EFTs for Heavy Hidden-Flavour Molecules

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Outline

Part 1: Formulation of the EFT for hidden-charm meson antimeson molecules

Part 2: The EFT in a finite volume

Part 1

Formulation of the EFT for hidden-charm meson antimeson molecules

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Phys.Rev. D87 (2013) 7, 076006

Introduction

- In this work, we study the possible consequences of Heavy Quark Symmetries (HQS) in relation with the existence of exotic mesonic molecules composed of a heavy-light meson and a heavy-light antimeson.

- These molecular systems were first theorized in the 70s (Voloshin, Okun; 1976)

- These molecular systems, not discarded by QCD, are not usually included in conventional quark models, where only mesons and baryons are allowed.

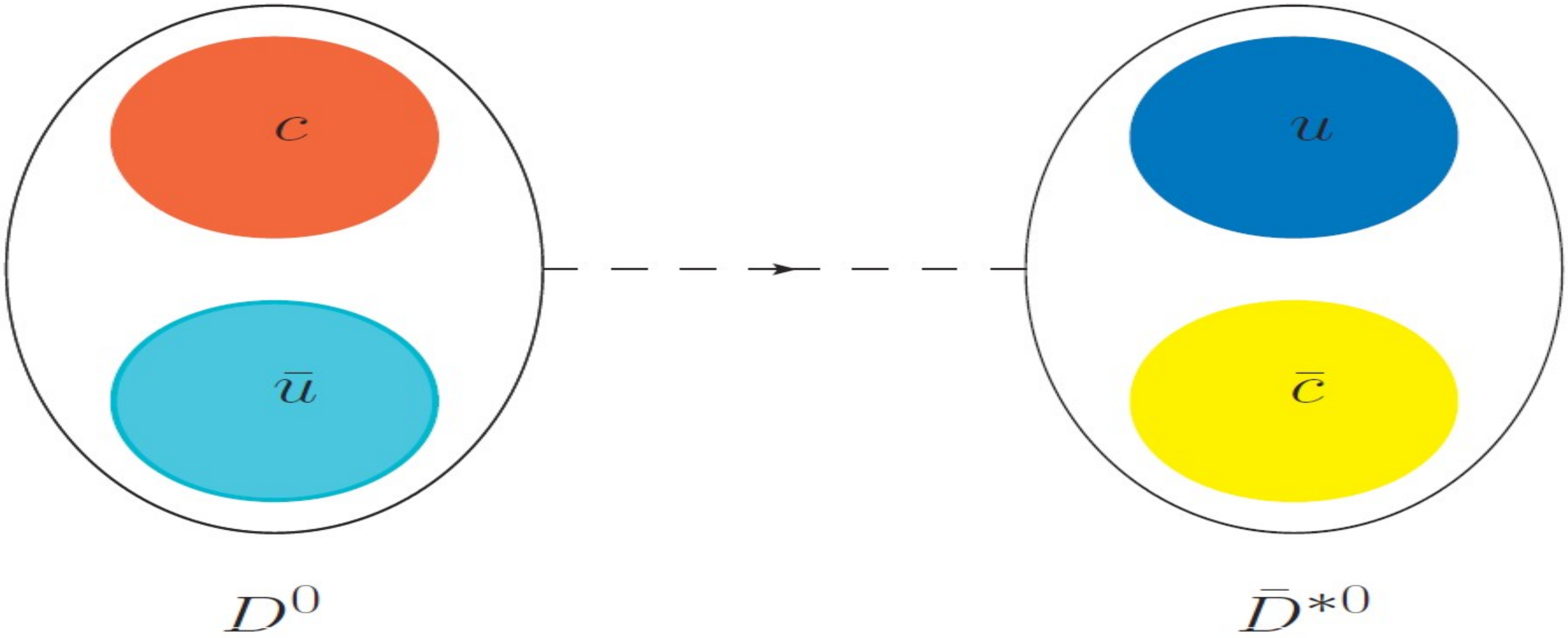
Introduction

- There have been numerous experimental resonances which have been given a molecular interpretation. Among all of them, the most important one is the *X(3872)* resonance. The molecular interpretation of this resonance has a capital importance in this work.

- Other important resonances with a mandatory exotic interpretation are the $Z_b(10610)/Z_b(10650)$.

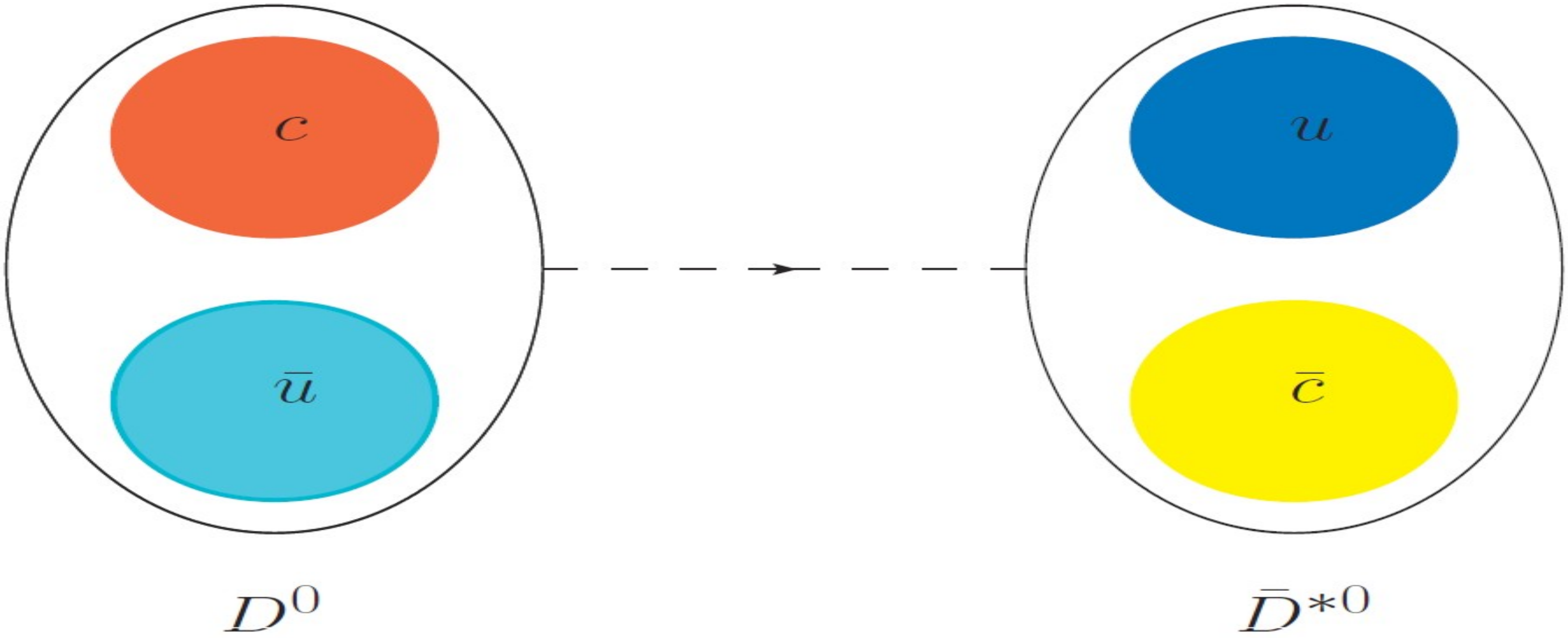
- Thanks to the high degree of symmetry these composite systems present, the formulation of an Effective Field Theory (EFT) that respects both HQS and chiral symmetry seems natural.

