Spectroscopy and structure of excited heavy baryons

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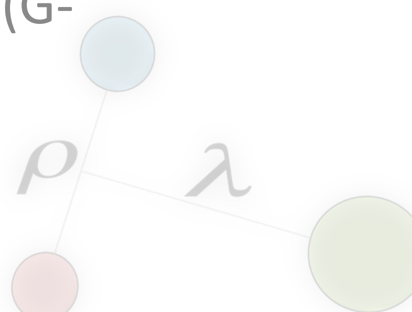


Outline

1. MOTIVATION

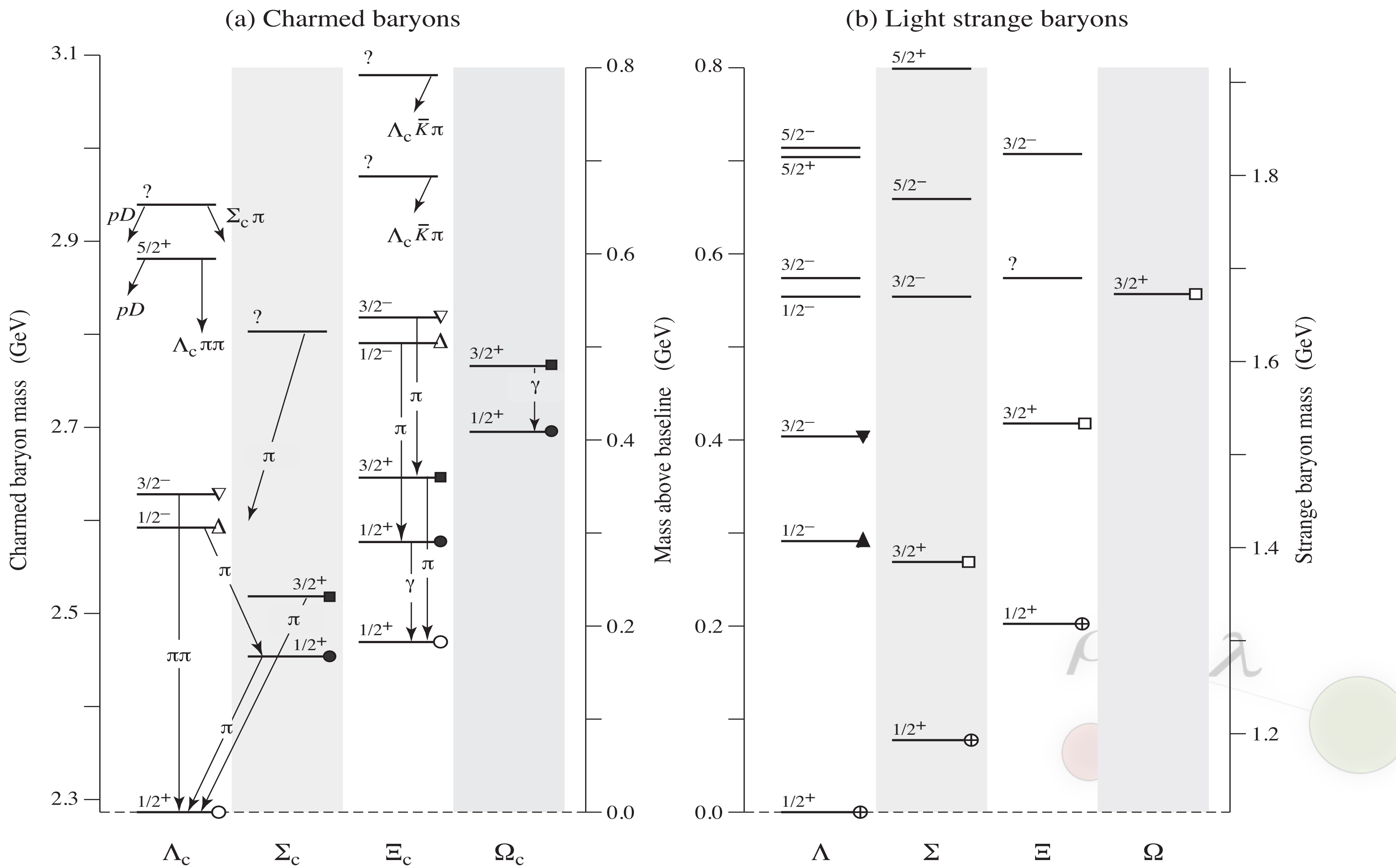
2. FORMARIZM
   - -Hamiltonian
   - -Gauss expansion method

3. λ-MODE AND ρ-MODE

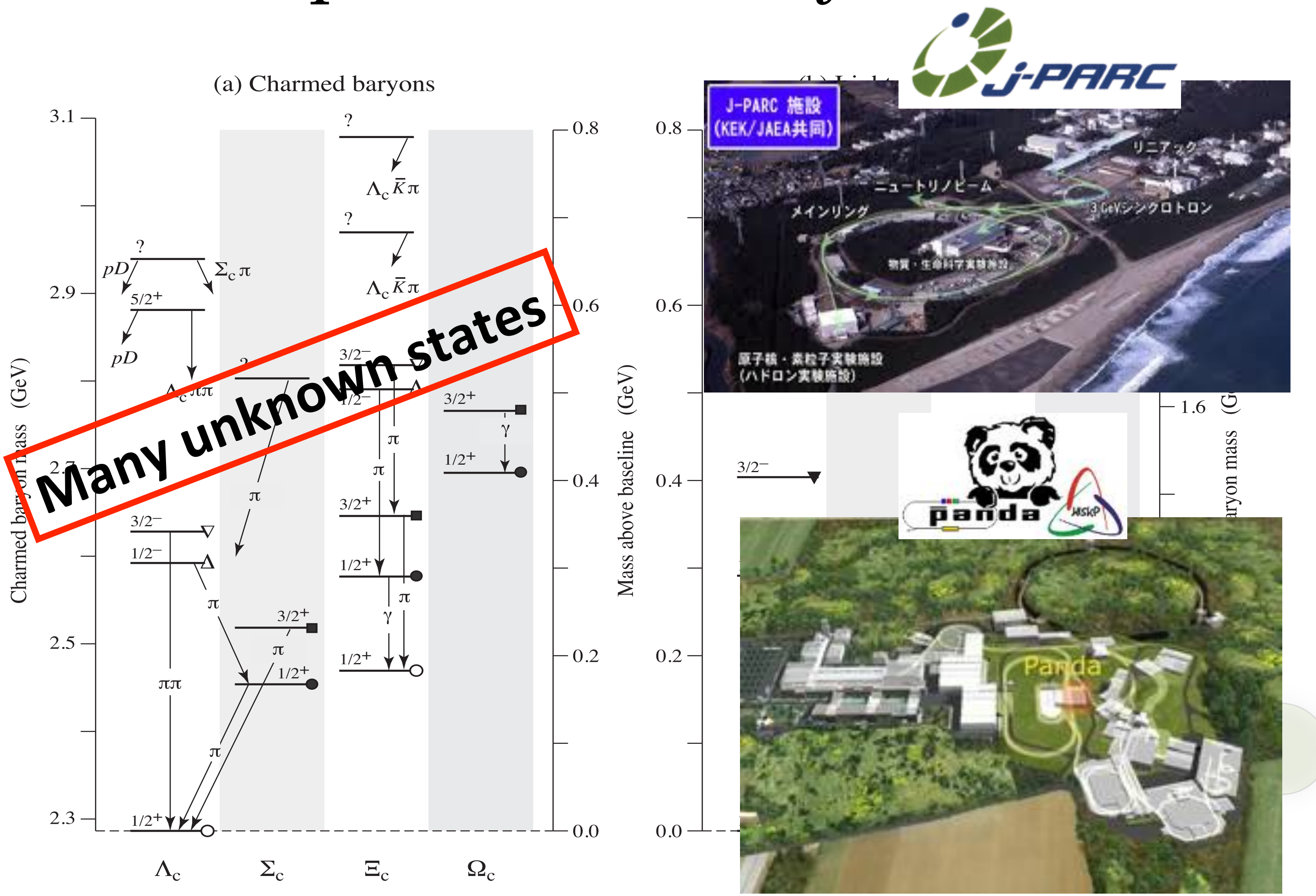
4. RESULT

5. SUMMARY

Mass spectra of Heavy baryon



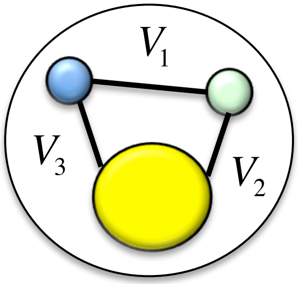
Mass spectra of Heavy baryon



Constituent quark model

Schrödinger equation

$$\left[T + \boxed{V_{conf}} + \boxed{V_{short}} - E\right]\left|\Psi_{JM}\right\rangle = 0$$



Potential

$$\boxed{V_{\rm con} = \sum_{i<j} \frac{b r_{ij}}{2} + C}$$

We introduce non-relativistic hamiltonian of three body system .
Hamiltonian guarantee heavy quark spin symmetry.

$$\begin{aligned}
V_{\rm short} = & \sum_{i<j} \Big[-\frac{2\alpha^{\rm coul}}{3 r_{ij}} + \frac{16\pi\alpha^{ss}}{9 m_i m_j} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \frac{\Lambda^2}{4\pi r_{ij}} \exp(-\Lambda r_{ij}) \\
& + \frac{\alpha^{\rm so}(1-\exp(-\Lambda r_{ij}))^2}{3{r_{ij}}^3} \\
& \times \left[\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + 4\frac{1}{m_i m_j}\right) \boldsymbol{L}_{ij} \cdot (\boldsymbol{S}_i + \boldsymbol{S}_j) + \left(\frac{1}{m_i^2} - \frac{1}{m_j^2}\right) \boldsymbol{L}_{ij} \cdot (\boldsymbol{S}_i - \boldsymbol{S}_j) \right] \\
& + \frac{2\alpha^{\rm ten}(1-\exp(-\Lambda r_{ij}))^2}{3 m_i m_j {r_{ij}}^3} \left(\frac{3(\boldsymbol{S}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{S}_j \cdot \boldsymbol{r}_{ij})}{{r_{ij}}^2} - \boldsymbol{S}_i \cdot \boldsymbol{S}_j \right) \Big] .
\end{aligned}$$

Parameters

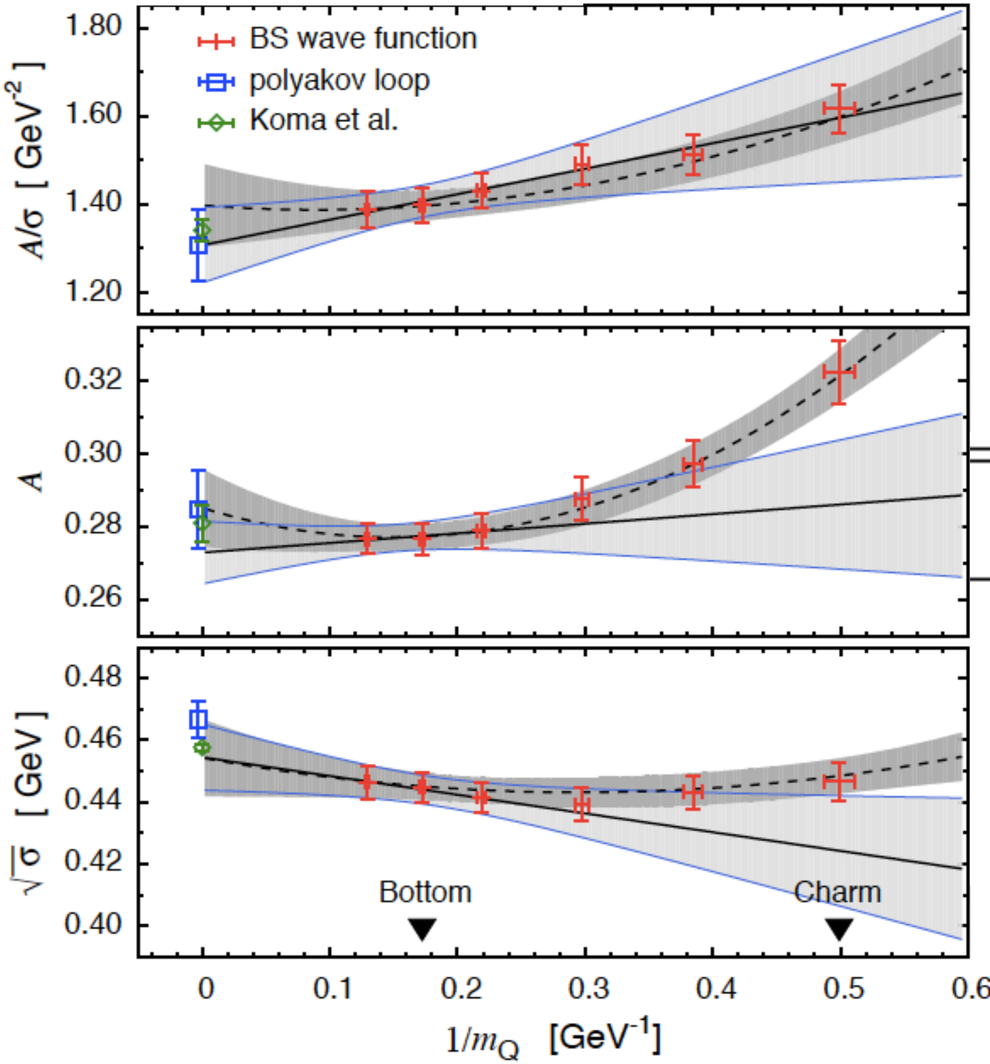
The parameters are determined from experimental data of strange baryons

m_q [MeV]	m_s [MeV]	m_c [MeV]	m_b [MeV]	b [GeV^2]	K [MeV]	$\alpha_{\rm ss}$	$\alpha_{\rm so}(=\alpha_{\rm ten})$	C [MeV]	Λ [fm^{-1}]
300	510	1750	5112	0.165	90	1.2	0.077	-1139	3.5

Constituent quark model

$$V_{q\bar{q}}(r) = -\frac{A}{r} + \sigma r + V_0,$$

Taichi Kawanai and Shoichi Sasaki.Phys.Rev.Lett.,107:091601,2011.



