Scaling violation and relativistic effective mass from quasielastic electron scattering: implications for neutrino reactions

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in collaboration with

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Introduction

- Our purpose in this work is to reanalyze the inclusive (e, e') scattering data from ^{12}C in terms of a new scaling variable (ψ^*) suggested by the interacting relativistic Fermi gas (RFG) with scalar and vector interactions ("the Walecka model"), which generate a relativistic effective mass (m_N^*) for the nucleons.
- By choosing $m_N^* = 0.8 m_N$ we will show that most of the data fall inside a region around the universal scaling function of the RFG. This, in turns, suggests a method to exclude the subset of data which are not dominated by the quasi-elastic process.
- The band of data around the universal scaling function of RFG can be generated with a Montecarlo simulation that reflects the genuine fluctuations in the effective mass.
- Finally, we transport this band into a theoretically predicted band for neutrino scattering cross section.

Scaling

- The idea behind the scaling phenomenon is to express inclusive scattering observables (cross sections or response functions) of weakly interacting probes off composite systems (nuclei, atoms...) in terms of elementary observables from the constituents of the composite system.
- The idea is to encode the maximum amount of information on the dynamics of the composite system in an *universal function* depending on the kinematic variables of the process and to use this function to predict behaviors in other kinematic regions.
- For inclusive scattering of weakly interacting probes, such electrons or neutrinos, the cross sections depend on two independent variables (ω, q), the energy and three-momentum transferred by the probe to the constituent. In this situation, *scaling of the first kind* is reached in an (ω, q) region if the cross sections depend only on a single variable $\psi = \psi(\omega, q)$.

SuperScaling

- One can go one step forward and wonder if *scaling of the second kind* is fulfilled as well. This would be the case if the *universal function* is *the same for different nuclear species*. This would lead us to SuperScaling.
- The idea is greatly appealing because of its simplicity and predictive usefulness, but it is theoretically well-motivated because a simple model such as RFG accomplishes all the requirements for superscaling [1] [2].
- Therefore, the final goal is to be able to write the nuclear inclusive cross section for a given process (electron scattering as an example) as:

$$\frac{d^2\sigma}{d\Omega_e\, d\omega} \sim F(\omega, q) \left(\frac{d^2\sigma}{d\Omega_e\, d\omega} \right)_{\text{single-nucleon}} \qquad (1)$$

with $F(\omega, q) = f(\psi(\omega, q))$ being function of a single variable ψ, which is a combination of the other two.

[1] W.M. Alberico et al, **Phys. Rev. C 38**, 1801 (1988)

[2] M.B. Barbaro et al, **Nucl. Phys. A643**, 137 (1998)

Evidences for scaling and superscaling [3]

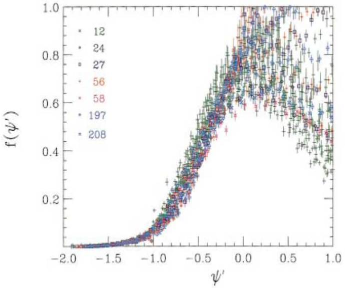

Figure: Scaling function $f(\psi')$ for all nuclei with $A \geqslant 12$ and all available kinematics.

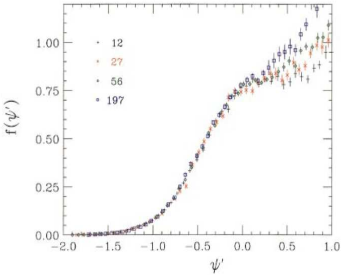

Figure: Scaling function $f(\psi')$ for ^{12}C, ^{27}Al, ^{56}Fe and ^{197}Au at the same kinematics ($q \approx 1$ GeV/c)

[3] Figures taken from reference T.W. Donnelly and I. Sick, **Phys. Rev. C 60**, 065502 (1999)