Meson-Antimeson Molecules



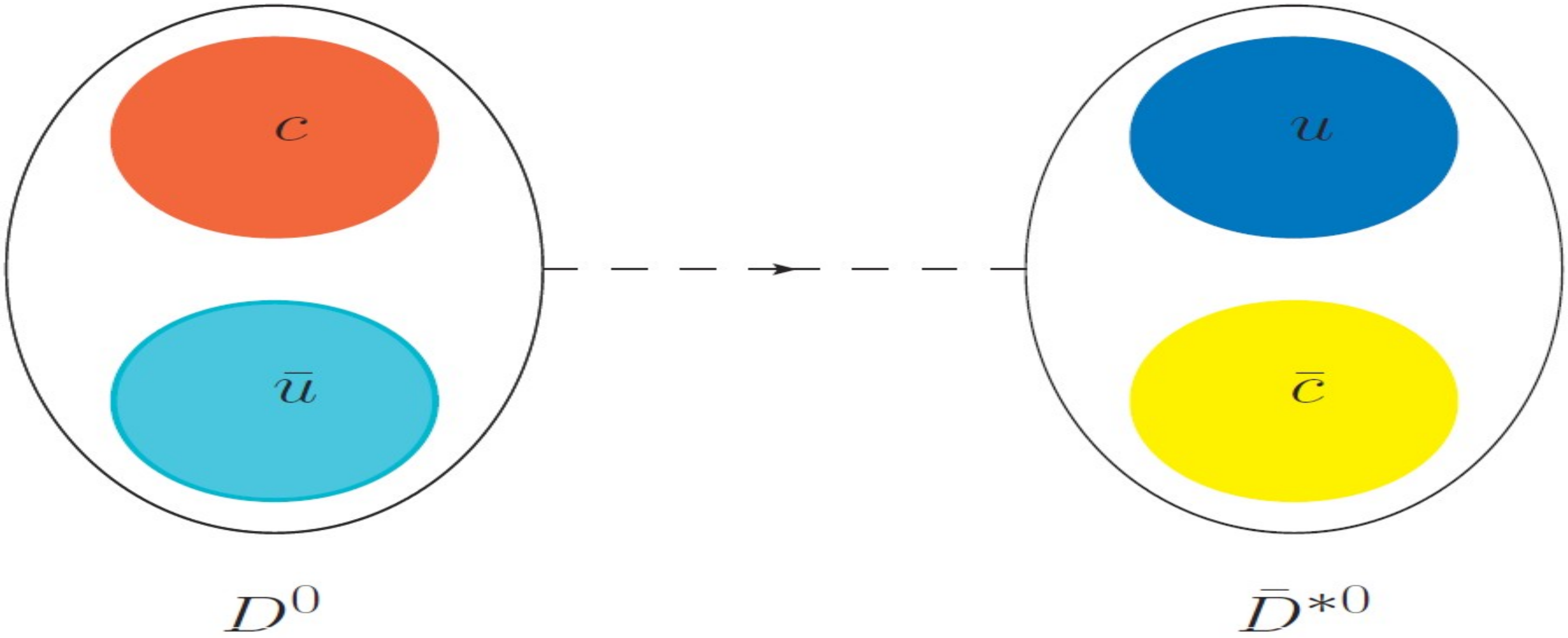
Diagramatic representation of a heavy meson-antimeson molecular system

Meson-Antimeson Molecules



- The mass of the heavy (anti-)quark in the (anti-)meson.
- The size of the mesons.

Meson-Antimeson Molecules



- The mass of the heavy (anti-)quark in the (anti-)meson.
- The size of the mesons.
- The meson-antimeson distance (order Λ_{QCD})
- The total momentum of the molecular system.

Symmetries

- Our approach for the study of heavy mesonic molecular systems will be based on,

  - **Heavy Quark Spin Symmetry (HQSS).** the dynamics is invariant under separate spin rotations of the heavy quark and antiquark.
  - **Heavy Flavour Symmetry (HFS).** Spectrum in the charm sector must be similar to the spectrum in the bottom sector.
  - **Heavy Antiquark-Diquark Symmetry (HADS).** Heavy diquark behaves as a heavy antiquark.

Symmetries

- Our approach for the study of heavy mesonic molecular systems will be based on,

    - **Chiral symmetry** contains pion exchange interactions.
    - **SU(3)-light flavour symmetry**: Heavy molecules also come in SU(3)-light flavour multiplets.

- HQS has a spin-flavour $SU(2N_h)$ symmetry.

