States of $\rho D^* \bar{D}^*$ with $J = 3$ and B meson weak decays to $J/\psi - f0(500)$ ($f0(980), \rho, \omega, \phi$)

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Outline

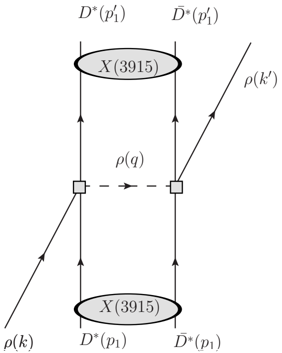
1. States of $\rho D^* \bar{D}^*$ with $J = 3$
2. B meson weak decays to $J/\psi - f0(500)$ ($f0(980), \rho, \omega, \phi$)

Introduction

- One of the important aims in the study of the strong interaction is to understand the nature and structure of hadronic resonances.
- The search for new resonances is a goal both in theories and experiments
- Recently the rich spectrum of hadronic resonances is studied actively from various viewpoints.
- The $\rho D^* \bar{D}^*$ system is new and has not been studied so far.
  $\Rightarrow$ the vector-vector interaction is found to be very strong ($J = 2$)
- The Faddeev equations under the Fixed Center Approximation (FCA) is an effective tool to deal with multi-hadron interaction.

The $\rho D^* \bar{D}^*$ three-body scattering

- A cluster of two bound particles ($D^* \bar{D}^*$ ($I = 0, J = 2$), $X(3915)$ or ρD^* ($I = 1/2, J = 2$), $D_2^*(2460)$)
- Third particle interacts with the cluster



Two body clusters

• $D^* \bar{D}^*$+coupled channel $\longrightarrow X(3915)$
R. Molina and E. Oset, Phys. Rev. D **80** (2009) 114013.
• ρD^*+coupled channel $\longrightarrow D_2^*(2460)$
R. Molina, H. Nagahiro, A. Hosaka and E. Oset, Phys. Rev. D **80** (2009) 014025.

$\rho D^*(\rho\bar{D}^*)$ and $D^*\bar{D}^*$ unitarized scattering amplitudes

$\rho - (D^*\bar{D}^*) \longrightarrow$ one needs t for $\rho D^*(\rho\bar{D}^*)$ and $D^*\bar{D}^*$

The Bethe-Salpeter equation in coupled channels

$$T = [1 - V\hat{G}]^{-1}(V), \quad (1)$$

The two meson loop function:

$$\widehat{G}_i(P) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_1 + i\epsilon} \frac{1}{(P-q)^2 - m_2 + i\epsilon} \quad (2)$$

the P is determined at the rest frame, $P = (\sqrt{s}, \vec{0})$

In the dimensional regularization scheme the loop function

$$\widehat{G}_i(\sqrt{s}) = \frac{1}{16\pi^2}\Bigg(a(\mu) + Log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} Log\frac{m_2^2}{m_1^2} + \frac{q_i}{\sqrt{s}}\Big(Log\frac{s - m_2^2 + m_1^2 + 2q_i\sqrt{s}}{-s + m_2^2 - m_1^2 + 2q_i\sqrt{s}} + Log\frac{s + m_2^2 - m_1^2 + 2q_i\sqrt{s}}{-s - m_2^2 + m_1^2 + 2q_i\sqrt{s}}\Big)\Bigg) \quad (3)$$

q_i determined at the center of mass frame

$$q_i = \frac{\sqrt{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)}}{2\sqrt{s}}. \quad (4)$$

μ, a scale parameter in this scheme. $a(\mu)$, the subtraction constant

The three body system

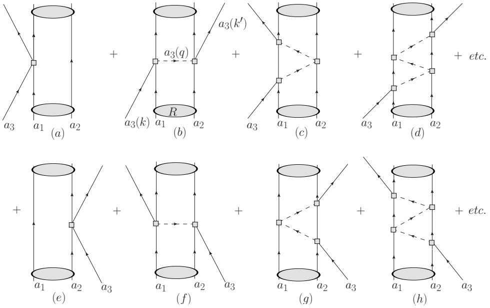


Figure: Diagrammatic representation of the fixed center approach.