Coulomb force strongly depend on quark mass.

$$\boxed{\alpha_{\text{coul}} = \frac{K}{\mu_{ij}}}$$

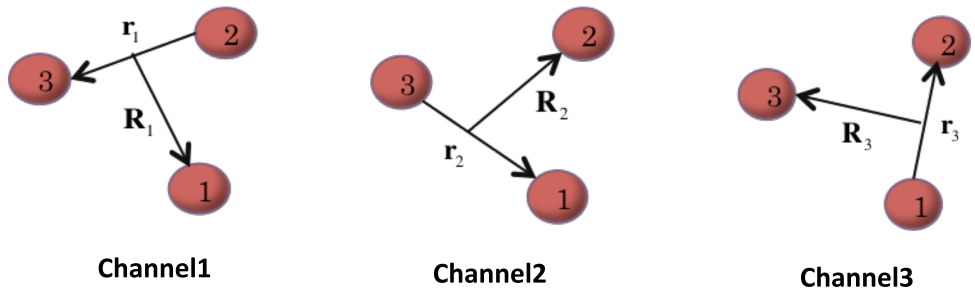
m_Q [GeV]	A	$\sqrt{\sigma}$ [GeV]	A/σ [GeV^{-2}]
2.00(5)	0.323(9)	0.447(6)	1.62(5)
2.60(5)	0.297(6)	0.443(5)	1.51(4)
3.36(6)	0.288(6)	0.439(5)	1.49(5)
4.57(7)	0.279(5)	0.441(5)	1.43(4)
5.80(7)	0.277(4)	0.445(5)	1.40(4)
7.71(8)	0.277(4)	0.446(5)	1.39(4)

Heavy quark mass dependence on color coulomb force and confinement force is suggested.

Gaussian Expansion Method

E. Hiyama, Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys. 51 (2003) 223

jacobi coodinate



Wave function

$$\Psi_{JM} = \Phi^{(C=1)}_{JM}(\mathbf{r}_1,\mathbf{R}_1) + \Phi^{(C=2)}_{JM}(\mathbf{r}_2,\mathbf{R}_2) + \Phi^{(C=3)}_{JM}(\mathbf{r}_3,\mathbf{R}_3)$$

$$\Phi^{(C)}_{JM} = \sum_{n_C l_C N_C L_C} A^{(C)}_{n_C l_C . N_C L_C} \left[\phi^G_{n_C l_C}(\mathbf{r}_C)\psi^G_{N_C L_C}(\mathbf{R}_C)\right]$$

Trial function

$$\phi^G_{nlm}(r) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}})$$

$$\psi^G_{NLM}(R) = N_{NL} R^L e^{-\lambda_N r^2} Y_{LM}(\hat{\mathbf{R}})$$

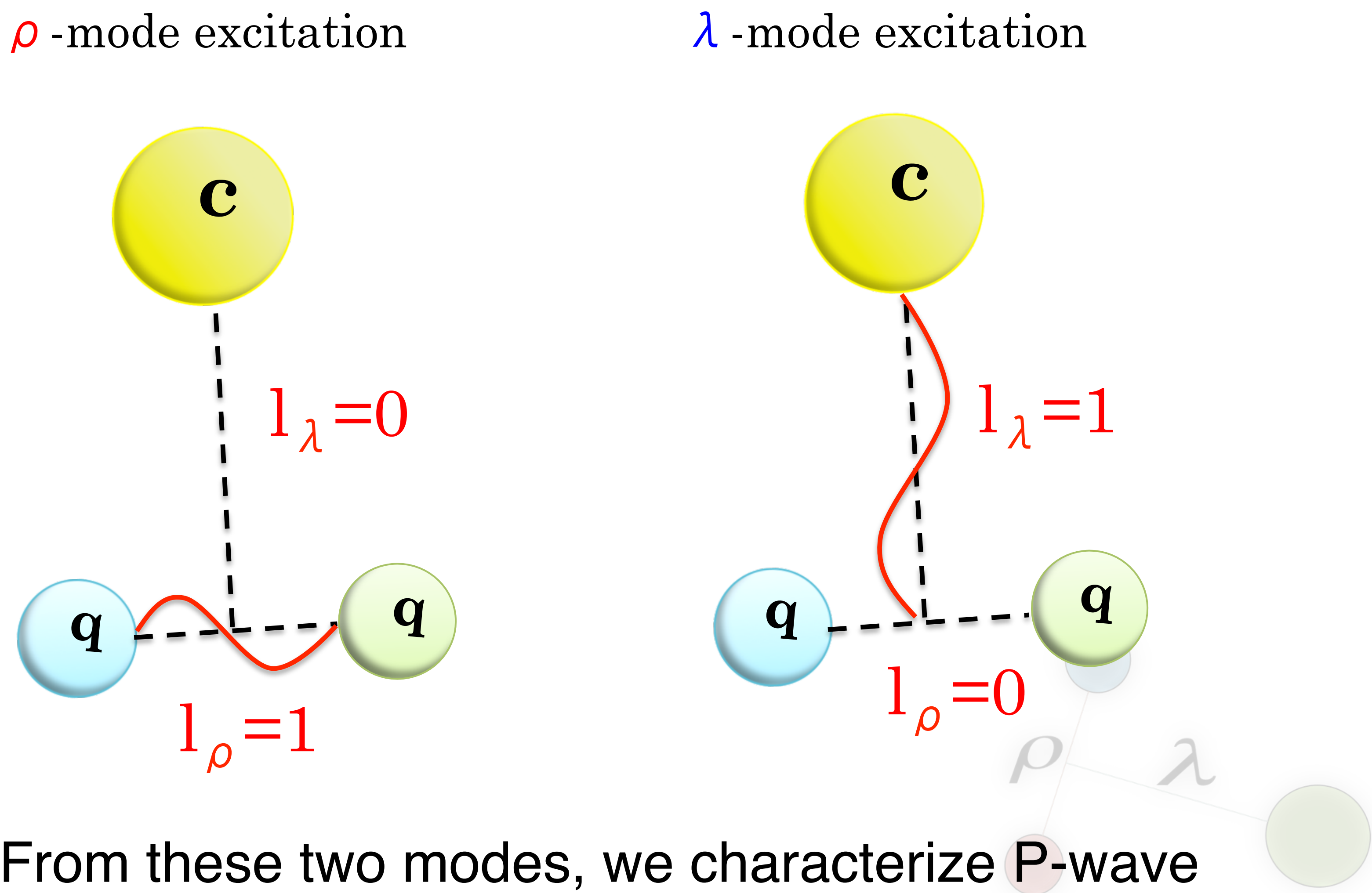
We use variational method to get the energy and wave function.
This type of bases enable us to solve the Schrödinger equation precisely.

Eigen value problem

$$\mathbf{Hc} = E\mathbf{Nc}$$

$$\begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1N} \\ H_{21} & H_{22} & \cdots & H_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ H_{nN} & H_{nN} & \cdots & H_{NN} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{pmatrix} = E \begin{pmatrix} N_{11} & N_{12} & \cdots & N_{1N} \\ N_{21} & N_{22} & \cdots & N_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ N_{nN} & N_{nN} & \cdots & N_{NN} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{pmatrix} \quad \left(\begin{array}{l} N_{ij} = \langle \phi^{(i)}_{JM} | \phi^{(j)}_{JM} \rangle \\ H_{ij} = \langle \phi^{(i)}_{JM} | H | \phi^{(j)}_{JM} \rangle \end{array} \right)$$

ρ -mode and λ -mode



From these two modes, we characterize P-wave state of the heavy baryons.

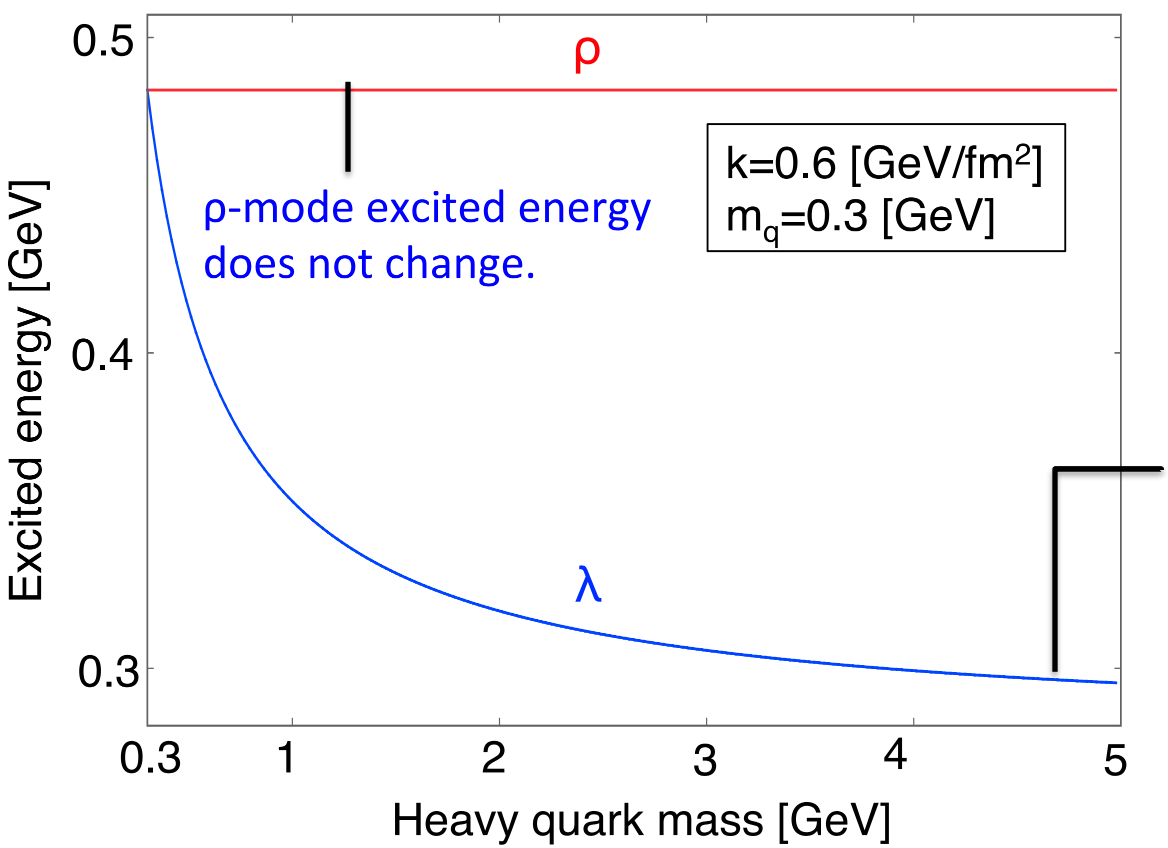
ρ -mode and λ -mode

Harmonic oscillator type potential

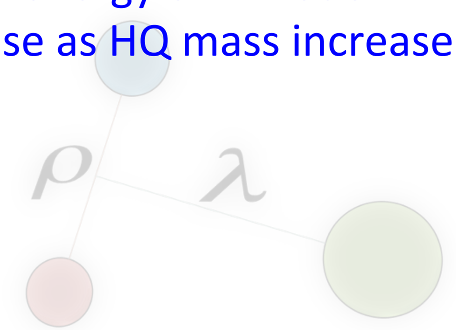
$$H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} \frac{3k}{2}|r_i - r_j|^2 = \frac{p_\rho^2}{2m_\rho} + \frac{p_\lambda^2}{2m_\lambda} + \frac{m_\rho {\omega_\rho}^2}{2}\rho^2 + \frac{m_\lambda {\omega_\lambda}^2}{2}\lambda^2$$

