The hadronic corrections to the Lamb shift in muonic hydrogen from ChPT and the proton radius
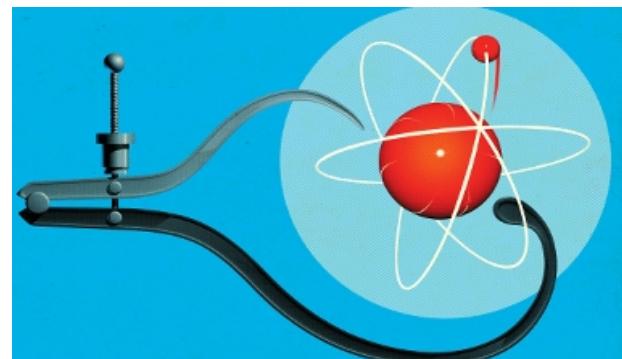
1st Hadron Spanish Network Days and Spanish-Japanese JSPS Workshop, IFIC, Valencia, 16th June 2015

Clara Peset

Outline

- Model independent prediction of the proton radius

- Effecive field theory framework: pNRQED

- Contributions to the Lamb shift

  - QED- like contributions

  - hadronic contributions (TPE)

- Summary

Definition of the proton radius

P+

Rutherford 1917-20

$\tau_p > 10^{34}$ *years*

QCD
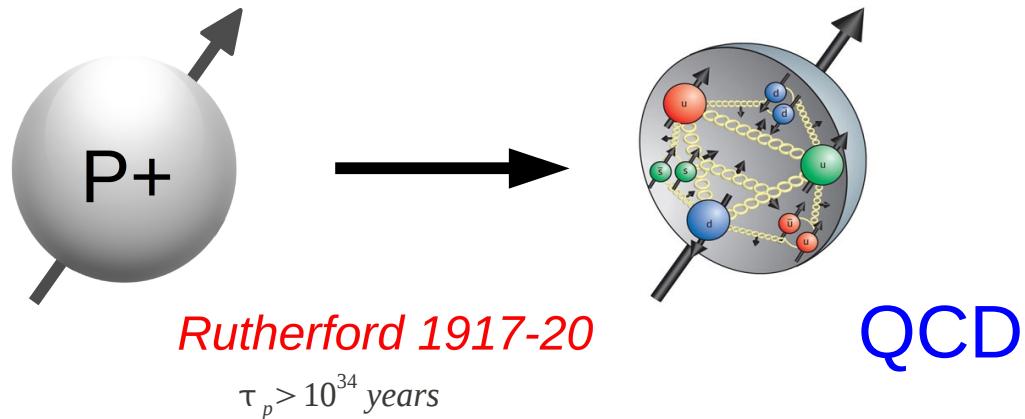
$$r_p^2 = 6 \frac{dG_{p,E}(q^2)}{dq^2}\bigg|_{q^2=0}$$

IR divergent!

NRQCD Lagrangian:

$$\delta\mathcal{L} = -e\frac{c_D^{(p)}}{m_p^2}N_p^\dagger \nabla \cdot \mathbf{E}N_p$$

Caswell, Lepage '86

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2)$$

$$c_D = 1 + 2F_2 + 8F_1'$$

$$r_{p,\overline{MS}}^2(\nu) = \frac{3}{4m_p^2}\left(c_D^{\overline{MS}}(\nu) - 1\right)$$

General expression:

$$c_D^{(p)}(\nu) - c_{D,point-like}^{(p)}(\nu) \equiv \frac{4}{3}m_p^2 r_p^2$$

The proton radius puzzle

Value from μ-H: $r_p = 0.84087(39)$ fm *(Science '13)*

⎤ 7.1 σ
⎦ away!

