Overview of transverse momentum distributions

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work in progress with U. D' Alesio (Cagliari), M.G. Echevarría (NIKHEF), A. Idilbi (Penn. State), S. Melis (Torino) and A. Vladimirov (Lund-> Regensburg)

And also:
arXiv: JHEP1 1(2014)098 with U. D' Alesio (Cagliari), M.G. Echevarría (NIKHEF), S. Melis (Torino)

and also EIS (Echevarria, Idilbi, Scimemi) FORMALISM:
PRD90 (2014) 014003, PLB726(2013) 795, JHEP1207 (2012) 002
EIS+A. Schafer, EPJC 73(2013)2636

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Topics and outline

* At hadron colliders the peaks of transverse momentum spectra are located at small qT/pT: these regions are affected by non-perturbative QCD effects. **We need a method to treat them.**

* Observables: Mw mass, Spin dependent observables, transverse momentum dependent observables, Bosons + jets

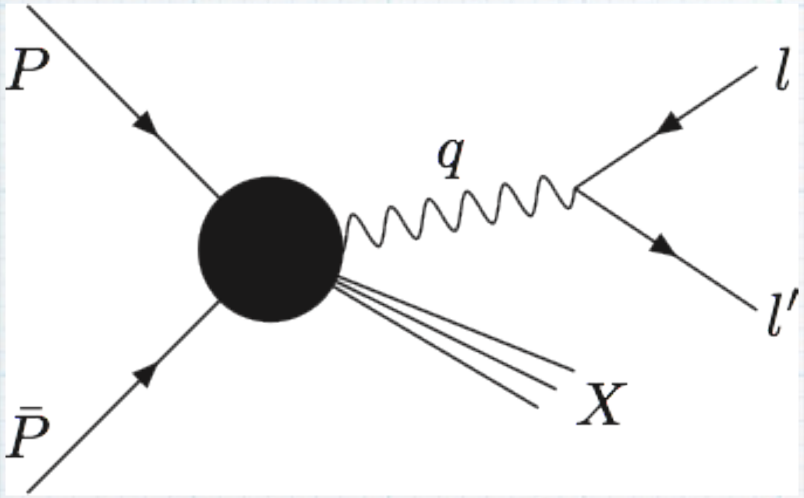
* Transverse momentum distributions involve non-perturbative QCD effects which go beyond the usual PDF formalism. New factorization theorem are required.

* TMD's are the fundamental non-perturbative objects to be used in factorization theorems in (un-)polarized Drell-Yan, SIDIS, e+e- to 2 jets (multi-jets?).

* Properties of TMD's:
  1) The evolution of all TMD's is universal (alike PDF and FF it is process independent)
  2)The evolution of all TMD's is spin independent and it is the same for TMDPDF and TMDFF

* **We can map all these non-perturbative effects fitting DY, SIDIS, ee data at low M: Here results for DY fit and some predictions for LHC**

Inclusive DY case



Collins, Soper, Sterman '82

$$q^2 = Q^2$$

$$\frac{d\sigma}{dQ^2} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1\, dx_2\, \mathcal{H}_{ij}(x_1, x_2, Q^2, \mu^2)\, f_{i/P}(x_1, \mu^2)\, f_{j/\bar{P}}(x_2, \mu^2)$$

Short Distance

Long Distance

$$f_{i/P}(0^+, y^-, 0_\perp) = \frac{1}{2}\sum_\sigma \langle P, \sigma | [\bar{\xi}_n W_n](0^+, y^-, 0_\perp) \frac{\bar{\not{n}}}{2} [W_n^\dagger \xi_n](0) | P, \sigma \rangle$$

- All non-perturbative information is encoded in the PDF
- We want to get more information on the nucleon structure exploring transverse momentum dependent cross section
- Consider $\frac{d\sigma}{dQ^2 dq_T^2}$, can we play the same game?

Naive extension of TMDPDF

The naive extension of the TMD does not work

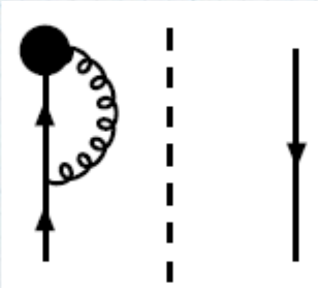
$$F_n^{naive}(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_\sigma \langle P,\sigma|\ \left[\bar{\xi}_n W_n\right](0^+, y^-, \vec{y}_\perp)\,\frac{\bar{n}\!\!\!/}{2}\,\left[W_n^\dagger \xi_n\right](0)\,|P,\sigma\rangle$$

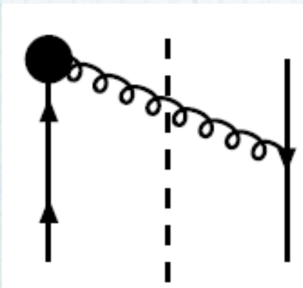
Transverse gauge links should be included EIS '11, but the core problem are rapidity divergences: at one loop

$$\tilde{F}_n^{naive} = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{2}{\varepsilon_{\rm UV}}\ln\frac{\Delta^+}{Q^2} + \frac{3}{2\varepsilon_{\rm UV}} - \frac{1}{4} + \frac{3}{2}L_T + 2L_T\ln\frac{\Delta^+}{Q^2}\right] - (1-x)\ln(1-x) - \mathcal{P}_{q/q}\ln\frac{\Delta^-}{\mu^2} - L_T\mathcal{P}_{q/q}\right\}$$

This quantity cannot be renormalized

Naive extensions of TMDPDF





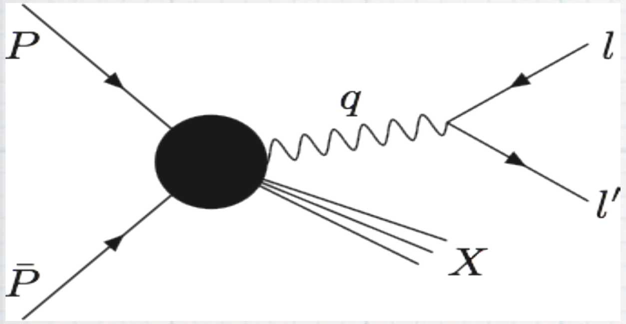
The problems appear when gluons and quarks are collinear

$$\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\frac{p^++k^+}{[k^+-i\varepsilon][(p+k)^2+i\varepsilon][k^2+i0]} \Longrightarrow \frac{1}{\varepsilon_{UV}}\times\int_0^1\frac{1}{t}$$

In the PDF the problem is solved combing virtual and real gluon emission. However this cancellation does not happen now for unintegrated distributions

This cancellation must hold in QCD because there are no rapidity divergences in the hadronic tensor

Energy scales: DY example



Q=M

$$q^2 = Q^2 \gg q_T^2$$

Both limits should be included in the phenomenological analysis

$q_T^2 \sim \Lambda_{QCD}^2$ ⇒ $\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$

$q_T^2 \gg \Lambda_{QCD}^2$ ⇒ $\tilde{M} = H(Q^2/\mu^2)\, \tilde{C}_n(b^2\mu^2, Q^2/\mu^2)\, \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2)\, f_n(x_n; \mu^2)\, f_{\bar{n}}(x_{\bar{n}}; \mu^2)$

The IR has to be regulated consistently in the theories above and below every matching scale in order to properly extract the matching (Wilson) coefficients.