Scaling in the Relativistic Fermi Gas

The double-differential inclusive (e, e') cross section in the LAB frame can be written as:

$$\frac{d^2\sigma}{d\Omega_e\, d\epsilon'} = \sigma_M \left\{ \left(\frac{Q^2}{\mathbf{q}^2}\right)^2 R_L(\omega, |\mathbf{q}|) + \left[\frac{1}{2}\left|\frac{Q^2}{\mathbf{q}^2}\right| + \tan^2\left(\frac{\theta_e}{2}\right)\right] R_T(\omega, |\mathbf{q}|)\right\} \quad (2)$$

where the longitudinal (L) and transverse responses (T) are given as particular components of the hadron tensor in the frame in which $\mathbf{q}$ defines the Z-axis,

$$R_L(\omega, |\mathbf{q}|) = W^{00}, \qquad R_T(\omega, |\mathbf{q}|) = W^{xx} + W^{yy} \quad (3)$$

$$\begin{aligned} W^{\mu\nu} &= \frac{3\mathcal{N} m_N^2}{4\pi k_F^3} \int \frac{d^3p}{E(\mathbf{p})E(\mathbf{p}+\mathbf{q})} \theta(k_F - |\mathbf{p}|)\theta(|\mathbf{p}+\mathbf{q}| - k_F) \\ &\times \delta\left(\omega - [E(\mathbf{p}+\mathbf{q}) - E(\mathbf{p})]\right) r^{\mu\nu}(P+Q, P) \end{aligned} \quad (4)$$

Scaling in the Relativistic Fermi Gas

with $r^{\mu\nu}(P+Q,P)$ being the single-nucleon response tensor, which can be expressed in a convenient and explicitly gauge-invariant fashion:

$$\begin{aligned} r^{\mu\nu}(P+Q,P) &= \left(-g^{\mu\nu}+\frac{Q^\mu Q^\nu}{Q^2}\right)W_1(\tau) \\ &+ \left(P^\mu-\frac{P\cdot Q}{Q^2}Q^\mu\right)\left(P^\nu-\frac{P\cdot Q}{Q^2}Q^\nu\right)\frac{W_2(\tau)}{m_N^2} \end{aligned} \tag{5}$$

and $\tau \equiv \frac{|Q^2|}{4m_N^2}$. In addition, if we wish to cast the results in terms of dimensionless variables, we can define:

$$\left.\begin{aligned} \kappa &\equiv \frac{|\mathbf{q}|}{2m_N} \\ \lambda &\equiv \frac{\omega}{2m_N} \end{aligned}\right\} \to \tau = \kappa^2-\lambda^2, \tag{6}$$

$$\begin{aligned} \eta &\equiv \frac{|\mathbf{p}|}{m_N}, \qquad \epsilon \equiv \frac{E(\mathbf{p})}{m_N} = \sqrt{1+\eta^2} \\ \eta_F &\equiv \frac{k_F}{m_N}, \qquad \epsilon_F = \sqrt{1+\eta_F^2} \end{aligned} \tag{7}$$

Scaling in the Relativistic Fermi Gas

With these variables, the responses take the simple form:

$$
\begin{aligned}
R_L &= \frac{3\mathcal{N}\kappa}{4m_N\tau\eta_F^3}(\epsilon_F-\Gamma)\,\theta(\epsilon_F-\Gamma)\,\{(1+\tau+\Delta)\,W_2(\tau)-W_1(\tau)\} \\
R_T &= \frac{3\mathcal{N}}{4m_N\kappa\eta_F^3}(\epsilon_F-\Gamma)\,\theta(\epsilon_F-\Gamma)\,(2W_1(\tau)+W_2(\tau)\Delta) \qquad (8)
\end{aligned}
$$

with Δ and Γ given by

$$
\begin{aligned}
\Delta &\equiv \frac{\tau}{\kappa^2}\left[\frac{1}{3}\left(\epsilon_F^2+\epsilon_F\Gamma+\Gamma^2\right)+\lambda\,(\epsilon_F+\Gamma)+\lambda^2\right]-(1+\tau) \\
\Gamma &\equiv \max\left[(\epsilon_F-2\lambda)\,,\gamma_-\equiv\kappa\sqrt{1+\frac{1}{\tau}}-\lambda\right] \qquad (9)
\end{aligned}
$$

