Novel finite-temperature effects on the $f_0(500)$ resonance and chiral symmetry restoration

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1. Large N expansion for LEQCD in the chiral limit and $T = 0$
2. Diagrammatics at $m_\pi = 0, 0 < T < T_\chi$ in the large N limit
3. Mass and width of the $f_0(500)$ pole and their relation to χSR
4. Conclusions

$1/N$ expansion (1)

$O(N+1)/O(N) \to$ nonlinear sigma model for massless pions:

$$\mathcal{L} = \frac{1}{2} g_{ab} \partial_\mu \pi^a \partial^\mu \pi^b. \tag{1}$$

Constraint and metric induced by the scalar fields:

$$\pi_a^2 + \sigma^2 = NF^2, \to f_\pi^2 = NF^2. \tag{2}$$

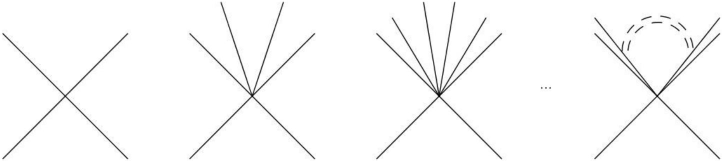
$$g_{ab} = \delta_{ab} + \frac{\pi_a \pi_b}{NF^2\left(1 - \frac{\pi_a^2}{NF^2}\right)}. \tag{3}$$

Refs:

A. Dobado, J. Morales, Phys. Rev. D **52**: 2878 (1995).
A. Dobado, A. Gómez Nicola, A. L. Maroto, J. R, Peláez, *Effective Lagrangians for the Standard Model*. Springer-Verlag Berlin Heidelberg (1997).

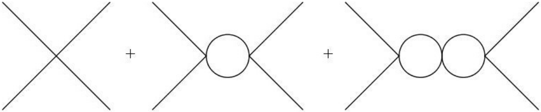
$1/N$ expansion (2)

Feynman rules and s-channel amplitude:



$\frac{s}{NF^2}$ $\qquad \frac{s}{(NF^2)^2}$ $\qquad \frac{s}{(NF^2)^3}$ $\qquad \cdots \qquad \frac{s}{(NF^2)^{N+1}}$

$A(s) =$



$+ \qquad + \qquad +\ldots$

$\boxed{\text{(Large } N \text{ limit)}}$

$$= \frac{s}{NF^2\left[1 + s\,J(s)/2F^2\right]} \to J(s) = -\frac{1}{(4\pi)^2}\left[N_\epsilon + \ln\left(\frac{\mu^2}{-s}\right)\right], N_\epsilon = \frac{2}{\epsilon} + \ln 4\pi - \gamma + 2.$$

$1/N$ expansion (3)

Renormalization and fitting to the s-channel phase shift:

$$\frac{s}{NF^2} \to \frac{s}{NF^2} G_0(s),\, G_0(s) = 1 + \sum_{k=1}^{\infty} g_{0,\,k} \left(\frac{s}{F^2}\right)^k.$$

$g_{0,\,k}$: Bare low energy constants (LEC's)

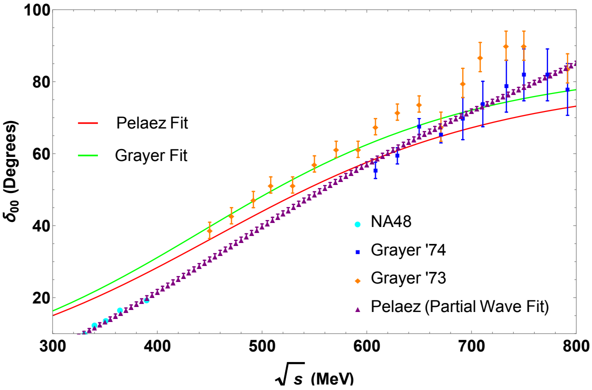
$$\Rightarrow A_R(s) = \frac{s\, G_R(s)}{NF^2\left[1 - \frac{s\, G_R(s)}{32\pi^2 F^2} \ln\left(\frac{\mu^2}{-s}\right)\right]},\, \frac{1}{G_R(s)} = \frac{1}{G_0(s)} - \frac{s\, N_\epsilon}{32\pi^2 F^2}. \quad (4)$$

