Chiral Corrections to The Masses and Axial Currents of Doubly Heavy Baryons

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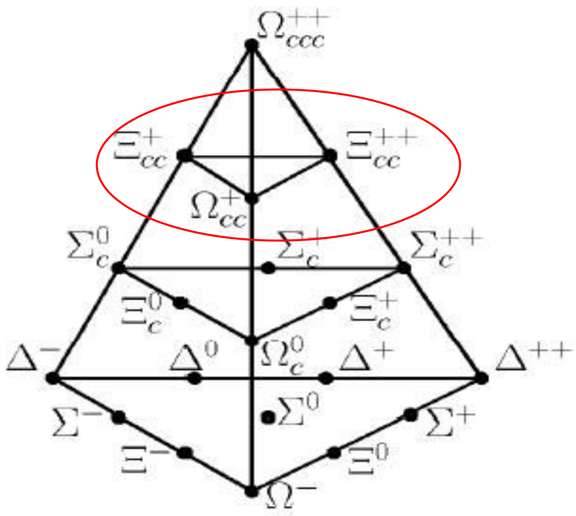
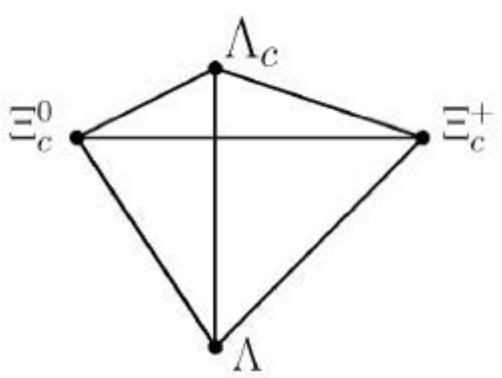
Introduction

Introduction





SELEX Collaboration

$\Lambda_c^+ K^- \pi^+ : \Xi_{cc}^+(3443) \quad \Xi_{cc}^+(3520)$

$pD^+ K^- / \Xi_c^+ \pi^+ \pi^- : \Xi_{cc}^+(3520)$

$\Lambda_c^+ K^- \pi^+ \pi^+ : \Xi_{cc}^{++}(3460) \quad \Xi_{cc}^{++}(3541) \quad \Xi_{cc}^{++}(3780)$

The results have not suported by FOCUS, BaBar, Belle, LHCb.

Introduction

$$\text{Quarkmodel: } M(\Xi_{cc}) = 3.48 \sim 3.74\,\text{GeV}$$
$$M(\Omega_{cc}) = 3.59 \sim 3.86\,\text{GeV}$$

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Phys. Rev. D70, 054022(2004)
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Eur. Phys. J. A32, 183(2007) [Erratum-ibid. A. 36, 119(2008)]
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$$\text{LQCD: } M(\Xi_{cc}) = 3.51 \sim 3.67\,\text{GeV}$$
$$M(\Omega_{cc}) = 3.68 \sim 3.76\,\text{GeV}$$

Phys. Rev. D64, 094509(2001)
PoS LATTICE 2008, 119(2008)
Phys. Rev. D81, 094505(2010)
PoS LATTICE 2012, 139(2012)
Phys. Rev. D86, 114501(2012)

Introduction

$$\Xi_{cc}^{++} = ccu, \Xi_{cc}^{+} = ccd, \Omega_{cc}^{+} = ccs$$

Heavy quark limit:

Heavy quarks can be seen as spectators

Chiral symmetry arises from the light quark

- Study the mass corrections using chiral perturbation theory
- Compare with the theoretical predictions
- Study the axial current corrections

Introduction

➢ QCD → Strong interaction

Asymptotic Freedom

Quark Confinement

Chiral Symmetry

Non-perturbative problem

Chiral perturbation theory

chiral symetry

power counting — the amplitudes are expanded by the low momentum

Introduction

Solving the power counting breaking problem

- ✓ Heavy-Baryon CHPT

  Baryon extremely heavy, as static source

- ✓ Infrared BChPT

  $$H = I + R = \int_0^1 \ldots = \int_0^\infty \ldots - \int_1^\infty \ldots$$

- ✓ Extended-on-mass-shell (EOMS)