Scaling in the Relativistic Fermi Gas

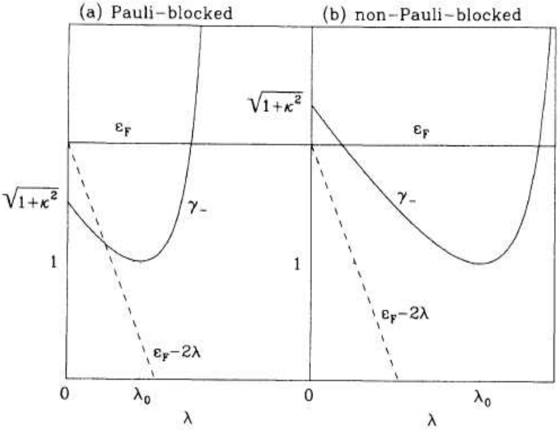

Figure: Behavior of γ_- as a function of λ in two regimes: (a) Pauli-blocked region ($\kappa < \eta_F$) and non-Pauli-blocked region ($\kappa > \eta_F$). It is also shown the line $\Gamma = \epsilon_F - 2\lambda$, which can be larger than γ_- for small λ in the Pauli-blocked regime.

Scaling in the Relativistic Fermi Gas

To map the factor $(\epsilon_F - \Gamma)$ of the response functions in a parabola in the variable λ for constant κ, we can define a generalized dimensionless scaling variable:

$$\psi \equiv \sqrt{\frac{\gamma_- - 1}{\epsilon_F - 1}} \begin{cases} +1, & \text{if} \quad \lambda \geqslant \lambda_0 \\ -1, & \text{if} \quad \lambda \leqslant \lambda_0 \end{cases} \tag{10}$$

where $\lambda_0 \equiv \frac{1}{2}\left[\sqrt{1+4\kappa^2} - 1\right]$ corresponds to the quasi-elastic peak. With this definition, we can easily see that if $\Gamma = \gamma_-$ (which is always correct for the non-Pauli-blocked region), then:

$$1 - \psi^2 = 1 - \frac{\gamma_- - 1}{\epsilon_F - 1} = \frac{\epsilon_F - \gamma_-}{\epsilon_F - 1} \tag{11}$$

Of course, this scaling variable[4] could have been defined with Γ instead of γ_- as it was originally. And indeed this will be our choice besides the change $m_N \to m_N^* = 0.8 m_N$, which we will justify later. We will call it the Pauli-blocked scaling variable ψ^*.

[4] W.M. Alberico et al, **Phys. Rev. C 38**, 1801 (1988)

Summary of scaling in the Relativistic Fermi Gas

- Within the framework of the Relativistic Fermi Gas model and with the aid of an adequate scaling variable ψ, we can write the double-differential cross section as:

$$\frac{d^2\sigma}{d\Omega\, d\epsilon'} = \frac{\mathcal{N}}{4m_N\kappa}\,\sigma_{\rm Mott}\, X(\theta,\tau,\psi;\eta_F)\, S_{RFG}(\psi;\eta_F) \qquad (12)$$

where $S_{RFG}(\psi;\eta_F) = \frac{3\xi_F}{\eta_F^3}\left(1-\psi^2\right)\theta(1-\psi^2)$ basically encodes the dynamical content of the Relativistic Fermi Gas (Fermi motion and Pauli blocking). This model, although simple, provides a picture of the nuclear system and it is independent of the electroweak probe.

- On the other hand, $X(\theta,\tau,\psi;\eta_F)$ contains the information on the interaction of the electroweak probe (electrons, neutrinos...) with the elementary constituents of the many-body system. And, furthermore, this function possesses the correct limiting behavior one would wish in the one-body problem, namely, the response of the single-nucleon to the electroweak probe when $\mathcal{N}=1$ and $\eta_F \to 0$.