Problems with different energy scales are more easily treated with EFT

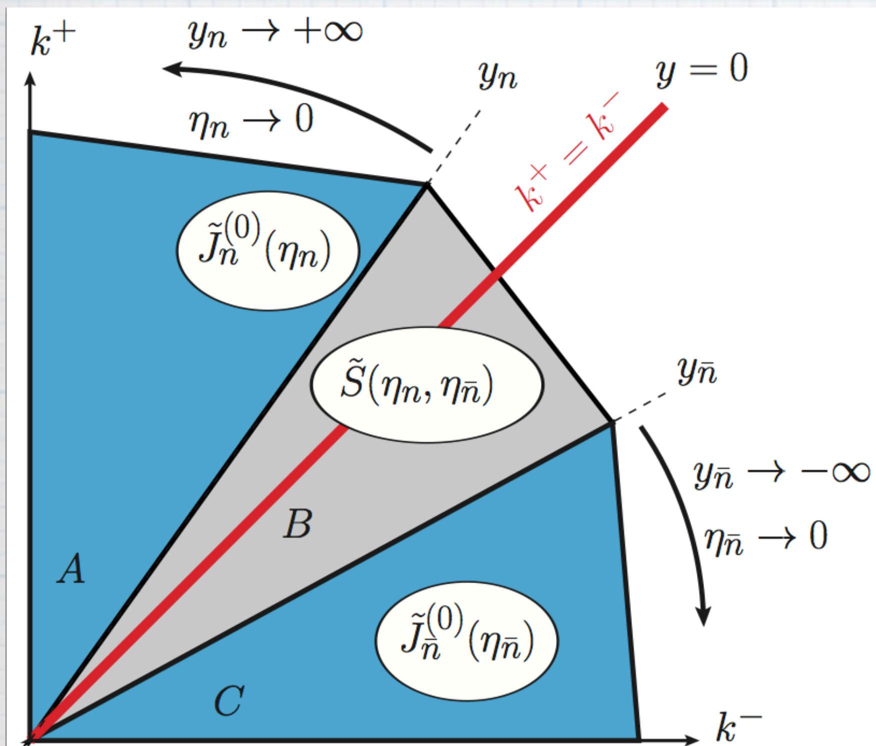
$$\text{QCD} \xrightarrow{q_T^2 \gg \Lambda_{QCD}^2} \text{SCET}_{q_T} \xrightarrow{q_T^2 \sim \Lambda_{QCD}^2} \text{SCET}_{\Lambda_{\text{QCD}}}$$

Modes in EFT

Using power counting we have collinear, anti-collinear, and soft sectors

$$H(Q^2)\,\tilde J_n^{(0)}(\eta_n)\,\tilde S(\eta_n,\eta_{\bar n})\,\tilde J_{\bar n}^{(0)}(\eta_{\bar n})$$

In EFT each mode belongs to a Hilbert space separate from the others.
To each mode correspond a different Lagrangian
Boosts mix soft and collinear modes (same invariant mass)



(+,-,perp)

$$k_n \sim Q(1,\lambda^2,\lambda) \quad \to \quad y \gg 0$$
$$k_{\bar n} \sim Q(\lambda^2,1,\lambda) \quad \to \quad y \ll 0$$
$$k_s \sim Q(\lambda,\lambda,\lambda) \quad \to \quad y \approx 0$$
$$\lambda \sim \frac{q_T}{Q}$$

$$n = (1,0,0,1)$$
$$\bar n = (1,0,0,-1)$$

$$\chi_n = W_n^\dagger \xi_n$$

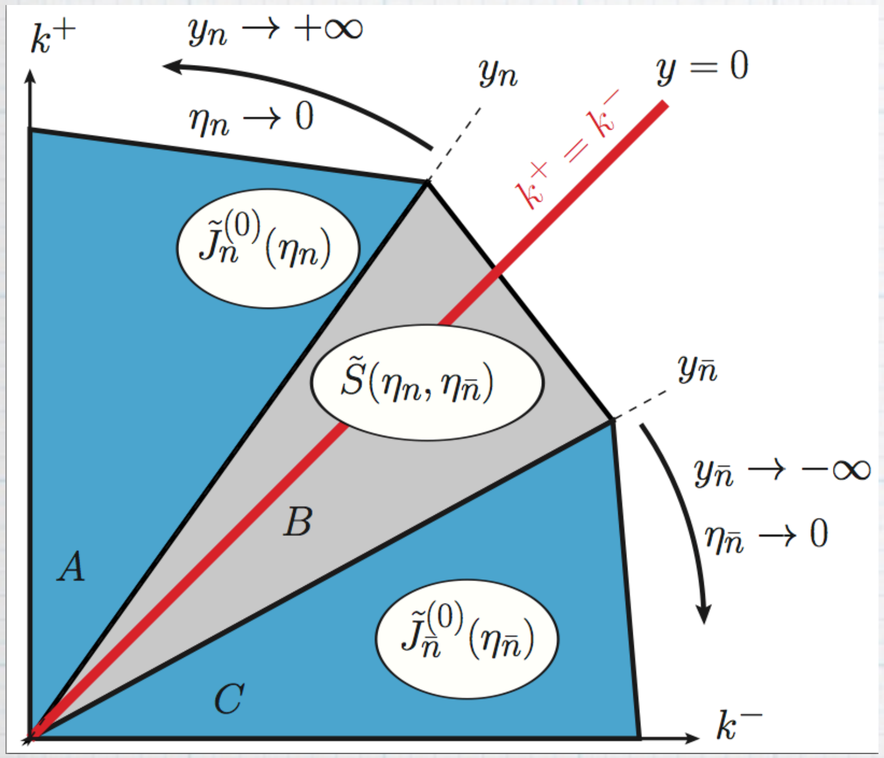
$$J_n^{(0)}(0^+,y^-,y_\perp) = \frac{1}{2}\sum_{\sigma_1}\langle N_1(P,\sigma_1)|\bar\chi_n(0^+,y^-,y_\perp)\frac{\bar{n}\!\!\!/}{2}\chi_n(0)|N_1(P,\sigma_1)\rangle$$

$$J^{(0)}_{\bar n}(y^+,0^-,y_\perp) = \frac{1}{2}\sum_{\sigma_1}\langle N_2(\bar P,\sigma_2)|\bar\chi_{\bar n}(0)\frac{n\!\!\!/}{2}\chi_{\bar n}(y^+,0^-,y_\perp)|N_2(\bar P,\sigma_2)\rangle$$

$$S(0^+,0^-,y_\perp) = \langle 0|\mathrm{Tr}\,\bar{\mathbf{T}}\left[S_n^{T\dagger}S_{\bar n}^{T}\right](0^+,0^-,y_\perp)\mathbf{T}\left[S_{\bar n}^{T\dagger}S_n^{T}\right](0)|0\rangle, \qquad \chi = W^{T\dagger}\xi$$

multipole expansion fixes arguments

Modes in EFT



Using power counting we have collinear, anti-collinear, and soft sectors

$$H(Q^2)\, \tilde{J}_n^{(0)}(\eta_n)\, \tilde{S}(\eta_n, \eta_{\bar{n}})\, \tilde{J}_{\bar{n}}^{(0)}(\eta_{\bar{n}})$$

In EFT each mode belongs to a Hilbert space separate from the others.
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$$k_n \sim Q(1, \lambda^2, \lambda) \quad \rightarrow \quad y \gg 0$$
$$k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda) \quad \rightarrow \quad y \ll 0$$
$$k_s \sim Q(\lambda, \lambda, \lambda) \quad \rightarrow \quad y \approx 0$$
$$\lambda \sim \frac{q_T}{Q}$$

$$J_n^{(0)}(0^+, y^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_1(P,\sigma_1) | \bar{\chi}_n(0^+, y^-, y_\perp) \frac{\not{\bar{n}}}{2} \chi_n(0) | N_1(P,\sigma_1) \rangle$$

$$J^{(0)}{}_{\bar{n}}(y^+, 0^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_2(\bar{P},\sigma_2) | \bar{\chi}_{\bar{n}}(0) \frac{\not{n}}{2} \chi_{\bar{n}}(y^+, 0^-, y_\perp) | N_2(\bar{P},\sigma_2) \rangle$$

$$S(0^+, 0^-, y_\perp) = \langle 0 | \mathrm{Tr}\, \overline{\mathbf{T}}\left[S_n^{T\dagger} S_{\bar{n}}^T\right](0^+, 0^-, y_\perp) \mathbf{T}\left[S_{\bar{n}}^{T\dagger} S_n^T\right](0) | 0 \rangle, \qquad \chi = W^{T\dagger} \xi$$