CODATA value
(2012): $r_p = 0.8775(51)$ fm

from Hydrogen spectroscopy and electron-proton scattering

Probability of the lepton being *within* the volume of the proton

$$\left(\frac{r_p}{a_B}\right)^3 = \left(\alpha\, m_r\, r_p\right)^3$$

$$m_\mu \approx 200\, m_e$$

8 million times larger for μH

(a)

2P$_{3/2}$
2P$_{1/2}$
8.4 meV
F=2
F=1
F=1
F=0

206 meV
50 THz
6 μm
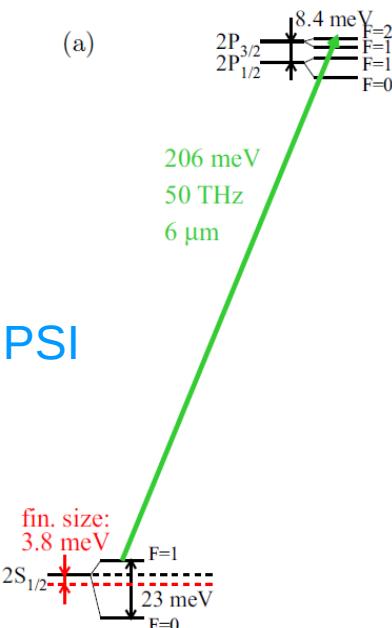
But also larger UNKNOWN hadronic effects ➤ Two Photon Exchange

PSI

LAMB SHIFT: $2S_{1/2} \rightarrow 2P_{3/2}$

displacement of nS energy levels

fin. size:
3.8 meV
2S$_{1/2}$
F=1
23 meV
F=0

EFT for bound states (μp)

Scales in bound states

Muonic hydrogen
(Coulomb interaction)

- Hard scale: $m_r \longrightarrow m_r$

- Soft scale: $|\boldsymbol{p}| \longrightarrow m_r \alpha$

- Ultrasoft scale: $E \longrightarrow m_r \alpha^2$

$\longrightarrow$ Well separated scales!

pNRQED

Scales in μH: $m_p \sim m_\rho, \quad m_\mu \sim m_\pi \sim m_r, \quad m_r \alpha \sim m_e$

Small expansion parameters: $\dfrac{m_\pi}{m_p} \sim \dfrac{m_\mu}{m_p} \approx \dfrac{1}{9}, \quad \dfrac{m_e}{m_r} \sim \dfrac{m_r \alpha}{m_r} \sim \alpha \approx \dfrac{1}{137}$

Energy levels:
$$E(\mu p) = \frac{-m_r \alpha^2}{2n^2}\left(1 + c_1 \alpha + c_2 \alpha^2 + c_3 \alpha^{3.}..\right)$$

$c_1 \sim c_1\left(\dfrac{m_\mu \alpha}{m_e}\right)$ pure QED, and $\quad c_n \sim \sum_{j=0}^{\infty} c_n^{(j)}\left(\dfrac{m_\pi}{m_p}\right)^j ; \quad c_n^{(j)} \sim c_n^{(j)}\left(\dfrac{m_r}{m_\mu}, \dfrac{m_\mu}{m_\pi}...\right)$

pNRQED

pNRQED is a theory of ultrasoft photons

$$\text{HBET} \longrightarrow \text{NRQED} \longrightarrow \text{pNRQED}$$

$$m_\pi, \Delta$$

The pNRQED lagrangian:

$$\mathcal{L}_{pNRQED} = \int d^3\mathbf{x}\, d^3\mathbf{X}\, S^\dagger(\mathbf{x}, \mathbf{X}, t) \left\{ i\partial_0 - \frac{\mathbf{p}^2}{m_r} + \frac{\mathbf{p}^4}{m_\mu^3} + \frac{\mathbf{p}^4}{m_p^3} - \frac{\mathbf{P}^2}{2M} \right.$$

$$\left. -V(\mathbf{x}, \mathbf{p}, \sigma_1, \sigma_2) + e\mathbf{x} \cdot \mathbf{E}(\mathbf{X}, t) \right\} S(\mathbf{x}, \mathbf{X}, t) - \int d^3\mathbf{x}\, \frac{d_1}{4} F_{\mu\nu} F^{\mu\nu}$$

The potential:

$$V = V^{(0)} + V^{(1)} + V^{(2)} + V^{(3)} + ...$$

$$V^{(i)} \propto \left(\frac{1}{m}\right)^i \qquad \text{+ expansions in the small parameters}$$

Matching coefficients

Hadronic contributions: $d_2^{had} \quad c_D^{had} \quad c_3^{had}$

EXAMPLE:

HBET: $$\delta\mathcal{L}_{(N,\Delta)l_\mu} = \frac{1}{m_p^2} c_{3,R}^{pl_\mu} \bar{N}_p \gamma^0 N_p \bar{l}_\mu \gamma_0 l_\mu$$

