Reanalysis of Lattice QCD spectra leading to the $D^*_{s0}(2317)$ and the $D_{s1}(2560)$

A. Martínez Torres, E. Oset, S. Prelovsek, A. Ramos

JHEP 1505 (2015) 153
(arXiv:1412.1706 [hep-lat])

Angels Ramos
(University of Barcelona)

Universitat de Barcelona

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Outline

- Experimental status of the $D^{*}_{s0}(2317)$ and the $D_{s1}(2560)$ mesons
- $c\bar{s}$ states or meson-meson molecules?
- The answer may come from Lattice QCD calculations
- Reanalysis of the works:

  C.B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, R.M. Woloshyn.
  Phys. Rev. D90 (2014) 3, 034510.; Phys. Rev. Lett. 111 (2013) 22, 222001.
- Conclusions

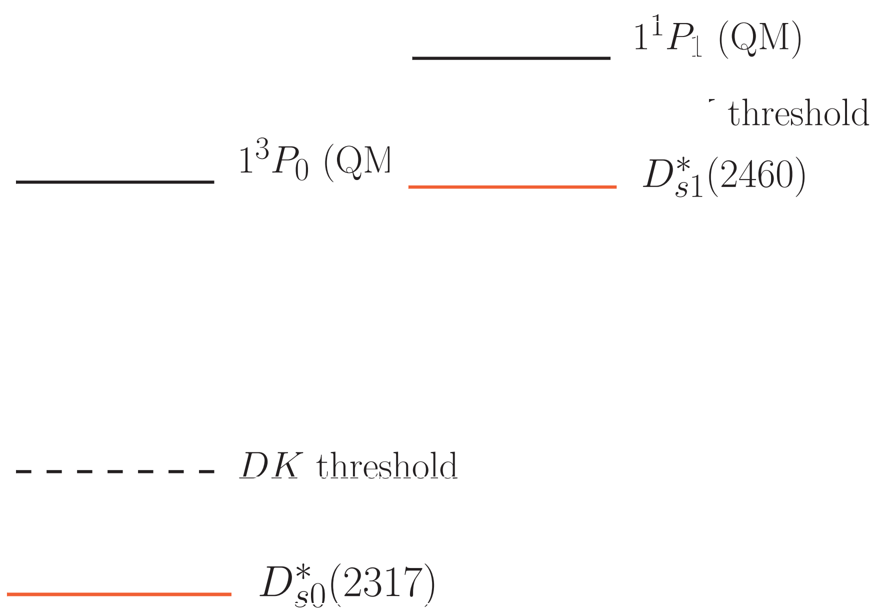
The $D^*_{s0}(2317)$ and $D_{s1}(2560)$ states

Discovered in 2003, the $D^*_{s0}(2317)$ and $D^*_{s1}(2560)$ states provide a challenge to conventional quark models

$D^*_{s0}(2317)$ $I(J^P)=0(0+)$ quark content: $c\bar{s}$

$D_{s1}(2560)$ $I(J^P)=0(1+)$ quark content: $c\bar{s}$

QM: S. Godfrey, N. Isgur, Phys.Rev.D 32, 189 (1985)



BaBar (B. Aubert et al.) Phys.Rev.Lett. 90, 242001 (2003)
CLEO (D. Besson et al.) Phys.Rev.D 68, 032002 (2003); 75, 119908(E) (2007)
Belle (P. Krokovny et al.) Phys.Rev.Lett. 91, 262002 (2003)

$c\bar{s}$ states or meson-meson molecules?