multipole expansion fixes arguments

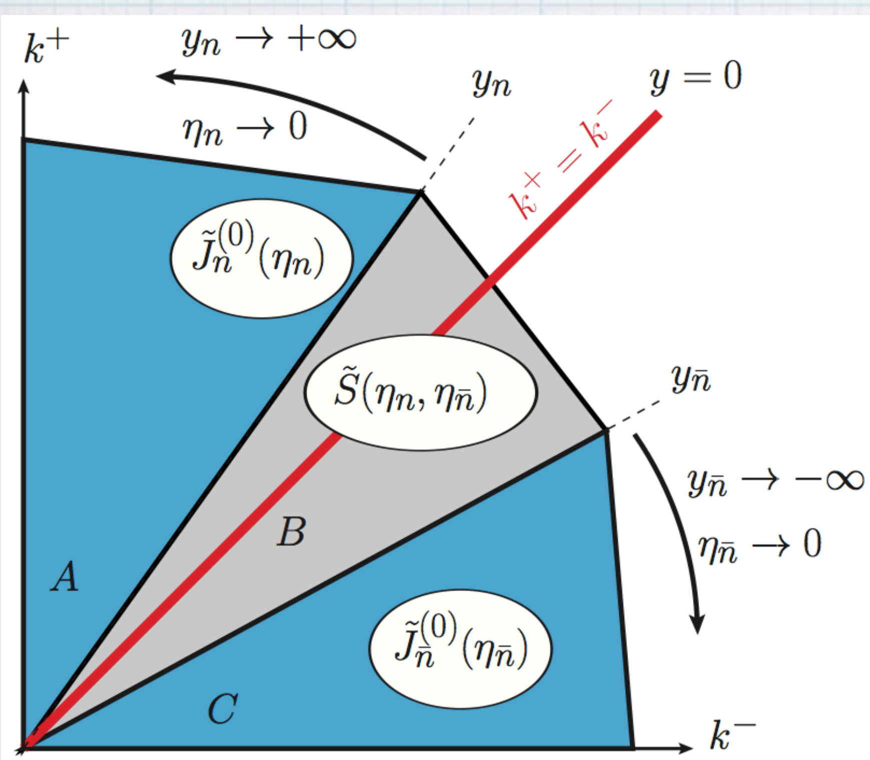
None of these sectors is well defined: rapidity divergences

Rapidity divergences

- Modes can be distinguished only by their rapidity, so need a rapidity regulator (Manohar, Stewart, 2006)
- All properties of TMD are regulator independent
- We performed our calculation on-the-light cone and using delta-regulator (Chiu, Fuhrer, Hoang Manohar, 2009). Checks with other regulators agree (Collins 2011, Chiu, Jain,Neill, Rothstein 2012, ..)

$$\frac{i(\not p + \not k)}{(p+k)^2 + i\Delta^-} \longrightarrow \frac{1}{k^- + i\delta^-}\,,\ \delta^- = \frac{\Delta^-}{p^+}$$

$$\frac{i(\not{\bar p} - \not k)}{(\bar p - k)^2 + i\Delta^+} \longrightarrow \frac{1}{-k^+ + i\delta^+}\,,\ \delta^+ = \frac{\Delta^+}{\bar p^-}$$



Rapidity divergences at one loop:

$$\tilde{J}^{(0)}_{n1}(\Delta^-) = \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{2}{\varepsilon^2_{\rm UV}} - \frac{2}{\varepsilon_{\rm UV}}\ln\frac{\Delta^-}{\mu^2} + \frac{3}{2\varepsilon_{\rm UV}} - \frac{1}{4} - \frac{2\pi^2}{12} - L_T^2 + \frac{3}{2}L_T - 2L_T\ln\frac{\Delta^-}{\mu^2}\right] - (1-x)\ln(1-x) - \mathcal{P}_{q/q}\ln\frac{\Delta^-}{\mu^2} - L_T\mathcal{P}_{q/q}\right\}$$

Pure collinear matrix elements are "per se" ill defined

- A well-defined TMDPDF should:

1. Be compatible with a factorization theorem.
2. Have no mixed UV/nUV divergencies, i.e., be renormalizable
3. Have a matching coefficient onto PDFs independent of nUV regulators.

Definition of TMD's

Positive and negative rapidity quanta can be collected into 2 different TMDs because of the splitting of the soft function: we can consistently split the soft radiation in the two sectors

$$\tilde{S}(b_T;\frac{Q^2\mu^2}{\Delta^+\Delta^-},\mu^2) = \tilde{S}_-\left(b_T;\zeta_F,\mu^2;\Delta^-\right)\tilde{S}_+\left(b_T;\zeta_D,\mu^2;\Delta^+\right)$$

$$\zeta_F = Q^2/\alpha \qquad \tilde{S}_-\left(b_T;\zeta_F,\mu^2;\Delta^-\right) = \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+},\alpha\frac{\Delta^-}{\bar{p}^-}\right)}$$

$$\zeta_D = \alpha\, Q^2$$

$$\tilde{S}_+\left(b_T;\zeta_D,\mu^2;\Delta^+\right) = \sqrt{\tilde{S}\left(\frac{1}{\alpha}\frac{\Delta^+}{p^+},\frac{\Delta^+}{\bar{p}^-}\right)}$$

$$\tilde{F}_n = \tilde{J}_n^{(0)}(\Delta^-)\sqrt{\tilde{S}(\Delta^-,\Delta^-)}$$

$$\tilde{F}_{\bar{n}} = \tilde{J}_{\bar{n}}^{(0)}(\Delta^+)\sqrt{\tilde{S}(\Delta^+,\Delta^+)}$$

Pure collinear

TMDPDF

$$\ln F_{ij}(x,\mathbf{b}_\perp,S;\zeta_F,\mu^2;\Delta^-) = \ln\tilde{\Phi}_{ij}^{(0)}(x,\mathbf{b},S;\mu^2;\Delta^-) + \ln\tilde{S}_-(b_T;\zeta_F,\mu^2;\Delta^-)$$

TMDFF

$$\ln D_{ij}(x,\mathbf{b}_\perp,S_h;\zeta_D,\mu^2;\Delta^+) = \ln\tilde{\Delta}_{ij}^{(0)}(x,\mathbf{b},S_h;\mu^2;\Delta^+) + \ln\tilde{S}_+(b_T;\zeta_D,\mu^2;\Delta^-)$$

Soft Function

The splitting of the soft functions works at all orders in PT: it is linear in logs of rapidity

$$\ln \tilde{S} = \mathcal{R}_s(b_T, \alpha_s) + 2D(b_T, \alpha_s)\, \ln\left(\frac{\Delta^+\Delta^-}{Q^2\mu^2}\right)$$

$$\ln \tilde{S}_- = \frac{1}{2}\mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln\left(\frac{(\Delta^-)^2}{\zeta_F \mu^2}\right),$$

$$\ln \tilde{S}_+ = \frac{1}{2}\mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln\left(\frac{(\Delta^+)^2}{\zeta_D \mu^2}\right)$$

Q-dependence of TMD's

$$\zeta_F = Q^2/\alpha$$
$$\zeta_D = \alpha\ Q^2$$

$$\frac{d}{d\ln\zeta_F} \ln \tilde{F}^{[\Gamma]}_{f/N}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) = -D(b_T; \mu^2)\,,$$

$$\frac{d}{d\ln\zeta_D} \ln \tilde{D}^{[\Gamma]}_{h/f}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) = -D(b_T; \mu^2)\,.$$