NRQED: $$\mathcal{L}_{Nl_\mu} = \frac{1}{m_p^2} c_{3,NR}^{pl_\mu} N_p^\dagger N_p l_\mu^\dagger l_\mu$$

pNRQED: $$V^{(2)}(r) = c_{3,NR}^{pl_\mu} \frac{\delta^{(3)}(\mathbf{r})}{m_p m_\mu}$$

$$c_{3,NR}^{pl_\mu} = c_{3,R}^{pl_\mu} + c_{3,point-like}^{pl_\mu} + c_3^{had}$$

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6)\text{meV}$$

CP, A. Pineda

"Pure" QED corrections

Theoretical equation for the Lamb Shift:

$$\Delta E_L = \boxed{206.0243(30)} - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}\left(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6\right)\text{meV}$$

CP, A. Pineda

μH QED leading contribution: ELECTRON VACUUM POLARIZATION
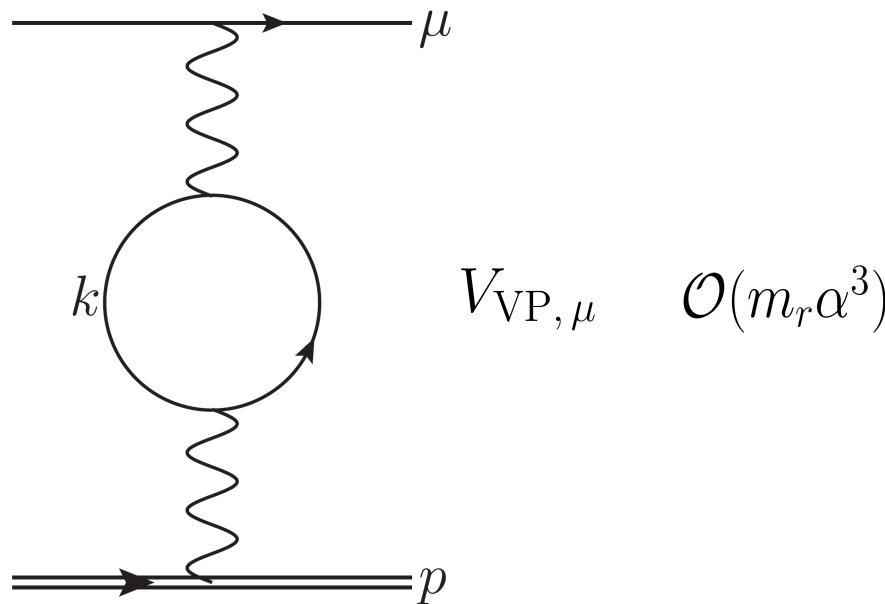
$$V_{\text{VP},\mu} \qquad \mathcal{O}(m_r \alpha^3)$$

"Pure" QED corrections

Theoretical equation for the Lamb Shift:

$$\Delta E_L = \boxed{206.0243(30)} - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}\left(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6\right)\text{meV}$$