- T1: all diagrams beginning with interaction in a_1 meson.
- T2: all diagrams beginning with interaction in a_2 meson.

$$T_1 = t_1 + t_1 G_0 T_2, \quad T_2 = t_2 + t_2 G_0 T_1, \quad T = T_1 + T_2,$$

For the normalization

The S-matrix for the single scattering which corresponds to (a) or (e) in Fig. 1

$$S_1^{(1)} = -it_1\frac{1}{\mathcal{V}^2}(2\pi)^4\delta^4(k+k_R-k^{'}-k_R^{'}) \times \frac{1}{\sqrt{2\omega_{a_3}}}\frac{1}{\sqrt{2\omega^{'}_{a_3}}}\frac{1}{\sqrt{2\omega_{a_1}}}\frac{1}{\sqrt{2\omega^{'}_{a_1}}}, \quad (5)$$

S-matrix for the double scattering, (b) or (f) in Fig. 1

$$S^{(2)} = -i(2\pi)^4\frac{1}{\mathcal{V}^2}\delta^4(k+k_R-k^{'}-k_R^{'}) \times \frac{1}{\sqrt{2\omega_{a_3}}}\frac{1}{\sqrt{2\omega^{'}_{a_3}}}\frac{1}{\sqrt{2\omega_{a_1}}}\frac{1}{\sqrt{2\omega^{'}_{a_1}}}\frac{1}{\sqrt{2\omega_{a_2}}}\frac{1}{\sqrt{2\omega^{'}_{a_2}}} \times \int\frac{d^3q}{(2\pi)^3}F_R(q)\frac{1}{q^{0^2}-\vec{q}^{\,2}-m_{a_3}^2+i\epsilon}t_1t_2, \quad (6)$$

The full three-body scattering is given by

$$S = -iT \frac{1}{\mathcal{V}^2}(2\pi)^4 \delta^4(k + k_R - k' - k'_R) \frac{1}{\sqrt{2\omega_{a_3}}} \frac{1}{\sqrt{2\omega'_{a_3}}} \frac{1}{\sqrt{2\omega_R}} \frac{1}{\sqrt{2\omega'_R}}. \quad (7)$$

Using the low energy reduction, $\sqrt{2\omega} \sim \sqrt{2m}$, we have

$$\tilde{t}_{1(2)} = \frac{2\, M_R}{2\, m_{a_{1(2)}}} t_{1(2)}. \quad (8)$$

Finally

$$T(s) = T_1 + T_2 = \frac{\tilde{t}_1(s_1) + \tilde{t}_2(s_2) + 2\tilde{t}_1(s_1)\tilde{t}_2(s_2)G_0}{1 - \tilde{t}_1(s_1)\tilde{t}_2(s_2)G_0^2}. \tag{9}$$

$$s_{1(2)} = (p_{a_3} + p_{a_1(a_2)})^2 = \left(\frac{\sqrt{s}}{M_R + m_{a_3}}\right)^2 (m_{a_3} + \frac{m_{a_1(a_2)}\, M_R}{m_{a_1} + m_{a_2}})^2 - \vec{P}^2_{a_2(a_1)} \tag{10}$$

where the approximate value of $\vec{P}_{a_2(a_1)}$ is given by

$$\frac{\vec{P}^2_{a_2(a_1)}}{2\, m_{a_2(a_1)}} \simeq B_{a_2(a_1)} \equiv \frac{m_{a_2(a_1)}\, M_R}{(m_{a_1} + m_{a_2})} \frac{(M_R + m_{a_3} - \sqrt{s})}{(M_R + m_{a_3})} \tag{11}$$

with $B_{a_2(a_1)}$ the binding energy of the particle $a_2(a_1)$.