Renormalized LEC's: $g_{R,\,k}(\mu) = 0 \to$ only two fit parameters (F and μ).

$$T_{I=0}(s, \cos\theta) = NA_R(s) + A_R(t) + A_R(u) \approx NA_R(s). \quad (5)$$

$1/N$ expansion (4)

δ_{00} fitted according to Grayer data: F and μ > massive case (A. Dobado, J. Morales, Phys. Rev. D **52**: 2878 -1995-).



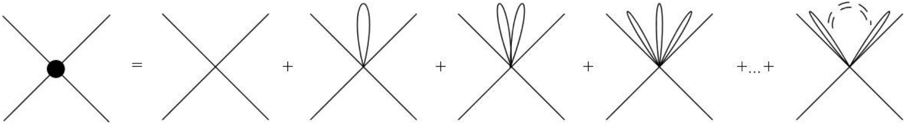
Grayer	Parameters
F (MeV)	63.16±1.62
μ (MeV)	1523.35±143.34
R^2	0.996

Peláez	Parameters
F (MeV)	65 (fixed)
μ (MeV)	1411.92±3.29
R^2	0.876

Refs: G. Grayer *et al*, Nucl. Phys. B **75**: 189-245 (1974); *ibid* **64**: 134-162 (1973); J. R. Batley *et al*, (NA48-2 Collaboration), Eur. Phys. J. C **70**: 635-657 (2010); R. García Martín, R. Kamiński, J. R. Peláez, J. Ruiz de Elvira, Phys. Rev. Lett. 107: 072001 (2011).

Finite-Temperature scattering amplitude (1)

Effective thermal vertex:



$$= \frac{s}{NF^2}\frac{1}{1 - I_\beta/F^2}$$

where $I_\beta = \frac{T^2}{12}$.

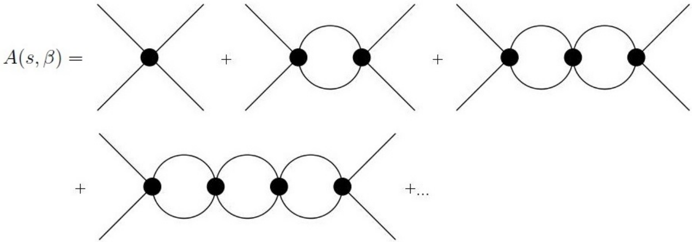
- **Null contribution at zero temperature**

Reminding that $m_\pi = 0$

$\Rightarrow$ **no need to renormalize the pion wavefunctions and their masses**

Finite-Temperature scattering amplitude (2)

Process:

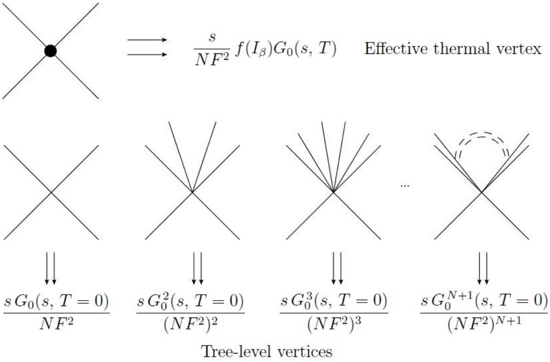


$$A(s,\beta)=\frac{s}{NF^2}\frac{f(I_\beta)}{1+\frac{s}{2F^2}f(I_\beta)J(s,\beta)};\, f(I_\beta)=\frac{1}{1-I_\beta/F^2}, \tag{6}$$

$$J(s,\beta)=J(s)+\overline{J}(s,\beta).$$

Renormalization

Redefinition of the effective thermal vertex and the tree-level vertices:



After building up $A(s,\beta)$:

$$\frac{1}{f(I_\beta) G_R(s, T)} = \frac{1}{G_R(s, T=0)} - \frac{I_\beta}{F^2}, \qquad (7)$$

where

$$\frac{1}{f(I_\beta) G_R(s, T)} = \frac{1}{f(I_\beta) G_0(s, T)} - \frac{s\, N_\epsilon}{2(4\pi)^2 F^2}.$$