CP, A. Pineda

Other contributions up to $\mathcal{O}(m_r \alpha^5)$
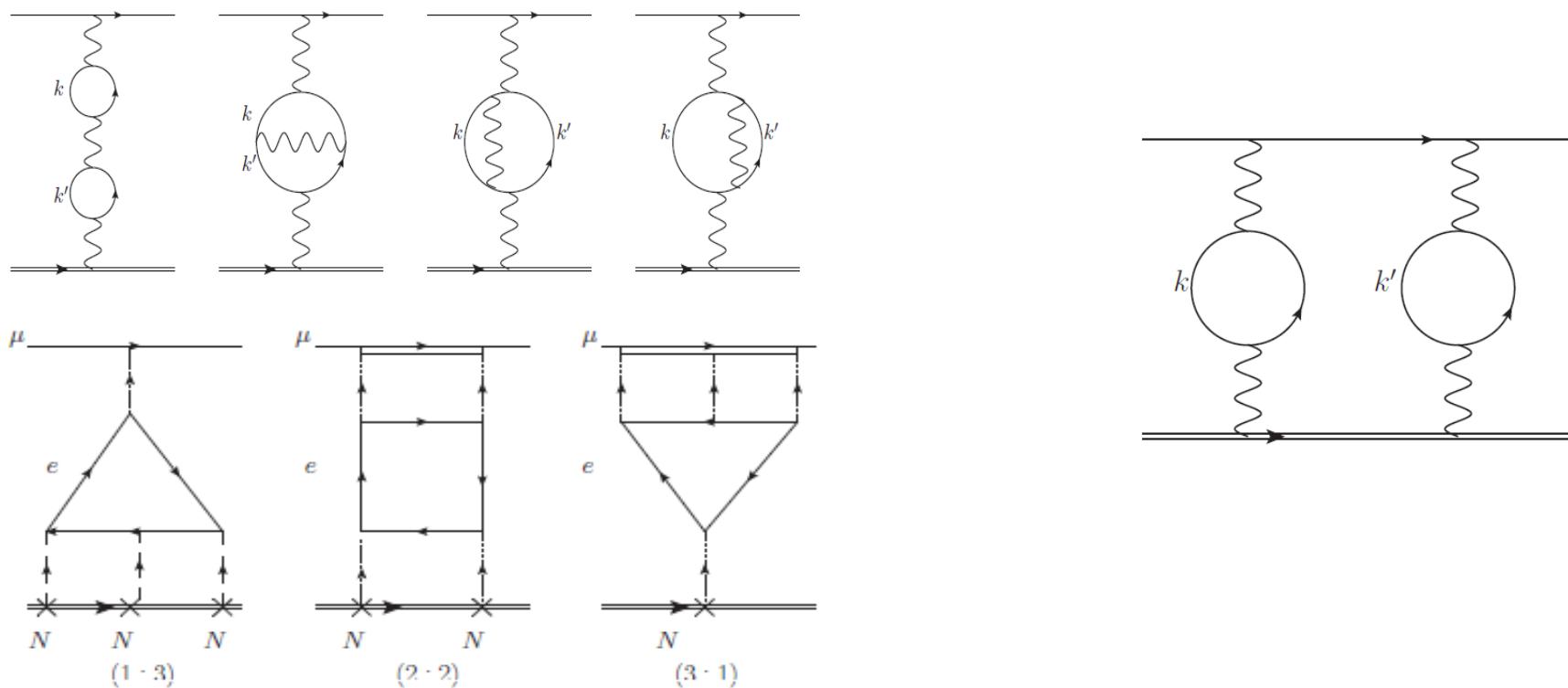

$(1 \cdot 3)$ $(2 \cdot 2)$ $(3 \cdot 1)$

"Pure" QED corrections

Theoretical equation for the Lamb Shift:

$$\Delta E_L = \boxed{206.0243(30)} - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}\left(m_\mu\alpha^5\frac{m_\mu^3}{m_\rho^3}, m_\mu\alpha^6\right)\text{meV}$$

