η–η' mixing, masses and pseudoscalar decay constants in $U(3)$ chiral effective field theory

J. A. Oller

Departamento de Física
Universidad de Murcia[1]

in collaboration with X.-K. Guo, Z-H. Guo and J.J.Sanz-Cillero
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Outline

Introduction

- η and η' decays are a rather old subject of study

- New revival of their multiple interests (both ex and th).

$\eta \to e^+e^-$ (Muon anomalous magnetic moment, comparison with SM, BSM, U boson, Astrophysical interest, etc.)

Related to $BR(\pi^0 \to e^+e^-)$, KTeV

Invisible decays (DM, BSM, ...)

Discrete symmetries C, CP, CPT

Spontaneous chiral symmetry breaking of QCD

Anomaly of QCD

Isospin breaking

SU(3) breaking

Gluonium content

Hadron-nucleon interactions

New types of matter: mesic nuclei (bound by strong forces)

Photoproduction of η. Baryonic resonances. . .

Vigorous experimental programs:
WASA-COSY, KLOE-2, BESIII, Crystal Ball/TAPS MAMI

But . . . we are not going to study the η, η' decays

Many recent LQCD simulations on η, η' properties

- η, η' masses: ETMC (2013), UKQCD(2012), HSC(2011),RBC/UKQCD(2010)
- K mass squared, RBC/UKQCD(2013)
- F_π, F_K, RBC/UKQCD(2013)
- F_K/F_π, BMW(2010)

The η, η' lattice simulations are analyzed in this work.

ChPT and N_C counting

In the chiral limit $m_u = m_d = m_s = 0$ the QCD Lagrangian is invariant under $U_L(3) \otimes U_R(3)$ symmetry.

$SU_L(3) \otimes SU_R(3) \to SU_V(3)$ is Spontaneously Broken. Goldstone bosons appear π, K, η

$U_V(1) \equiv U_{L+R}$ Conserved Baryon Number.

$U_A(1) \equiv U_{L-R}$ Neither Conserved nor Goldstone Boson.

Puzzle:
Goldstone mode: There should be an η_1 with a mass $< \sqrt{3} m_\pi$ Weinberg PRD'75 but η is much heavier.

The ninth axial singlet current has an anomalous divergence Adler PR'69 Fujikawa PRD'80

$$J_5^{\mu\,(0)} = \bar{q}\gamma_\mu\gamma_5 q$$
$$\partial_\mu J_5^{\mu\,(0)} = \frac{g^2}{16\pi^2}\frac{1}{N_c} Tr_c(G_{\mu\nu}\widetilde{G}^{\mu\nu})$$

Large N_c QCD $N_c \to \infty$, $g^2 N_c \to constant$
't Hooft NPB'74, Witten NPB'79

$U_L(N_F) \otimes U_R(N_F) \to U_{L+R}(N_F)$
Entire Nonet of Goldstone bosons results.

Coleman, Witten PRL'80
Knecht, de Rafael PLB'98

The explicit breaking of chiral symmetry due to quark masses and the $U_A(1)$ anomaly is treated perturbatively.

Combined power expansion in the light quark masses and $1/N_c$

This formalism is set up in
Di Vecchia, Veneziano NPB'80
Rosenzweig, Schechter, Trahern PRD'80
Witten Ann.Phys.'80
The Leading Order in $1/N_c$ and the Derivative Expansion was worked out.