Walecka model QHD-I (Quantum Hadrodynamics)

The building blocks of this model are the nucleon doublet field $\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$ and two neutral and isoscalar mesons, one of them is scalar (σ) and the other one is vector (ω^μ). The Lagrangian density for this model is given by:

$$\begin{aligned} \mathcal{L} &= \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_N\right)\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\,\omega_\mu\,\omega^\mu \\ &- g_\omega\bar{\psi}\gamma^\mu\psi\,\omega_\mu + g_\sigma\bar{\psi}\psi\,\sigma \end{aligned} \qquad (13)$$

where $F^{\mu\nu} \equiv \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$.

The field equations for this model can be obtained from the Euler-Lagrange ones and these are:

$$\left(\partial^\mu\partial_\mu + m_\sigma^2\right)\sigma = g_\sigma\,\bar{\psi}\psi \qquad (14)$$

$$\partial_\nu F^{\nu\mu} + m_\omega^2\,\omega^\mu = g_\omega\,\bar{\psi}\gamma^\mu\psi \qquad (15)$$

$$\left[\gamma^\mu\left(i\partial_\mu - g_\omega\,\omega_\mu\right) - \left(m_N - g_\sigma\,\sigma\right)\right]\psi = 0 \qquad (16)$$

Mean Field Theory (MFT) description of the model

If we consider the situation in which we have a system of B baryons in a large box of volume V and we are in the rest frame of the matter, i.e, the baryon current $B^\mu = (\rho_B, \mathbf{B}) = \bar{\psi}\gamma^\mu\psi$ has $\mathbf{B} = 0$. If the baryon density B/V increases, the sources increase as well; and if these are large enough, one would expect to substitute the meson fields by their expectation values:

$$\sigma \rightarrow \langle\sigma\rangle \equiv \sigma_0, \qquad \omega^\mu \rightarrow \langle\omega^\mu\rangle \equiv (\omega_0, 0) \tag{17}$$

Since we are restricting ourselves to stationary situation and uniform system, σ_0 and ω_0 are constants completely independent of space and time. And since the matter is at rest, the three-vector field $\vec{\omega} = 0$.

Mean Field Theory (MFT) description of the model

We can substitute these meson fields on the Lagrangian density to obtain the mean-field Lagrangian:

$$\mathcal{L}_{\mathrm{MFT}} = \bar{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m_N)\,\psi - \frac{1}{2}m_{\sigma}^2\,\sigma_0^2 + \frac{1}{2}m_{\omega}^2\,\omega_0^2 - g_{\omega}\,\bar{\psi}\gamma^0\psi\,\omega_0 + g_{\sigma}\,\bar{\psi}\psi\,\sigma_0 \tag{18}$$

Only the fermion field has to be quantized, and we can particularize the previous Dirac equation to our MFT problem:

$$\left[i\gamma^{\mu}\partial_{\mu} - g_{\omega}\,\gamma^0\,\omega_0 - (m_N - g_{\sigma}\,\sigma_0)\right]\psi(t,\mathbf{x}) = 0 \tag{19}$$

We can see here that the effect of the scalar field is a shift in the baryon mass from m_N to $m_N^* \equiv m_N - g_{\sigma}\,\sigma_0$, and that of the vector field is a shift in the energy spectrum.