- HQET eigenstates are "would-be" hadrons composed by a heavy quark with light antiquarks and gluons, which, assuming SU(3) light-flavour symmetry, will be described into triplets, e.g. $D = (D^0, D^+, D_s)$

EFT Lagrangian

- At Leading Order, the most general potential that respects HQSS takes the form,

$$
\begin{aligned}
V_4 \;=\; & + \frac{C_A}{4}\, Tr\left[\bar{H}^{(Q)} H^{(Q)} \gamma_\mu\right] Tr\left[H^{(\bar{Q})} \bar{H}^{(\bar{Q})} \gamma^\mu\right] + \\
& + \frac{C_A^\lambda}{4}\, Tr\left[\bar{H}_a^{(Q)} \lambda_{ab}^i H_b^{(Q)} \gamma_\mu\right] Tr\left[H_c^{(\bar{Q})} \lambda_{cd}^i \bar{H}_d^{(\bar{Q})} \gamma^\mu\right] + \\
& + \frac{C_B}{4}\, Tr\left[\bar{H}^{(Q)} H^{(Q)} \gamma_\mu \gamma_5\right] Tr\left[H^{(\bar{Q})} \bar{H}^{(\bar{Q})} \gamma^\mu \gamma_5\right] + \\
& + \frac{C_B^\lambda}{4}\, Tr\left[\bar{H}_a^{(Q)} \lambda_{ab}^j H_b^{(Q)} \gamma_\mu \gamma_5\right] Tr\left[H_c^{(\bar{Q})} \lambda_{cd}^j \bar{H}_d^{(\bar{Q})} \gamma^\mu \gamma_5\right]
\end{aligned}
$$

- It only depends on four Low Energy Constants. From now on, we refer to the LECs as C_{0a}, C_{0b}, C_{1a} and C_{1b}.

Lippmann-Schwinger Equation

- Once we have determined V, we find bound states by solving the LSE equation for each spin, isospin and charge-conjugation sector:

$$T = V + V\ G\ T$$

$$\langle \vec{p}\ |T|\vec{p}\ '\rangle = \langle \vec{p}\ |V|\vec{p}\ '\rangle + \int d^3\vec{k} \frac{\left\langle \vec{p}\ |V|\vec{k}\right\rangle \left\langle \vec{k}|T|\vec{p}\ '\right\rangle}{E - m_1 - m_2 - \frac{k^2}{2\mu}}$$

- Bound states of this model will appear as poles in the T-matrix.
- Ultraviolet divergences are regularized/renormalized introducing a Gaussian regulator Λ:

$$\langle \vec{p}|V|\vec{p}\ '\rangle = V(\vec{p}, \vec{p}\ ') = v\ e^{-\vec{p}^2/\Lambda^2}\ e^{-\vec{p}'^2/\Lambda^2} \quad \Rightarrow \quad G = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{e^{-2\vec{k}^2/\Lambda^2}}{E-m_1-m_2-\frac{k^2}{2\mu}}$$

Determination of the LECs

- To determine the LECs, we have made use of the following assumptions.

  - X(3917) is a $D^*\bar{D}^*$ bound state with $J^{PC} = 0^{++}$.
  - Y(4140) is a $D^*_s\bar{D}_s^*$ bound state with $J^{PC} = 0^{++}$.
  - X(3872) is $D\bar{D}^*$ bound state with $J^{PC} = 1^{++}$.
  - The fourth condition will be obtained from the "isospin violation" observed in the X(3872) decays.

Isospin Violation in the X(3872)

- It has been difficult to understand the X(3872) decay widths in different isospin channels. In fact, if X(3872) had a well defined isospin, it would be hard to accomodate the following experimental ratio:

$$\frac{\mathcal{B}(X(3872) \to J/\psi\ \overbrace{\pi^+\pi^-\pi^0}^{\omega})}{\mathcal{B}(X(3872) \to J/\psi\ \underbrace{\pi^+\pi^-}_{\rho})} \simeq 0.8 \pm 0.3.$$

- To explain this ratio there are two different scenarios:
  - X(3872) isospin is well defined but then isospin is not conserved in these strong decays.
  - X(3872) isospin is not well defined and strong interactions conserve isospin.

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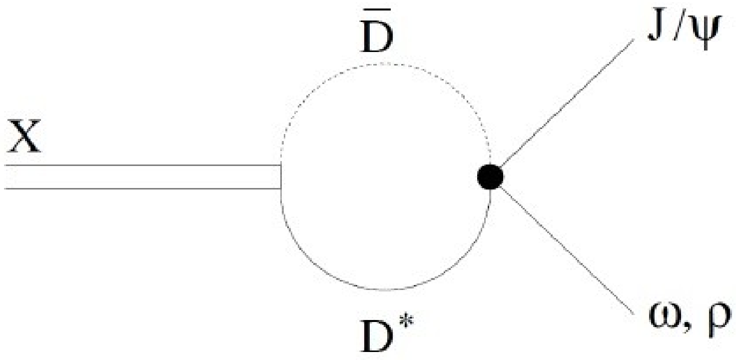
  - **X(3872) isospin is not well defined and strong interactions conserve isospin.**

Isospin Violation in the X(3872)

- A more detailed analysis of the X(3872) decays (Hanhart et al. 2012) gives the ratio between the amplitudes of the decays taking into account the different widths of the intermediate vector bosons ρ and ω:

$$R_X = \frac{\mathcal{M}(X \to J/\psi\ \rho)}{\mathcal{M}(X \to J/\psi\ \omega)} = 0.26^{+0.08}_{-0.05}$$

- In our effective model, these processes are described by the following diagram.



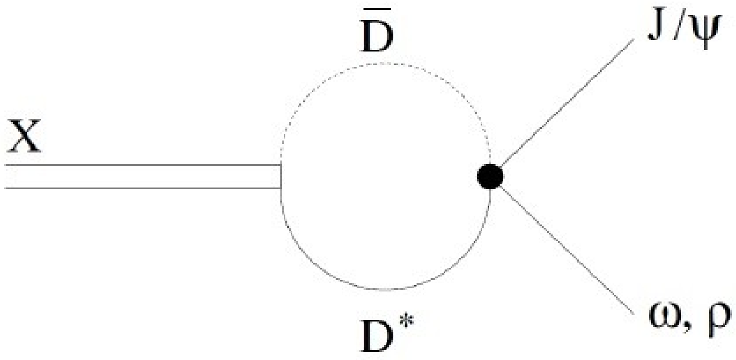
Isospin Violation in the X(3872)

- Then, the same ratio R_X is given by,

$$R_X = \frac{\mathcal{M}(X \to J/\psi\ \rho)}{\mathcal{M}(X \to J/\psi\ \omega)} = \frac{g_\rho}{g_\omega}\left(\frac{\hat{\psi}_1 - \hat{\psi}_2}{\hat{\psi}_1 + \hat{\psi}_2}\right)$$