The G_0 function

$$G_0 = \frac{1}{2M_R} \int \frac{d^3q}{(2\pi)^3} F_R(q) \frac{1}{{q^0}^2 - \vec{q}^{\,2} - m_{a_3}^2 + i\epsilon}. \qquad (12)$$

The energy of the propagator q^0 is determined at the three-body rest frame

$$q^0 = \frac{s + m_{a_3}^2 - M_R^2}{2\sqrt{s}} \qquad (13)$$

The form factor

$$\begin{aligned} F_R(q) &= \frac{1}{\mathcal{N}} \int_{\substack{p<k_{max} \\ |\vec{p}-\vec{q}|<k_{max}}} d^3p \, \frac{1}{2\omega_{a_1}(\vec{p}\,)} \, \frac{1}{2\omega_{a_2}(\vec{p}\,)} \, \frac{1}{M_R - \omega_{a_1}(\vec{p}\,) - \omega_{a_2}(\vec{p}\,)} \\ &\times \left(\frac{1}{2\omega_{a_1}(\vec{p}-\vec{q}\,)}\right) \left(\frac{1}{2\omega_{a_2}(\vec{p}-\vec{q}\,)}\right) \frac{1}{M_R - \omega_{a_1}(\vec{p}-\vec{q}\,) - \omega_{a_2}(\vec{p}-\vec{q}\,)} \end{aligned} \quad (14)$$

with the normalization $\mathcal{N}$

$$\mathcal{N} = \int_{p<k_{max}} d^3p \left[\frac{1}{2\omega_{a_1}(\vec{p}\,)} \, \frac{1}{2\omega_{a_2}(\vec{p}\,)} \, \frac{1}{M_R - \omega_{a_1}(\vec{p}\,) - \omega_{a_2}(\vec{p}\,)}\right]^2 \quad (15)$$

The cutoff $k_{max} = 1200$ MeV

Results

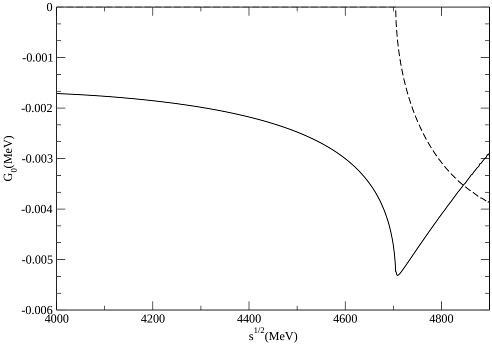


Figure: Real (solid line) and imaginary (dashed line) parts of G_0 function in $\rho(D^*\bar{D}^*)$

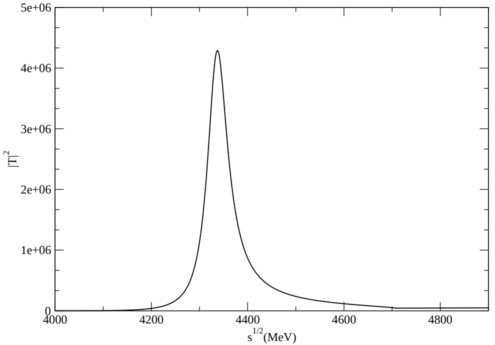


Figure: Modulus squared of the $\rho(D^* \bar{D}^*)$ scattering amplitude with total isospin $I = 1$. [M. Bayar, X. L. Ren and E. Oset, Eur. Phys. J. A **51** (2015) 5, 61.]

$\Rightarrow$ A clear peak at $\sqrt{s} = 4338$ MeV about 360 MeV below the threshold of the $X(3915) - \rho$ system, $\Gamma \sim 50$ MeV.
$\Rightarrow$ The vector-vector interaction in $J = 2$ is indeed very strong.
$m_{D^*} + m_\rho - m_{D_2^*} = 320$ MeV,
$\Rightarrow$ extra 40 MeV binding.