Mean Field Theory (MFT) description of the model

We can look for plane-wave solutions of the Dirac equation in this MFT approximation:

$$\psi^{(+)}_{\mathbf{k}\lambda}(t,\mathbf{x}) = U(\mathbf{k},\lambda)e^{i\mathbf{k}\cdot\mathbf{x}-i\epsilon_+(\mathbf{k})t}, \qquad \psi^{(-)}_{\mathbf{k}\lambda}(t,\mathbf{x}) = V(\mathbf{k},\lambda)e^{-i\mathbf{k}\cdot\mathbf{x}-i\epsilon_-(-\mathbf{k})t} \tag{20}$$

Substituting these possible solutions in the Dirac equation, we can obtain the corresponding Dirac equations in momentum representation:

$$[\mathbf{k}\cdot\vec{\alpha} + m_N^*\beta]\, U(\mathbf{k},\lambda) = [\epsilon_+(\mathbf{k}) - g_\omega\,\omega_0]\, U(\mathbf{k},\lambda) \tag{21}$$

$$[-\mathbf{k}\cdot\vec{\alpha} + m_N^*\beta]\, V(\mathbf{k},\lambda) = [\epsilon_-(-\mathbf{k}) - g_\omega\,\omega_0]\, V(\mathbf{k},\lambda) \tag{22}$$

with $\vec{\alpha} = \gamma^0\vec{\gamma}$ and $\beta = \gamma^0$ being the usual Dirac matrices.
Eqs. (21) and (22) look like the free Dirac equation of a fermion of mass $m_N^* = m_N - g_\sigma\,\sigma_0$ with "energy" eigenvalues $E^*_\pm(\mathbf{k}) = \begin{cases} \epsilon_+(\mathbf{k}) - g_\omega\,\omega_0 \\ \epsilon_-(-\mathbf{k}) - g_\omega\,\omega_0 \end{cases}$

Mean Field Theory (MFT) description of the model

Therefore, we can interpret $U(\mathbf{k},\lambda)$ and $V(\mathbf{k},\lambda)$ as free spinors of a fermion of mass m_N^* obeying the Dirac equation with "energy" eigenvalues $E^*_\pm(\mathbf{k}) = \pm\sqrt{\mathbf{k}^2 + m_N^{*2}}$. And it is this $E^*(\mathbf{k})$ which enters in the normalization of the new spinors.
So we can write the hadron tensor as:

$$\begin{aligned} W^{\mu\nu} &= \frac{V}{(2\pi)^3}\int d^3p\,\frac{(m_N^*)^2}{E^*(\mathbf{p})E^*(\mathbf{p}+\mathbf{q})}\theta(k_F-|\mathbf{p}|)\theta(|\mathbf{p}+\mathbf{q}|-k_F) \\ &\times\ \delta\left(\omega-[E^*(\mathbf{p}+\mathbf{q})-E^*(\mathbf{p})]\right)2\,w^{\mu\nu}_{s.n.}(\mathbf{p}',\mathbf{p}) \end{aligned} \quad (23)$$

where the single-nucleon tensor is given by:

$$w^{\mu\nu}_{s.n.}(\mathbf{p}',\mathbf{p}) = \frac{1}{2}\sum_{s\,s'} J^{\mu *}(\mathbf{p}',\mathbf{p})J^{\nu}(\mathbf{p}',\mathbf{p}) \quad (24)$$

Here, $J^{\mu *}$ is the electroweak current matrix element between free positive energy Dirac spinors with mass m_N^*.

Mean Field Theory (MFT) description of the model

In the case of electron scattering, the electromagnetic current matrix element is:

$$J^{\mu}_{s'\,s}(\mathbf{p}',\mathbf{p}) = \bar{u}_{s'}(\mathbf{p}')\ \left[F_1(Q^2)\gamma^{\mu} + i\frac{F_2(Q^2)}{2m_N}\sigma^{\mu\nu}Q_{\nu}\right]\ u_s(\mathbf{p}) \qquad (25)$$