Being ψ_1 and ψ_2 an average of the neutral and charged $D\bar{D}^*$ wave functions in the vicinities of the origin and,

$$g_\omega = \mathcal{M}_\omega(D\bar{D}^*(I=0) \to J/\psi\ \omega) \qquad g_\rho = \mathcal{M}_\rho(D\bar{D}^*(I=1) \to J/\psi\ \rho)$$



Gamermann et al., PRD81(2010) 014029)

Isospin Violation in the X(3872)

- Assuming SU(3) light flavour symmetry and the OZI rule (ss pair creation is suppressed) hold, we get

$$g_\omega = g_\rho$$

$$\Rightarrow R_X = \frac{\mathcal{M}(X \to J/\psi\ \rho)}{\mathcal{M}(X \to J/\psi\ \omega)} = \left(\frac{\hat{\psi}_1 - \hat{\psi}_2}{\hat{\psi}_1 + \hat{\psi}_2}\right) = 0.26^{+0.08}_{-0.05}$$

- If we now assume a vanishing $D\bar{D}^*$ interaction in the isovector sector, it is found that the ratio R_X only depends on the mass of the resonance and the thresholds of the different channels (Gamermann et al., PRD81(2010) 014029) and takes the value: $R_X \sim 0.13$.

Isospin Violation in the X(3872)

- We improve on this, and consider a non-vanishing I = 1 interaction that we fit to the value for R_X reported by Hanhart et al. Hence, we work with a coupled channel (neutral and charged channels) contact potential:

$$V_{coupled} = \frac{1}{2}\begin{pmatrix} V_0 + V_1 & V_0 - V_1 \\ V_0 - V_1 & V_0 + V_1 \end{pmatrix}$$

being V_0 and V_1 the well defined isospin potentials with I = 0 and I = 1, respectively.

- Therefore, the experimental ratio of the ρ and ω decay widths of the X(3872) provides further constraints to the counter-terms.

Predictions of HQSS partners.

- Thus, we have determined the value of the four different counter-terms and we can predict a whole family of resonances related (SU(3) and HQSS partners) to the X(3872), X(3915) and Y(4140).

- We have used two different values of the regulator Λ to account for the regulator-dependence of the predictions. The two values chosen has been 0.5 and 1.0 GeV to guaranty that the involved momenta are reasonably smaller than the heavy quark mass. The relation between the Gaussian regulator and the counter-terms C(Λ) is very similar to RGE of other field theories. Predictions (bound states), though, should be Λ-independent

Predictions of HQSS partners.

- The I = 0 without the hidden-strangeness sector,

J^{PC}	$\mathrm{H\bar{H}}$	$^{2S+1}L_J$	V_C	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp [7]	Threshold [MeV]
0^{++}	$D\bar{D}$	$^1S_0$	C_{0a}	3709^{+9}_{-10}	3715^{+12}_{-15}	–	3734.5^*
1^{++}	$D^*\bar{D}$	$^3S_1$	$V_{coupled}$	Input	Input	3871.6	3871.8/3879.9
1^{+-}	$D^*\bar{D}$	$^3S_1$	$C_{0a} - C_{0b}$	3815^{+16}_{-17}	3821^{+23}_{-26}	–	3875.9^*
0^{++}	$D^*\bar{D}^*$	$^1S_0$	$C_{0a} - 2\,C_{0b}$	Input	Input	3917	4017.3^*
1^{+-}	$D^*\bar{D}^*$	$^3S_1$	$C_{0a} - C_{0b}$	3955^{+16}_{-17}	3958^{+24}_{-27}	3942	4017.3^*
2^{++}	$D^*\bar{D}^*$	$^5S_2$	$V_{coupled}$	$4013^{\dagger\dagger}_{-9}$	$4013^{\dagger\dagger}_{-12}$	–	4014.0/4020.6

Predictions of HQSS partners.

- The I = 0 with hidden-strangeness sector,

J^{PC}	$H\bar{H}$	$^{2S+1}L_J$	V_C	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp [7]	Threshold [MeV]
0^{++}	$D_s\bar{D}_s$	$^1S_0$	$\frac{1}{2}(C_{0a}+C_{1a})$	3924^{+10}_{-13}	3928^{+9}_{-19}	–	3937.0
1^{++}	$D_s^*\bar{D}_s$	$^3S_1$	$\frac{1}{2}(C_{0a}+C_{1a}+C_{0b}+C_{1b})$	–	–	–	
1^{+-}	$D_s^*\bar{D}_s$	$^3S_1$	$\frac{1}{2}(C_{0a}+C_{1a}-C_{0b}-C_{1b})$	4035^{+23}_{-25}	4040^{+33}_{-39}	–	4080.8
0^{++}	$D_s^*\bar{D}_s^*$	$^1S_0$	$\frac{1}{2}(C_{0a}+C_{1a}-2C_{0b}-2C_{1b})$	Input	Input	4140	4224.6
1^{+-}	$D_s^*\bar{D}_s^*$	$^3S_1$	$\frac{1}{2}(C_{0a}+C_{1a}-C_{0b}-C_{1b})$	4177^{+23}_{-25}	4180^{+35}_{-40}	–	4224.6
2^{++}	$D_s^*\bar{D}_s^*$	$^5S_2$	$\frac{1}{2}(C_{0a}+C_{1a}+C_{0b}+C_{1b})$	–	–	–	

New Predictions!

Predictions of HQSS partners.

- The I = ½ sector,

J^{PC}	$\mathrm{H\bar{H}}$	$^{2S+1}L_J$	V_C	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp [7]	Threshold [MeV]
0^+	$D_s^+\bar{D}^-$	$^1S_0$	C_{1a}	$3835.8^{+2.3}_{-7.3}$	$3837.7^{+0.4}_{-8.1}$	–	3838.1
1^+	$D_s\bar{D}^*$,$D_s^*\bar{D}$	$^3S_1$	V_s	3949^{+20}_{-21}	3957^{+22}_{-32}	–	$3977.15^\dagger$, $3979.55^\dagger$
0^+	$D_s^*\bar{D}^*$	$^1S_0$	$C_{1a} - 2\,C_{1b}$	4056^{+31}_{-35}	4061^{+45}_{-54}	–	$4120.9^\dagger$
1^+	$D_s^*\bar{D}^*$	$^3S_1$	$C_{1a} - C_{1b}$	4091^{+19}_{-22}	4097^{+24}_{-33}	–	$4120.9^\dagger$
2^+	$D_s^*\bar{D}^*$	$^5S_2$	$C_{1a} + C_{1b}$	–	–	–	

$$V_s = \begin{pmatrix} C_{1A} & -C_{1B} \\ -C_{1B} & C_{1A} \end{pmatrix}$$

New Predictions!