$$\omega_\rho = \sqrt{\frac{3k}{2m_\rho}} \quad \omega_\lambda = \sqrt{\frac{2k}{m_\lambda}}.$$

$$\frac{\omega_\lambda}{\omega_\rho} = \sqrt{\frac{1}{3}(1 + 2m_q/m_Q)}$$



Excited energy of λ-mode
decrease as HQ mass increase



The number of λ and ρ -mode

flavor	l	L	I	s	S	mode	J
Λ_Q	0	1	1	0	1/2	$\lambda_{1/2}$	$1/2^-, 3/2^-$
	1	0	1	1	1/2	$\rho_{1/2}$	$1/2^-, 3/2^-$
	1	0	1	1	3/2	$\rho_{3/2}$	$1/2^-, 3/2^-, 5/2^-$
Σ_Q	0	1	1	1	1/2	$\lambda_{1/2}$	$1/2^-, 3/2^-$
	0	1	1	1	3/2	$\lambda_{3/2}$	$1/2^-, 3/2^-, 5/2^-$
	1	0	1	0	1/2	$\rho_{1/2}$	$1/2^-, 3/2^-$
Ξ_Q	0	1	1	0	1/2	$\lambda_{1/2}$	$1/2^-, 3/2^-$
	1	0	1	1	1/2	$\rho_{1/2}$	$1/2^-, 3/2^-$
	1	0	1	1	3/2	$\rho_{3/2}$	$1/2^-, 3/2^-, 5/2^-$
	0	1	1	1	1/2	$\lambda_{1/2}$	$1/2^-, 3/2^-$
	0	1	1	1	3/2	$\lambda_{3/2}$	$1/2^-, 3/2^-, 5/2^-$
	1	0	1	0	1/2	$\rho_{1/2}$	$1/2^-, 3/2^-$
Ξ_{QQ}	0	1	1	1	1/2	$\lambda_{1/2}$	$1/2^-, 3/2^-$
	0	1	1	1	3/2	$\lambda_{3/2}$	$1/2^-, 3/2^-, 5/2^-$
	1	0	1	0	1/2	$\rho_{1/2}$	$1/2^-, 3/2^-$
Ω_{QQ}	0	1	1	1	1/2	$\lambda_{1/2}$	$1/2^-, 3/2^-$
	0	1	1	1	3/2	$\lambda_{3/2}$	$1/2^-, 3/2^-, 5/2^-$
	1	0	1	0	1/2	$\rho_{1/2}$	$1/2^-, 3/2^-$
Ω_{QQQ}	0	1	1	1	1/2	$\lambda_{1/2}$	$1/2^-, 3/2^-$
	1	0	1	0	1/2	$\rho_{1/2}$	$1/2^-, 3/2^-$