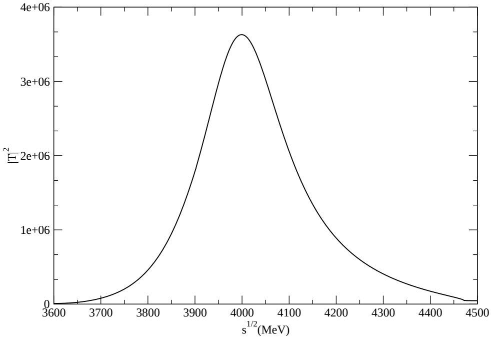


Figure: Modulus squared of the $\bar{D}^*(\rho D^*)$ scattering amplitude with total isospin $I = 0$.

$\Rightarrow$ A peak at $\sqrt{s} = 4000$ MeV about 470 MeV below the $D_2^*(2460)$ and $\bar{D}^*$ threshold, $\Gamma \sim 250$ MeV.
$\Rightarrow$ the D^* with a ρ would be bound by 320 MeV,
$\Rightarrow$ the D^* with the $\bar{D}^*$ by about 63 MeV

$\Rightarrow$ **Why 470 MeV!! and $\Gamma \sim 250$ MeV !! It is not clear.**

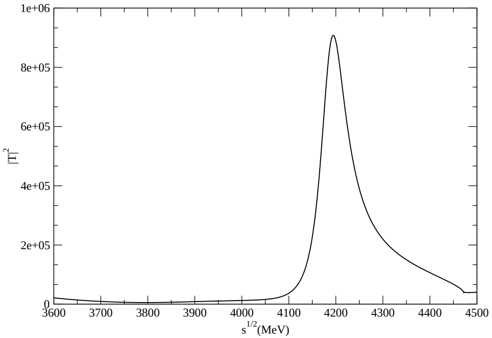


Figure: Modulus squared of the $\bar{D}^*(\rho D^*)$ scattering amplitude with total isospin $I = 1$.

$\Rightarrow$ A peak at $\sqrt{s} = 4195$ MeV about 270 MeV below the $D_2^*(2460)$ and $\bar{D}^*$ threshold, $\Gamma \sim 60$ MeV.

Conclusion

- The $\rho D^* \bar{D}^*$ three-body amplitude with $J = 3$ by means of the fixed center approach
- ρ-$D^* \bar{D}^*$ $\Rightarrow$ a clear and narrow peak around 4340 MeV.
- $\bar{D}^*$-$D^* \rho$ $\Rightarrow$ more uncertain

  an $I = 0$ state around 4000 MeV

  an $I = 1$ state around 4200 MeV

  ($m_{\bar{D}^*} > m_\rho$, the FCA is less reliable!!)

  $\Rightarrow$ our results should be taken as indicative

B^0 and B_s^0 decays into J/ψ plus a scalar or vector meson

Introduction

- The nature of the scalar mesons is the topic of the permanent debate
- The light scalars $f_0(500)$, $f_0(980)$, $a_0(980)$, $\kappa(800)$ have been associated to $q\bar{q}$ structures. N. A. Tornqvist, Z. Phys. C **68**, 647 (1995)
- These mesons are considered as $(q)^2(\bar{q})^2$ tetraquarks.

  R. L. Jaffe, Phys. Rev. D **15**, 281 (1977)., L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. Lett. **93**, 212002 (2004)
- They are considered as a mixture of $q\bar{q}$ and tetraquarks

  A. H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D **79**, 074014 (2009).
- In the χ PT, they come from the interaction of pseudoscalar mesons using input from Chiral lagrangian.

  J. A. Oller and E. Oset, Nucl. Phys. A **620** (1997) 438, J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. **80** (1998) 3452, J. R. Pelaez and G. Rios, Phys. Rev. Lett. **97** (2006) 242002
- It is clear that the debate is not over.