Predictions of HQSS partners.

- The I = 1 sector

J^{PC}	$H\bar{H}$	$^{2S+1}L_J$	V_C	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp [7]	Threshold [MeV]
0^{++}	$D^+\bar{D}^0$	$^1S_0$	C_{1a}	$3732.5^{+2.0}_{-6.9}$	$3734.3^{+0.2}_{-6.9}$	–	3734.5
1^{++}	$D^*\bar{D}$	$^3S_1$	$V_{coupled}$	–	–	–	
1^{+-}	$D^*\bar{D}$	$^3S_1$	$C_{1a} - C_{1b}$	3848^{+15}_{-17}	3857^{+15}_{-22}	–	3875.9*
0^{++}	$D^*\bar{D}^*$	$^1S_0$	$C_{1a} - 2C_{1b}$	3953^{+24}_{-26}	3960^{+31}_{-37}	–	4017.3*
1^{+-}	$D^*\bar{D}^*$	$^3S_1$	$C_{1a} - C_{1b}$	3988^{+15}_{-17}	3995^{+17}_{-23}	–	4017.3*
2^{++}	$D^*\bar{D}^*$	$^5S_2$	$V_{coupled}$	–	–	–	

New Predictions!

Conclusions

- Light flavour symmetry and HQSS in heavy meson-antimeson systems, along with the determination of four LECs, provides a systematic study of a whole family of hidden charm molecules.
- Uncertainties quoted in tables only account for the approximate nature of HQSS (~15% error) and those induced by the errors of R_X.
- Pion exchanges and coupled channels should be considered. However, according to previous studies, these effects are small and smaller than those expected from HQSS breaking terms.

Part 2

The EFT in a finite volume

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Phys.Rev. D88 (2013) 1, 014510

LQCD

- Since QCD is non-perturbative at low energies, perturbative methods cannot be directly used. LQCD computes path integrals in a finite volume. This formalism allows the analysis of QCD at low energies.

- There exists a connection between LQCD with the infinite volume real world. The *Lüscher method* [C.Mat.Phys., 105,153('86); NP,B354,531('91)] translates energy levels calculated in LQCD to hadron-hadron phase shifts of binding energy. (See A. Ramos talk)

- This method was generalized and simplified in [Döring et al., EPJA47, 139 (2011)].

(Generalized) Lüscher approach

- In a finite box (with periodic bound conditions), momenta are quantized.

$$\vec{q} = \frac{2\pi}{L}\vec{n},\ \vec{n} \in \mathbb{Z}^3$$

- It is possible to rewrite the amplitude in the box by replacing the integrals by sums (see e.g. [Döring, Meiβner, Oset, Rusetsky, EPJ,A47, 139 (2011)]). In our EFT model,

$$T^{-1}(E) = V^{-1}(E) - G(E)$$

$$G = \int_{|\vec{q}|<\Lambda} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} \frac{e^{-2(q^2-k^2)/\Lambda^2}}{E - m_1 - m_2 - \frac{\vec{q}^2}{2\mu} + i0^+}$$

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- It is possible to rewrite the amplitude in the box by replacing the integrals with sums (see e.g. [Döring, Meiβner, Oset, Rusetsky, EPJ,A47, 139 (2011)]). In our EFT model,

$$\widetilde{T}^{-1}(E) = V^{-1}(E) - \widetilde{G}(E)$$

$$\widetilde{G}(E) = \frac{1}{L^3}\sum_{\vec{q}} \frac{e^{-2(\vec{q}^{\,2}-k^2)/\Lambda^2}}{E - m_1 - m_2 - \vec{q}^{\,2}/2\mu}$$

(Generalized) Lüscher approach

- Therefore, the energy levels in a finite volume are given by, $\widetilde{T}^{-1}(E_n) = 0$

- The relation of the finite volume amplitude with its infinite volume counter-part reads then (notice the explicit dependence on the cutoff),

$$T^{-1}(E_n) = V^{-1} - G = \widetilde{G} - G \propto e^{-2i\delta(E_n)}$$

- The Lüscher formula is recovered when $\Lambda \to \infty$:

$$\sqrt{4\pi}\mathcal{Z}_{00}(1, \hat{k}^2) = -\frac{L}{2\pi}\frac{(2\pi)^3}{2\mu}\delta G_L(E), \quad \hat{k}^2 = \frac{k^2 L^2}{(2\pi)^2}$$

$$\delta G_L = \lim_{\Lambda\to\infty} \widetilde{G} - G$$

(Generalized) Lüscher approach

- A very useful way to compute the Lüscher function is then obtained.