The Q-dependence of the TMD is dictated by the soft function: spin independent

One loop results for TMDPDF (DY case)

$$\tilde{M} = H(Q^2/\mu^2)\tilde{F}_n(x_n, b^2; Q^2, \mu^2)\tilde{F}_{\bar{n}}(x_{\bar{n}}, b^2; Q^2, \mu^2)$$

$$\tilde{F}_n = \tilde{J}_n^{(0)}(\Delta^-)\sqrt{\tilde{S}(\Delta^-, \Delta^-)}$$

$$\tilde{F}_{\bar{n}} = \tilde{J}_{\bar{n}}^{(0)}(\Delta^+)\sqrt{\tilde{S}(\Delta^+, \Delta^+)}$$

$$\tilde{F}_{n1} = \frac{\alpha_s C_F}{2\pi}\Big\{ \delta(1-x)\Big[\frac{1}{\varepsilon_{UV}^2} - \frac{1}{\varepsilon_{UV}}\ln\frac{Q^2}{\mu^2} + \frac{3}{2\varepsilon_{UV}}$$
→ NO Mixed divergences

$$-\frac{1}{2}L_T^2 + \frac{3}{2}L_T - L_T\ln\frac{Q^2}{\mu^2} - \frac{\pi^2}{12}\Big] + (1-x) - L_T P_{q/q}$$
→ Matching coeff. to PDF

$$-P_{q/q}\ln\frac{\Delta^-}{\mu^2} - \frac{1}{4}\delta(1-x) - (1-x)[1+\ln(1-x)] \Big\}$$
→ PDF

D-resummation

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s) \qquad D(b;\mu) = \sum_{n=1}^{\infty} d_n(L_\perp)\left(\frac{\alpha_s}{4\pi}\right)^n$$

	LL	NLL	NNLL		
$d_1(L_\perp) =$	$d_1^{(1)} L_\perp$	$+ d_1^{(0)}$			
$d_2(L_\perp) =$	$d_2^{(2)} L_\perp^2$	$+ d_2^{(1)} L_\perp$	$+ d_2^{(0)}$		
$d_3(L_\perp) =$	$d_3^{(3)} L_\perp^3$	$+ d_3^{(2)} L_\perp^2$	$+ d_3^{(1)} L_\perp$	$+ d_3^{(0)}$	
$d_4(L_\perp) =$	$d_4^{(4)} L_\perp^4$	$+ d_4^{(3)} L_\perp^3$	$+ d_4^{(2)} L_\perp^2$	$+ d_4^{(1)} L_\perp$	$+ d_4^{(0)}$
$d_5(L_\perp) =$	$\ldots$				

$$D\left(b;Q_i\right) = D\left(b;\mu_b\right) + \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\bar{\mu}}\Gamma_{\text{cusp}}; \qquad \mu_b = 2e^{-\gamma_E}/b$$

Landau pole

$$D(b;Q_i) = -\frac{\Gamma_0}{2\beta_0}\ln\frac{\alpha_s(Q_i)}{\alpha_s(\mu_b)} \longrightarrow D(b;Q_i) = -\frac{\Gamma_0}{2\beta_0}\ln(1-X)$$

$$\alpha_s(\mu_b) = \alpha_s(Q)/(1-X)$$

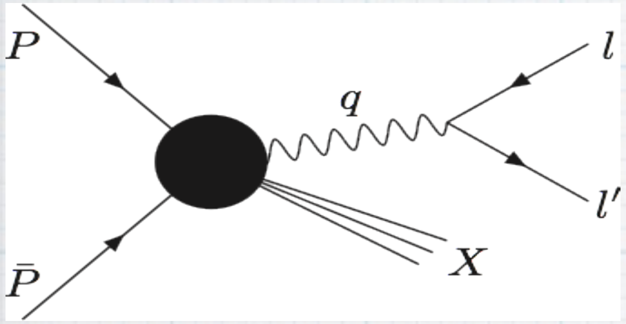
The perturbative expansion of the D is valid when logs are small

$$\mu \sim q_T \sim 1/b$$

Outside this region we have to resum the D, and finally one gets to the pure non-perturbative part of D. Is the NP part dominant?

If the answer is yes we are almost lost ..

DY, SIDIS, ee-> 2j, TMD's and energy scales



* Transverse momentum distributions involve non-perturbative QCD effects which go beyond the usual PDF formalism. New factorization theorem are required. (Collins '11, Echevarría-Idilbi-S. '12)

$$q^2 = Q^2 \gg q_T^2$$ Q=M=dilepton invariant mass

$q_T^2 \gg \Lambda_{QCD}^2$ ⇒ $$\tilde{M} = H(Q^2/\mu^2)\, \tilde{C}_n(b^2\mu^2, Q^2/\mu^2)\, \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2)\, f_n(x_n;\mu^2)\, f_{\bar{n}}(x_{\bar{n}};\mu^2)$$

Example: Vector boson (Tevatron, LHC) and Higgs production at LHC (up to a certain precision, qT>5-10 GeV..),
Some DIS data from HERA

$q_T^2 \sim \Lambda_{QCD}^2$ ⇒ $$\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

Example: DY Tevatron experiments (E288: Q=4-15GeV, qt<2 GeV)
no (usable) DIS data... waiting for EIC..

Issues: Can we understand Compass DY-DIS results in this formalism (Q=1-2 GeV)? (Hermes, Compass, JLAB) $Q^2 \not\gg \Lambda_{QCD}^2 \quad q_T^2 \sim \Lambda_{QCD}^2$

Construction of unpolarized TMDs

- Take the asymptotic limit (High Q, qT) of each TMDPDF

PDF

2-loop matching of PDFs:
Florence (Catani et al.), Zurich (Gehrmann. et al)

$$\tilde{F}_{q/N}(x, b_T; \zeta, \mu) = \left(\frac{\zeta}{\mu_b}\right)^{-D(b_T;\mu)} \sum_j \int_x^1 \frac{dz}{z} \tilde{C}^{Q}_{q\leftarrow j}(x/z, b_T; \mu_b, \mu) f_{j/N}(x, \mu) M(x, b, \zeta)$$

OPE to PDF, valid ONLY for qT›› Λ_{QCD}

Process independent
Non-perturbative correction

This construction formally recovers the perturbative limit.
However the second matching is not true at low Q!!

Scales and Theoretical errors:

$Q^2 \gg q_T^2 \gg \Lambda_{QCD}^2$ Perturbative regime: 3 scales $\zeta,\ \mu,\ \mu_b$ de Florian, Catani, Ferrera, Grazzini, .. Chiu, Jain, Neill, Rothstein, Vaidya,..

$Q^2 \gg q_T^2 \sim \Lambda_{QCD}^2$ TMD regime: currently studied 2 scales ζ, μ Then $\mu_b = 2/(e^{2\gamma} b)$ defines the TMD scheme (WORK in PROGRESS!!)

Construction of unpolarized TMDs

- Take the asymptotic limit (High Q, qT) of <u>each</u> TMDPDF

2-loop matching of PDFs:
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- Exponentiation of part of the coefficient and complete resummation of the logs in the exponent
(Kodaira, Trentadue 1982, Becher, Neubert Wilhelm 2011)

$$\tilde{C}_{q\leftarrow j}(x, \vec{b}_\perp, \mu) \equiv \exp(h_\Gamma - h_\gamma) \hat{C}_{q\leftarrow j}(x, \vec{b}_\perp, \mu)$$

$$\frac{dh_\Gamma}{d\ln\mu} = \Gamma_{cusp} L_\perp$$

$$\frac{dh_\gamma}{d\ln\mu} = \gamma^V$$

$$h_\Gamma^R(b, \mu) = \int_{\alpha_s(1/\hat{b})}^{\alpha_s(\mu)} d\alpha' \frac{\Gamma^F_{cusp}(\alpha')}{\beta(\alpha')} \int_{\alpha_s(1/\hat{b})}^{\alpha'} \frac{d\alpha}{\beta(\alpha)}.$$

Same resummation as for the D

finally write a(1/b) in terms of a(mu) and fix mu=Qi.