Herrera-Siklody, Latorre, Pascual, Taron NPB'97
Generalization of Gasser, Leutwyler Ann.Phys.'84, NPB'85 from $SU_L(3) \otimes SU_R(3)$ to $U_L(3) \otimes U_R(3)$ at $\mathcal{O}(p^4)$

S.Z.Jiang, F.J.Ge, Q. Wang, PRD'14
$U_L(N_F) \otimes U_R(N_F)$ at $\mathcal{O}(p^6)$

Combined chiral and $1/N_C$ expansion

δ-expansion: $p^2 \sim m_q \sim 1/N_c \sim \delta$

Chiral loops start to contribute at NNLO

LO: $\mathcal{O}(\delta^0) = \{\mathcal{O}(N_C p^2)\ ,\ \mathcal{O}(N_C^0 p^0)\}$

$$\mathcal{L}^{(\delta^0)} = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \frac{F^2 M_0^2}{12}\left[\ln\det u - \ln\det u^\dagger\right]^2$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

$$u = \exp\left(\frac{i\Phi}{\sqrt{2}F}\right)\ ,\ u_\mu = i u^\dagger \partial_\mu U u^\dagger\ ,\ \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

Di Vecchia, Veneziano NPB'80 , Rosenzweig, Schechter, Trahern PRD'80 , Witten Ann.Phys.'80

NLO: $\mathcal{O}(\delta) = \{\mathcal{O}(N_C p^4)\ ,\ \mathcal{O}(N_C^0 p^2)\}$

$$\mathcal{L}^{(\delta)} = L_5 \langle u_\mu u^\mu \chi_+ \rangle + \frac{1}{2} L_8 \langle \chi_+ \chi_+ + \chi_- \chi_- \rangle + \frac{F^2 \Lambda_1}{12} \partial^\mu X \partial_\mu X + \frac{F^2 \Lambda_2}{12} X \langle \chi_- \rangle \qquad X = 2 \log \det u = i\sqrt{6}\eta_0/F$$

Kaiser, Leutwyler, EPJC'00

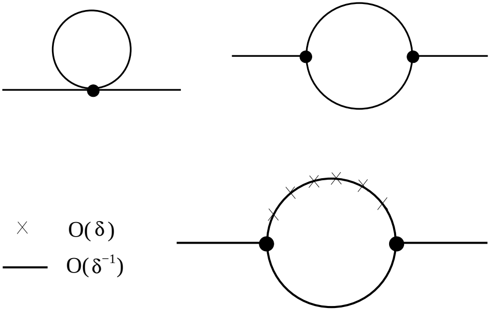
NNLO: $\mathcal{O}(\delta^2) = \{\mathcal{O}(N_C^{-1} p^2)\ ,\ \mathcal{O}(N_C^0 p^4)\ ,\ \mathcal{O}(N_c p^6)\}$

$$\mathcal{L}^{(\delta^2)} = \frac{F^2 v_2^{(2)}}{4} X^2 \langle \chi_+ \rangle + L_4 \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle + L_6 \langle \chi_+ \rangle \langle \chi_+ \rangle + L_7 \langle \chi_- \rangle \langle \chi_- \rangle + L_{18} \langle u_\mu \rangle \langle u^\mu \chi_+ \rangle + L_{25} X \langle \chi_+ \chi_- \rangle + C_{12} \langle h_{\mu\nu} h^{\mu\nu} \chi_+ \rangle + C_{14} \langle u_\mu u^\mu \chi_+ \chi_+ \rangle + C_{17} \langle u_\mu \chi_+ u^\mu \chi_+ \rangle + C_{19} \langle \chi_+ \chi_+ \chi_+ \rangle + C_{31} \langle \chi_- \chi_- \chi_+ \rangle \ .$$

Herrera-Siklody, Latorre, Pascual, Taron NPB'97; Bijnens, Colangelo, Ecker, JHEP'99

η-η' Mixing

Technical Point. For calculating one-loop diagrams



1) First diagonalize at LO: η_B, $\eta'_B \to \bar{\eta}$, $\bar{\eta}'$

$$\begin{pmatrix} \bar{\eta} \\ \bar{\eta}' \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \cdot \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

2) Mixing happens at NLO and higher orders.