In the case of neutrino scattering, the weak charged current matrix element has vector and axial-vector contributions:

$$J^{\mu}_{W}(\mathbf{p}',\mathbf{p}) = V^{\mu}(\mathbf{p}',\mathbf{p}) - A^{\mu}(\mathbf{p}',\mathbf{p}) \qquad (26)$$

$$V^{\mu}_{s'\,s}(\mathbf{p}',\mathbf{p}) = \bar{u}_{s'}(\mathbf{p}')\ \left[F_1^V(Q^2)\gamma^{\mu} + i\frac{F_2^V(Q^2)}{2m_N}\sigma^{\mu\nu}Q_{\nu}\right]\ u_s(\mathbf{p}) \qquad (27)$$

$$A^{\mu}_{s'\,s}(\mathbf{p}',\mathbf{p}) = \bar{u}_{s'}(\mathbf{p}')\ \left[G_A(Q^2)\gamma^{\mu}\gamma_5 + \frac{G_P(Q^2)}{2m_N}Q^{\mu}\gamma_5\right]\ u_s(\mathbf{p}) \qquad (28)$$

M^*-scaling analysis

Following the lines of the scaling analysis for the relativistic Fermi Gas with the scaling variable ψ, we can define an analogous one with the replacement of m_N by m_N^* everywhere in the formulas.

$$\left.\begin{array}{l} \kappa^* \equiv \frac{|\mathbf{q}|}{2m_N^*} \\ \lambda^* \equiv \frac{\omega}{2m_N^*} \end{array}\right\} \to \tau^* = \kappa^{*2} - \lambda^{*2}, \tag{29}$$

$$\begin{aligned} \eta^* &\equiv \frac{|\mathbf{p}|}{m_N^*}, \qquad \epsilon^* \equiv \frac{E^*(\mathbf{p})}{m_N^*} = \sqrt{1+\eta^{*2}} \\ \eta_F^* &\equiv \frac{k_F}{m_N^*}, \qquad \epsilon_F^* = \sqrt{1+\eta_F^{*2}} \end{aligned} \tag{30}$$

And the nuclear response functions can be written in the factorized form:

$$R_K = G_K\, f_{\mathrm{RFG}}(\psi^*) \qquad f_{\mathrm{RFG}}(\psi^*) = \frac{3}{4}\left(1-\psi^{*2}\right)\theta(1-\psi^{*2}) \tag{31}$$

$$G_K = \Lambda\left(Z\, U_K^p + N\, U_K^n\right) \qquad \text{with} \qquad \Lambda = \frac{\epsilon_F^* - 1}{m_N^*\, \kappa^*\, \eta_F^{*3}} \tag{32}$$

M^*-scaling analysis

And U_K ($K = L, T, T', ...$) are the single nucleon response functions which can be found, for instance, in Ref. *J.E. Amaro et al,* **Phys. Rev. C71** *(2005) 065501*.

As we do not change the current operator, the only way for m_N^* to appear is through the nucleon spinors when summing over spin polarizations. This is equivalent to take the formulas for the single nucleon response functions from the above reference as they stand there but defining all the kinematic variables with respect to the effective mass instead of the free one m_N, including the electric and magnetic form factors as well.

This implies that even although F_1 and $\frac{F_2}{m_N}$ are not modified in the medium, the electric (G_E) and magnetic (G_M) Sach's form factors are modified in the medium. This is because the nucleon mass enters in their definition in two places: in one of them the mass must be changed but not in the other.

This can be easily seen with the example of the electric form factor G_E:

$$G_E = F_1 - \tau F_2 = F_1 - m_N \tau \left(\frac{F_2}{m_N} \right) \tag{33}$$

$\frac{F_2}{m_N}$ does not change, so we can redefine it in terms of a re-scaled form factor F_2^* provided that they are related as follows:

$$F_2^* = \frac{m_N^*}{m_N} F_2 \tag{34}$$

Therefore, eq. (33) can be rewritten as:

$$G_E = F_1 - m_N \tau \left(\frac{F_2^*}{m_N^*} \right) = F_1^* - \frac{m_N^*}{m_N} \tau^* F_2^* \neq F_1^* - \tau^* F_2^* \equiv G_E^* \tag{35}$$

where in the second step we have used that F_1 is not modified (i.e, $F_1^* = F_1$), and the relation between τ^* and τ, which goes inversely with respect to their squared masses, namely:

$$\frac{\tau^*}{\tau} = \frac{m_N^2}{m_N^{*2}} \tag{36}$$

Results

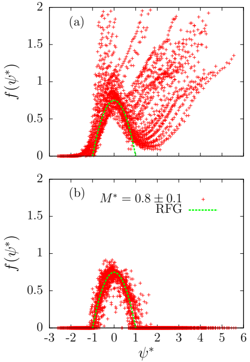

Figure: Top panel (a): M^*-scaling analysis of the experimental data of ^{12}C as a function of the scaling variable ψ^* for $m_N^* = 0.8 m_N$ and comparison with the RFG parabola. Bottom panel (b): RFG Monte Carlo simulation of QE data with a Gaussian distribution of relativistic effective mass quotient around $M^* = \frac{m_N^*}{m_N} = 0.8 \pm 0.1$. The Fermi momentum is $k_F = 225$ MeV/c

$$f_{\rm exp} = \frac{\left(\frac{d\sigma}{d\Omega'\, d\epsilon'}\right)_{\rm exp}}{\sigma_{\rm Mott}\left(v_L\, G_L + v_T\, G_T\right)} \tag{37}$$

Results

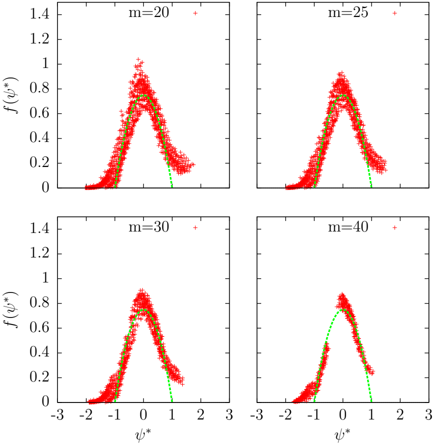

Figure: Experimental data selection in terms of the scaling variable ψ^*, obtained with different choices of the number m of points inside a circle of radius $r = 0.1$.

Results

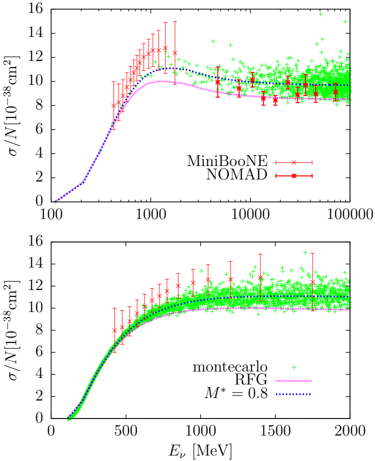

Figure: Total QE neutrino cross section off ^{12}C per neutron as a function of the neutrino energy for different relativistic effective masses generated in a Monte Carlo simulation around $M^* = 0.8 \pm 0.1$. The experimental points are from NOMAD and MiniBooNE. The axial dipole mass is $M_A = 1$ GeV.

Concluding remarks

- We have reanalyzed the inclusive (e, e') scattering data off ^{12}C in terms of a new scaling variable ψ^* as a generalization of the Relativistic Fermi Gas model to include scalar and vector interactions in a minimal way.
- Scaling violations or departures from the expected RFG universal-shaped parabola can be mimicked by the standard statistical fluctuations in the determination of the relativistic effective mass, as seen in the Monte Carlo simulation.
- This uncertainty band which reflects the scaling deviations can be translated into a theoretically predicted error band for other related reactions with electroweak probes such as Charged Current neutrino reactions. And this uncertainty band can help to reconcile different data sets from different experiments that, in principle, seemed incompatible.
- Furthermore, in this model gauge invariance and PCAC are automatically fulfilled, so there is no need to restore them afterwards. Besides, there is no need to worry about any ω-shift applied in previous studies to make the data to scale better, and this phenomenological energy shift does not have a well understood theoretical origin.