Λ_Q

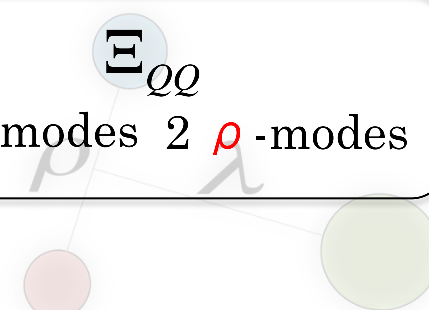
2 λ -modes 5 ρ -modes

Σ_Q

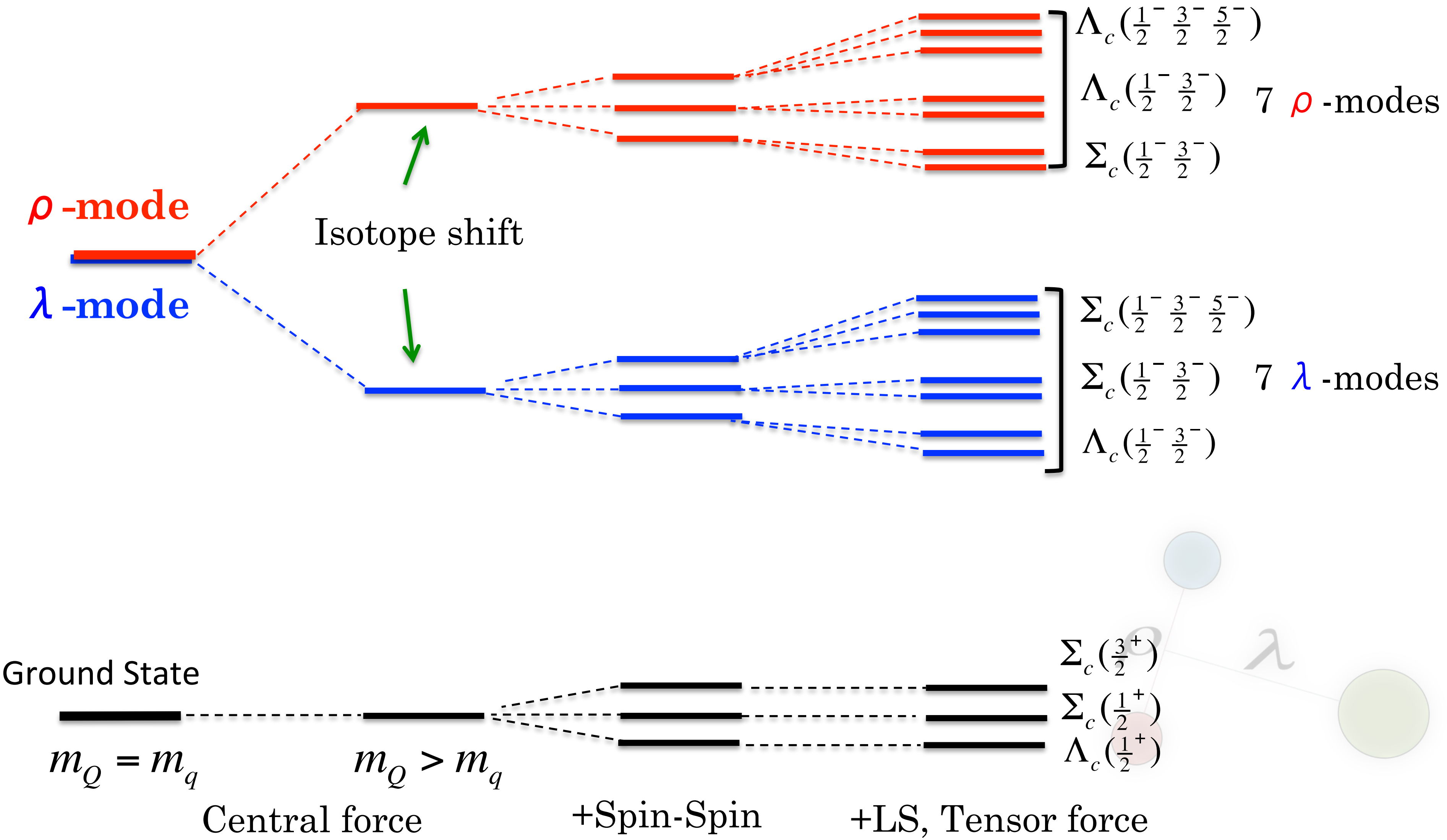
5 λ -modes 2 ρ -modes

Ξ_{QQ}

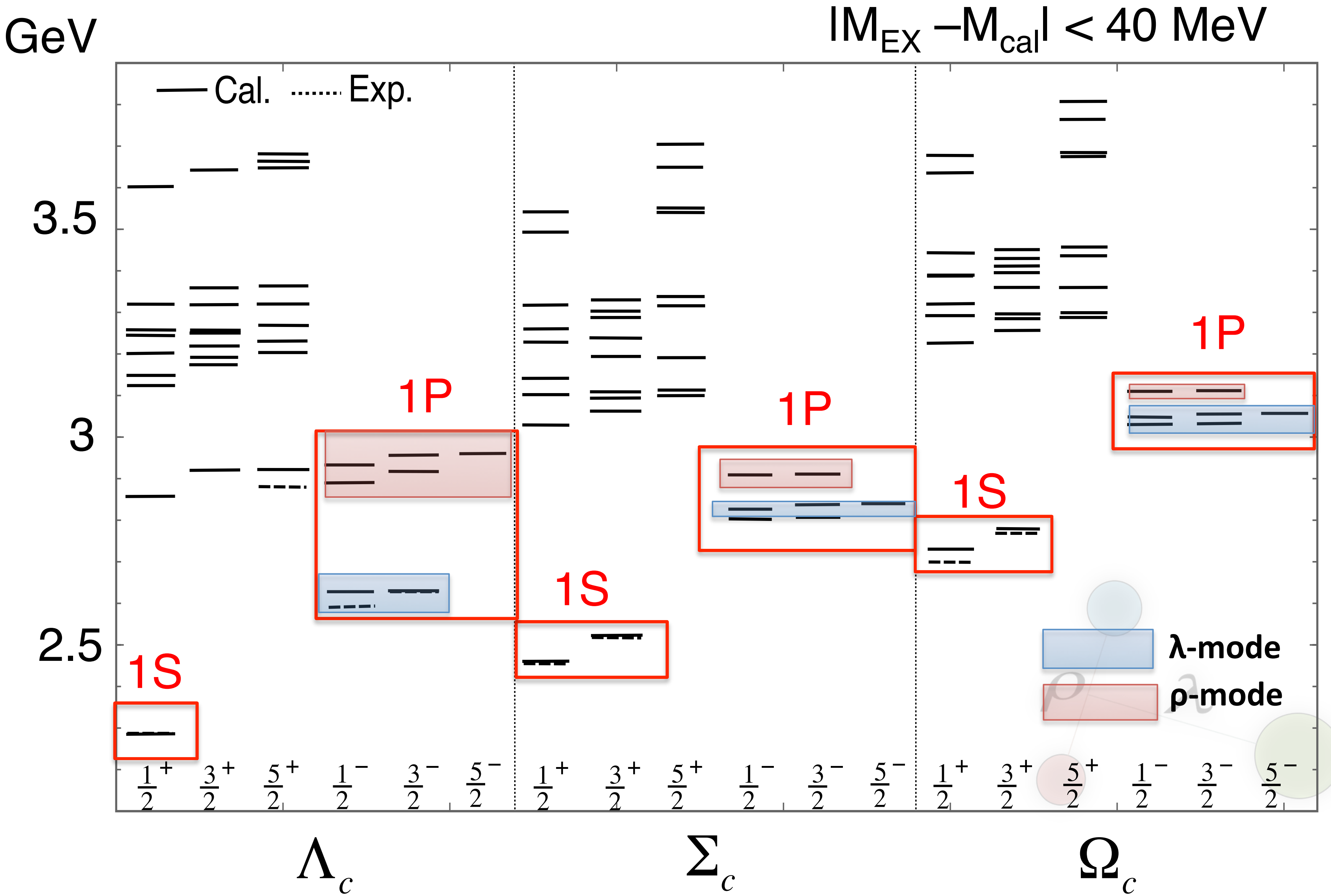
5 λ -modes 2 ρ -modes

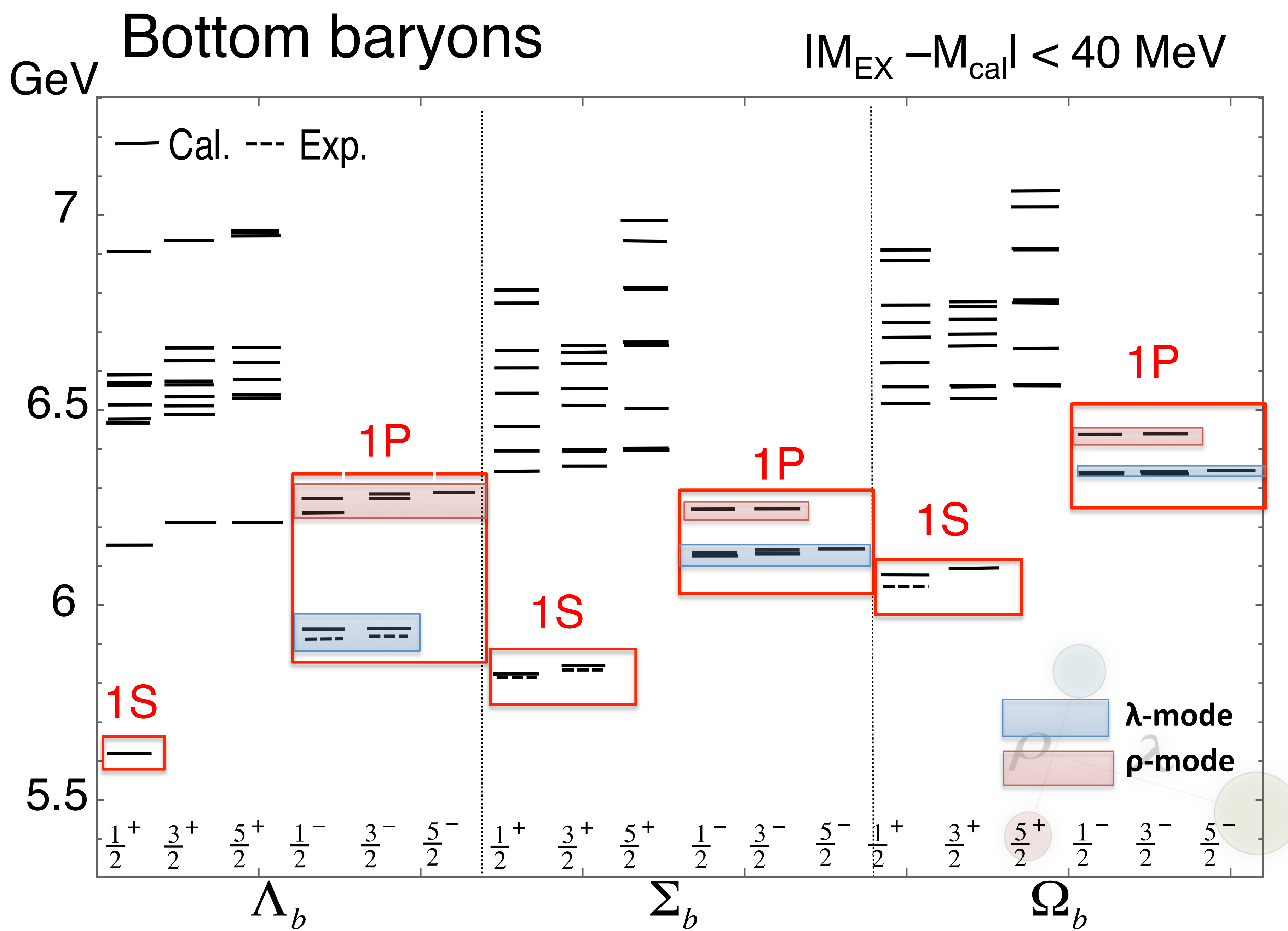


Level structure of P-wave singly heavy baryon



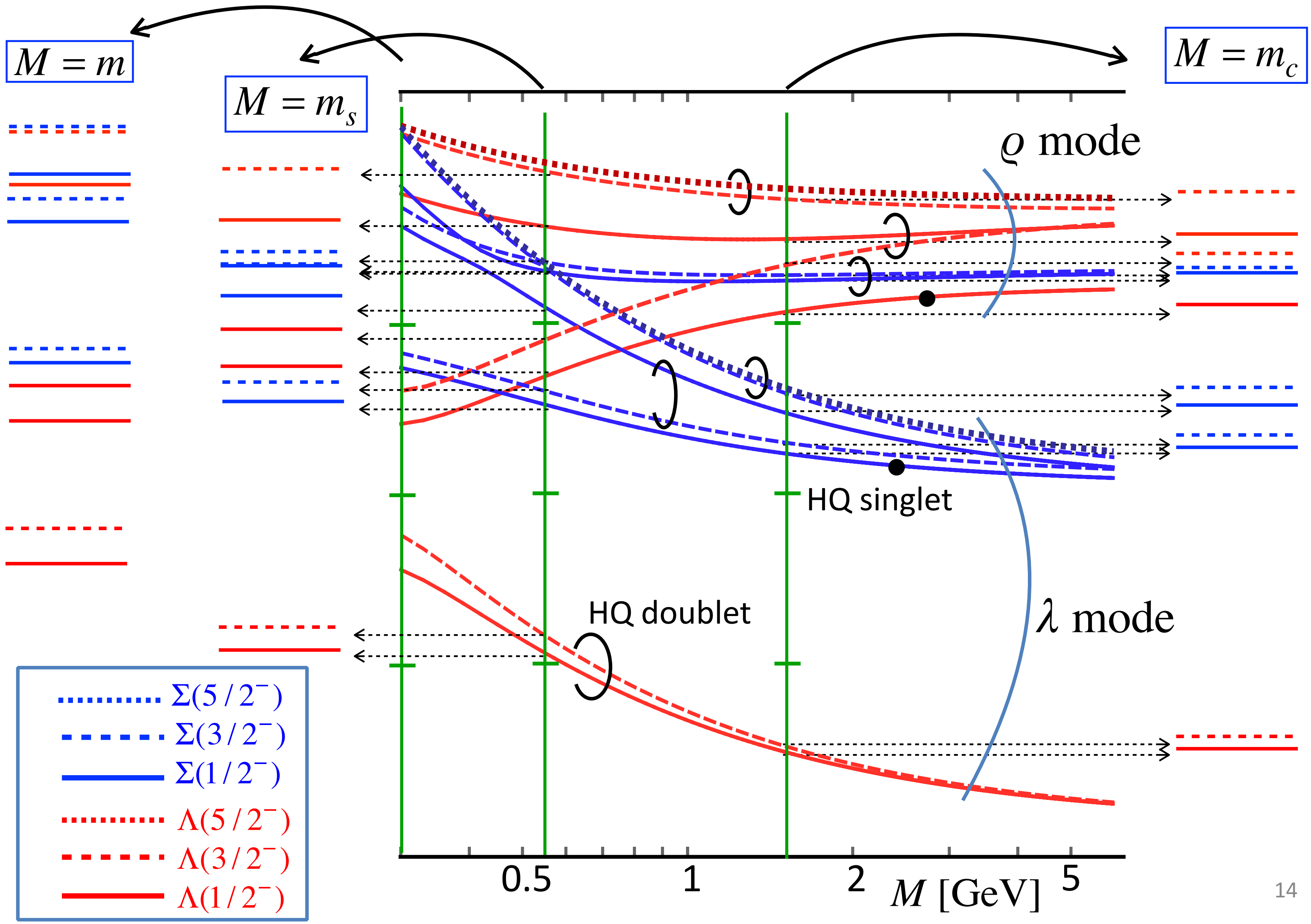
Charmed baryons



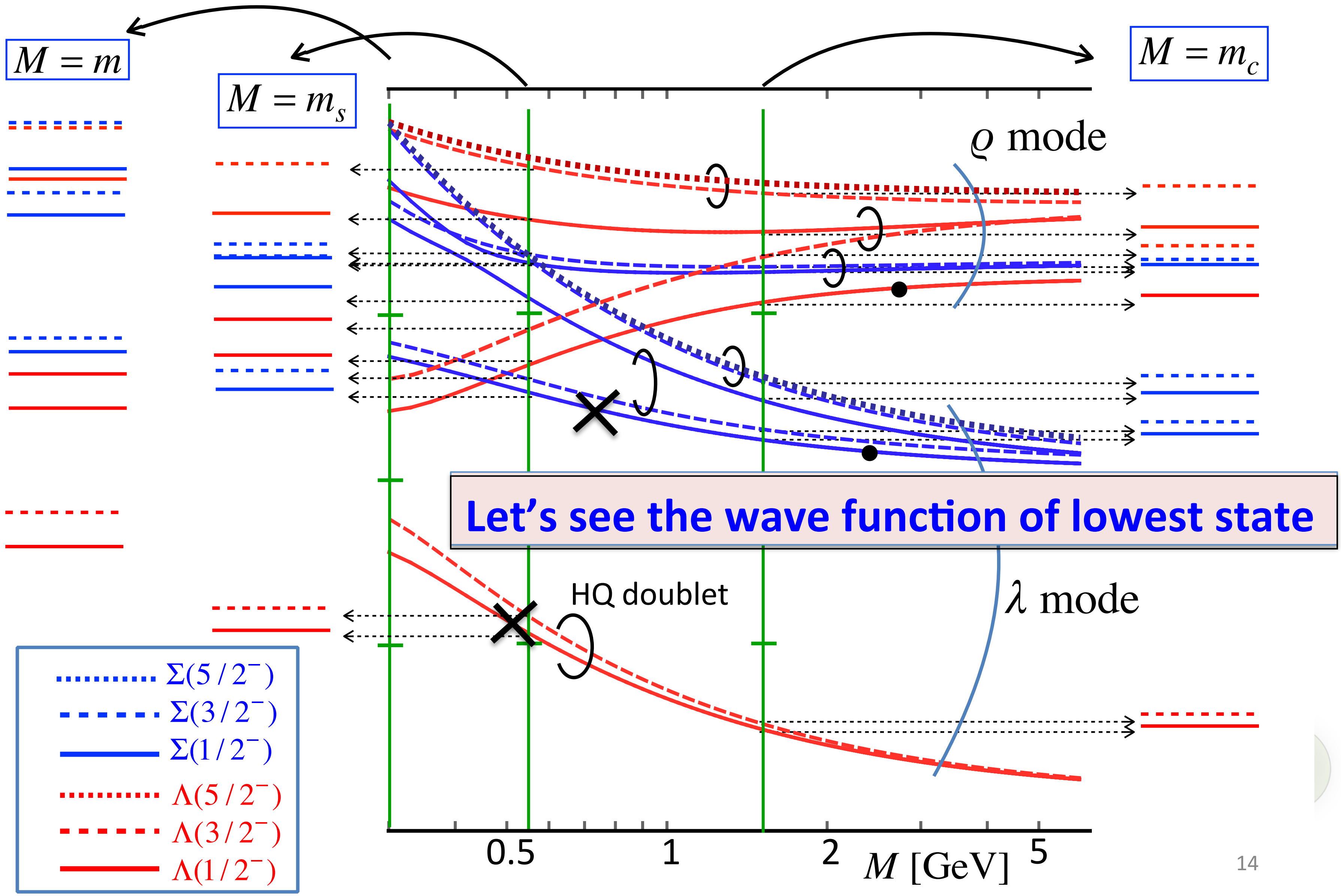


Bottom baryons
|MEX −Mcal| < 40 MeV
GeV
Cal.
Exp.
7
6.5
6
5.5
1P
1S
λ-mode
ρ-mode
1/2 +
3/2 +
5/2 +
1/2 −
3/2 −
5/2 −
Λb
Σb
Ωb