A lot of theoretical evidence has been accumulated in the literature to interpret the $D^*_{s0}(2317)$ and the $D_{s1}(2560)$ as being meson-meson bound states of DK and D^*K, respectively

T. Barnes, F.E. Close, H.J. Lipkin, Phys.Rev.D 68, 054006 (2003)
E. van Beveren, G. Rupp, Phys.Rev.Lett. 91, 012003 (2003)
D. Gamermann, E. Oset, D. Strottman, M.J. Vicente-Vacas, Phys.Rev.D76, 074016 (2007)
D. Gamermann, E. Oset, Eur.Phys.J.A 40, 119 (2007)
E.E. Kolomeitsev and M.F.M. Lutz, Phys.Lett.B 582, 39 (2004)
F.K. Guo, P.N. Shen, H.C. Chiang, R.G. Ping, B.S. Zou, Phys.Lett.B 641, 278 (2006)
M. Altenbuchinger, L.S. Geng, W. Weise, Phys.Rev.D 89, 014026 (2014)
F.K. Guo, C. Hanhart, U.G. Meissner, Phys.Rev.Lett. 102, 242004 (2009); Eur.Phys.J.A 40, 171 (2009)
P. Wang and X.G. Wang, Phys.Rev.D 86, 014030 (2012)

Most of the models rely on extensions of the SU(3) chiral lagrangian (at lowest order or next-to-leading order) to incorporate the charm sector

Unitarization naturally gives rise to the production of the $D^*_{s0}(2317)$ as a DK bound state and the $D_{s1}(2560)$ as a D^*K bound state.

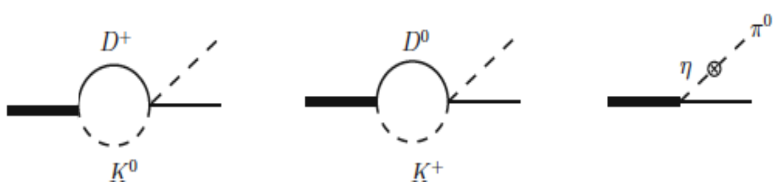
These models also give the scattering length for DK and D^*K scattering

Can one obtain experimental confirmation of the molecular nature of the $D^*_{s0}(2317)$ and the $D_{s1}(2560)$?

- From DK scattering (unlikely in present facilities)

- From strong or radiative decays

see e.g. D. Gamermann, L.R. Dai and E.Oset, Phys.Rev.C 76, 055205 (2007)
M.F.M. Lutz, M. Soyeur, Nucl.Phys.A 813, 14 (2008)
A. Faessler, T. Gutsche, V.E. Lyubovitskij, Y.L. Ma, Phys.Rev.D 77, 114013 (2008)
M. Cleven, H. Griesshammer, F.K. Guo, C. Hanhart, U.G. Meissner, Eur.Phys.J.A 50, 149 (2014)



Strong decays (isospin violating processes)
Γ_{strong} ~ 100 keV

Radiative e.m. decays
Γ_γ ~ 1-10 keV

- From Lattice data ← our approach

We have reanalyzed recent lattice data to learn on the nature of the $D^*_{s0}(2317)$ and the $D_{s1}(2560)$

Lattice QCD is a provides a non-perturbative framework to calculate hadron properties, employing the QCD lagrangian and discretizing space-time in a box.

It has been applied to D_s spectroscopy:

Lattice simulations provide discrete energy levels (which are extracted from correlations, $C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle$, between sets of interpolating operators) .

- Using only quark-antiquark interpolating operators:
  ➔ Energies of D^*_{s0} and D_{s1} too large, above DK, D^*K thresholds

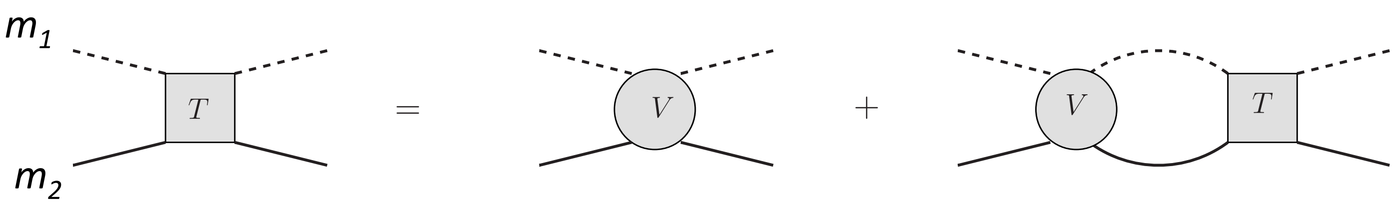
- Recently, operators for DK scattering states were included in the operator basis:
  ➔ masses of D^*_{s0} and D_{s1} compatible with experiment