- Up to NLO: Formalism in Jamin, Pich, JAO, NPB'00

$$\mathcal{L}_C = \frac{1}{2}\partial_\mu \bar{\eta}^T \mathcal{K} \partial^\mu \bar{\eta} - \frac{1}{2}\bar{\eta}^T \mathcal{M} \bar{\eta} \ , \ \mathcal{K}^T = \mathcal{K} \ , \ \mathcal{M}^T = \mathcal{M} \ , \ \bar{\eta}^T = (\bar{\eta}, \bar{\eta}') \ ,$$

$$\bar{\eta} = \mathcal{R}\mathcal{Z}^{1/2}\eta_K \ , \ \mathcal{R}^T\mathcal{R} = I \ , \ \mathcal{Z}^{1/2} = \mathrm{diag}(z_1, z_2)$$

First diagonalization:

$$\mathcal{Z}^{1/2}\mathcal{R}\mathcal{K}\mathcal{R}^T\mathcal{Z}^{1/2} = I_2 \ , \ \hat{\mathcal{M}} = \mathcal{Z}^{1/2}\mathcal{R}^T\mathcal{M}\mathcal{R}\mathcal{Z}^{1/2}$$

$$\bar{\mathcal{L}} = \frac{1}{2}\partial_\mu \eta_K^T \partial^\mu \eta_K - \frac{1}{2}\bar{\eta}_K^T \hat{\mathcal{M}} \bar{\eta}_K$$

Second diagonalization:

$$\eta = \mathcal{O}\bar{\eta}_K \ , \ \mathcal{O}^T\hat{\mathcal{M}}\mathcal{O} = \mathcal{M}_D \ , \ \mathcal{O} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

- Up to NNLO. Higher derivatives terms (HDT) prop. to C_{12}:

$$\mathcal{L}_C = \frac{1}{2}\partial_\mu \eta_B^T \mathcal{K} \partial^\mu \eta_B - \frac{1}{2}\eta_B^T \mathcal{M} \eta_B + \frac{1}{2}\partial_\mu \partial_\nu \eta_B^T \bar{\mathcal{K}}_2 \partial^\mu \partial^\nu \eta_B \ ,$$
$$\mathcal{K}_2 = \begin{pmatrix} \delta_1 & \delta_3 \\ \delta_3 & \delta_2 \end{pmatrix}$$

HDT are removed perturbatively up to NNLO:

$$\bar{\eta} \to \bar{\eta} - \frac{\delta_1}{2}\Box\bar{\eta} - \frac{\delta_2}{2}\Box\bar{\eta}' \ , \ \ \bar{\eta}' \to \bar{\eta}' - \frac{\delta_2}{2}\Box\bar{\eta} - \frac{\delta_3}{2}\Box\bar{\eta}'$$

$$\begin{aligned}\mathcal{L} =& \frac{\delta_1}{2}\,\partial_\mu\partial_\nu\overline{\eta}\partial^\mu\partial^\nu\overline{\eta} + \frac{\delta_2}{2}\,\partial_\mu\partial_\nu\overline{\eta}'\partial^\mu\partial^\nu\overline{\eta}' + \delta_3\,\partial_\mu\partial_\nu\overline{\eta}\partial^\mu\partial^\nu\overline{\eta}' \\ &+ \frac{1+\delta_{\overline{\eta}}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta} + \frac{1+\delta_{\overline{\eta}'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}' + \delta_k\,\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}' \\ &- \frac{m^2_{\overline{\eta}}+\delta_{m^2_{\overline{\eta}}}}{2}\overline{\eta}\,\overline{\eta} - \frac{m^2_{\overline{\eta}'}+\delta_{m^2_{\overline{\eta}'}}}{2}\overline{\eta}'\overline{\eta}' - \delta_{m^2}\,\overline{\eta}\,\overline{\eta}'\,,\end{aligned}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_\delta & -\sin\theta_\delta \\ \sin\theta_\delta & \cos\theta_\delta \end{pmatrix} \begin{pmatrix} 1+\delta_A & \delta_B \\ \delta_B & 1+\delta_C \end{pmatrix} \begin{pmatrix} \overline{\eta} \\ \overline{\eta}' \end{pmatrix}$$