- The B_s^0 decays into J/ψ plus $f_0(500)$ or $f_0(980)$ receive attention both from experiment and theory.
$\Longrightarrow$ In LHCb, a peak is observed for the $f_0(980)$ in the B_s^0 decay while no appreciable signal is seen for the $f_0(500)$ [R. Aaij *et al.* [LHCb Collaboration], Phys. Lett. B **698**, 115 (2011)]
confimed by the Belle J. Li *et al.* [Belle Collaboration], Phys. Rev. Lett. **106**, 121802 (2011) ,
CDF T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. D **84**, 052012 (2011) ,
D0 V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **85**, 011103 (2012) ,
again LHCb RAaij *et al.* [LHCb Collaboration], Phys. Rev. D **86**, 052006 (2012), R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. D **89**, 092006 (2014)
$\Longrightarrow$ The $\bar{B}^0$ into J/ψ and $\pi^+\pi^-$ is investigated, and a signal is seen for the $f_0(500)$ production while only a very small fraction is observed for the $f_0(980)$ production RAaij *et al.* [LHCb Collaboration], Phys. Rev. D **87**, no. 5, 052001 (2013)

Formalism for scalar meson production

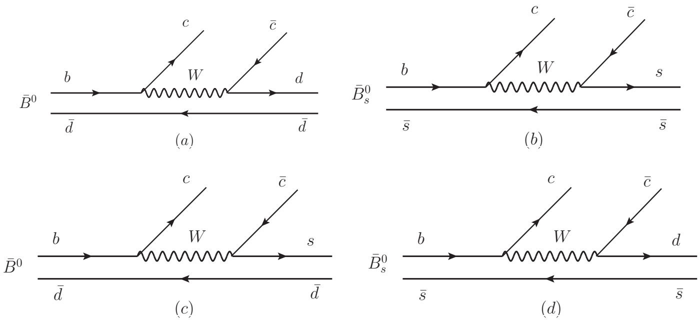


Figure: Diagrams for the decay of $\bar{B}^0$ and $\bar{B}^0_s$ into J/ψ and a primary $q\bar{q}$ pair. (a) Cabbibo suppressed $\bar{B}^0$ decay, (b) Cabbibo favored $\bar{B}^0_s$ decay, (c) Cabbibo favored $\bar{B}^0$ decay, (d) Cabbibo suppressed $\bar{B}^0_s$ decay.

$\Longrightarrow$ production of scalar mesons, $f_0(500)$ and $f_0(980) \Rightarrow \pi^+\pi^-$ production
$\Longrightarrow \pi K$ for the case of the κ.
⋇ The $q\bar{q}$ pair must hadronize. The $q\bar{q}$ matrix M;

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} \quad (16)$$

has the property: $M \cdot M = M \times (\bar{u}u + \bar{d}d + \bar{s}s)$

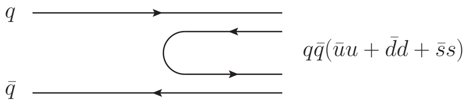


Figure: Schematic representation of the hadronization $q\bar{q} \rightarrow q\bar{q}(\bar{u}u + \bar{d}d + \bar{s}s)$.

In terms of mesons

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \qquad (17)$$

$$\begin{aligned} d\bar{d}(\bar{u}u + \bar{d}d + \bar{s}s) &\equiv (\phi\cdot\phi)_{22} = \pi^-\pi^+ + \frac{1}{2}\pi^0\pi^0 - \frac{1}{\sqrt{3}}\pi^0\eta + K^0\bar{K}^0 + \frac{1}{6}\eta\eta, \\ s\bar{s}(\bar{u}u + \bar{d}d + \bar{s}s) &\equiv (\phi\cdot\phi)_{33} = K^-K^+ + K^0\bar{K}^0 + \frac{4}{6}\eta\eta, \qquad (18) \\ s\bar{d}(\bar{u}u + \bar{d}d + \bar{s}s) &\equiv (\phi\cdot\phi)_{32} = K^-\pi^+ - \frac{1}{\sqrt{2}}\bar{K}^0\pi^0 - \frac{1}{\sqrt{6}}\eta\bar{K}^0, \\ d\bar{s}(\bar{u}u + \bar{d}d + \bar{s}s) &\equiv (\phi\cdot\phi)_{23} = \pi^-K^+ - \frac{1}{\sqrt{2}}K^0\pi^0 - \frac{1}{\sqrt{6}}\eta K^0. \end{aligned}$$