  Power counting breaking terms are subtracted
  The LECs are redefined

Formalism

Lagrangians

$\mathrm{CH}: \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ $\quad$ P : Parity $\quad$ C : Charge conjugation

$$\mathcal{L}^{(1)} = \bar{\psi}(i\not{D} - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu)\psi, \tag{13}$$

$$\begin{aligned}\mathcal{L}^{(2)} = {} & c_1\bar{\psi}\langle\chi_\pm\rangle\psi - \left\{\frac{c_2}{8m^2}\bar{\psi}\langle u_\mu u_\nu\rangle\{D^\mu, D^\nu\}\psi + h.c.\right\} \\ & -\left\{\frac{c_3}{8m^2}\bar{\psi}\{u_\mu, u_\nu\}\{D^\mu, D^\nu\}\psi + h.c.\right\} + \frac{c_4}{2}\bar{\psi}\langle u^2\rangle\psi \\ & + \frac{c_5}{2}\bar{\psi}u^2\psi + \left\{\frac{ic_6}{4}\bar{\psi}\sigma^{\mu\nu}[u_\mu, u_\nu]\psi + h.c.\right\} + c_7\bar{\psi}\hat{\chi}_+\psi \\ & + \frac{c_8}{8m}\bar{\psi}\sigma^{\mu\nu}f^+_{\mu\nu}\psi + \frac{c_9}{8m}\bar{\psi}\sigma^{\mu\nu}\langle f^+_{\mu\nu}\rangle\psi\end{aligned} \tag{14}$$

$$\begin{aligned}\mathcal{L}^{(3)} = {} & \bar{\psi}\left\{\frac{h_1}{2}\gamma^\mu\gamma_5\langle\chi_+\rangle u_\mu + \frac{h_2}{2}\gamma^\mu\gamma_5\{\hat{\chi}_+, u_\mu\} + \frac{h_3}{2}\gamma^\mu\gamma_5\langle\hat{\chi}_+ u_\mu\rangle\right. \\ & \left. + \ldots\right\}\psi,\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{L}^{(4)} = {} & e_1\bar{\psi}\langle\chi_+\rangle\langle\chi_+\rangle\psi + e_2\bar{\psi}\hat{\chi}_+\langle\chi_+\rangle\psi + e_3\bar{\psi}\langle\hat{\chi}_+\hat{\chi}_+\rangle\psi \\ & + e_4\bar{\psi}\hat{\chi}_+\hat{\chi}_+\psi + e_5\bar{\psi}\langle\chi_-\rangle\langle\chi_-\rangle\psi + e_6\bar{\psi}\hat{\chi}_-\langle\chi_-\rangle\psi \\ & + e_7\bar{\psi}\langle\hat{\chi}_-\hat{\chi}_-\rangle\psi + e_8\bar{\psi}\hat{\chi}_-\hat{\chi}_-\psi + \ldots.\end{aligned} \tag{16}$$

Power Counting

- the number of independent loop momenta N_L,
- the number of internal pion lines I_M,
- the number of pion vertices N_{2n}^M originating from $\mathcal{L}_{2n}$,
- the total number of pion vertices $N_M = \sum_{n=1}^{\infty} N_{2n}^M$,
- the number of internal nucleon lines I_B,
- the number of baryonic vertices N_n^B originating from $\widehat{\mathcal{L}}_{\pi N}^{(n)}$,
- and the total number of baryonic vertices $N_B = \sum_{n=1}^{\infty} N_n^B$.

$$D = 4N_L - 2I_M - I_B + \sum_{n=1}^{\infty} 2nN_{2n}^M + \sum_{n=1}^{\infty} nN_n^B.$$

The Heavy-Baryon Formulation

$\boxed{\text{The baryon is very heavy:}}$ $p_\mu = mv_\mu + l_\mu \qquad v \cdot l \ll m.$

$\boxed{\text{Define:}}$ $\psi = e^{-imv\cdot x}(H + h)$

$$\begin{aligned}\exp iZ[\eta,\bar{\eta},v,a,s,p] &= \int [d\psi][d\bar{\psi}][du]\exp\{i[S + \int d^4x(\bar{\eta}\psi + \bar{\psi}\eta)]\} \\ &= \int [dH][d\bar{H}][du]\Delta_h \exp i\{S' + \int d^4x(\bar{R}H + \bar{H}R)\}\end{aligned}$$

$$S' = \int d^4x \bar{H}(\mathcal{A} + (\gamma_0\mathcal{B}^\dagger\gamma_0)\mathcal{C}^{-1}\mathcal{B})H$$