Negative parity states — p-wave excitations - $1/2^-$, $3/2^-$

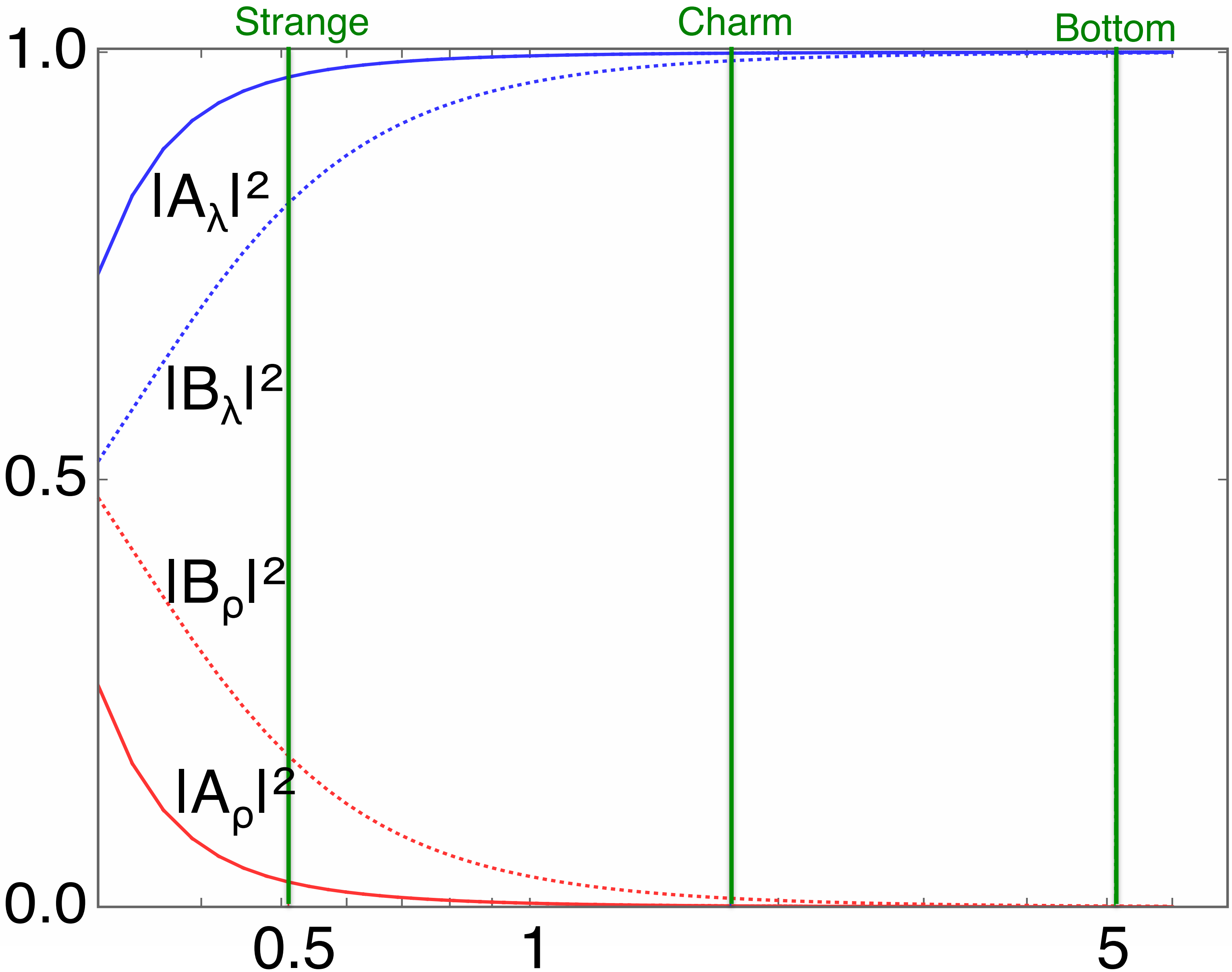


Negative parity states — p-wave excitations - 1/2⁻, 3/2⁻



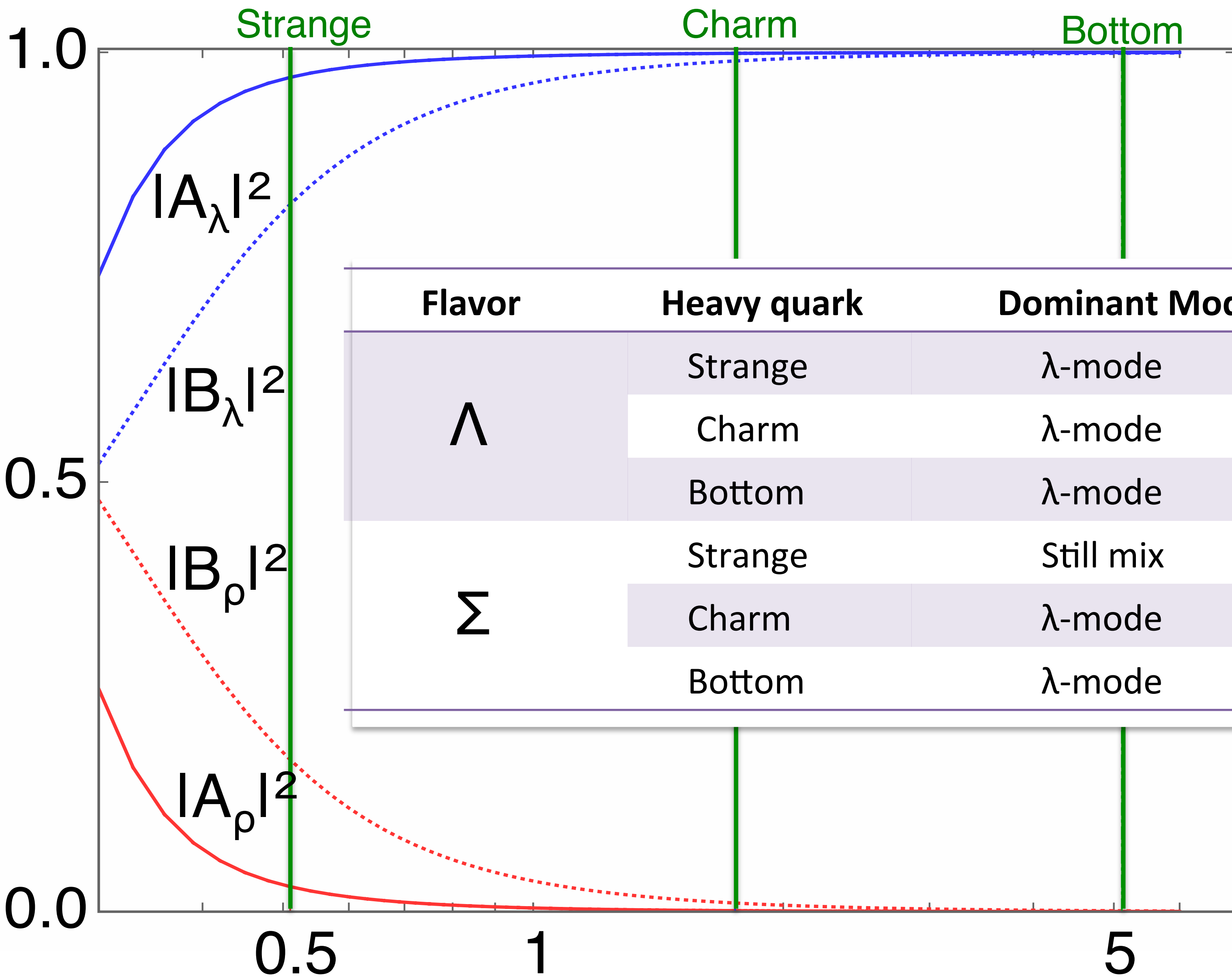
- Wave function of P-wave state

$$\left|\Lambda_Q\right\rangle = A_\lambda(m_Q)\left|\lambda\right\rangle + A_\rho(m_Q)\left|\rho\right\rangle \quad \left|\Sigma_Q\right\rangle = B_\lambda(m_Q)\left|\lambda\right\rangle + B_\rho(m_Q)\left|\rho\right\rangle$$



- Wave function of P-wave state

$$\left|\Lambda_Q\right\rangle = A_\lambda(m_Q)\left|\lambda\right\rangle + A_\rho(m_Q)\left|\rho\right\rangle \quad \left|\Sigma_Q\right\rangle = B_\lambda(m_Q)\left|\lambda\right\rangle + B_\rho(m_Q)\left|\rho\right\rangle$$



Flavor	Heavy quark	Dominant Mode
Λ	Strange	λ-mode
	Charm	λ-mode
	Bottom	λ-mode
Σ	Strange	Still mix
	Charm	λ-mode
	Bottom	λ-mode

Summary

- **Mass Spectra of heavy baryons**

We predict baryon spectra.

→ our calculated result reproduce experimental data within 40 MeV

→we find that λ and ρ-mode well separate in the P-wave states of heavy baryons

We saw the heavy quark mass dependence of mass spectra for the P-wave state.

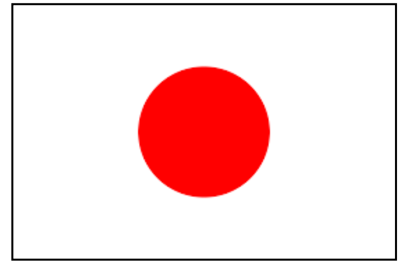
→as heavy quark mass increase, the difference of excitation energy of λ and ρ-mode become large. It is consistent with naïve discussion of harmonic oscillator.

- **Structure of P-wave single heavy baryons**

We saw the heavy quark mass dependence of the state of P-wave single heavy baryon

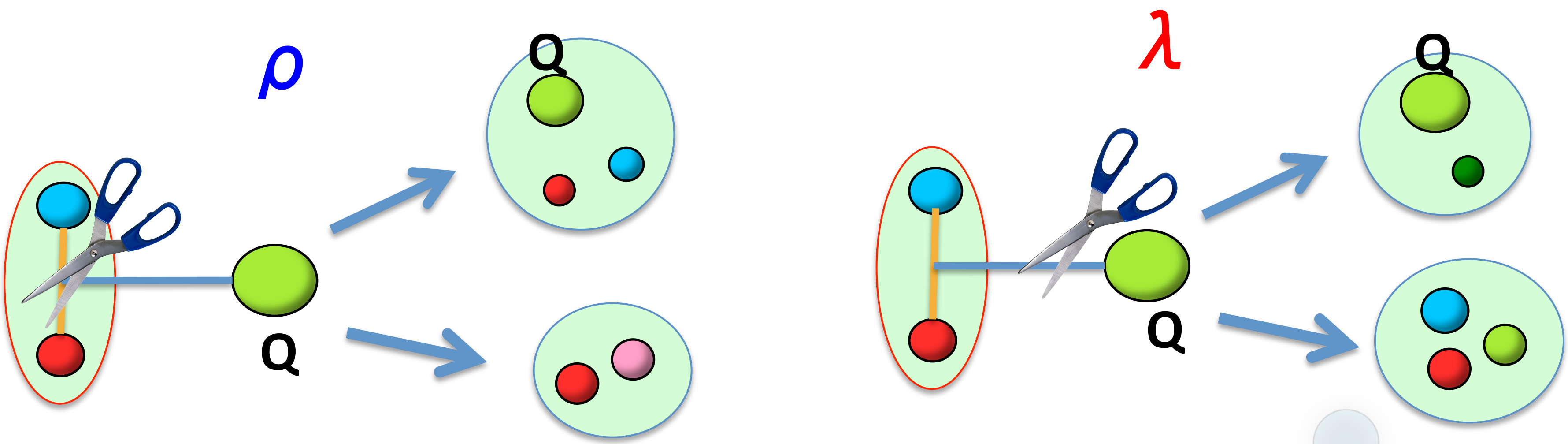
→The Probability of λ and ρ-mode rapidly change as heavy quark mass increase.

We find P-wave single heavy baryon take pure λ-mode for lowest lying states.



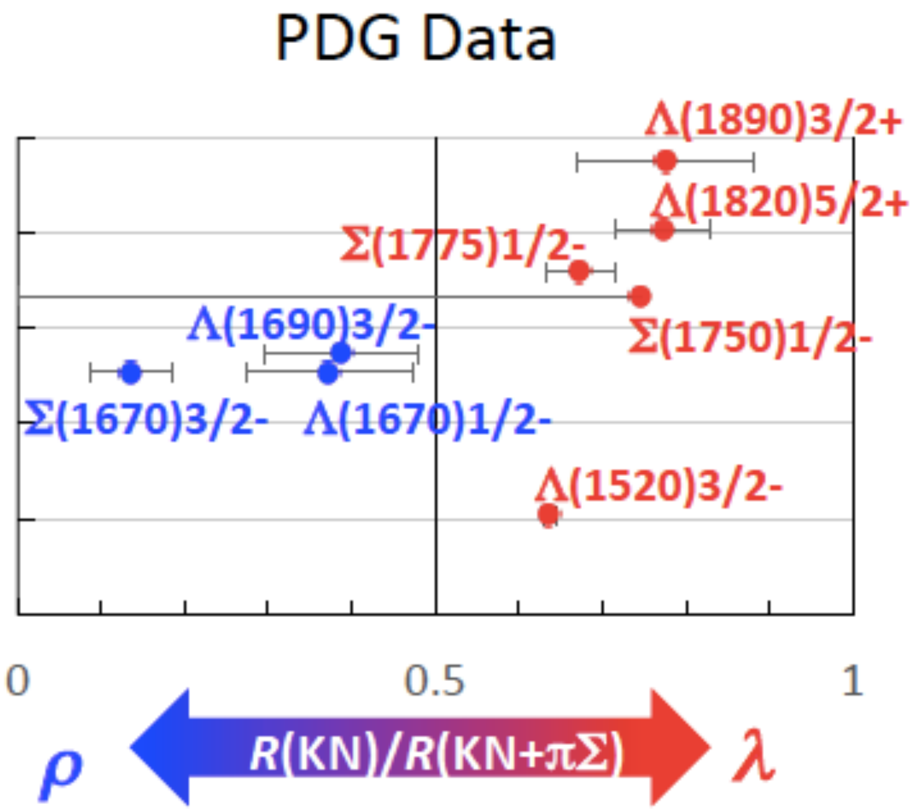
Thank you for your attention!