D. Mohler, C.B. Lang, L. Leskovec, S. Prelovsek and R.M. Woloshyn, Phys.Rev.Lett. 111, 222001 (2013)
C.B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, R.M. Woloshyn, Phys.Rev.D 90, 034510 (2014)

Unitarized Chiral Perturbation Theory in a finite volume

see e.g. M. Döring, U.G. Meissner, E. Oset, A. Rusetsky, Eur.Phys.J. A 47, 139 (2011)

Scattering amplitude (T-matrix) in infinite volume:



(Bethe-Salpeter equation in its factorized form)

$$T = V + V\,G\,T$$ if two channels (e.g , $\pi\pi, K\bar{K}$), then V is a 2x2 matrix

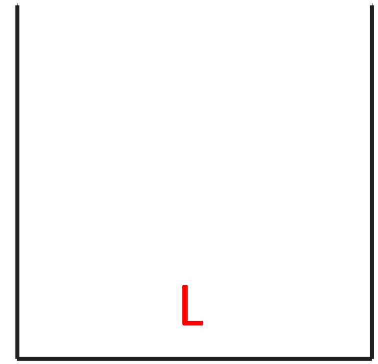
$$T = [V^{-1} - G]^{-1}$$

and

$$G_j = \int^{|\boldsymbol{q}|<q_{\max}} \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} \frac{1}{2\omega_1(\boldsymbol{q})\,\omega_2(\boldsymbol{q})} \; \frac{\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q})}{E^2 - (\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q}))^2 + i\epsilon}$$

$$\omega_{1,2}(\boldsymbol{q}) = \sqrt{m_{1,2}^2 + \boldsymbol{q}^2}$$

Scattering amplitude (T-matrix) in a box:



$$q = \frac{2\pi}{L} n, \quad n \in \mathbb{Z}^3$$

Momentum eigenstates in the box, periodical boundary conditions

$$T = [V^{-1} - G]^{-1} \longrightarrow \tilde{T} = [V^{-1} - \tilde{G}]^{-1}$$

$$L^3 \int d^3q \rightarrow \sum_{\boldsymbol{q}}$$

$$T = V + V\,G\,T$$

$$\tilde{G}_j = \frac{1}{L^3} \sum_{\boldsymbol{q}}^{|\boldsymbol{q}|<q_{\max}} \frac{1}{2\omega_1(\boldsymbol{q})\,\omega_2(\boldsymbol{q})} \frac{\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q})}{E^2 - (\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q}))^2}$$

Eigenstates in the box:

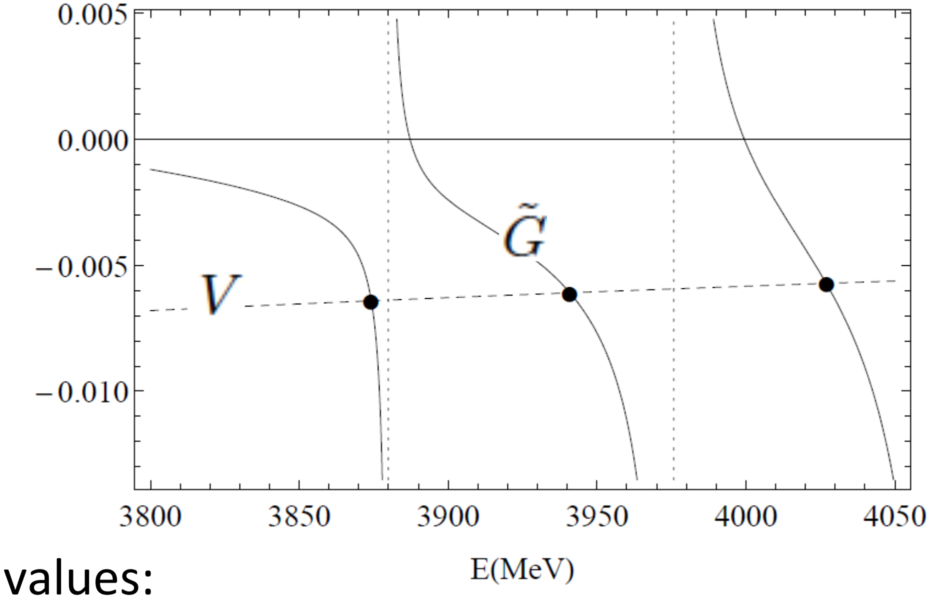
one channel (to simplify)