The Heavy-Baryon Formulation

$$\mathcal{L}' = \bar{H}(T_{(1)} + T_{(2)} + T_{(3)} + T_{(4)} + \ldots)H,$$

$$\begin{aligned}
T_{(1)} &= i(v \cdot D) + g_A S_v \cdot u, \\
T_{(2)} &= c_1\langle\chi_+\rangle + \frac{c_2}{2}\langle(v \cdot u)^2\rangle + c_3(v \cdot u)^2 + \frac{c_4}{2}\langle u^2\rangle \\
&\quad + \frac{c_5}{2}u^2 + \frac{c_6}{2}[S_v^{\mu}, S_v^{\nu}][u_\mu, u_\nu] + c_7\hat{\chi}_+ \\
&\quad + \frac{2}{m}(S_v \cdot D)^2 - \frac{ig_A}{2m}\{S_v \cdot D, v \cdot u\} \\
&\quad - \frac{g_A^2}{8m}(v \cdot u)^2 + \ldots,
\end{aligned}$$

The Heavy-Baryon Formulation

$$
\begin{aligned}
T_{(3)} \;&=\; h_1 S_v^\mu \langle \chi_+ \rangle u_\mu + h_2 S_v^\mu \{\hat{\chi}_+, u_\mu\} + h_3 S_v^\mu \langle \hat{\chi}_+ u_\mu \rangle \\
&\quad - \frac{i}{m^2} S_v \cdot D v \cdot D S_v \cdot D + \ldots,
\end{aligned}
$$

$$
\begin{aligned}
T_{(4)} \;&=\; e_1 \langle \chi_+ \rangle \langle \chi_+ \rangle + e_2 \hat{\chi}_+ \langle \chi_+ \rangle + e_3 \langle \hat{\chi}_+ \hat{\chi}_+ \rangle + e_4 \hat{\chi}_+ \hat{\chi}_+ \\
&\quad + e_5 \langle \chi_- \rangle \langle \chi_- \rangle + e_6 \hat{\chi}_- \langle \chi_- \rangle + e_7 \langle \hat{\chi}_- \hat{\chi}_- \rangle + e_8 \hat{\chi}_- \hat{\chi}_- \\
&\quad + \frac{c_7}{m^2} S_v \cdot D \hat{\chi}_+ S_v \cdot D + \frac{c_1}{m^2} S_v \cdot D \langle \chi_+ \rangle S_v \cdot D \\
&\quad - \frac{1}{2m^3} S_v \cdot D (v \cdot D)^2 S_v \cdot D + \ldots .
\end{aligned}
$$

Baryon mass corrections

Mass corrections (next-to-next-to leading order)

$$iG = \frac{i}{v\cdot p - m_0 - \Sigma(\eta,\xi)}$$
$$= \frac{iZ_N}{v\cdot p - m - Z_N\bar{\Sigma}(\eta,\xi)}$$

$$\eta = v\cdot p - m$$
$$\xi = (p - mv)^2$$
$$\Sigma'(0,0) = \left.\frac{\partial\Sigma(\eta,\xi)}{\partial\eta}\right|_{(\eta,\xi)=(0,0)}$$

$$\boxed{\begin{aligned} m &= m_0 + \Sigma(0,0) \\ Z_N &= \frac{1}{1-\Sigma'(0,0)} \end{aligned}}$$