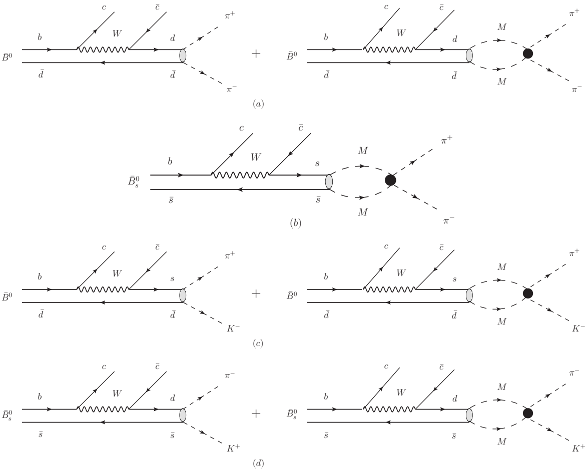


Figure: Diagrammatic representations of $\pi^+\pi^-$, π^+K^- and π^-K^+ via direct plus rescattering mechanisms in $\bar{B}^0$ and $\bar{B}^0_s$ decays.

The amplitudes for $\pi^+\pi^-$ and πK production are given by

$$
\begin{aligned}
t(\bar{B}^0 \to J/\psi \pi^+\pi^-) &= V_P V_{cd} (1 + G_{\pi^+\pi^-} t_{\pi^+\pi^- \to \pi^+\pi^-} + \frac{1}{2}\frac{1}{2} G_{\pi^0\pi^0} t_{\pi^0\pi^0 \to \pi^+\pi^-} \\
&\quad + G_{K^0\bar{K}^0} t_{K^0\bar{K}^0 \to \pi^+\pi^-} + \frac{1}{6}\frac{1}{2} G_{\eta\eta} t_{\eta\eta \to \pi^+\pi^-}), \\
t(\bar{B}_s^0 \to J/\psi \pi^+\pi^-) &= V_P V_{cs} (G_{K^+K^-} t_{K^+K^- \to \pi^+\pi^-} + G_{K^0\bar{K}^0} t_{K^0\bar{K}^0 \to \pi^+\pi^-} \\
&\quad + \frac{4}{6}\frac{1}{2} G_{\eta\eta} t_{\eta\eta \to \pi^+\pi^-}), \\
t(\bar{B}^0 \to J/\psi \pi^+ K^-) &= V_P V_{cs} (1 + G_{\pi^+K^-} t_{\pi^+K^- \to \pi^+K^-} \\
&\quad - \frac{1}{\sqrt{2}} G_{\pi^0\bar{K}^0} t_{\pi^0\bar{K}^0 \to \pi^+K^-} - \frac{1}{\sqrt{6}} G_{\eta\bar{K}^0} t_{\eta\bar{K}^0 \to \pi^+K^-}),
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
t(\bar{B}_s^0 \to J/\psi \pi^- K^+) &= V_P V_{cd} (1 + G_{\pi^-K^+} t_{\pi^-K^+ \to \pi^-K^+} \\
&\quad - \frac{1}{\sqrt{2}} G_{\pi^0K^0} t_{\pi^0K^0 \to \pi^-K^+} - \frac{1}{\sqrt{6}} G_{\eta K^0} t_{\eta K^0 \to \pi^-K^+}),
\end{aligned}
$$