CP, A. Pineda

Relativistic contributions up to $\mathcal{O}(m_r\alpha^5)$
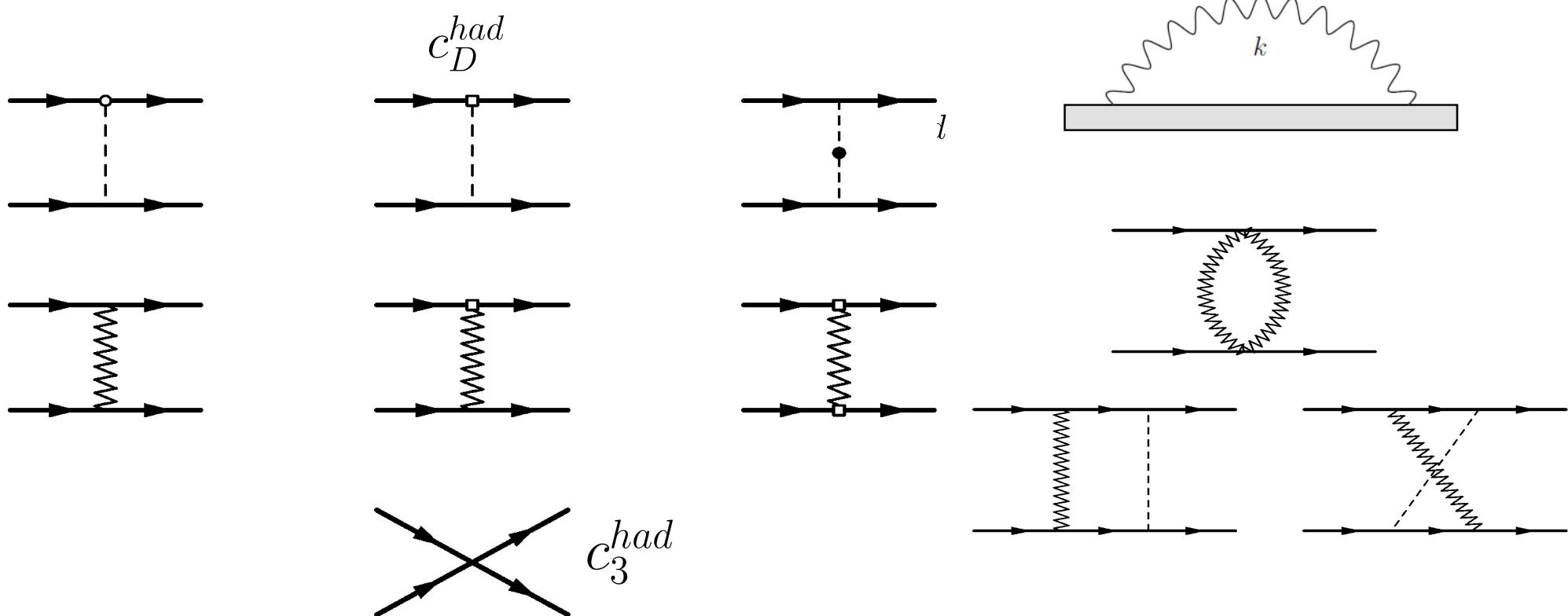
Summary and error estimates

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6)\text{meV}$$

CP, A. Pineda

$\mathcal{O}(m_r \alpha^3)$	$V_{\text{VP}}^{(0)}$	205.00745
$\mathcal{O}(m_r \alpha^4)$	$V_{\text{VP}}^{(0)}$	1.50795
$\mathcal{O}(m_r \alpha^4)$	$V_{\text{VP}}^{(0)}$	0.15090
$\mathcal{O}(m_r \alpha^5)$	$V_{\text{VP}}^{(0)}$	0.00752
$\mathcal{O}(m_r \alpha^5)$	$V_{LbL}^{(0)}$	-0.00089
$\mathcal{O}(m_r \alpha^4 \times \frac{m_\mu^2}{m_p^2})$	$V^{(2)} + V^{(3)}$	0.05747
$\mathcal{O}(m_r \alpha^5)$	$V_{\text{soft}}^{(2)}/\text{ultrasoft}$	-0.71903
$\mathcal{O}(m_r \alpha^5)$	$V_{\text{VP}}^{(2)}$	0.01876
$\mathcal{O}(m_\mu \alpha^6 \times \ln(\frac{m_\mu}{m_e}))$	$V^{(2)}; c_D^{(\mu)}$	-0.00127
$\mathcal{O}(m_\mu \alpha^6 \times \ln \alpha)$	$V_{\text{VP}}^{(2)}; c_D^{(\mu)}$	-0.00454

Hadronic Effects

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6)\text{meV}$$

CP, A. Pineda

$\mathcal{O}(m_r \alpha^4 \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-5.1975\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r \alpha^5 \times m_r^2 r_p^2)$	$V_{\text{VP}}^{(2)}; c_D^{(p)}; r_p^2$	$-0.0283\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r \alpha^6 \ln\alpha \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-0.0014\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r \alpha^5 \times \frac{m_r^2}{m_\rho^2})$	$V_{\text{VP}_{\text{had}}}^{(2)}; d_2^{\text{had}}$	$0.0111(2)$
$\mathcal{O}(m_r \alpha^5 \times \frac{m_r^2}{m_\rho^2}\frac{m_\mu}{m_\pi})$	$V^{(2)}; c_3^{\text{had}}$	$0.0344(125)$

$$\delta V^{had} = \frac{D_d^{had}}{m_p^2}\delta^{(3)}(\mathbf{r}) \qquad D_d^{had} \equiv -c_3^{had} - 16\pi\alpha d_2^{had} + \frac{\pi\alpha}{2}c_D^{had}$$