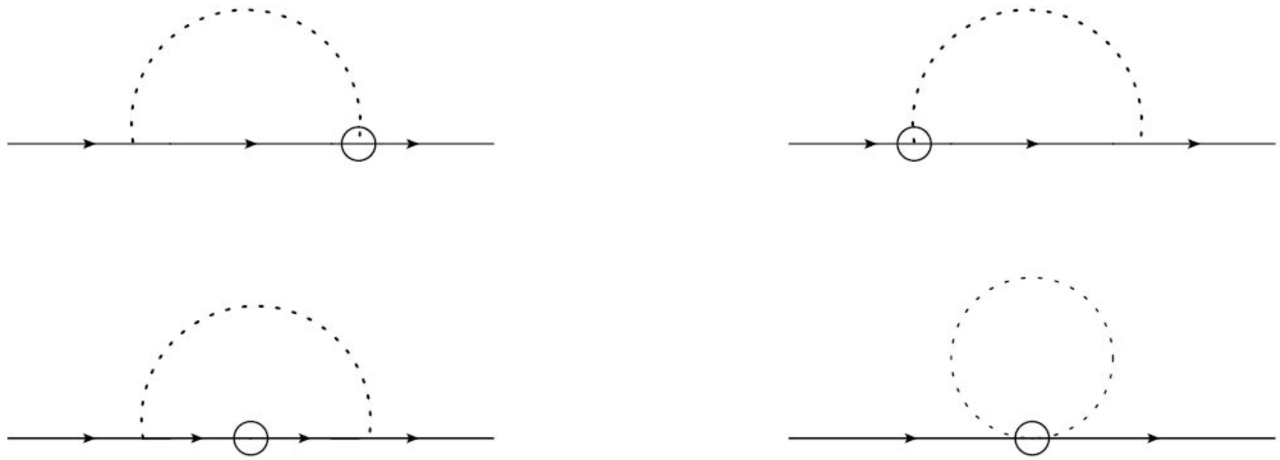
Axial current corrections

Axial currents obtained from $\mathcal{L}'_{(1,3)}$

$$A^{k,\mu} = \frac{\partial \mathcal{L}}{\partial r_k^\mu} - \frac{\partial \mathcal{L}}{\partial l_k^\mu}$$

Matrix elements considering one loop contribution

$$\langle B_d | A^{k,\mu} | B_a \rangle$$



Results

Baryon mass corrections

$$\begin{aligned}\Sigma^{(b)}_{B,P} &= iC^{(b)}_{BP}\int \frac{d^4q}{(2\pi)^4}\left[\frac{g_A}{F_{P0}}S_v\cdot q\right]\frac{i}{v\cdot(k-q)+i\epsilon}\\ &\quad\times \frac{i}{q^2-M_P^2+i\epsilon}\left[-\frac{g_A}{F_{P0}}S_v\cdot q\right]\\ &= -C^{(b)}_{BP}\frac{g_A^2}{(4\pi F_{P0})^2}\left\{\frac{v\cdot k_u}{4}\left([3M_P^2-2(v\cdot k_u)^2]\right.\right.\\ &\quad\left.\times\left[R+\ln\left(\frac{M_P^2}{\mu^2}\right)\right]-2[M_P^2-(v\cdot k_u)^2]\right)\\ &\quad\left.+[M_P^2-(v\cdot k_u)^2]^{3/2}\arccos\left(-\frac{v\cdot k_u}{M_P}\right)\right\},\end{aligned}$$

$$\Sigma^{(b)}_B = -\{2c_1\langle\chi\rangle + 2c_7\hat{\chi}_{ii} - \frac{2}{m}S^{\mu}_{v}S^{\nu}_{v}k_\mu k_\nu\}$$

$$\Sigma^{(f)}_B = -\frac{1}{m^2}(S_v\cdot k)(v\cdot k)(S_v\cdot k)$$

They keep the power counting!

Baryon mass corrections

Heavy diquark symmetry

J. Hu and T. Mehen, PRD.73.054003

$$\mathcal{L} = \mathrm{Tr}[T_a^\dagger (iD_0)_{ba} T_b] - g\mathrm{Tr}[T_a^\dagger T_b \vec{\sigma} \cdot \vec{A}_{ba}] + \cdots$$

$$T_{a,i\beta} = \sqrt{2}\left(\Xi^*_{a,i\beta} + \frac{1}{\sqrt{3}} \Xi_{a,\gamma} \sigma^i_{\gamma\beta}\right)$$

Comparing with our Lagrangians we get

$$\boxed{g_A = g = 0.6}$$

c_7 and m_0 still unknown

The lattice data

C. Alexandrou et al., PRD 86, 114501

m_π	Δ_{m_π}	m_c	Δ_{m_c}	$m_{\Xi_{cc}^{++}}$	$\Delta_{m_{\Xi_{cc}^{++}}}$	$m_{\Xi_{cc}^{+}}$	$\Delta_{m_{\Xi_{cc}^{+}}}$
0.262	0.001	0.479223	0.00855755	3.1678	0.0636153	3.1678	0.0636153
0.262	0.001	0.599028	0.0106969	3.5836	0.0710402	3.58008	0.0709773
0.2925	0.0018	0.479223	0.00684604	3.1967	0.0597619	3.20515	0.0570637
0.2925	0.0018	0.563791	0.00805416	3.50678	0.0585538	3.50678	0.0641917
0.2925	0.0018	0.591981	0.00845687	3.6139	0.0600841	3.59699	0.0626614
0.2925	0.0018	0.64836	0.00926229	3.7915	0.0626211	3.78304	0.0681382
0.2925	0.0018	0.732929	0.0104704	4.07057	0.0666079	4.06212	0.072125
0.3032	0.0016	0.532118	0.00597885	3.39668	0.0492508	3.38782	0.046934
0.3032	0.0016	0.554289	0.00622797	3.5275	0.0507206	3.49202	0.050322
0.3032	0.0016	0.598632	0.00672621	3.63614	0.0497241	3.60953	0.047208
0.3032	0.0016	0.665147	0.00747356	3.84899	0.05655	3.8379	0.0519911
0.377	0.0009	0.532118	0.00597885	3.44103	0.0453147	3.3989	0.0470585
0.377	0.0009	0.598632	0.00672621	3.65388	0.0521406	3.62284	0.0495746
0.377	0.0009	0.665147	0.00747356	3.87337	0.0523897	3.81794	0.0562012

