1}(\sigma) - \tanh^{-1}\left(\frac{\sigma}{\sqrt{1+\sigma^2}}\right)$

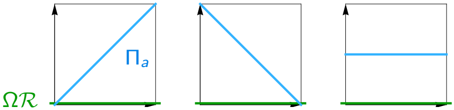
Axions & String Theory: Example

reviews: Blumenhagen-Cvetič-Langacker-Shiu ('05); Blumenhagen-Körs-Lüst-Stieberger ('06); Ibañez-Uranga ('12)

e.g. Type IIA D6-branes on $CY_3/\Omega\mathcal{R} \rightsquigarrow \Sigma^i = \Sigma^i_+ + \Sigma^i_-$

$$\int_{\Sigma^i_-} C_{(5)} \wedge F \neq 0 \quad \rightsquigarrow \quad \text{Stückelberg coupling for } a^i$$

$T^6/\Omega\mathcal{R}$ with 4 $\Omega\mathcal{R}$-even 3-cycles $\underbrace{\Sigma_+^{i=0,1,2,3}}_{\text{4 axions } a^i}$ and 4 $\Omega\mathcal{R}$-odd 3-cycles $\Sigma_-^{i=0,1,2,3}$



$$\Pi_a = \underbrace{\Sigma^0_+ - \Sigma^3_+}_{\Downarrow} + \underbrace{\Sigma^1_- - \Sigma^2_-}_{\Downarrow}$$

$$\begin{array}{cc} a^0 F_a \wedge F_a & (da^1 - A_a)^2 \\ -a^3 F_a \wedge F_a & (da^2 + A_a)^2 \end{array}$$

An Effective Action...

w/ Shiu-Ye 1503.01015, 1503.02965 [hep-th]

- String Theory compactifications $\rightsquigarrow$ 4d EFT with mixing axions

$$\mathcal{S}_{axion}^{\rm eff} = \int \left[\frac{1}{2} \sum_{i,j=1}^{N} \mathcal{G}_{ij} (\mathrm{d}a^i - k^i A) \wedge \star_4 (\mathrm{d}a^j - k^j A) - \frac{1}{8\pi^2} \left(\sum_{i=1}^{N} r_i a^i \right) \mathrm{Tr}(G \wedge G) + \mathcal{L}_{gauge} \right]$$

- 2 types of kinetic mixing

  (1) metric mixing: $\mathcal{G}_{ij}$ is not diagonal
  (2) $U(1)$ mixing: $k^i \neq 0$ for some $i \in \{1, \dots, N\}$
  gauged axions: $a^i \to a^i + k^i \chi, \quad A \to A + d\chi$

- $\mathrm{Tr}(G \wedge G)$-term associated to non-Abelian gauge group
  $\rightsquigarrow$ collective periodicity: $\sum_{i=1}^{N} r_i a^i \simeq \sum_{i=1}^{N} r_i a^i + 2\pi$

- axions couple to D-brane instantons
  $\rightsquigarrow$ individual periodicity: $a^i \to a^i + 2\pi \nu^i$, $\nu^i \in \mathbb{Z}$
  note: effective contributions of D-brane instantons to $\mathcal{S}_{axion}^{\rm eff}$ is model-dependent
  see e.g. Ibáñez-Uranga (2007, 2012), Blumenhagen-Cvetič-Kachru-Weigand (2009)

...for Mixing Axions

w/ Shiu-Ye 1503.01015, 1503.02965 [hep-th]

$$\left.\begin{array}{l}\text{To determine axion decay constants}\\ \text{To figure out axions eaten by } A_{U(1)}\end{array}\right\} \Longrightarrow \begin{array}{c}\text{Diagonalise}\\ \text{kinetic terms}\end{array}$$

$$\begin{array}{ccc}\text{eigenbasis} & & \text{eigenbasis}\\ \text{for kinetic terms} & \neq & \text{for potentials}\end{array}$$

$$\Downarrow$$

axionic directions

with large f_a?

Note: different from N-flation Dimopoulos-Kachru-McGreevy-Wacker (2005), KNP-alignment with large N Choi-Kim-Yung (2014), Kinematic alignment (with RMT) Bachlechner-Long-McAllister (2014/15), Junghans (2015)

for N axions: $f_{\rm eff} \sim N^p f$ with p $\geq 1/2$

2 Mixing Axions

- minimal set-up: 2 axions + 1 $U(1)$ + 1 Non-Abelian gauge group
- 1 axion eaten by $U(1)$ gauge boson, $\perp$ axion ξ with decay constant:

$$f_\xi = \frac{\sqrt{\lambda_+\lambda_-}M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+k^+r_2+\lambda_-k^-r_1\right)+\sin\frac{\theta}{2}\left(\lambda_-k^-r_2-\lambda_+k^+r_1\right)}$$

with $\lambda_\pm$ eigenvalues of $\mathcal{G}_{ij}$ and $M_{st}\equiv\sqrt{\lambda_+(k^+)^2+\lambda_-(k^-)^2}$

$$\cos\theta = \frac{\mathcal{G}_{11}-\mathcal{G}_{22}}{\lambda_+-\lambda_-}, \quad \sin\theta = \frac{2\mathcal{G}_{12}}{\lambda_+-\lambda_-}, \qquad \begin{pmatrix} k^+ \\ k^- \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & -\cos\frac{\theta}{2} \end{pmatrix}\begin{pmatrix} k^1 \\ k^2 \end{pmatrix}$$

- Contour plot representation of f_ξ (in units $\sqrt{\mathcal{G}_{11}}$)

$2k^1 = k^2 = 2r_1 = 2r_2$ $\quad$ $k^1 = 2k^2 = r_1 = 2r_2$ $\quad$ $k^1 = -2k^2 = r_1 = 2r_2$