They are found from:

$$V^{-1}(E) - \tilde{G}(E) = 0$$

solutions:

$E_1, E_2, E_3, \ldots \rightarrow \{E_i\}$



Therefore, for the discretized eigenvalues:

$$T(E_i) = [V^{-1}(E_i) - G(E_i)]^{-1} = \left[\tilde{G}(E_i) - G(E_i)\right]^{-1}$$

recalling $T(E) = \dfrac{-8\pi E}{p\cot\delta(p) - ip}$

Equivalent to Lüscher!!

$$\rightarrow \quad p_i \cot\delta(p_i) = -8\pi E_i \left\{\tilde{G}(E_i) - \left(G(E_i) + \frac{ip_i}{8\pi E_i}\right)\right\}$$

[above threshold]
phase-shifts for the box eigenvalues

using the effective-range expansion $p\cot\delta(p) = -\dfrac{1}{a} + \dfrac{1}{2} r p^2 + \mathcal{O}(p^4)$
and continuing analytically below threshold: $p \rightarrow i\tilde{p}$

$$\rightarrow \quad -\frac{1}{a} - \frac{1}{2} r\tilde{p}_i^2 + \cdots = -8\pi E_i \left\{\tilde{G}(E_i) - \left(G(E_i) - \frac{\tilde{p}_i}{8\pi E_i}\right)\right\}$$

[below threshold]

These are the type of expressions used in the analysis of

D. Mohler, C.B. Lang, L. Leskovec, S. Prelovsek and R.M. Woloshyn, Phys.Rev.Lett. 111, 222001 (2013)
C.B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, R.M. Woloshyn, Phys.Rev.D 90, 034510 (2014)

e.g. $D^*_{s0}(2317)$ case

A bound state below threshold emerges when using DK interpolators

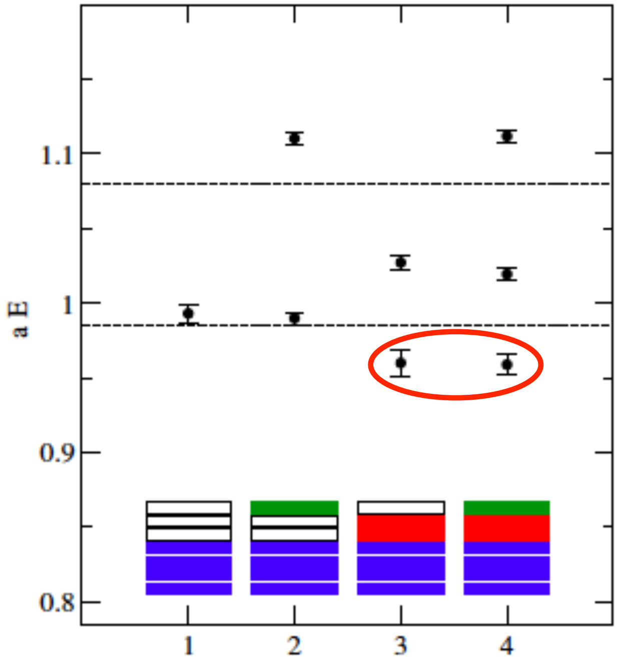


FIG. 3 (color online). A_1^+: Effective energies as obtained for various subsets of operators for ensemble (2). The horizontal broken lines indicate the positions of $D(0)K(0)$ and $D(1)K(-1)$ in the noninteracting case. The boxes indicate the operators (listed in the Appendix) considered in each case [blue: $\bar{q}q$, red: $D(0)K(0)$, green: $D(1)K(-1)$].