Thermal unitarity

$$a_{00}(s,\beta) = \frac{1}{64\pi}\int_0^1 NA_R(s,\beta)P_0(\cos\theta)d(\cos\theta). \tag{8}$$

$$\mathrm{Im}\left\{\frac{1}{a_{00}(s,\beta)}\right\} = -\frac{1}{\pi}\left[\pi + 16\pi^2\mathrm{Im}\left\{\overline{J}(s,\beta)\right\}\right] = -\left[1 + 2n\left(\frac{\sqrt{s}}{2}\right)\right].$$

Since $\sigma_\beta(\sqrt{s}) = 1 + 2n(\sqrt{s}/2) \to n(p) = 1/\left[\exp(\beta p) - 1\right]$ (chiral limit),

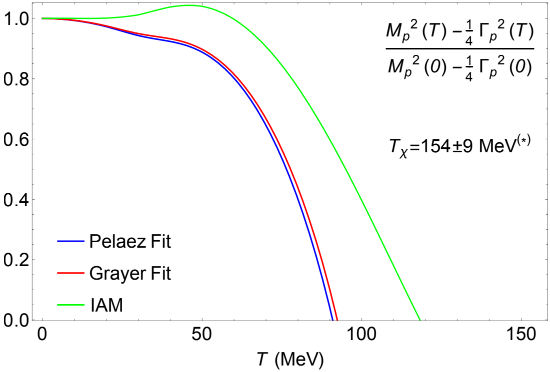
$$\boxed{\mathrm{Im}\left\{\frac{1}{a_{00}(s,\beta)}\right\} = -\sigma_\beta(\sqrt{s}).} \tag{9}$$

Refs:

A. Gómez Nicola, F. J. Llanes-Estrada, J. R. Peláez, Phys. Lett. B **550**:55-64 (2002); A. Gómez Nicola, J. R. Peláez, A. Dobado, F. J. Llanes-Estrada, AIP Conf.Proc. **660**: 156-169 (2003). arXiv: hep-ph/0212121 (perturbative thermal unitarity).

Chiral symmetry restoration ($\sqrt{s} = M_P(T) - i\Gamma_P(T)/2$)

$\chi_S(T) \propto 1/\mathrm{Re}\{s\}$ in the limit $p = 0^\dagger$ **(Gómez Nicola's talk.)**



Plot	$T_C(MeV)$
Peláez	90.89
Grayer	92.33
IAM	118.23

$M_P(0)$ (MeV)	$\Gamma_P(0)$ (MeV)
447.68	576.17
438.81	536.47
406.20	522.70
$440^{(PDG)}$	$490^{(PDG)}$

Refs:

† A. Gómez Nicola, J. Ruiz de Elvira, R. Torres Andrés, Phys.Rev.D **88**: 076007 (2013).
(*) A. Bazavov *et al* (HotQCD Collaboration), Phys. Rev. D **85**: 054503 (2012).
(PDG): Dobado *et al* (as listed in PDG 2014).

- Although we work in the Chiral Limit, our analysis of ellastic pion scattering at finite temperature in the large N expansion grants a description of the $f_0(500)$ pole thermal dependence that is consistent with previous works*.
- We show that thermal unitarity holds exactly for the sum of all perturbative orders when considering finite-temperature effects.
- In the Chiral Limit, the behavior of $\chi_S^{-1}(T)$ (saturated by the σ pole) is consistent with a second-order phase transition (as seen in the lattice**).
    - The obtained critical temperature is not far from the expected lattice values (about $0.8\,T_\chi$).

Refs:

* A. Gómez Nicola, J. Ruiz de Elvira, R. Torres Andrés, Phys.Rev.D **88**: 076007 (2013) ; A. Gómez Nicola, J. R. Peláez, J. Ruiz de Elvira, Phys. Rev. D **87**: 016001 (2013).
** A. Bazavov *et al* (HotQCD Collaboration), Phys. Rev. D **85**: 054503 (2012).