Decay pattern



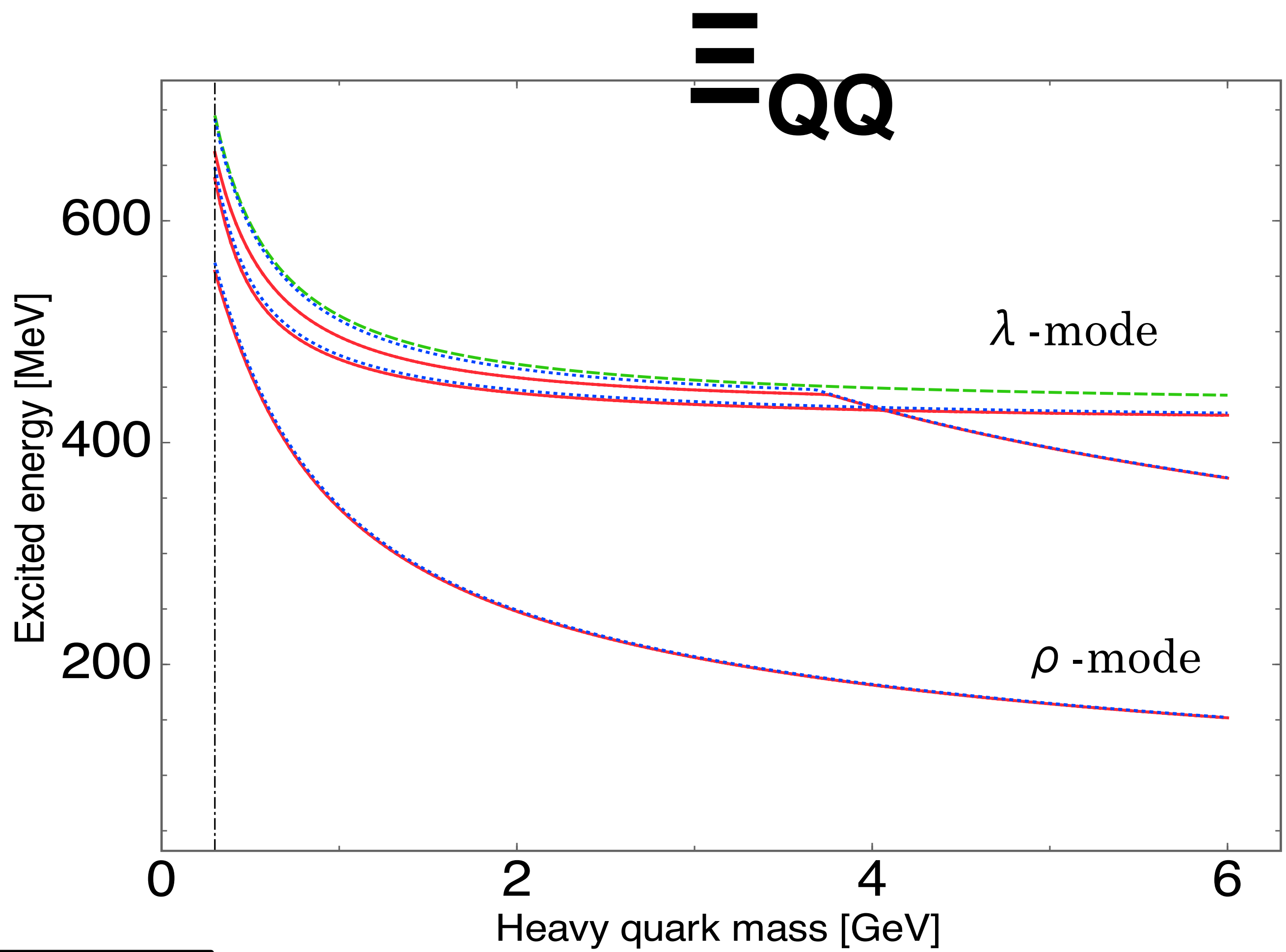
$\Gamma(\Sigma_Q\pi)>\Gamma(pD)$ $\Gamma(\Sigma_Q\pi)<\Gamma(pD)$

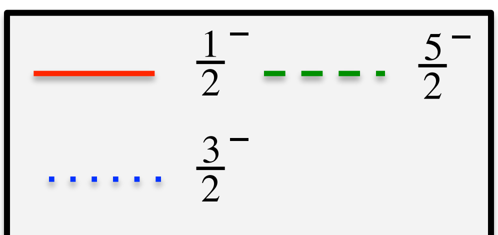
Decay pattern

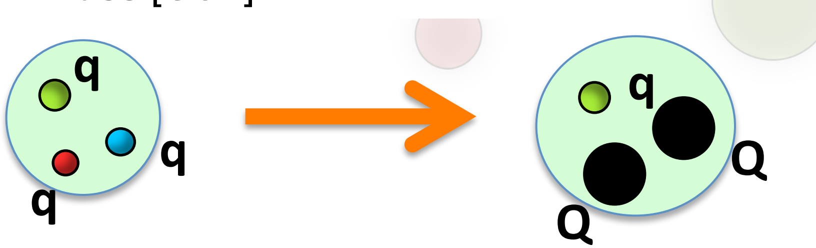


State	J^P	$P_\lambda : P_\rho$
Λ(1520)	3/2-	0.97:0.03
Λ(1670)	1/2-	0.065:0.935
Λ(1690)	3/2-	0.032:0.968
Λ(1890)	3/2+	0.99:0.001
Λ(1820)	5/2+	0.99:0.001
Σ(1750)	3/2-	0.11:0.89
Σ(1670)	3/2-	0.79:0.21

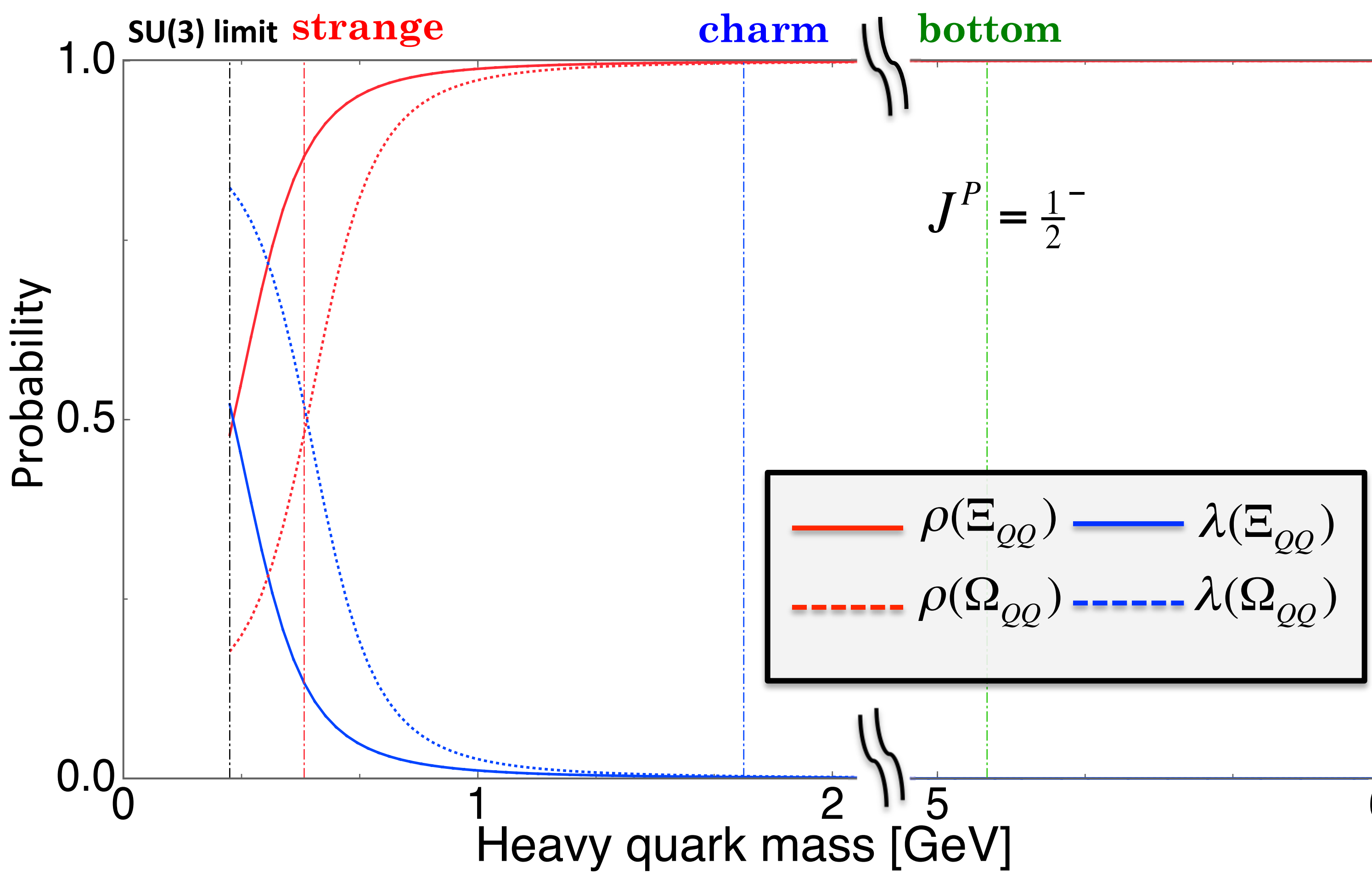
Quark mass dependence of Excited energy







Quark mass dependence of Probability

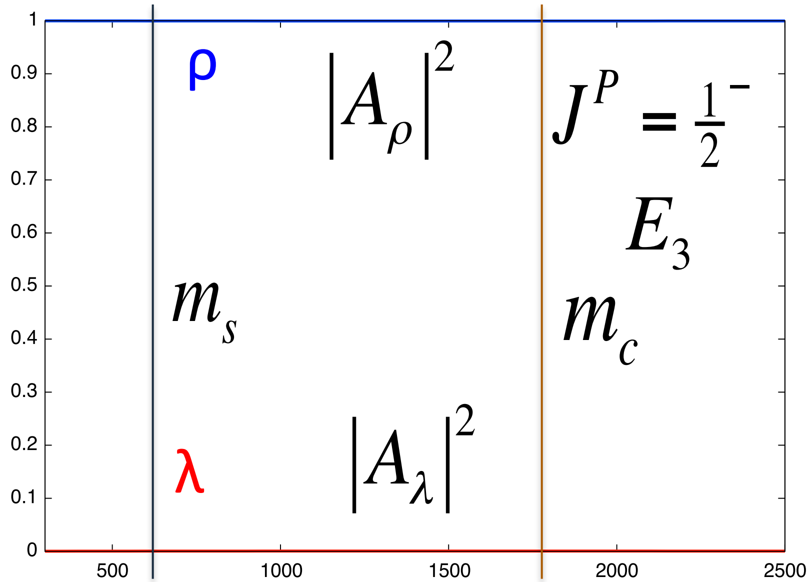


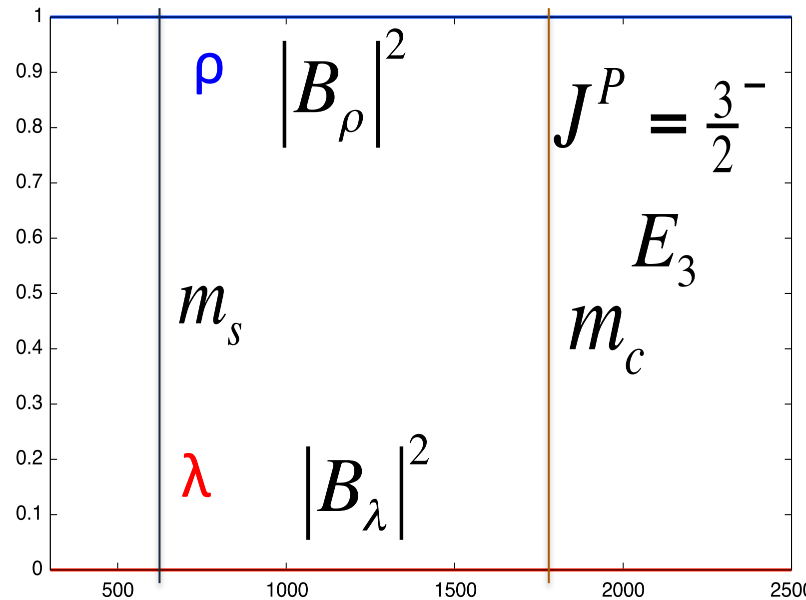
Ξ_{QQ}, Ω_{QQ}

- Wave function of P-wave state

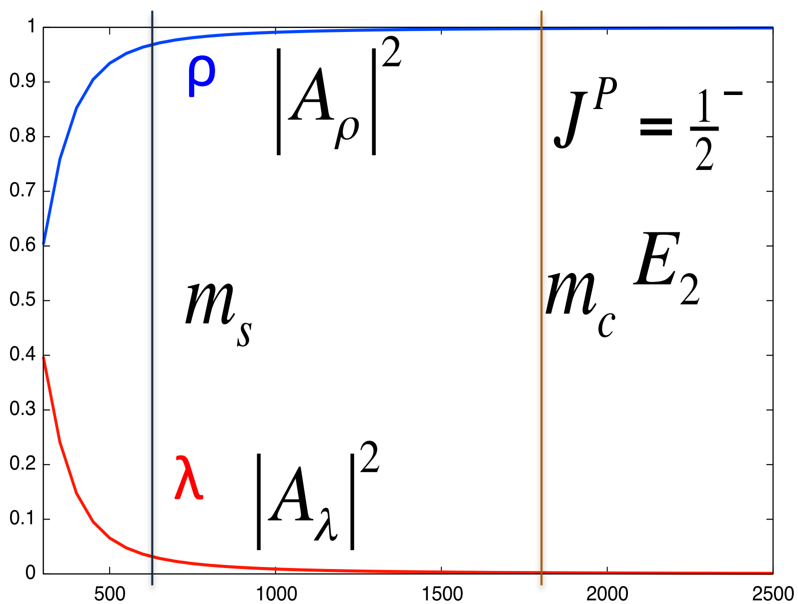
$$\left|\Lambda_Q\right\rangle = A_\lambda(m_Q)\left|\lambda\right\rangle + A_\rho(m_Q)\left|\rho\right\rangle \qquad \left|\Sigma_Q\right\rangle = B_\lambda(m_Q)\left|\lambda\right\rangle + B_\rho(m_Q)\left|\rho\right\rangle$$