All coefficients are calculated up to NNLO in δ-counting U(3) ChPT

• We also give the relation to the popular two-mixing angle scheme:

$$\left(\begin{array}{c}\eta\\ \eta'\end{array}\right)=\frac{1}{F}\left(\begin{array}{cc}F_8\,\cos\theta_8 & -F_0\,\sin\theta_0\\ F_8\,\sin\theta_8 & F_0\,\cos\theta_0\end{array}\right)\left(\begin{array}{c}\eta_8\\ \eta_0\end{array}\right).$$

$$\begin{array}{ll}F_\eta^8=\cos\theta_8 F_8 & F_{\eta'}^8=\sin\theta_8 F_8\\ F_\eta^0=-\sin\theta_0 F_0 & F_{\eta'}^0=\cos\theta_0 F_0\end{array}\qquad \langle 0|A_\mu^a|\eta^{(\prime)}\rangle = ip_\mu F^a_{\eta^{(\prime)}}$$

• F_K and F_π are also studied

Some previous (partial) lower-order calculations:

LO in δ-expansion, Georgi, PRD'94

LO in p^2 and NLO in $1/N_c$, Peris, PLB'94.

NLO in p^2, LO in $1/N_c$, Gerard, Kou, PLB'05; Degrande, Gerard JHEP'09; Mathieu, Vento PLB'10.

NLO in p^2 and $1/N_c$, Herrera-Siklody *et al.*, PLB'98

One-loop calculation+resonance exchange (partial NNLO), Z.H.Guo,Oller, PRD'11

FKS formalism, Feldmann,Kroll,Stech,PRD'98

etc

Numerical Results

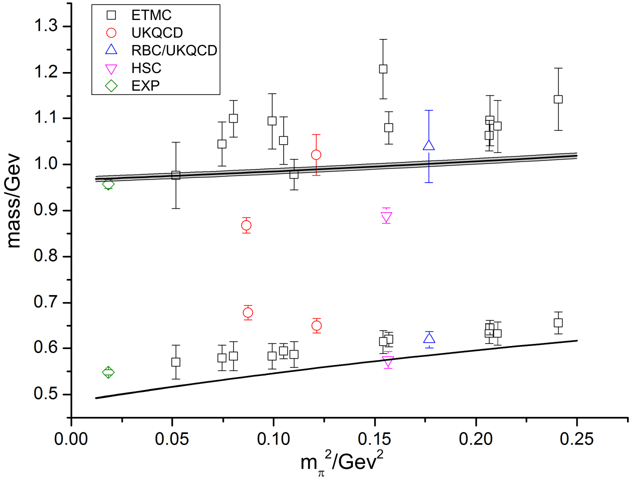
- Leading order:

One free parameter, $M_0 = 836 \pm 8$ MeV

$m_\eta = 496.4 \pm 1.3$, $m_{\eta'} = 970 \pm 6$ MeV
$\theta = -18.9^{\mathrm{o}} \pm 0.3^{\mathrm{o}}$

$$
\begin{aligned}
m^2_{\overline{\eta}} &= \frac{M_0^2}{2} + \overline{m}_K^2 - \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^4}}{2} \\
m^2_{\overline{\eta}'} &= \frac{M_0^2}{2} + \overline{m}_K^2 + \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^4}}{2} \\
\sin\theta &= -\left(\sqrt{1 + \frac{\left(3M_0^2 - 2\Delta^2 + \sqrt{9M_0^4 - 12M_0^2\Delta^2 + 36\Delta^4}\right)^2}{32\Delta^4}}\right)^{-1} \\
\Delta &= \overline{m}_K^2 - \overline{m}_\pi^2 \ .
\end{aligned}
$$

η-η' masses:



NLO and NNLO Fits

Fitted observables:

1. LQCD + experimental values:
   $m_\eta,\ m_{\eta'},\ m_K,\ F_\pi,\ F_K,\ F_K/F_\pi$
2. Phenomenological values: Y.H.Chen, Z.H.Guo, B.S.Zou, PRD91,014010 $F_8,\ F_0,\ \theta_8,\ \theta_0$