Supposing

$m_0 = \tilde{m}_0 + 2m_c + \alpha/m_c + O(1/m_c^2)$ respecting the heavy quark expansion

and fitting the lattice data, we get

c_7	$\tilde{m}_0$	α	χ^2_{dof}	$m_{\Xi_{cc}}$	$m_{\Omega_{cc}}$
0.6	3.314	−0.518	1.0	$3.710^{+0.096}_{-0.100}$	$3.045^{+0.096}_{-0.100}$
0.3	3.363	−0.505	0.8	$3.690^{+0.095}_{-0.099}$	$3.297^{+0.095}_{-0.099}$
0.0	3.450	−0.510	0.7	$3.677^{+0.095}_{-0.099}$	$3.557^{+0.095}_{-0.099}$
−0.1	3.472	−0.509	0.6	$3.672^{+0.095}_{-0.099}$	$3.642^{+0.095}_{-0.099}$
−0.2	3.460	−0.488	0.6	$3.665^{+0.093}_{-0.097}$	$3.726^{+0.093}_{-0.097}$
−0.3	3.517	−0.506	0.5	$3.661^{+0.095}_{-0.099}$	$3.813^{+0.095}_{-0.099}$
−0.4	3.541	−0.506	0.5	$3.655^{+0.095}_{-0.099}$	$3.898^{+0.095}_{-0.099}$
−0.5	3.562	−0.503	0.4	$3.650^{+0.095}_{-0.098}$	$3.983^{+0.095}_{-0.098}$
−1.0	3.552	−0.427	0.4	$3.618^{+0.089}_{-0.092}$	$4.405^{+0.089}_{-0.092}$
−2.0	3.900	−0.484	0.1	$3.567^{+0.093}_{-0.097}$	$5.261^{+0.093}_{-0.097}$
−4.0	4.351	−0.458	0.4	$3.457^{+0.091}_{-0.095}$	$6.966^{+0.091}_{-0.095}$
−6.0	4.801	−0.431	1.6	$3.347^{+0.089}_{-0.092}$	$8.671^{+0.089}_{-0.092}$

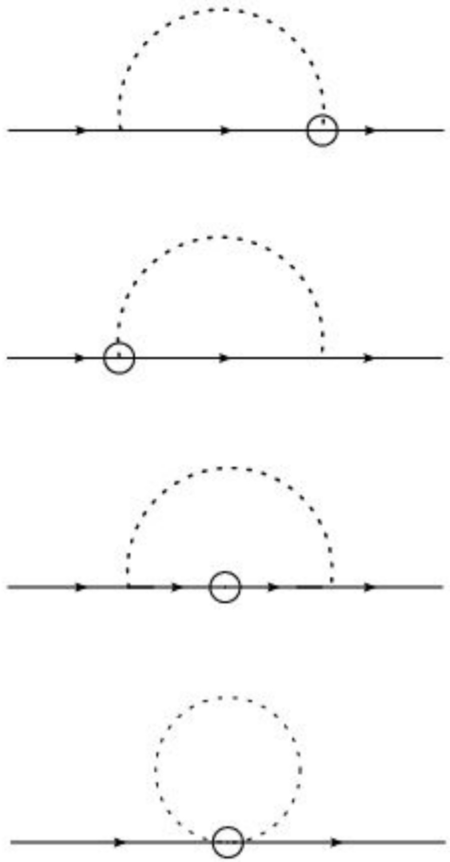
Quark model :

$$M(\Omega_{cc}) = 3.59 \sim 3.86\,\mathrm{GeV}$$

LQCD :