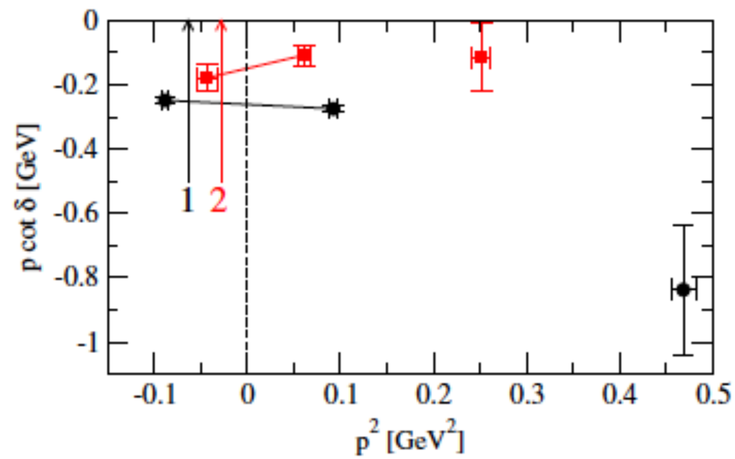


FIG. 4 (color online). Effective range fits for A_1^+, cf. Table VIII. Ensemble (1) black dots, ensemble (2) red squares; the vertical arrows give the positions of the bound state for ensembles (1) and (2), see Table VIII, the dashed line indicates the threshold.

From Lüscher formula and using the two lowest energies:

$$a = -1.33(20)\ \text{fm}$$
$$r = 0.27(17)\ \text{fm}$$

Our reanalysis exploits the advantages of using an auxiliary potential

1. Given the lattice levels (E1, E2, E3), one can find a potential that fulfills

$$V^{-1}(E_i) - \tilde{G}(E_i) = 0 \quad \text{for } i = 1, 2, 3$$

Use the three levels (instead of two)

(the shape of the potential can be inspired by chiral unitary theories)

2. One can then determine the bound state from the infinite volume T-matrix

$$T = V + V\,G\,T$$

(if wanted, one could also obtain phase-shift for ALL energy levels)

3. One can also obtain the amount of meson-meson components (compositeness) of the bound state

Compositeness of states

1. Energy independent potential

<u>One meson-meson channel:</u> $$T = \frac{V}{1-VG} = \frac{1}{V^{-1}-G}$$

Around a pole: $$T \sim \frac{g^2}{s-s_0}$$

Hence: $$g^2 = \lim_{s\to s_0}(s-s_0)T = \lim_{s\to s_0}(s-s_0)\,\frac{1}{V^{-1}-G} = \frac{1}{-\frac{\partial G}{\partial s}}$$

➔ $$-g^2\frac{\partial G}{\partial s} = 1$$

Sum-rule indicating that the bound state is entirely made of the meson-meson component considered
(extends Weinberg's compositeness condition to larger binding energies)
S. Weinberg, Phys.Rev.137, B672 (1965)

<u>Generalization to various meson-meson channels:</u> $$\sum_i\left(-g_i^2\frac{\partial G_i}{\partial s}\right) = 1$$

From $$\int d^3p \mid \langle p \mid \Psi_i\rangle \mid^2 = g_i^2\frac{\partial G_i}{\partial E}$$

[D. Gamermann, J. Nieves, E. Oset, E. Ruiz-Arriola, Phys.Rev.D81, 014029 (2010)]

it is clear that $P_i = -g_i^2\frac{\partial G_i}{\partial s}$ represents the probability of having channel *i* in the wave-function of the bound state

2. Energy dependent potential

The sum-rule for an energy dependent potential is generalized as:

[T.Hyodo, Int. J. Mod. Phys. A28, 1330045 (2013)]

$$-\sum_i g_i^2 \frac{\partial G_i}{\partial s} - \sum_{i,j} g_i g_j G_i \frac{\partial V_{i,j}}{\partial s} G_j = 1$$

strength of meson-meson components

strength going to other components (either q-qbar or omitted meson-meson channels)