Logs are minimized with the choice
mu=Qi=Q0+qT

DATA FIT: 1.0

Objectives:

1)TMD evolution kernel is basically model independent when Q>4-5 GeV

2)Preliminary model for TMD's

3) (Theoretical and fit) Error understanding

4)Perform analysis at NNLL

Theoretical settings

* Matching scale of TMDPDF to PDF at Qi=2 GeV+qT, at NNLL
* Hard coefficient with π^2 resummation (Ahrens, Becher, Lin Yang, Neubert '08)
* Checked both NLL and NNLL
* Several sets of PDF checked (MSTW, CTEQ)
* Checked several form of non-perturbative models: gaussian, exponential, Q-dependence, ...
* Non-perturbative input

$$M_q(x, \vec{b}, Q_i) = \exp[-\lambda_1 b](1 + b^2\lambda_2 + \dots)$$

Order	γ	Γcusp	C	D
LL	-	α	tree	-
NLL	α	α^2	tree	α
NNLL	α^2	α^3	α	α^2
NNNLL	α^3	α^4	α^2	α^3

To be fully included in upgraded version (at present only scale dependent part)

$\alpha_s L_\perp \sim 1$

Naive attempts

Aybat, Collins , Qiu,
Rogers; Aybat, Rogers;
Anselmino, Boglione,Melis

EIS

Known pieces: C for unpolarized TMD
from Catani et al. '12,
Gehrmann, Luebbert. Lin Yang '12, '14

Experimental Data

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
points	32	16	41	9
$\sqrt{s}$	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
σ	248 ± 11 pb	221 ± 11.2 pb	256 ± 15.2 pb	255.8 ± 16.7 pb

Z, run I:

Becher, Neubert, Wilhelm 2011:
ad hoc model for these data at low qT

Catani et al. 2009: Minimal Subtraction

Z, run I and low energy data
BLNY-RESBOS: model for everything

	E288 200	E288 300	E288 400	R209
points	35	35	49	6
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	62 GeV
E_{beam}	200 GeV	300 GeV	400 GeV	-
Beam/Target	p Cu	p Cu	p Cu	p p
M range used	4-9 GeV	4-9 GeV	5-9 and 10.5-14 GeV	5-8 and 11-25 GeV
Other kin. var	y=0.4	y=0.21	y=0.03	
Observable	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$d\sigma/dq_T^2$

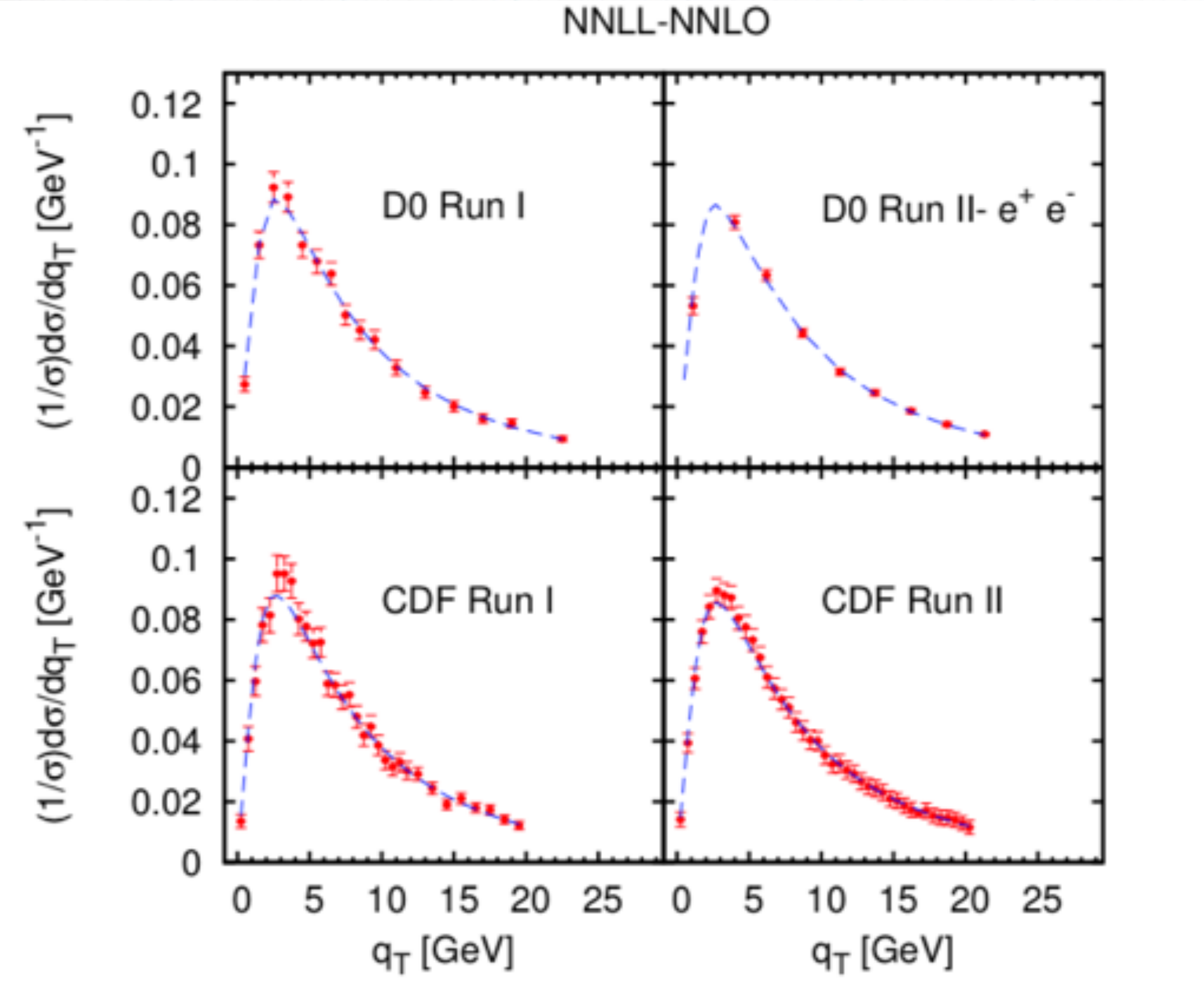
Expected to be insensitive to Landau pole region
Factorization hypothesis hold

Opportunity for ATLAS/CMS: unexplored measurement of DY

$$\frac{d\sigma}{dm_{\ell\ell}dq_T dy}$$

with $10 \text{ GeV} \simeq m_{\ell\ell} \simeq 70 \text{ GeV}$

Results at NNLL: Z production



Z-boson data are (fairly) sensitive to functional non-perturbative form (gaussian vs exponential) and (poorly) sensitive just to λ_1.
In order to fix it we need the global fit