- **NLO** Free parameters: $L_5,\ L_8,\ \Lambda_1,\ \Lambda_2$

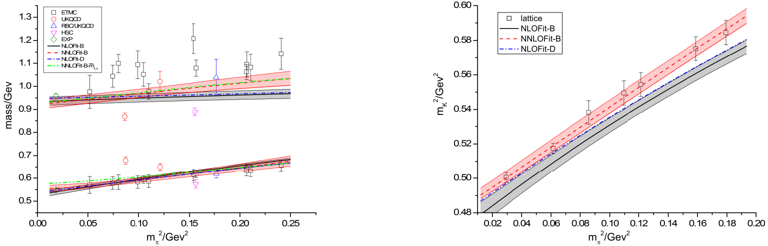
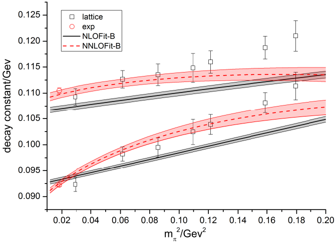


Figure: Left: m_η and $m_{\eta'}$. Right: m_K^2



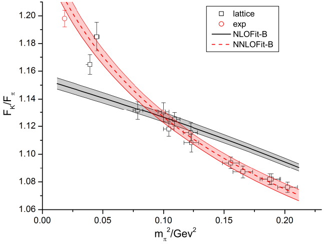


Figure: Left: F_π and F_K. Right: F_K/F_π

	NLOFit-A	NLOFit-B
$\chi^2/(\text{d.o.f})$	481.2/(76-5)	477.7/(76-6)
M_0 (MeV)	835.7*	767.3±31.5±32.3
F (MeV)	92.1±0.2±0.6	92.1±0.2±0.6
$10^3 \times L_5$	1.45±0.02±0.30	1.47±0.02±0.29
$10^3 \times L_8$	1.00±0.07±0.10	1.08±0.05±0.04
Λ_1	0.02±0.05±0.06	-0.09±0.08±0.02
Λ_2	0.25±0.06±0.02	0.14±0.07±0.03

	Input	NLOFit-A	NLOFit-B
F_0 (MeV)	118.0 ±16.5	104.9±2.9±0.3	99.7±3.6±1.6
F_8 (MeV)	133.7 ±11.1	113.2±0.3±4.4	113.5±0.3±4.2
θ_0 (°)	-11.0 ±3.0	-7.2±2.1±1.3	-10.6±2.4±0.1
θ_8 (°)	-26.7 ±5.4	-21.5±2.2±3.9	-25.4±2.6±2.3
$m_s/\widehat{m}$	27.5 ±3.0	22.6±0.8±0.6	21.9±0.6±1.2

Statistical+Systematic errors. Systematic errors are estimated by using F_π in ChPT expressions for each fit

- **NNLO Fits:**
  - m_K^2, F_π, F_K and F_K/F_π are not reproduced at NLO accurately
  - One-loop ChPT contributions start at NNLO
  - Eleven new free parameters: $v_2^{(2)}$, L_4, L_6, L_7, L_{18}, L_{25}, C_{12}, C_{14}, C_{17}, C_{19}, C_{31}
  - Constraints:
    1. $M_0 = 836$ MeV (LO value)
    2. $v_2^{(2)}$ only enters in the combination $M_0^2 + 6v_2^{(2)}(2m_K^2 + m_\pi^2)$, $v_2^{(2)} \to 0$.
    3. $|\Lambda_1| < 0.4$, $|\Lambda_2| < 0.7$ (Natural-value estimates)
    4. L_{18}, $L_{25} \to 0$ (They do not appear in SU(3) ChPT and only affects η-η' mixing)
    5. $C_i \to \alpha C_i^{th}$, with C_i^{th} from Dyson-Schwinger calculations of S. Z. Jiang, Y. Zhang, C. Li and Q. Wang in Phys. Rev. D 81, 014001 (2010) [Fit A], 1502.05087 [hep-ph] [Fit B]
  - Instead of 11 new free parameters → 4 new free parameter