Nicely illustrated by eliminating one channel (in a two-channel problem) in favor of an effective energy dependent potential. [F. Aceti, L.R. Dai, L.S. Geng, E. Oset, Y. Zhang, Eur.Phys.J.A 50, 57 (2014)]

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{12} & 0 \end{pmatrix}, \quad \rightarrow \quad T_{11} = \frac{V_{11} + V_{12}^2 G_2}{1 - (V_{11} + V_{12}^2 G_2) G_1} = \frac{V_{\text{eff}}}{1 - V_{\text{eff}} G_1}$$

one-channel problem with an energy-dependent $V_{\text{eff}} = V_{11} + V_{12}^2 G_2$

$$g_1^2 = \lim_{s \to s_0} (s - s_0) T_{11} = \lim_{s \to s_0} (s - s_0) \frac{1}{V_{\text{eff}}^{-1} - G_1} = \frac{1}{\frac{\partial V_{\text{eff}}^{-1}}{\partial s} - \frac{\partial G_1}{\partial s}}$$

Probability of channel 1

$$-g_1^2 \frac{\partial G_1}{\partial s} + g_1^2 \frac{\partial V_{\text{eff}}^{-1}}{\partial s} = 1$$

Probability of channel eliminated

Reanalisis of the Lattice Data using the auxiliary potential

A. Martínez Torres, E. Oset, S. Prelovsek, A. Ramos, JHEP 1505, 153 (2015) (arXiv:1412.1706 [hep-lat])

Lattice Data:

C.B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, R.M. Woloshyn.
Phys. Rev. D90 (2014) 3, 034510.; Phys. Rev. Lett. 111 (2013) 22, 222001.

	Ensemble (2)
$N_L^3 \times N_T$	$32^3 \times 64$
N_f	$2+1$
a[fm]	0.0907(13)
L[fm]	2.90(4)
Lm_π	2.29(10)
#configs	196

$$m_\pi = 156 \text{ MeV}$$
$$m_K = 504(1) \text{ MeV}$$

$$E_{D(D^*)}(\vec{p}) = M_1 + \frac{\vec{p}^2}{2M_2} - \frac{(\vec{p}^2)^2}{8M_4^3} \qquad m_{D(D^*)} = M_1$$

	D meson	D^* meson
M_1 (MeV)	1639	1788
M_2 (MeV)	1801	1969
M_4 (MeV)	1936	2132

Using DK and D^*K interpolators, 3 energy levels are found: E_1, E_2, E_3

	KD channel	KD^* channel
E_1 (MeV)	2086 (34)	2232 (33)
E_2 (MeV)	2218 (33)	2349 (34)
E_3 (MeV)	2419 (36)	2528 (53)

1. One channel analysis with a potential of the type:

$$V = \alpha + \beta(s - s_{\rm th}) \qquad s_{\rm th} = (M_{D^{(*)}} + M_K)^2$$

➔ a best fit is conducted to the lattice energies to determine the parameters α,β of the potential

$V^{-1}(E_i) - \tilde{G}(E_i) = 0 \quad \text{for } i = 1, 2, 3,$ i.e. $\tilde{T} = \dfrac{1}{V^{-1} - \tilde{G}}$ has poles at the lattice levels

$$\tilde{G} = G + \lim_{q_{\rm max}\to\infty} \left[\frac{1}{L^3} \sum_{q_i}^{q_{\rm max}} I(\vec{q}_i) - \int_{q<q_{\rm max}} \frac{d^3q}{(2\pi)^3} I(\vec{q}) \right]$$

$$I(\vec{q}) = \frac{\omega_1(\vec{q}) + \omega_2(\vec{q})}{2\omega_1(\vec{q})\omega_2(\vec{q})\,[P^2 - (\omega_1(\vec{q}) + \omega_2(\vec{q}))^2 + i\epsilon]}$$