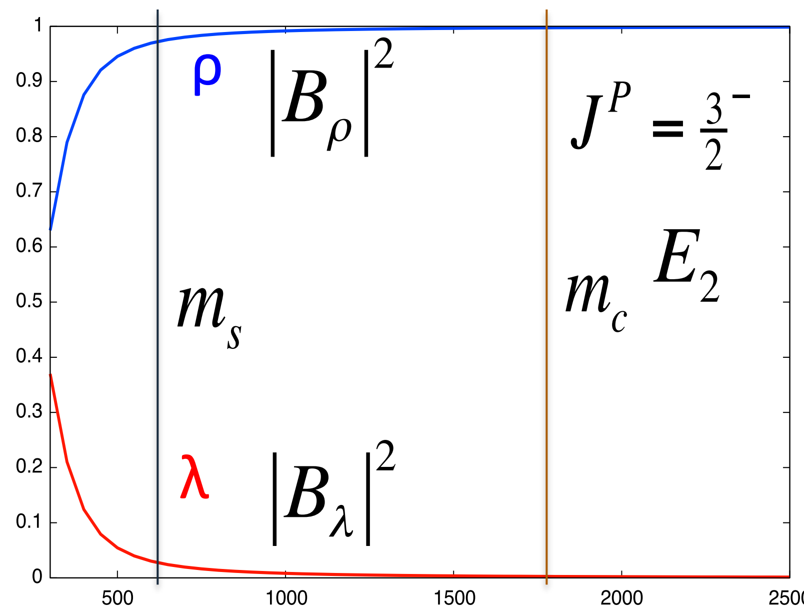
3rd



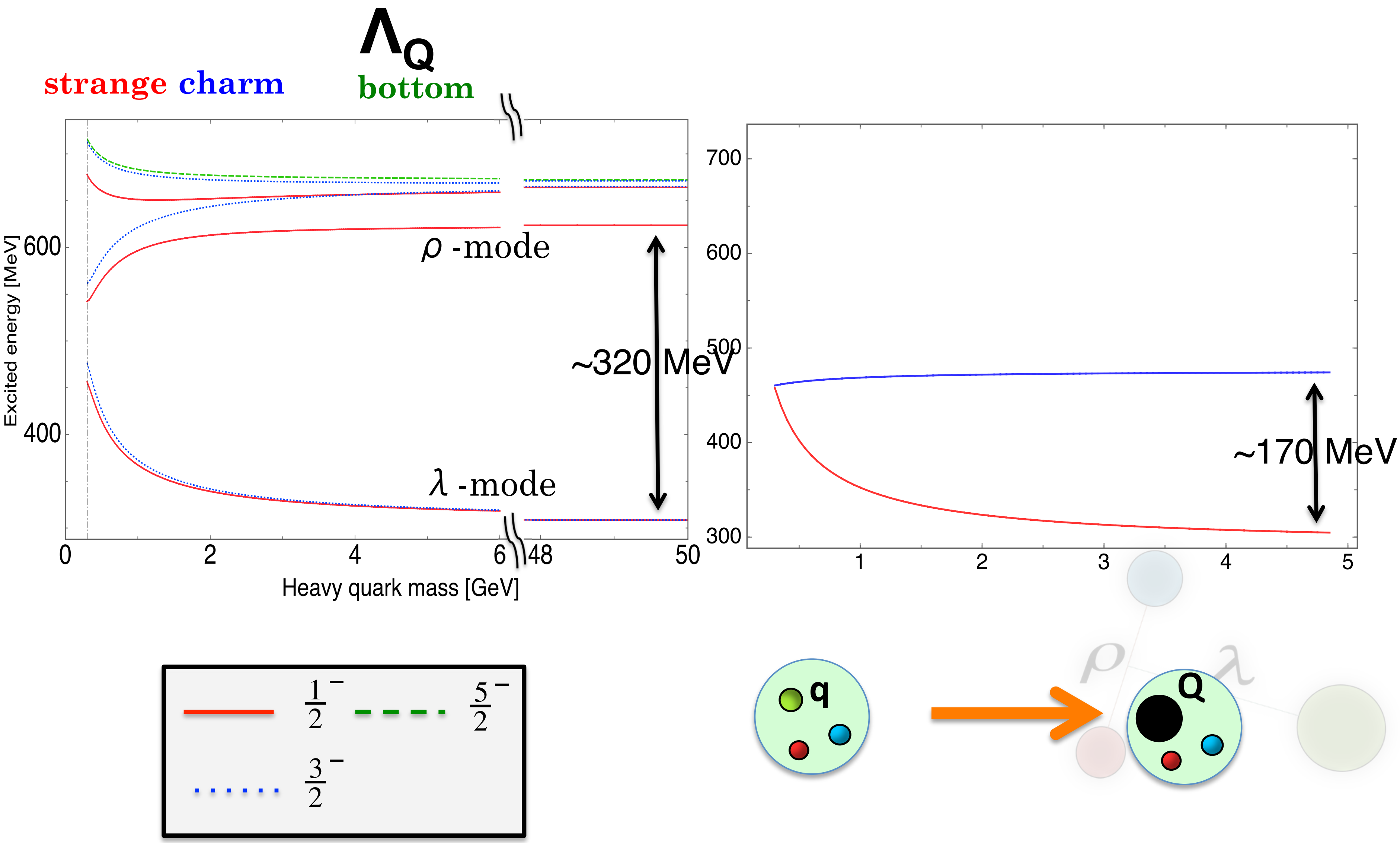


2nd

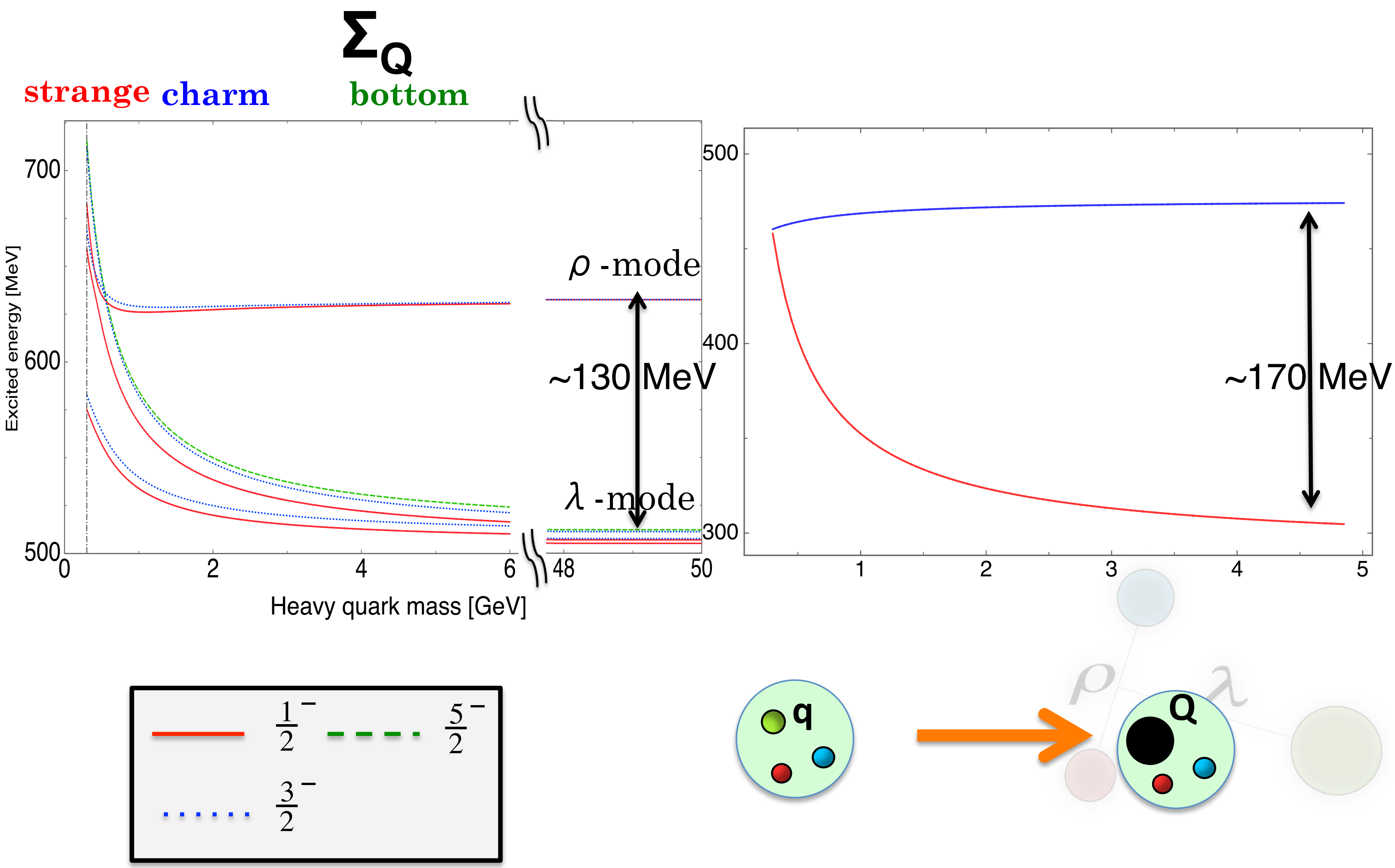




Quark mass dependence of Excited energy



Quark mass dependence of Excited energy



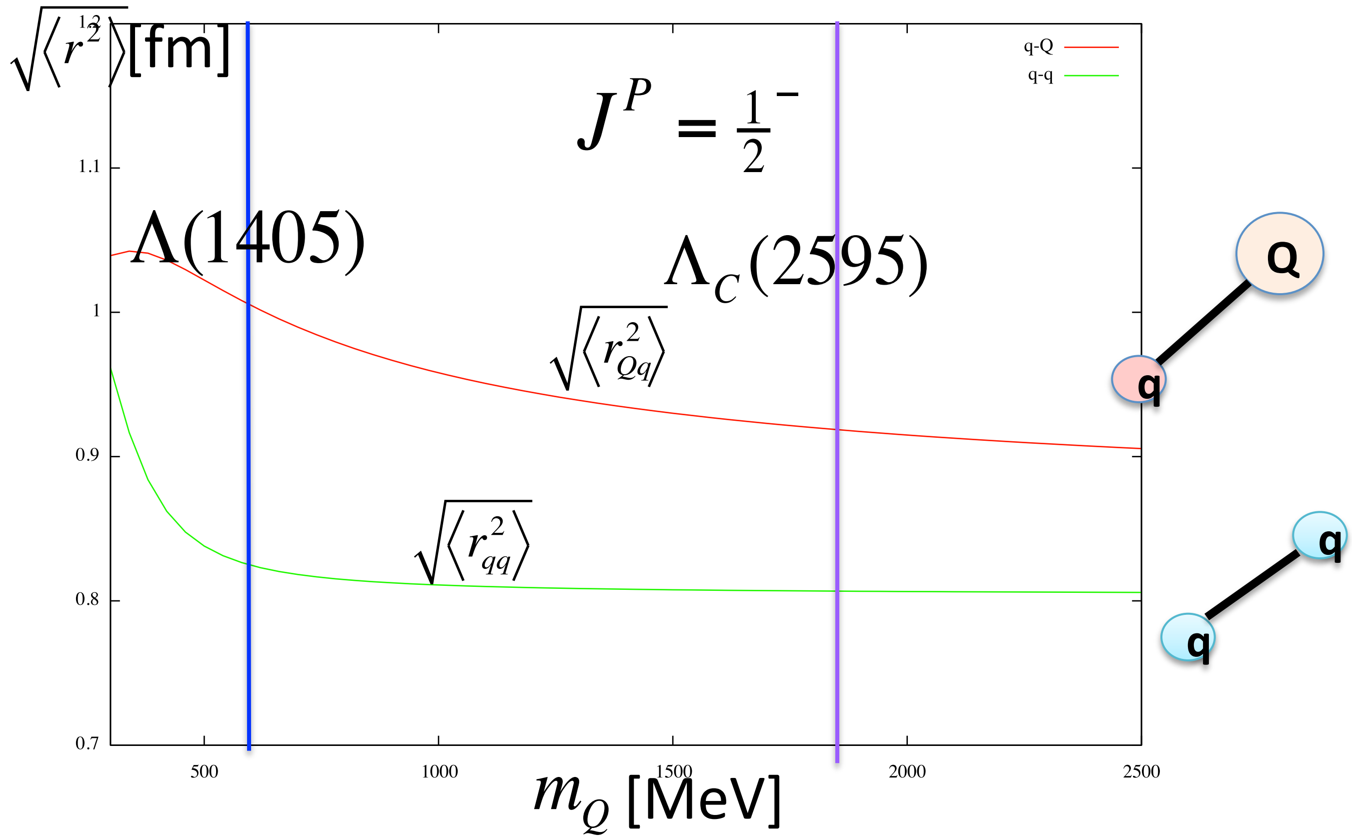
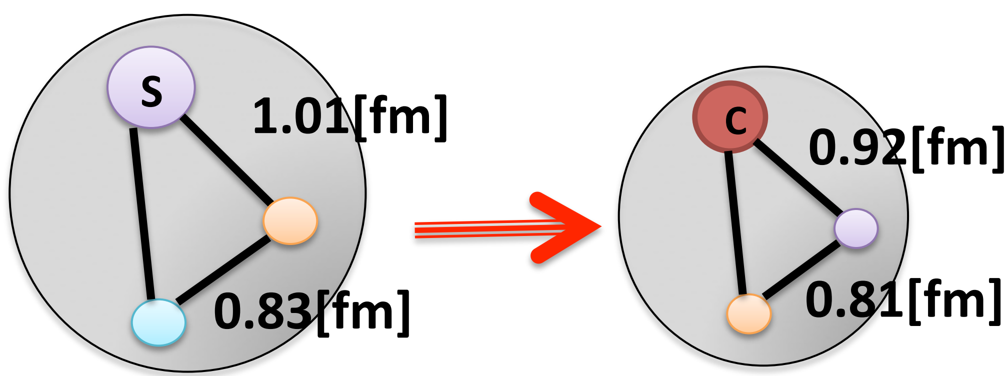
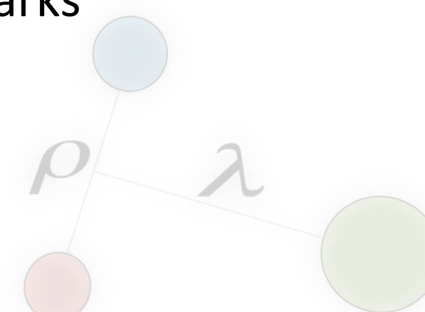


図6 The length between constituent quarks of Λ(1/2⁻)





Angular momentum

$\Lambda\left(\frac{1}{2}^+\right)$

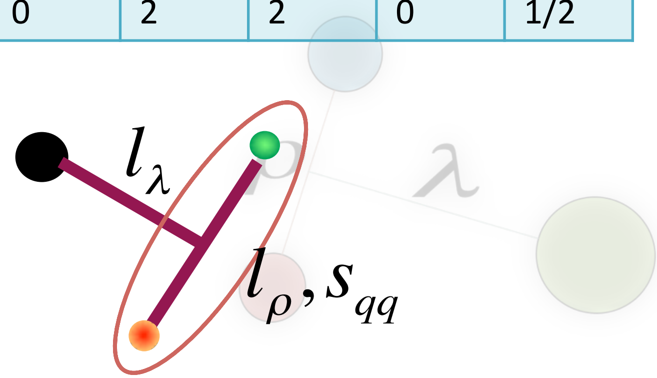
N	lρ	lλ	L	s_qq	S
1	0	0	0	0	1/2
2	1	1	0	1	1/2
3	1	1	1	1	1/2
4	1	1	1	1	3/2
5	1	1	2	1	3/2

$\Lambda\left(\frac{3}{2}^+\right)$

N	lρ	lλ	L	s_qq	S
1	1	1	0	1	3/2
2	1	1	1	1	1/2
3	1	1	1	1	3/2
4	1	1	2	1	1/2
5	1	1	2	1	3/2
6	2	0	2	0	1/2
7	0	2	2	0	1/2

$\Lambda\left(\frac{5}{2}^+\right)$

N	lρ	lλ	L	s_qq	S
1	1	1	1	1	3/2
2	1	1	2	1	1/2
3	1	1	2	1	3/2
4	2	0	2	0	1/2
5	0	2	2	0	1/2



Angular momentum

$\Sigma\left(\frac{1}{2}^+\right)$

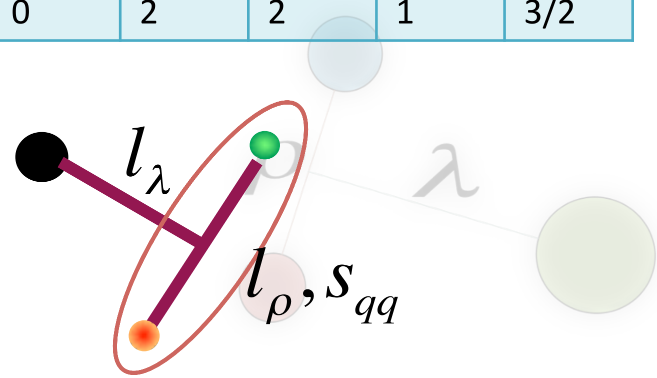
N	lρ	lλ	L	s_qq	S
1	0	0	0	1	1/2
2	1	1	0	0	1/2
3	1	1	1	0	1/2