Formalism for vector meson production

The amplitudes associated to figure 1

$$t_{\bar{B}^0 \to J/\psi \rho^0} = -\frac{1}{\sqrt{2}} \tilde{V}'_P \, V_{cd} , \quad t_{\bar{B}^0 \to J/\psi \omega} = \frac{1}{\sqrt{2}} \tilde{V}'_P \, V_{cd} , \quad t_{\bar{B}^0_s \to J/\psi \phi} = \tilde{V}'_P \, V_{cs} ,$$
$$t_{\bar{B}^0 \to J/\psi \bar{K}^{*0}} = \tilde{V}'_P \, V_{cs} , \quad t_{\bar{B}^0_s \to J/\psi K^{*0}} = \tilde{V}'_P \, V_{cd} , \qquad (20)$$

The width for $J/\psi V$ vector decay:

$$\Gamma_{V_i} = \frac{1}{8\pi} \frac{1}{m^2_{\bar{B}^0_i}} \left| t_{\bar{B}^0_i \to J/\psi V_i} \right|^2 p_{J/\psi} . \qquad (21)$$

By taking as input the branching ratio of $\bar{B}^0_s \to J/\psi\phi$,

$$BR(\bar{B}^0_s \to J/\psi\phi) = (10.0^{+3.2}_{-1.8}) \times 10^{-4}, \quad (22)$$

The branching ratios [M. Bayar, W. H. Liang and E. Oset, Phys. Rev. D **90** (2014) 11, 114004.]

$$\begin{aligned} BR(\bar{B}^0 \to J/\psi\rho^0) &= (2.63^{+0.84}_{-0.47}) \times 10^{-5}, \\ BR(\bar{B}^0 \to J/\psi\omega) &= (2.63^{+0.84}_{-0.47}) \times 10^{-5}, \\ BR(\bar{B}^0 \to J/\psi\bar{K}^{*0}) &= (9.57^{+3.1}_{-1.7}) \times 10^{-4}, \\ BR(\bar{B}^0_s \to J/\psi K^{*0}) &= (5.51^{+1.7}_{-1.0}) \times 10^{-5}. \end{aligned} \quad (23)$$

The experimental numbers [J. Beringer *et al.* [PDG], Phys. Rev. D **86**, 010001 (2012)] :

$$\begin{aligned} BR(\bar{B}^0 \to J/\psi\rho^0) &= (2.58 \pm 0.21) \times 10^{-5}, \\ BR(\bar{B}^0 \to J/\psi\omega) &= (2.3 \pm 0.6) \times 10^{-5}, \\ BR(\bar{B}^0 \to J/\psi\bar{K}^{*0}) &= (1.34 \pm 0.06) \times 10^{-3}, \\ BR(\bar{B}^0_s \to J/\psi K^{*0}) &= (4.4 \pm 0.9) \times 10^{-5}. \end{aligned} \quad (24)$$

BABAR Collaboration, (B. Aubert *et al.* PRD **76**, 092004 (2007).)

$$BR(\bar{B}^0 \to J/\psi\bar{K}^{*0}) = (1.33^{+0.22}_{-0.21}) \times 10^{-3} \quad (25)$$

Results

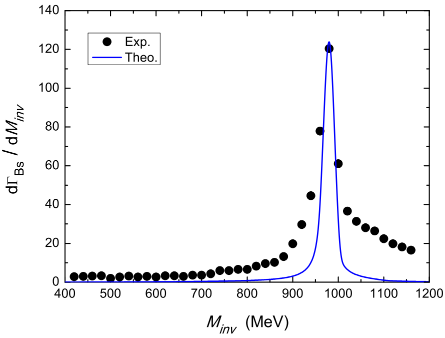
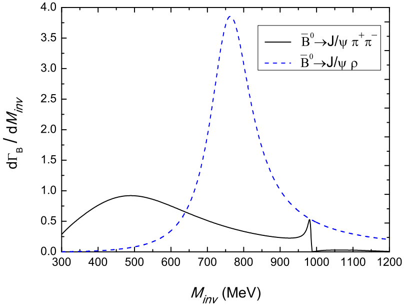


Figure: $\pi^+\pi^-$ invariant mass distribution for the $\bar{B}_s^0 \to J/\psi\pi^+\pi^-$ decay, with arbitrary normalization [W. H. Liang and E. Oset, Phys. Lett. B **737**, 70 (2014)]. Data from [R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. D **89**, 092006 (2014)].