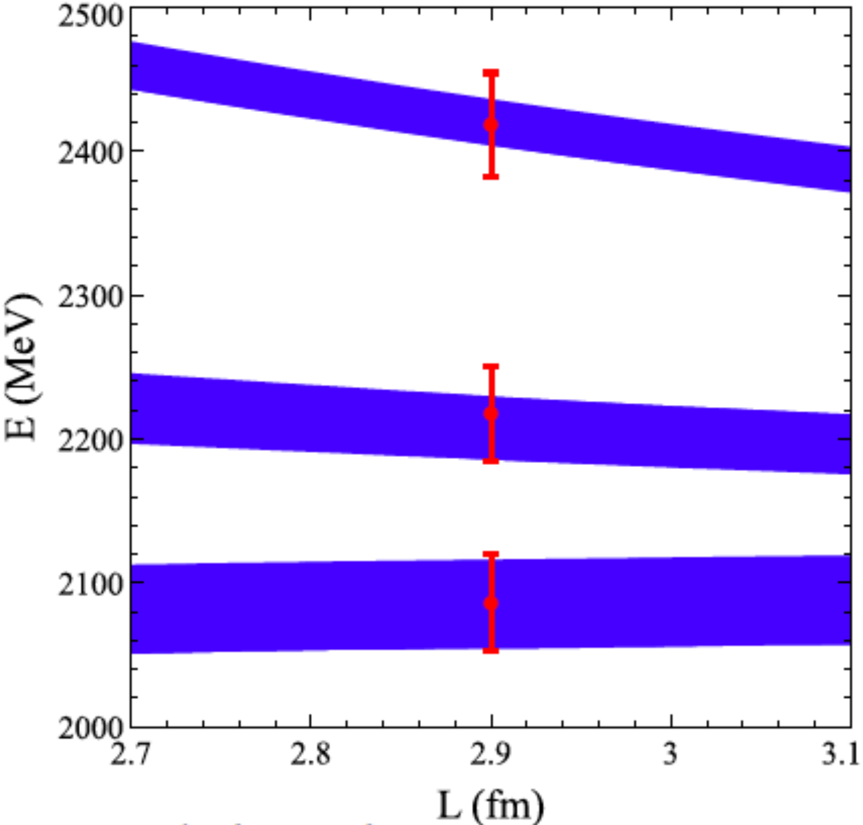
➔ Once V is determined, we solve the T-matrix in the continuum to obtain the bound state, scattering length, effective range...

$$T = [V^{-1} - G]^{-1}$$

➔ Coupling constants are evaluated and the probability of the bound-state to be in the meson-meson channel considered is determined