Definition of the proton radius

Hadronic Effects

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6)\text{meV}$$

CP, A. Pineda

$\mathcal{O}(m_r \alpha^4 \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-5.1975\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r \alpha^5 \times m_r^2 r_p^2)$	$V_{\text{VP}}^{(2)}; c_D^{(p)}; r_p^2$	$-0.0283\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r \alpha^6 \ln \alpha \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-0.0014\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r \alpha^5 \times \frac{m_r^2}{m_\rho^2})$	$V_{\text{VP}_{\text{had}}}^{(2)}; d_2^{\text{had}}$	$0.0111(2)$
$\mathcal{O}(m_r \alpha^5 \times \frac{m_r^2}{m_\rho^2}\frac{m_\mu}{m_\pi})$	$V^{(2)}; c_3^{\text{had}}$	$0.0344(125)$

F. Jegerlehner

$$\delta V^{had} = \frac{D_d^{had}}{m_p^2}\delta^{(3)}(\mathbf{r})$$

$$D_d^{had} \equiv -c_3^{had} - 16\pi\alpha d_2^{had} + \frac{\pi\alpha}{2}c_D^{had}$$

Hadronic Vacuum polarization:
Obtained from DR

Hadronic Effects

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6)\text{meV}$$

CP, A. Pineda

$\mathcal{O}(m_r \alpha^4 \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-5.1975\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r \alpha^5 \times m_r^2 r_p^2)$	$V_{\text{VP}}^{(2)}; c_D^{(p)}; r_p^2$	$-0.0283\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r \alpha^6 \ln \alpha \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-0.0014\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r \alpha^5 \times \frac{m_r^2}{m_\rho^2})$	$V_{\text{VP}_{\text{had}}}^{(2)}; d_2^{\text{had}}$	$0.0111(2)$
$\mathcal{O}(m_r \alpha^5 \times \frac{m_r^2}{m_\rho^2}\frac{m_\mu}{m_\pi})$	$V^{(2)}; c_3^{\text{had}}$	$0.0344(125)$

$$\delta V^{had} = \frac{D_d^{had}}{m_p^2}\delta^{(3)}(\mathbf{r}) \qquad D_d^{had} \equiv -c_3^{had} - 16\pi\alpha d_2^{had} + \frac{\pi\alpha}{2}c_D^{had}$$

$$c_3^{\text{had}} = c_3^{\text{Born}} + c_3^{\text{pol}} \sim \alpha^2 \frac{m_\mu}{m_\pi}\left(1 + \#\frac{m_\pi}{\Delta} + ...\right) + \mathcal{O}\left(\alpha^2 \frac{m_\mu}{\Lambda_{\text{QCD}}}\right)$$

Large-Nc limit: we expect a large contribution from Δ(1232)

c_3^{had}: Born/Zemach contribution

$$c_{3,\text{Born}}^{pl_i} = 4(4\pi\alpha)^2 M_p^2 m_{l_i} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{1}{\mathbf{q}^6} G_E^{(0)} G_E^{(2)}(-\mathbf{q}^2) = \frac{\pi}{3}\alpha^2 M_p m_\mu \langle r^3 \rangle_{(2)}$$

Zemach third momentum

Zemach momenta:

EFT:
CP, Pineda

FITS:

	$\langle r^3 \rangle$	$\langle r^4 \rangle$	$\langle r^5 \rangle$	$\langle r^6 \rangle$	$\langle r^7 \rangle$	$\langle r^3 \rangle_{(2)}$
π	0.4980	0.6877	1.619	5.203	20.92	0.9960
$\pi\&\Delta$	0.4071	0.6228	1.522	4.978	20.22	0.8142
Dipole	0.7706	1.083	1.775	3.325	7.006	2.023
Kelly	0.9838	1.621	3.209	7.440	19.69	2.526
Distler et al	1.16(4)	2.59(19)(04)	8.0(1.2)(1.0)	29.8(7.6)(12.6)	$---$	2.85(8)

large dependence on the fitted function & large difference with EFT

Born energy shift:

CP, Pineda

μeV	DR Pachucki Carlson et al	HBET (π)	$(\pi\&\Delta)$
ΔE_{Born}	23.2(1.0) 24.7(1.6)	10.1(5.1)	8.3(4.3)

we would expect less difference with the DR analysis

c_3^{had}: **Polarizability**

$$T_S^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) S_1(\rho, q^2) + \frac{1}{M_p^2} \left(p^\mu - \frac{M_p\rho}{q^2}q^\mu\right) \left(p^\nu - \frac{M_p\rho}{q^2}q^\nu\right) S_2(\rho, q^2)$$
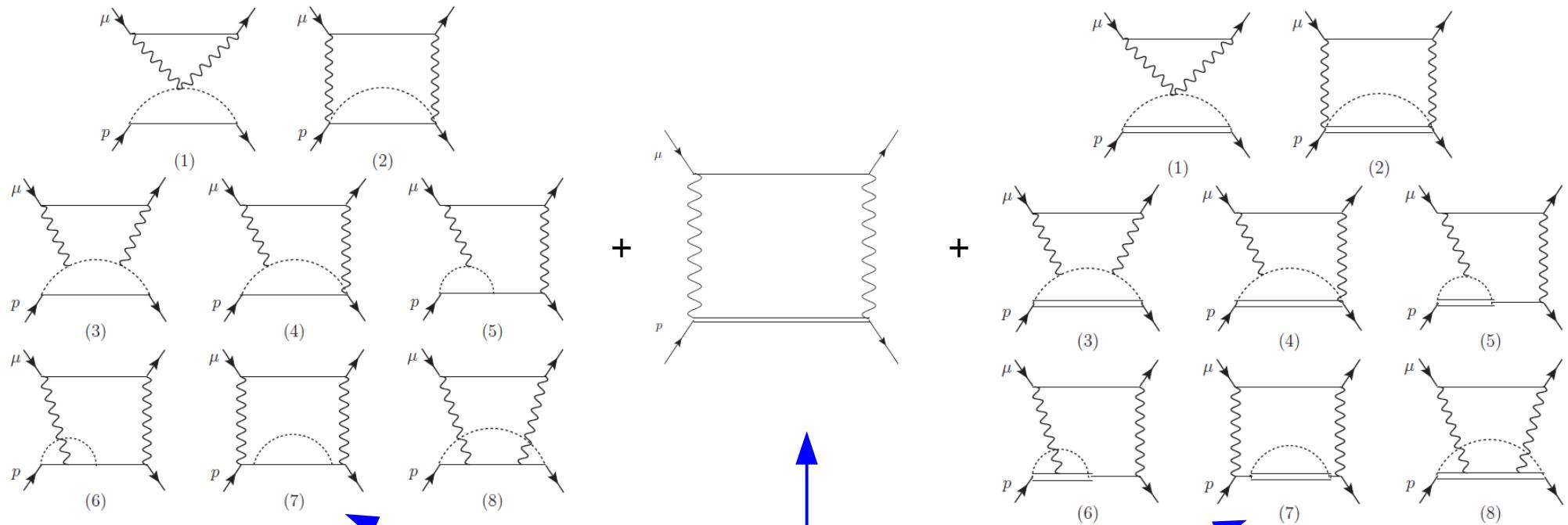
$$\Delta E_{\text{pol}} = \frac{c_{3,\text{pol}}^{pl_\mu}}{M_p^2}\frac{1}{\pi}\left(\frac{m_r\alpha}{2}\right)^3 = 18.51\,(\pi-\text{loop}) - 1.58\,(\Delta-\text{tree}) + 9.25\,(\pi\Delta-\text{loop}) = 26.2(10.0)\,\mu\text{eV}$$

CP, Pineda

c_3^{had}: **Polarizability**

Different results for the polarizability contribution:

(μeV)	DR + Model				BχPT (π)	HBET (π)	$(\pi\&\Delta)$
ΔE_{pol}	12(2)	11.5	7.4(2.4)	15.3(5.6)	$8.2(^{+1.2}_{-2.5})$	18.5(9.3)	26.2(10.0)

Pachucki

Martynenko

Carlson et al

Gorchtein et al

Alarcon et al

CP, Pineda

corrections to HBET are suppressed by $\dfrac{m_\mu}{m_\rho}$

The polarizability contribution from EFT is larger than the one computed using combinations of DR and different models