4	2	0	2	1	3/2
5	0	2	2	1	3/2

$\Sigma\left(\frac{3}{2}^+\right)$

N	lρ	lλ	L	s_qq	S
1	0	0	0	1	3/2
2	1	1	1	0	1/2
3	1	1	2	0	3/2
4	2	0	2	1	1/2
5	2	0	2	1	3/2
6	0	2	2	1	1/2
7	0	2	2	1	3/2

$\Sigma\left(\frac{5}{2}^+\right)$

N	lρ	lλ	L	s_qq	S
1	1	1	2	0	1/2
2	2	0	2	1	1/2
3	2	0	2	1	3/2
4	0	2	2	1	1/2
5	0	2	2	1	3/2



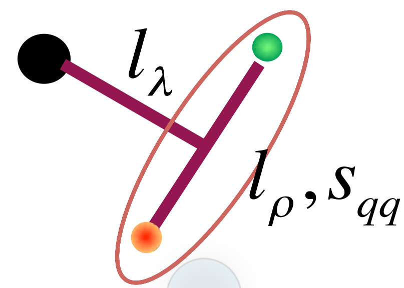
Angular momentum

$$\Lambda\left(\frac{1}{2}^-, \frac{3}{2}^-\right)$$

N	lρ	lλ	L	s_qq	S
1	0	1	1	0	1/2
2	1	0	1	1	1/2
3	1	0	1	1	3/2

$$\Lambda\left(\frac{5}{2}^-\right)$$

N	lρ	lλ	L	s_qq	S
1	1	0	1	1	3/2



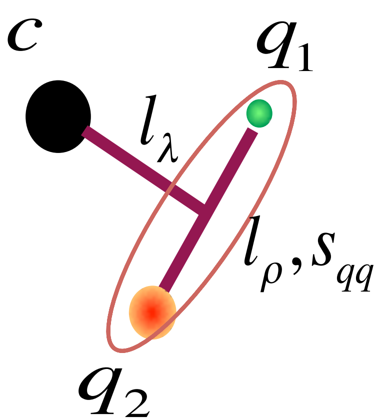
$$\Sigma\left(\frac{1}{2}^-, \frac{3}{2}^-\right)$$

N	lρ	lλ	L	s_qq	S
1	1	0	1	0	1/2
2	0	1	1	1	1/2
3	0	1	1	1	3/2

$$\Sigma\left(\frac{5}{2}^-\right)$$

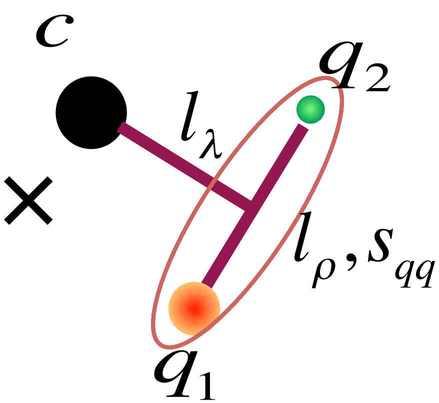
N	lρ	lλ	L	s_qq	S
1	0	1	1	1	3/2

Anti-Symmetrization



Exchange q_1 and q_2

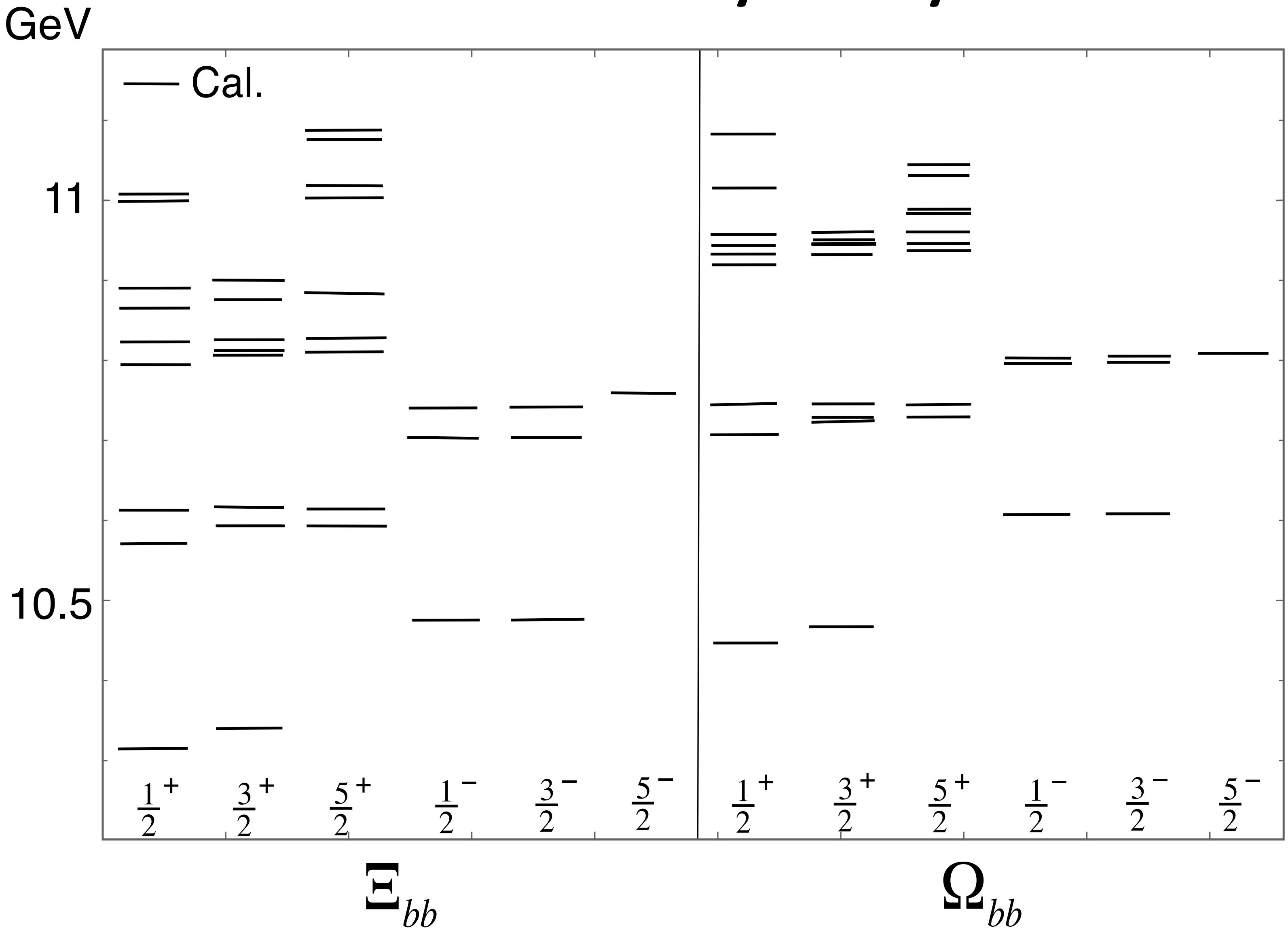
$$(-1)^{s+l+t} \times$$



Because of Pauli principal

$$(-1)^{s+l+t+1} = -1$$

Double heavy baryons



Double heavy baryons