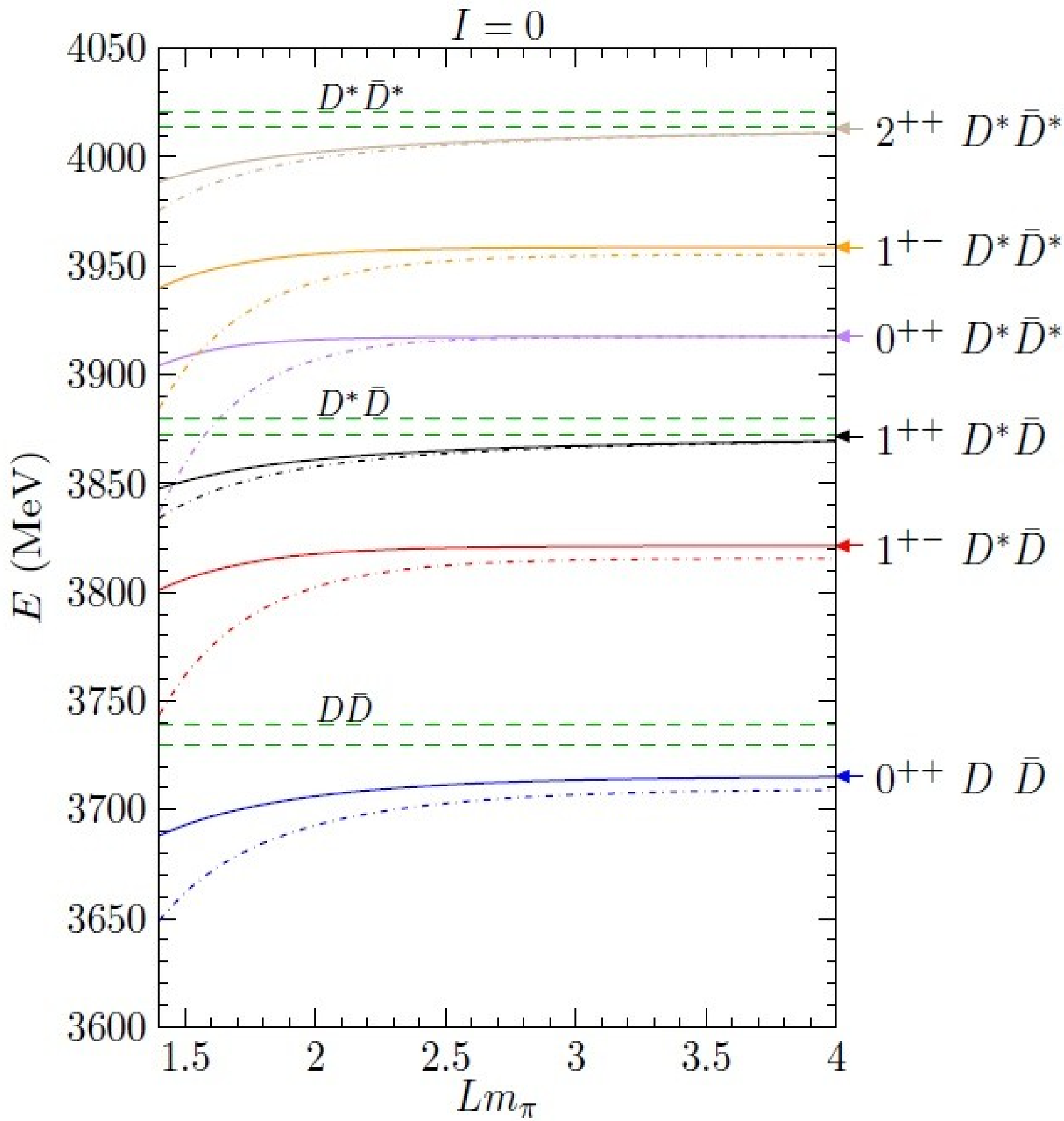
$$\delta G(E;\Lambda) = \tilde{G}(E;\Lambda) - G(E;\Lambda) = \left(\frac{1}{L^3}\sum_{\vec{q}} - \int\frac{d^3\vec{q}}{(2\pi)^3}\right)\frac{e^{-2(\vec{q}^{\,2}-k^2)/\Lambda^2}}{\frac{\vec{k}^{\,2}}{2\mu}-\frac{\vec{q}^{\,2}}{2\mu}+i0^+}$$

$$= \underbrace{\left(\frac{1}{L^3}\sum_{\vec{q}} - \int\frac{d^3\vec{q}}{(2\pi)^3}\right)\frac{e^{-2(\vec{q}^{\,2}-k^2)/\Lambda^2}-1}{\frac{\vec{k}^{\,2}}{2\mu}-\frac{\vec{q}^{\,2}}{2\mu}+i0^+}}_{\delta G_\Lambda(E;\Lambda)} + \underbrace{\left(\frac{1}{L^3}\sum_{\vec{q}} - \int\frac{d^3\vec{q}}{(2\pi)^3}\right)\frac{1}{\frac{\vec{k}^{\,2}}{2\mu}-\frac{\vec{q}^{\,2}}{2\mu}+i0^+}}_{\delta G_L(E)}$$

- For a finite Λ,

$$\delta G(E;\Lambda) = \delta G_L(E) + \frac{24\mu}{(2\pi)^{3/2}}\frac{e^{-\frac{\Lambda^2 L^2}{8}}}{\Lambda L^2}\left[1 + \frac{2(k^2L^2-2)}{L^2\Lambda^2} + \mathcal{O}\left(\Lambda^{-4}\right)\right] + \cdots$$

The EFT in a finite box



- Attractive potentials generate energy levels. Are they bound states?

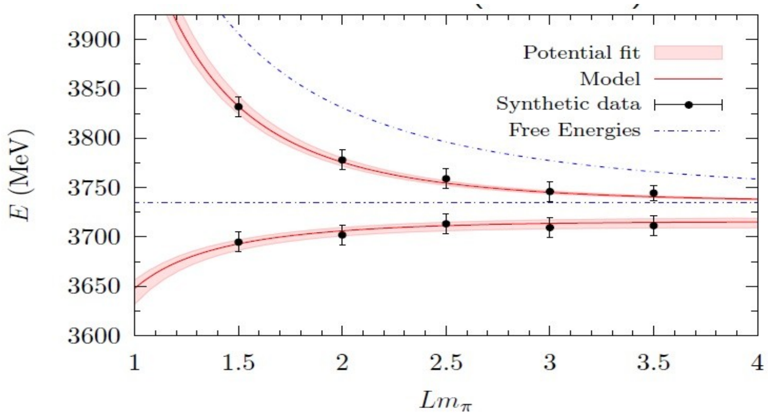
- There are some cases where the answer is clear but others are more uncertain.

- Algorithms to analyze the energy levels are then required.