Total TPE energy shift:

$$\Delta E_{\text{TPE}} = \Delta E_{\text{Born}} + \Delta E_{\text{pol}} = 28.59(\pi) + 5.86(\pi\&\Delta) = 34.4(12.5)\mu eV$$

(LO)   (NLO)

CP, Pineda

The total contribution of TPE agrees with DR results better than when arbitrarily split into Born and polarizability pieces

Summary

We can theoretically predict the Lamb shift in a <u>model independent</u> way in an EFT framework

from μH: $\quad r_p = 0.8412\,(15)\,\text{fm}$

CP, A. Pineda

CODATA value
(from H): $\quad r_p = 0.8775\,(51)\,\text{fm}$

$6.8\,\sigma$ away!

- The hadronic contributions are the main source of uncertainty, although it is not enough to account for the discrepancy with ep

- The main radius-independent hadronic contribution is the **<u>Two Photon Exchange</u>**

   - The EFT approach gives Born & polarizability contributions which are quite different from the ones obtained by DR plus different models

   - Both the agreements and the disagrements should be further understood.

- In conclusion, **the proton radius puzzle survives the EFT analysis**

Hadronic corrections

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}(m_\mu\alpha^5\frac{m_\mu^3}{m_\rho^3}, m_\mu\alpha^6)\text{meV}$$

CP, Pineda

Scheme dependent $c_D^{(p)} \circ 1 + \frac{4}{3}r_p^2 m_p^2 + \dot{c}\,\alpha + \ldots$

$$\delta\mathcal{L} = -e\frac{c_D^{(p)}}{m_p^2}N_p^\dagger\nabla\cdot\mathbf{E}N_p$$
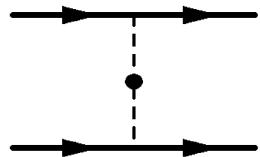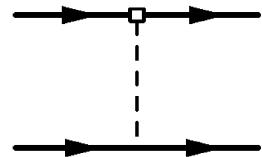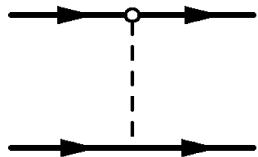
d_2^{had}
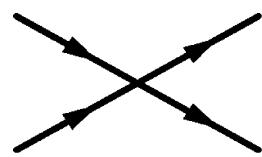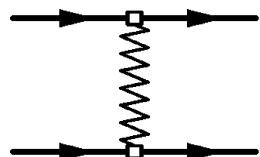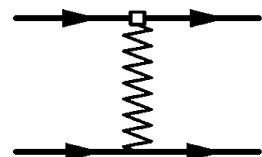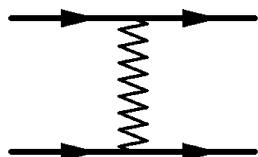
c_3^{had}

Zemach momenta

Even momenta:

$$G_E(-\mathbf{k}^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \mathbf{k}^{2n} \int_0^{\infty} dr (4\pi) r^{2n} \rho_e(r) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \mathbf{k}^{2n} \langle r^{2n} \rangle$$

Odd momenta:

$$\langle r^{2k+1} \rangle = \frac{\pi^{3/2} \Gamma[2+k]}{\Gamma[-1/2-k]} 2^{4+2k} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{\mathbf{q}^{2(2+k)}} \left[G_E(-\mathbf{q}^2) - \sum_{n=0}^{k} \frac{\mathbf{q}^{2n}}{n!} \left(\frac{d}{d\mathbf{q}^2} \right)^n G_E(-\mathbf{q}^2) \Big|_{\mathbf{q}^2=0} \right]$$

Polarizability energy shift

$$
\begin{aligned}
c_3^{pl_i} \;=\;& -e^4 M_p m_{l_i} \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^4} \frac{1}{k_E^4 + 4m_{l_i}^2 k_{0,E}^2} \left\{ (3k_{0,E}^2 + \mathbf{k}^2) S_1(ik_{0,E}, -k_E^2) - \mathbf{k}^2 S_2(ik_{0,E}, -k_E^2) \right\} \\
& + \mathcal{O}(\alpha^3) \,. \hspace{10cm} (4.1)
\end{aligned}
$$