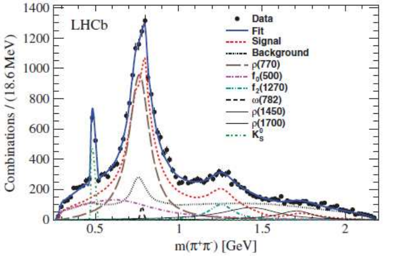


FIG. 13 (color online). Fit projection of $m(\pi^+\pi^-)$ showing the different resonant contributions in the best model.

Figure: Left: $\pi^+\pi^-$ invariant mass distributions for the $\bar{B}^0 \to J/\psi\pi^+\pi^-$ (S wave) (solid line)[W. H. Liang and E. Oset, Phys. Lett. B **737**, 70 (2014)] and $\bar{B}^0 \to J/\psi\rho$, $\rho \to \pi^+\pi^-$ (P wave) decays, with arbitrary normalization. [M. Bayar, W. H. Liang and E. Oset, Phys. Rev. D **90** (2014) 11, 114004.] Right: [R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. D **90**, 012003 (2014)]

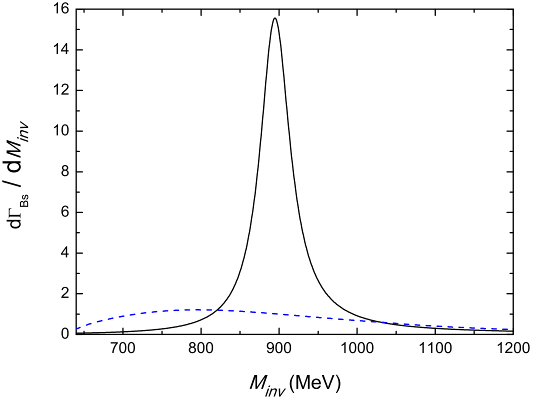


Figure: $\pi^- K^+$ invariant mass distributions for the $\bar{B}^0_s \to J/\psi K^{*0}$, $K^{*0} \to \pi^- K^+$ (solid line) and $\bar{B}^0_s \to J/\psi \kappa$, $\kappa \to \pi^- K^+$ (dashed line), with arbitrary normalization.

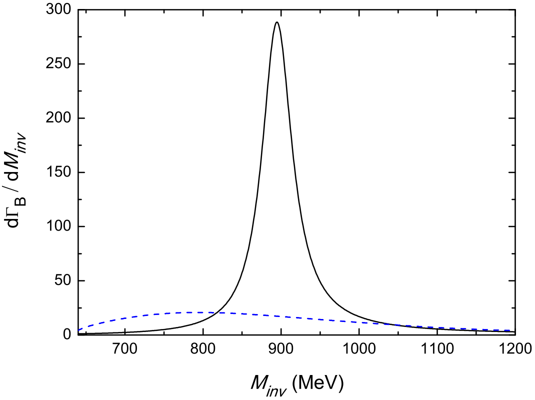


Figure: $\pi^+ K^-$ invariant mass distributions for the $\bar{B}^0 \to J/\psi \bar{K}^{*0}$, $\bar{K}^{*0} \to \pi^+ K^-$ (solid line) and $\bar{B}^0 \to J/\psi \bar{\kappa}$, $\bar{\kappa} \to \pi^+ K^-$ (dashed line), with arbitrary normalization.

Conclusion

- The nature of hadrons,
  $\Rightarrow$ The vector mesons stand as largely $q\bar{q}$ states
  $\Rightarrow$ The low-lying scalar mesons are dynamically generated states from the meson-meson interaction
- The B decay into the J/ψ scalar (and J/ψ vector) greatly support this picture
- We obtained a remarkable agreement with experimental results