INVERSE PROBLEM

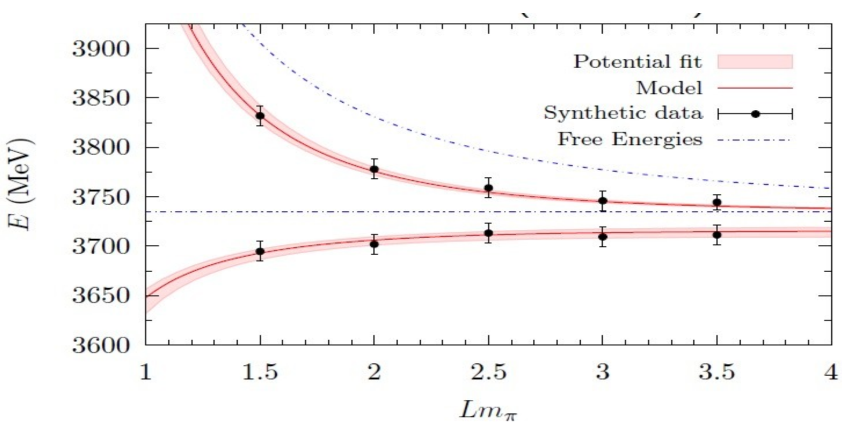
Inverse Analysis

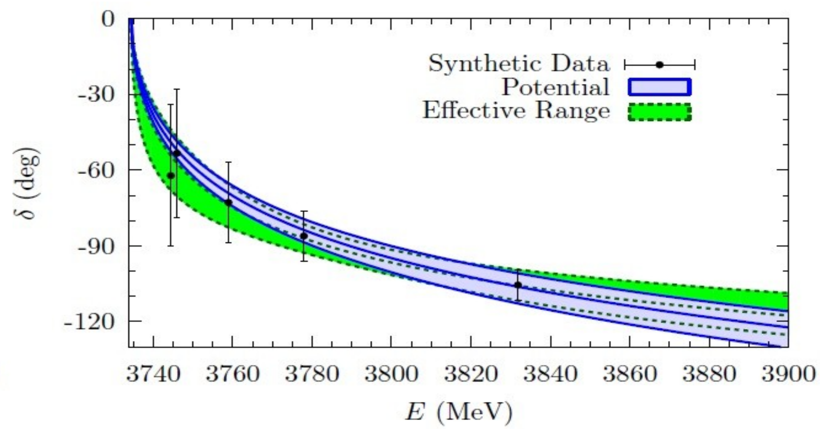
- We generate "synthetic" levels of energy.



- Three algorithms are tested in two cases, the $D\bar{D}$ with $J^{PC} = 0^{++}$ and the $D^*\bar{D}^*$ with $J^{PC} = 2^{++}$:
  - The phase shift analysis (level above threshold).
  - A potential fit (above and below threshold).
  - An effective range analysis (above and below threshold).

I.A.: Phase Shifts $(D\bar{D}, 0^{++})$



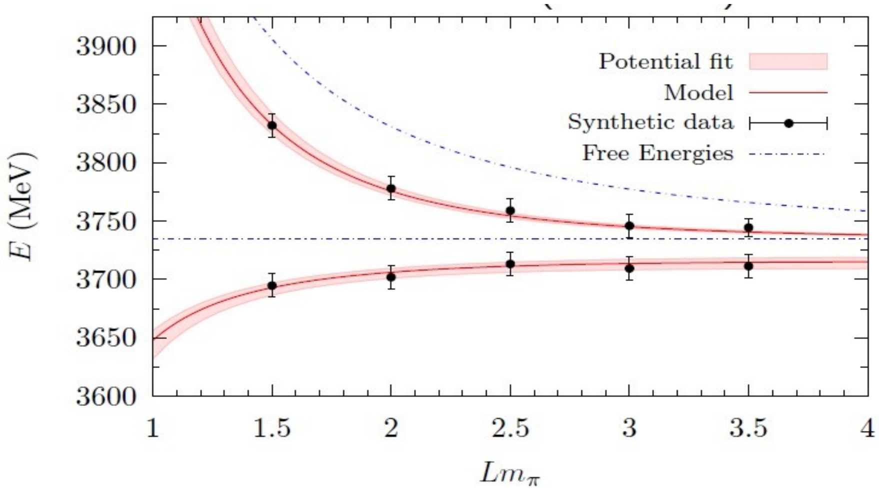


- Lüscher method transforms energy levels (E_n) into phase shifts $\delta(E_n)$

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r k^2 = -\frac{2\pi}{\mu} \lim_{\Lambda \to \infty} \mathrm{Re}\left(\widetilde{G}(E) - G(E)\right) = \frac{4}{\sqrt{4\pi} L} \mathcal{Z}_{00}(1, \hat{k}^2)$$

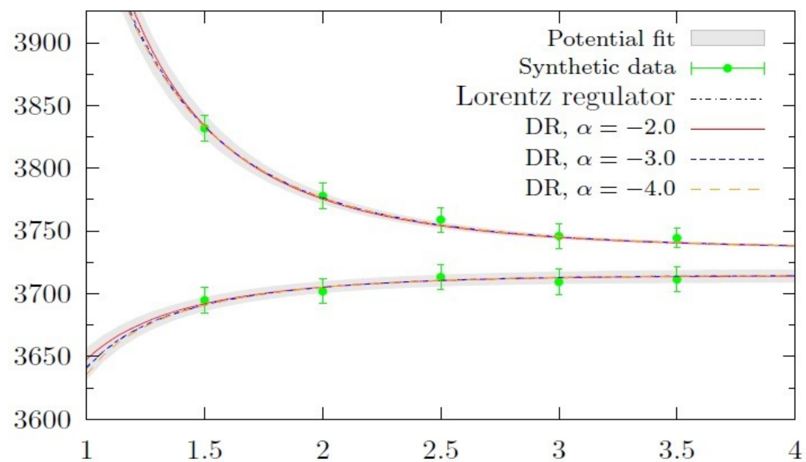
Observable	Analysis	Theory
a (fm)	$1.6^{+1.0}_{-0.5}$	1.38
r (fm)	0.53 ± 0.18	0.52
M (MeV)	3721^{+10}_{-25}	3715

I.A.: Potential fit ($D\bar{D}$, 0^{++})



Observable	Analysis	Theory
C_{0a} (fm^2)	$-1.08^{+0.19}_{-0.29}$	-1.024
Λ (GeV)	0.97 ± 0.13	1.00
M (MeV)	3715^{+3}_{-6}	3715

More accurate predictions!



Similar results with different regulators!