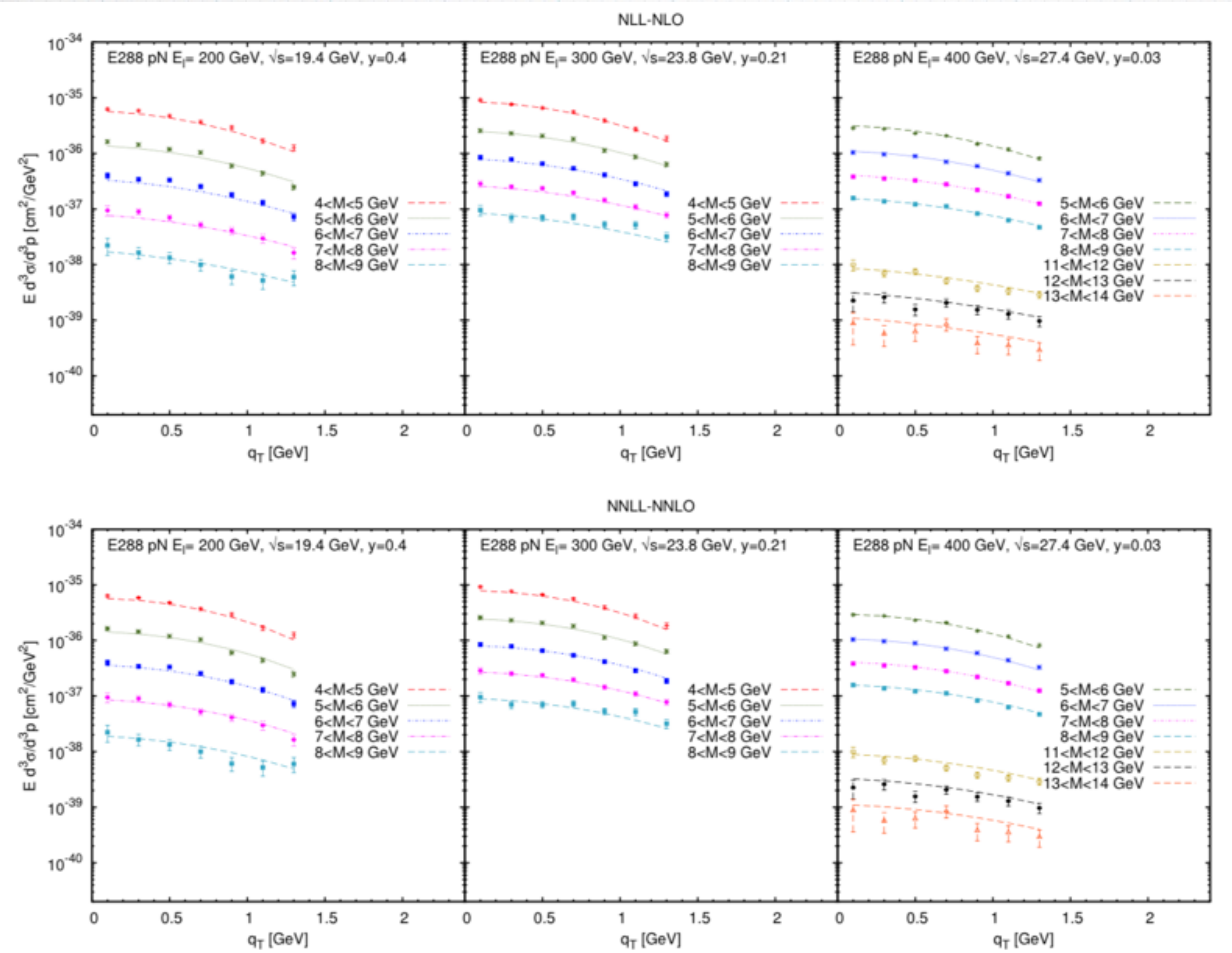
DYNNLO: Catani, Grazzini '07, Catani, Cieri, Ferrera, de Florian, Grazzini '09

Data: $$\frac{1}{\sigma_{exp}}\left(\frac{d\sigma}{dq_T}\right)_{exp}$$

Theory: $$\frac{1}{\sigma_{th}}\left(\frac{d\sigma}{dq_T}\right)_{th}$$

Message:
One cannot fix the NP part of TMD's just looking at Z-boson production:
Extrapolating parameters from Z to W may not be accurate enough.

Results at NNLL



Exp. Normalization
NE288, NR209
deduced from the fit.

Total: 4 parameters

Results

MSTW08

		NNLL, NNLO	NLL, NLO
	points	χ^2/points	χ^2/points
	223	1.10	1.48
E288 200	35	1.53	2.60
E288 300	35	1.50	1.12
E288 400	49	2.07	1.79
R209	6	0.16	0.25
CDF Run I	32	0.74	1.31
D0 Run I	16	0.43	1.44
CDF Run II	41	0.30	0.62
D0 Run II	9	0.61	2.40

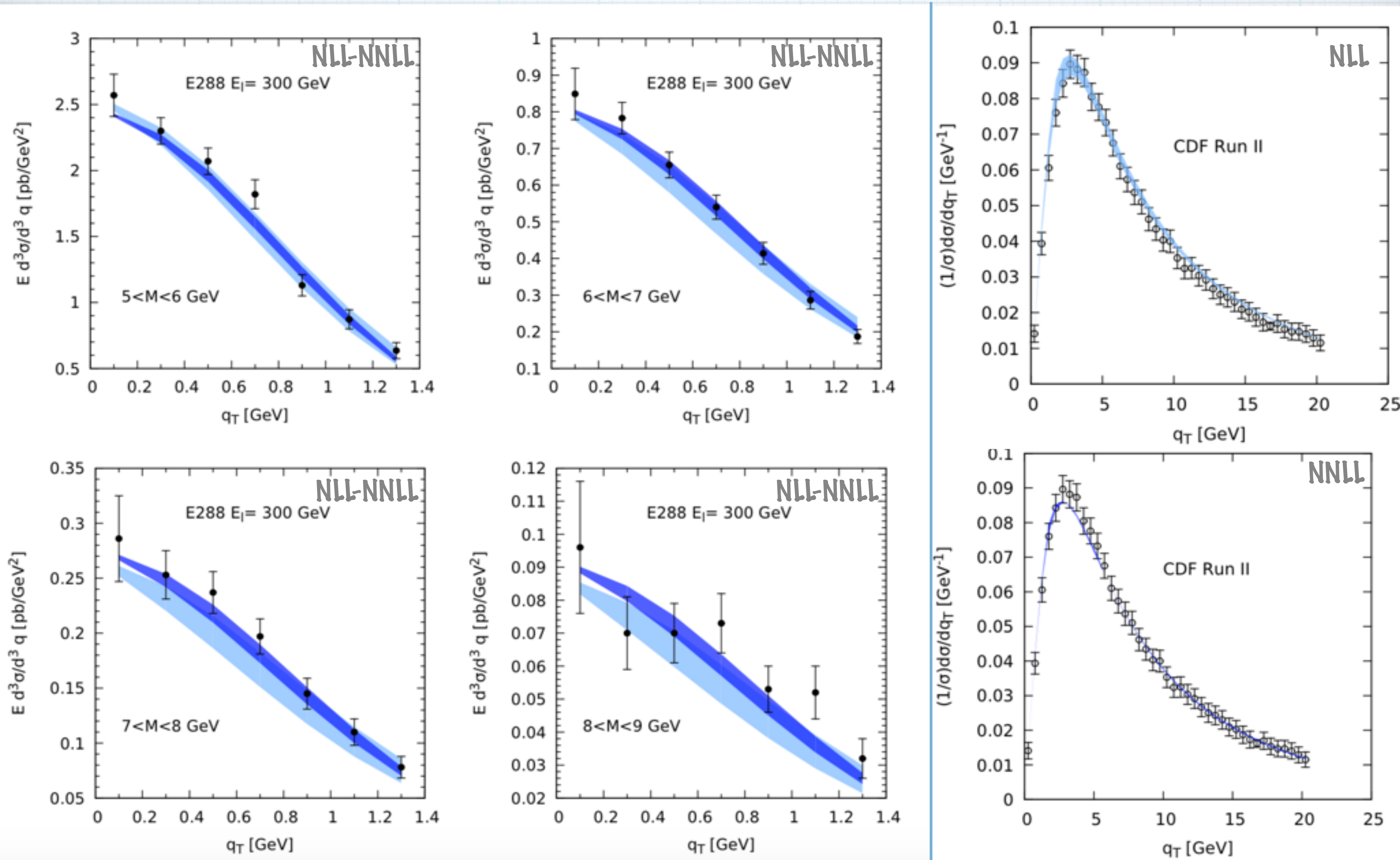
NLL	223 points	χ^2/d.o.f. $= 1.51$
	$\lambda_1 = 0.26^{+0.05_{th}}_{-0.02_{th}} \pm 0.05_{stat}$ GeV	$\lambda_2 = 0.13 \pm 0.01_{th} \pm 0.03_{stat}$ GeV^2
	$N_{E288} = 0.9^{+0.2_{th}}_{-0.1_{th}} \pm 0.04_{stat}$	$N_{R209} = 1.3 \pm 0.01_{th} \pm 0.2_{stat}$
NNLL	223 points	χ^2/d.o.f. $= 1.12$
	$\lambda_1 = 0.33 \pm 0.02_{th} \pm 0.05_{stat}$ GeV	$\lambda_2 = 0.13 \pm 0.01_{th} \pm 0.03_{stat}$ GeV^2
	$N_{E288} = 0.85 \pm 0.01_{th} \pm 0.04_{stat}$	$N_{R209} = 1.5 \pm 0.01_{th} \pm 0.2_{stat}$