$$M(\Omega_{cc}) = 3.68 \sim 3.76\,\mathrm{GeV}$$

Axial current corrections



$$
\begin{aligned}
\langle B_d | A^{k,\mu} | B_a \rangle = & \Bigg\{ g_A 2T^k_{ad} \Bigg[1 - \sum_j \frac{g_A^2}{(4\pi F_0)^2} \Bigg(\frac{3M^2}{4} (R + \ln \frac{M^2}{\mu^2}) \\
& + \frac{1}{2} M^2 \Bigg) (C_{\phi_{aj}\phi_{ja}} + C_{\phi_{dj}\phi_{jd}}) \frac{1}{2} + h_1 \langle \chi_+ \rangle 2T^k_{ad} \\
& + h_2 \sum_i [2\chi_{ai} T^k_{id} + 2T^k_{ai} \chi_{id}] + h_3 2 \langle \chi_+ T^k \rangle \\
& + \sum_{bc} \Sigma^{current}_{bc} \Bigg\} \bar{u}_a S^\mu_v u_d .
\end{aligned}
$$

$$\equiv g^A_{ad} \bar{u}_d S^\mu_v u_a$$

Axial current corrections

$$\Sigma^{current} = \Sigma^{current}_{(1)} + \Sigma^{current}_{(2)} + \Sigma^{current}_{(3)} + \Sigma^{current}_{(4)}$$

$$\begin{aligned}
\Sigma^{current}_{(1)} &= \Sigma^{current}_{(2)} = 0, \\
\Sigma^{current}_{(3)} &= -\frac{ig_A}{4F_0^2}[-C_{\phi_{ab}\phi_{bc}T^k_{cd}} + 2C_{\phi_{ab}T^k_{bc}\phi_{cd}} - C_{T^k_{ab}\phi_{bc}\phi_{cd}}] \\
&\quad \times \frac{M^2}{(4\pi)^2}\left[-R + \ln\frac{\mu^2}{M^2}\right]\bar{u}_a S^\mu_v u_d
\end{aligned}$$

$$\begin{aligned}
\Sigma^{current}_{(4)} &= -\frac{g_A^3}{6F_0^2}C_{\phi_{ba}}C_{\phi_{ab}}\delta_{cd}\delta_{ab}T^k_{bc}\Big\{-2(v\cdot k)[-4\frac{1}{32\pi^2}(R-\frac{2}{3}) \\
&\quad \times(v\cdot k)\frac{v\cdot k}{8\pi^2}(1-2\ln\frac{M}{\mu}) - \frac{1}{4\pi^2}\sqrt{M^2-(v\cdot k)^2} \\
&\quad \times \arccos\frac{-v\cdot k}{M}] + [M^2-(v\cdot k)^2][-4\frac{1}{32\pi^2}(R-\frac{2}{3}) \\
&\quad + \frac{1}{8\pi^2}(1-2\ln\frac{M}{\mu}) + \frac{v\cdot k}{2\pi^2}[M^2-(v\cdot k)^2]^{-1/2} \\
&\quad \times \arccos\frac{-v\cdot k}{M} - \frac{1}{4\pi^2}] - 2M^2(\frac{1}{32\pi^2}(R-\frac{2}{3}) \\
&\quad + \frac{1}{16\pi^2}\ln\frac{M}{\mu})\Big\}
\end{aligned}$$

$$\boxed{h_2 = \frac{1}{\lambda^2}, h_3 = \frac{1}{\lambda^2}}$$

Tree level: $g^{1+i2}_{\Xi^{++}_{cc}\Xi^{+}_{cc}} = 4.8, g^{4+i5}_{\Xi^{++}_{cc}\Omega^{+}_{cc}} = 4.8$

One loop level: $g^{1+i2}_{\Xi^{++}_{cc}\Xi^{+}_{cc}} = 4.0, g^{4+i5}_{\Xi^{++}_{cc}\Omega^{+}_{cc}} = 7.39$

Summary

- **The mass corrections of doubly heavy baryons are calculated in the frame of heavy-baryon CHPT.**

- **We also use heavy diquark symmetry to determine the coupling; Then we fit the lattice data of Ξ_{cc} mass. Supposing c_7~-0.2GeV^{-1}, we could obtain the Ω_{cc} mass which is agree with the quark model and LQCD predictions.**

- **The corrections to the axial currents are studied; we hope that it could provide some information to further experiments.**

¡Muchas gracias!