- Lorentzian Regulator:

$$e^{-2(q^2-k^2)/\Lambda^2} \Rightarrow \left(\frac{k^2+\Lambda^2}{q^2+\Lambda^2}\right)^2$$

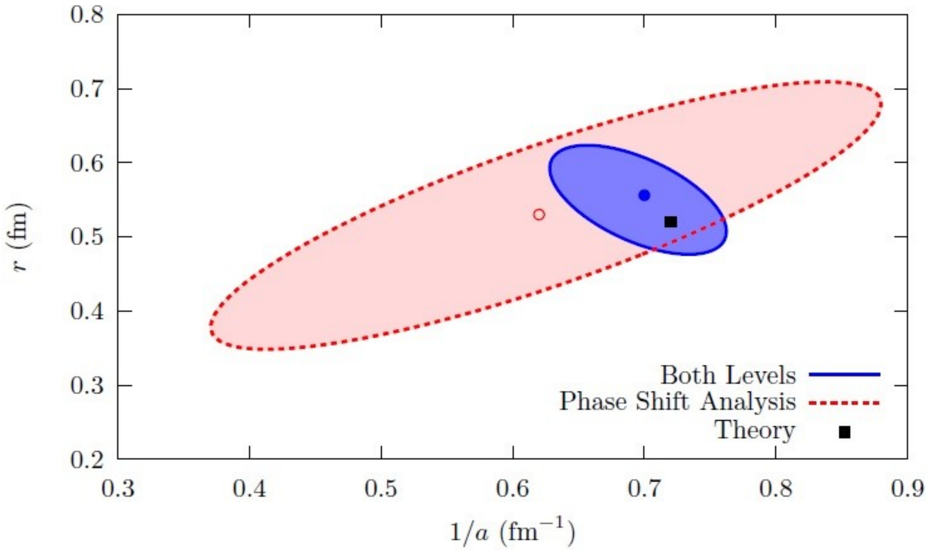
- Relativistic amplitude, once subtracted dispersion relation

$$V = a + bk^2,$$
$$16\pi^2 G = \alpha + \log\frac{m^2}{\mu^2} - \sigma(s)\log\frac{\sigma(s)-1}{\sigma(s)+1}$$

I.A.: Effective Range ($D\bar{D}$, 0^{++})

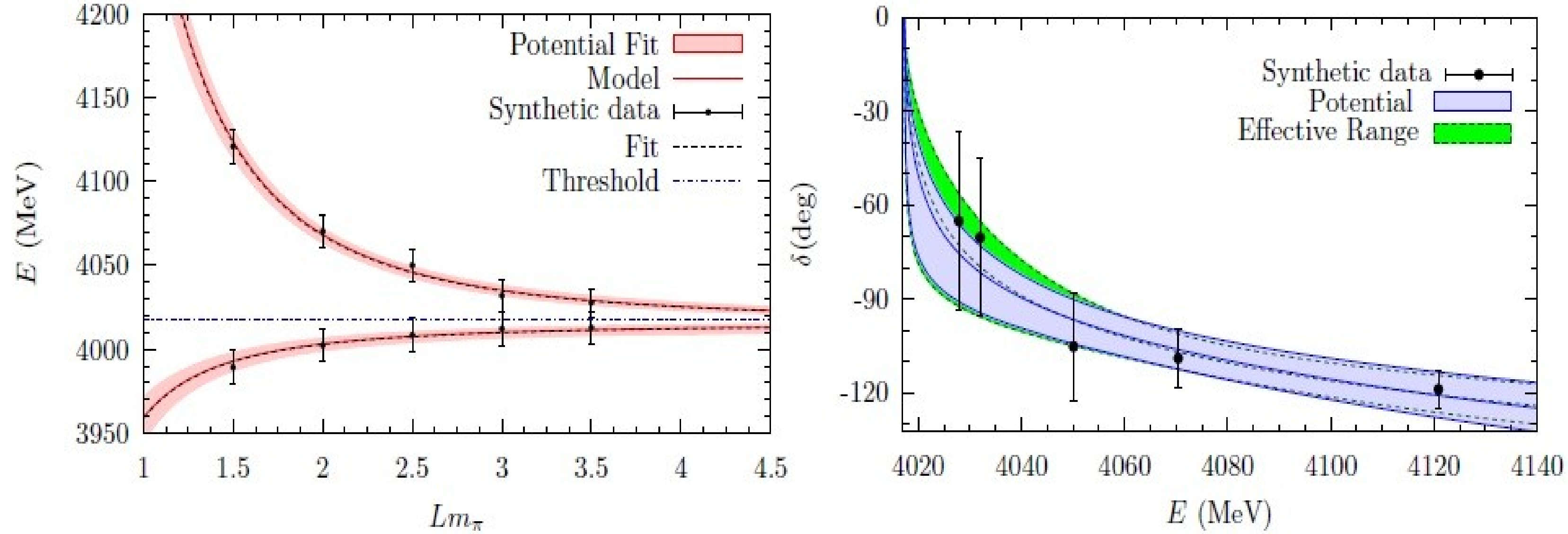
- We parameterize the amplitude as,

$$T^{-1} = -\frac{1}{a} + \frac{1}{2} r k^2$$



Phase shift and Eff. Range				Potential		
Par.	Phases	Eff. Range	Theory	Par.	Analysis	Theory
a (fm)	$1.6^{+1.0}_{-0.5}$	$1.43^{+0.16}_{-0.13}$	1.38	C_{0a} (fm^2)	$-1.08^{+0.19}_{-0.29}$	-1.024
r (fm)	0.53 ± 0.18	0.56 ± 0.07	0.52	Λ (GeV)	0.97 ± 0.13	1.00
M (MeV)	3721^{+10}_{-25}	3716^{+4}_{-5}	3715	M (MeV)	3715^{+3}_{-6}	3715

Inverse Analysis: ($D^*\overline{D}^*$, 2^{++})



Phase shift and Eff. Range				Potential		
Par.	Phases	Eff. Range	Theory	Par.	Analysis	Theory
a (fm)	$2.4^{+2.4}_{-1.2}$	$2.9^{+2.0}_{-0.9}$	3.0	C_0 (fm^2)	$-0.71^{+0.19}_{-0.39}$	-0.73
r (fm)	0.67 ± 0.19	0.64 ± 0.15	0.58	Λ (GeV)	1.20 ± 0.24	1.00
M (MeV)	4013^{+4}_{-18}	$4014.2^{+2.3}_{-4.8}$	4014.6	M (MeV)	$4014.3^{+2.3}_{-5.4}$	4014.6

Conclusions II

- The interaction in a finite volume produces energy levels (above and below threshold). These predictions can be tested in LQCD.
- We have studied the inverse problem: analyze the generated energy levels with different methods. Standard phase-shifts analysis, potential analysis, effective range analysis. Particular emphasis is done in the error analysis.
- ER and potential analyses work best (though ER may be limited to near threshold energies).
- We focus on two I = 0 different channels: $D\bar{D}$ with $J^{PC} = 0^{++}$ and $D^*\bar{D}^*$ with $J^{PC} = 2^{++}$.
- An efficient method to compute the Lüscher function is also presented.