	NNLO-A	NNLO-B
χ^2/(d.o.f)	212.4/(76-9)	231.9/(76-9)
F (MeV)	81.7±1.5±5.3	80.8±1.6±6.1
$10^3 \times L_5$	0.60±0.11±0.52	0.45±0.12±0.78
$10^3 \times L_8$	0.25±0.07±0.31	0.30±0.06±0.30
Λ_1	-0.003±0.060±0.093	-0.04±0.06±0.13
Λ_2	0.08±0.11±0.20	0.14±0.10±0.40
$10^3 \times L_4$	-0.12±0.06±0.19	-0.09±0.06±0.23
$10^3 \times L_6$	-0.05±0.04±0.02	0.03±0.03±0.02
$10^3 \times L_7$	0.26±0.05±0.06	0.36±0.05±0.12
α	-0.59±0.09±0.18	-0.76±0.08±0.44

	Input	NNLO-A	NNLO-B
F_0 (MeV)	118.0 ±16.5	108.0±1.5±3.6	109.1±1.3±5.9
F_8 (MeV)	133.7 ±11.1	124.7±1.2±8.7	126.5±1.2±11.8
θ_0 (Degree)	-11.0 ±3.0	-6.8±1.1±2.6	-6.8±0.9±3.7
θ_8 (Degree)	-26.7 ±5.4	-26.8±1.1±0.2	-27.9±1.0±1.4
$m_s/\widehat{m}$	27.5 ±3.0	27.0±0.6±0.4	29.4±0.4±0.6
F_q (MeV)	106.0 ± 11.1*	92.8±1.1±1.2	92.7±1.0±1.0
F_s (MeV)	143.8 ± 16.5*	136.4±1.5±10.0	139.0±1.4±14.9
θ_q (°)	34.5 ± 5.4*	36.4±1.4±0.2	35.8±1.2±0.3
θ_s (°)	36.0 ± 4.2*	37.8±0.9±1.5	37.1±0.8±1.1

- F is smaller at NNLO
- Chiral loops+$\mathcal{O}(p^6)$ LECs $\longrightarrow$ L_5 and L_8 are smaller than at NLO and compatible with two-loop SU(3) ChPT determinations
- Vanishing of the scalar form factors in the large N_C limit for $s \to \infty$ Jamin,Pich,JAO, NPB'02

$$L_{(5,8)} = 2L_8 - L_5$$
$$L_{(5,8)} \simeq 0 \to L_8 \simeq \frac{1}{2} L_5$$

- Roca,JAO, EPJA'07 Scalar contributions to pseudoscalar masses

$$L_{(4,6)} = 2L_6 - L_4$$
$$L_{(5,8)} + 6L_{(4,6)} = 0 \to L_{(4,6)} \simeq 0$$
$$L_6 \simeq \frac{1}{2} L_4$$

Reflection of scalar-dynamics dominance

Conclusions

- $\mathcal{O}(\delta^2)$ δ-expansion U(3) ChPT is used to study η, η' masses and mixing
- F_π, F_K and F_K/F_π are also reproduced together with m_K^2
- It provides rather accurate extrapolation of LQCD results in m_π
- In order to achieve this a full NNLO calculation is required
- Determination of LECs with relatively small errors:

$$\begin{aligned}
\Lambda_1 &= -0.04 \pm 0.06 \pm 0.13 \\
\Lambda_2 &= 0.14 \pm 0.10 \pm 0.40 \\
10^3 \times L_4 &= -0.09 \pm 0.06 \pm 0.23 \\
10^3 \times L_6 &= 0.03 \pm 0.03 \pm 0.02 \\
10^3 \times L_7 &= 0.36 \pm 0.05 \pm 0.12
\end{aligned}$$

- This theory could be also accurate to study pseudoscalar decays, and other processes involving η and η'.