CTEQ10

		NNLL, NNLO	NLL, NLO
	points	χ^2/points	χ^2/points
	223	0.96	1.79
E288 200	35	1.58	2.61
E288 300	35	1.09	1.10
E288 400	49	1.17	2.43
R209	6	0.20	0.35
CDF Run I	32	0.83	1.55
D0 Run I	16	0.48	1.79
CDF Run II	41	0.38	0.79
D0 Run II	9	1.036	3.28

NLL	223 points	χ^2/dof $= 1.79$
	$\lambda_1 = 0.28 \pm 0.05_{stat}$ GeV	$\lambda_2 = 0.14 \pm 0.04_{stat}$ GeV^2
	$N_{E288} = 1.02 \pm 0.04_{stat}$	$N_{R209} = 1.4 \pm 0.2_{stat}$
NNLL	223 points	χ^2/dof $= 0.96$
	$\lambda_1 = 0.32 \pm 0.05_{stat}$ GeV	$\lambda_2 = 0.12 \pm 0.03_{stat}$ GeV^2
	$N_{E288} = 0.99 \pm 0.05_{stat}$	$N_{R209} = 1.6 \pm 0.3_{stat}$

Scale dependence:bands



Work in progress 1

Data analysis/fits:

Full inclusion of two loop results (NNLL/N3LL)

Scale dependences

Improved non-perturbative inputs for weak boson productions

LHC results

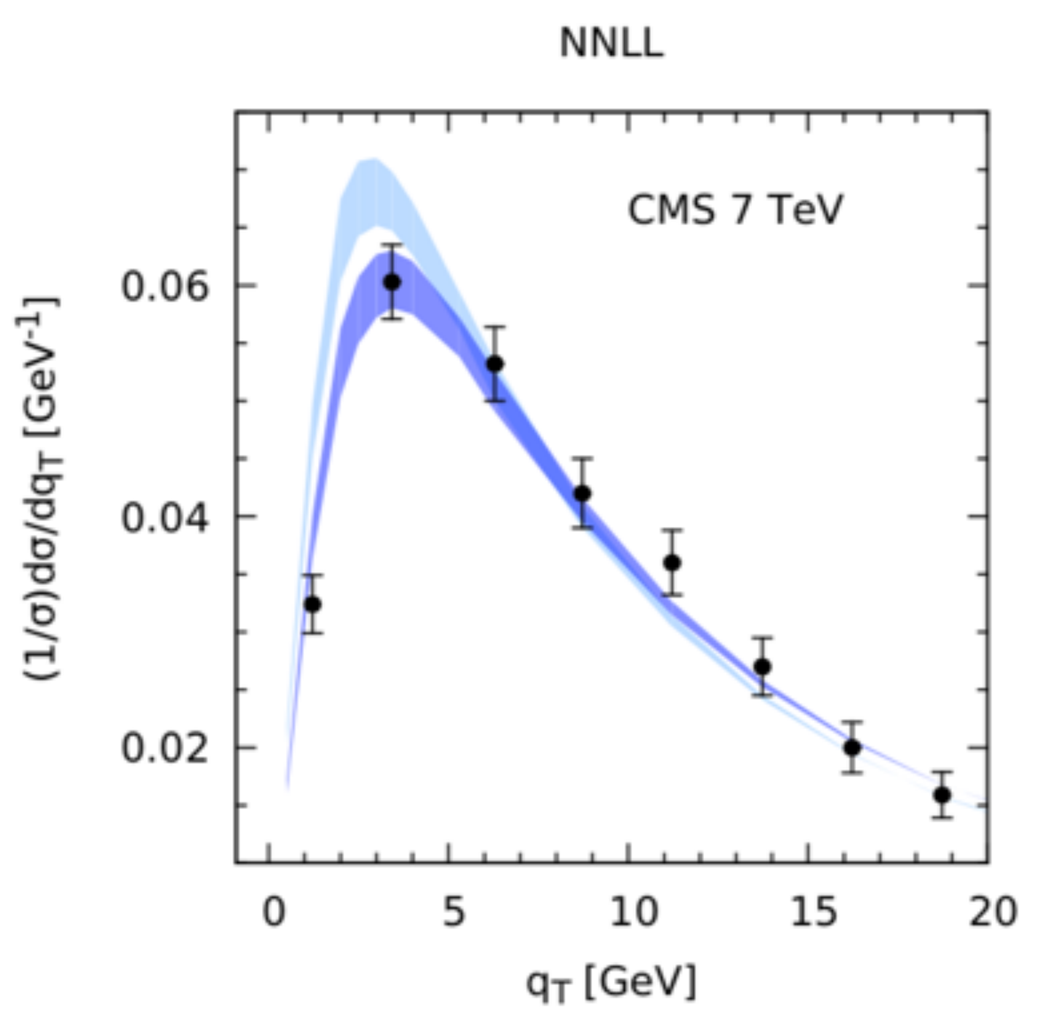
Work in progress 2

Universality of TMDs: TMD fragmentations at 2 loops

we want achieve the same perturbative precision for PDF and FF

Ingredients:
Soft function at 2 loops
Transverse momentum dependent collinear functions at 2 loops

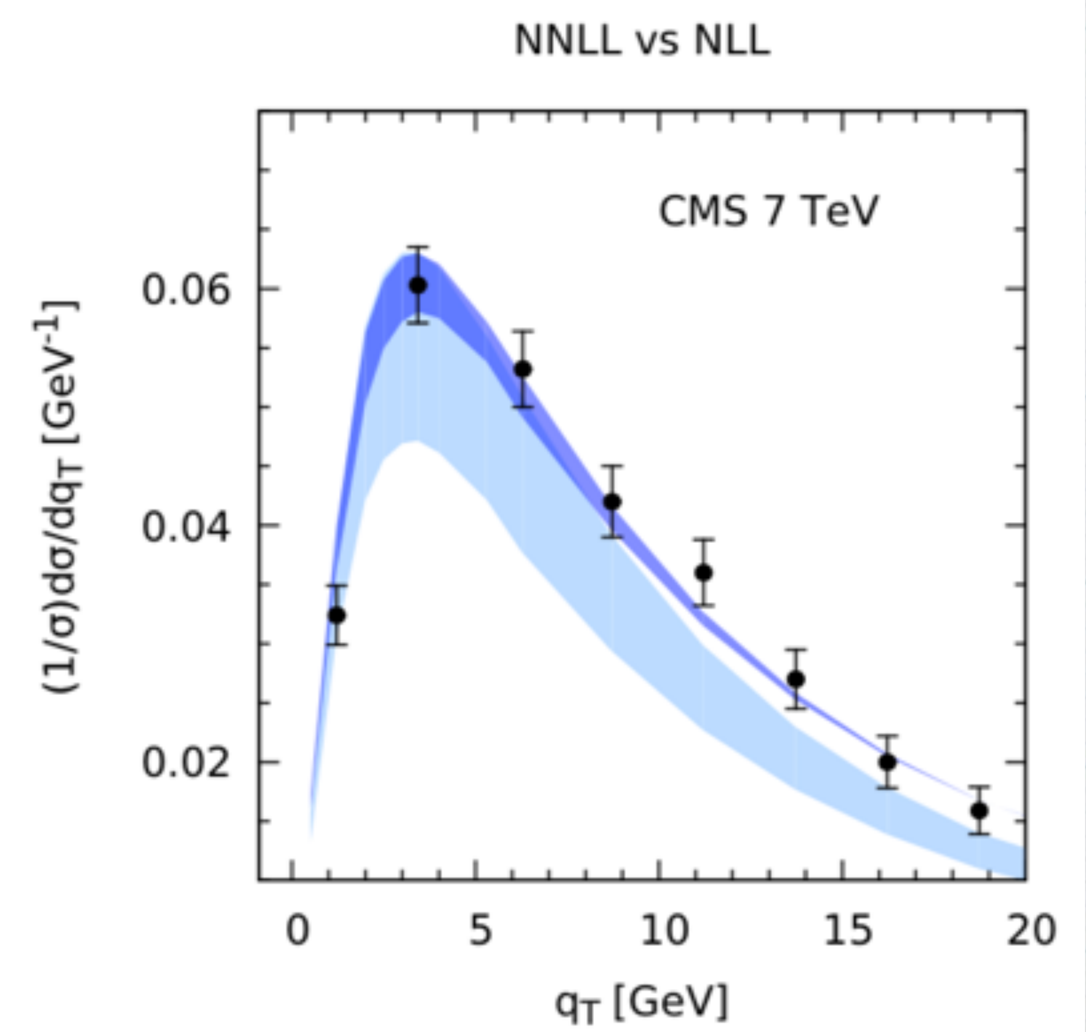
Predictions for CMS



Pure-perturbative vs complete TMDs at NNLL

Very large bins!! (not shown)

NLL vs NNLL for complete TMDs: scale dependence



CMS goes at smaller values of Bjorken x than TeVatron: broader bands

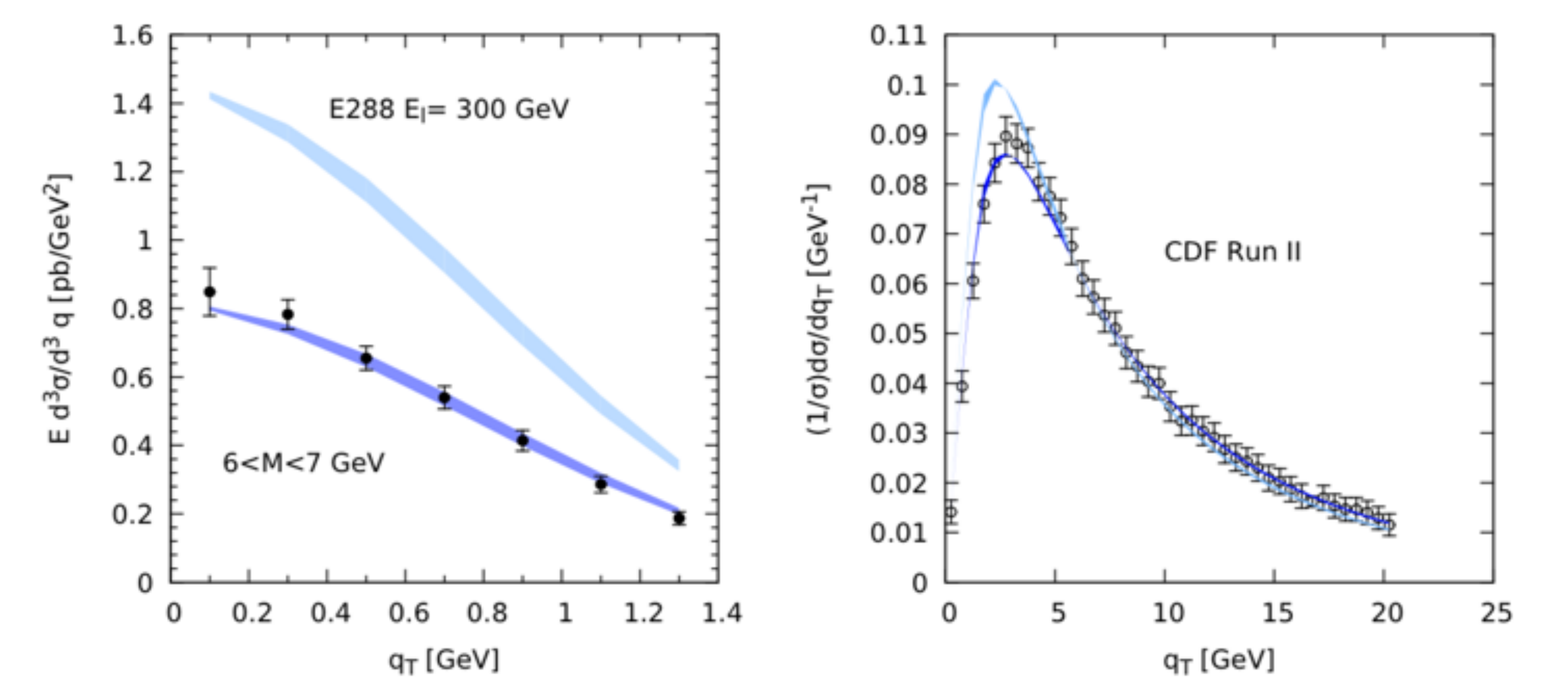
Conclusions

- The correct measurement of non-perturbative effects in transverse momentum dependent observables requires the use of TMDs on very different energy spectrum (We want to use TMDPDF in the same way as PDF, as far as possible).
- Golden energy range for TMDs, Q>2-3 GeV, qT<<Q. LHC, ee collider (Belle, Bes) and EIC can provide a huge development of the field
- The evolution of TMD's should be used at highest available order (here NNLL, expandable up to NNLL'/ N^3LL, in progress)
- TMD's are universal (the same for SIDIS, DY, ee-> 2 j) . Two loop calculations in progress for TMDFF.
- The evolution of TMDPDF and TMDFF is the same and spin independent.
- Non-perturbative QCD effects should be included in high precision LHC observables (TMDs): Frontier of QCD precision.
- Analysis of spin dependent observables including evolution is starting now. A lot to do!!

Thanks!!.. and enjoy Valencia!

Back-up slides

Model dependence



Non-perturbative inputs necessary for the peak region in Z-production: **Consistency between DY and Z data**

Theoretical arguments suggest also a non-perturbative Q-dependence of the evolution kernel (check RESBOS).
We test

$$M_q(x,b,Q) = \exp[-\lambda_1 b]\,(1+\lambda_2 b^2+...)\left(\frac{Q^2}{Q_0^2}\right)^{-\lambda_3 b^2/2}$$

Model dependence

$Q_0 = 2.0$ GeV $+ q_T$		NNLL	NLL
λ_1		$0.29 \pm 0.04_{\rm stat}$ GeV	$0.27 \pm 0.06_{\rm stat}$ GeV
λ_2		$0.170 \pm 0.003_{\rm stat}$ GeV^2	$0.19 \pm 0.06_{\rm stat}$ GeV^2
λ_3		$0.030 \pm 0.01_{\rm stat}$ GeV^2	$0.02 \pm 0.01_{\rm stat}$ GeV^2
N_{E288}		$0.93 \pm 0.01_{\rm stat}$	$0.98 \pm 0.06_{\rm stat}$
N_{R209}		$1.5 \pm 0.1_{\rm stat}$	$1.3 \pm 0.2_{\rm stat}$
χ^2		180.1	375.2
	points	χ^2/points	χ^2/points
	223	0.81	1.68
	points	χ^2/dof	χ^2/dof
	223	0.83	1.72
E288 200	35	1.35	2.28
E288 300	35	0.98	1.22
E288 400	49	1.05	2.33
R209	6	0.27	0.40
CDF Run I	32	0.70	1.50
D0 Run I	16	0.41	1.77
CDF Run II	41	0.25	0.76
D0 Run II	9	0.82	3.2

No significative improvement:

1- Resummation in the evolution kernel greatly reduce TMD model dependence

2- The bulk of non-perturbative QCD corrections is scale independent