$$P_i = -g_i^2 \frac{\partial G_i}{\partial s}$$

DK



$D_{s0}^*(2317)$

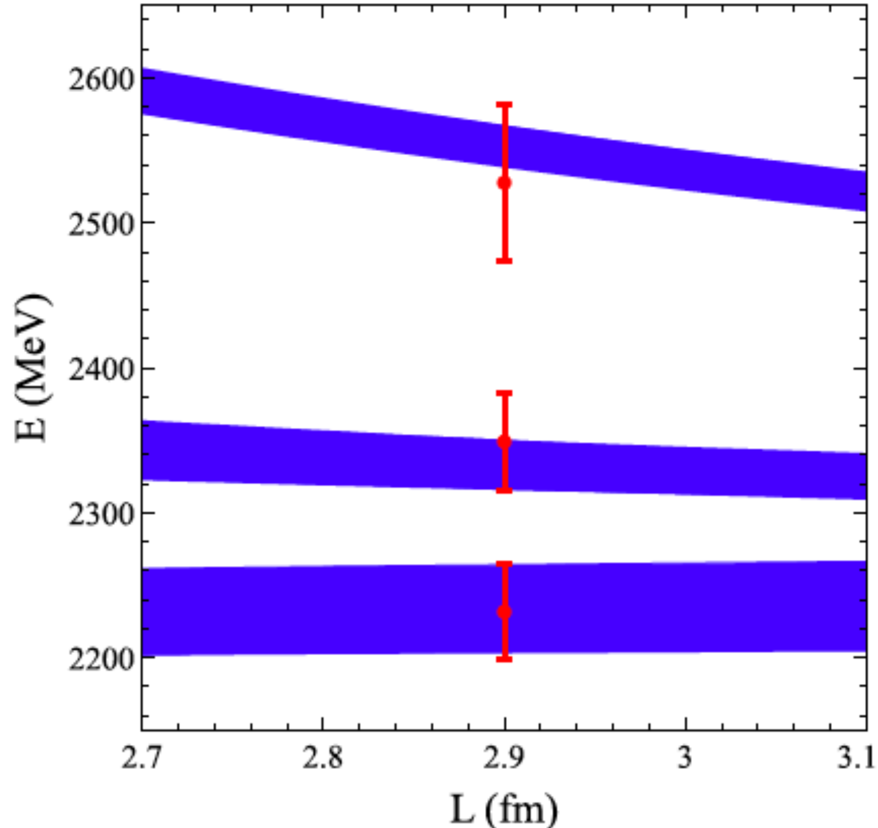
$$B(KD) = 46 \pm 21 \text{ MeV}$$

$$P(KD) = (76 \pm 12)\ \%,$$

$$a_0 = -1.2 \pm 0.6 \text{ fm},$$

$$r_0 = 0.04 \pm 0.16 \text{ fm}$$

D^*K



$D_{s1}(2460)$

$$B(KD^*) = 52 \pm 22 \text{ MeV}$$

$$P(KD^*) = (53 \pm 17)\ \%$$

$$a_0 = -0.9 \pm 0.3 \text{ fm},$$

$$r_0 = -0.3 \pm 0.4 \text{ fm}$$

2. Simulating possible genuine q-qbar components

A near threshold level was found when only $c\bar{s}$ interpolators where used. The possible presence of a genuine component is simulated by adding a CDD pole to the potential,

see e.g. A. Martinez-Torres, L.R. Dai, C. Koren, D. Jido, E. Oset, Phys.Rev.D 85, 014027 (2012)

$$V = \alpha + \beta(s - s_{\text{th}}) + \frac{\gamma^2}{s - M^2_{\text{CDD}}}$$

Most of the solutions preferred a large M_{CDD} value, 300 MeV away from threshold, indicating that lattice energies do not favor a CDD component, or at least not a significant one.

However, the solutions become more dispersed (we accept this as a source of systematic uncertainty)

$$B(KD) = 29 \pm 15 \text{ MeV}$$
$$P(KD) = (67 \pm 14)\ \%$$
$$a_0 = -1.4 \pm 0.4 \text{ fm}$$
$$r_0 = -0.2 \pm 0.4 \text{ fm}$$

$$B(KD^*) = 37 \pm 23 \text{ MeV}$$
$$P(KD^*) = (61 \pm 26)\ \%$$
$$a_0 = -1.3 \pm 0.6 \text{ fm}$$
$$r_0 = -0.1 \pm 0.2 \text{ fm}$$

3. Two-channel analysis

Chiral unitary models studying these states found the ηD_s and ηD^*_s components to be relevant (about 20%) in the $D^*_{s0}(2317)$ and the $D_{s1}(2560)$ states, respectively.

➔ We attempt a two-channel analysis of these states

KD, ηD_s for the $D^*_{s0}(2317)$

KD^*, ηD^*_s for the $D_{s1}(2560)$

With only 3 energies and two-channels the coupled channel potential can only be taken energy-independent.

By construction, this choice would saturate the sum-rule with only meson-meson components.

But we could learn about relative probabilities of the two channel.

➔ We do not find a suitable fit to data

(actually this is not a surprising result as the lattice simulations did not incorporate ηD_s and ηD^*_s interpolators, and these components are probably inhibited)

4. Evaluation of systematic uncertainties

- ✓ Range effects ➔ Much smaller than statistical uncertainties

q_{max} (MeV)	770	875	1075	1275	Average
B(MeV)	34.2	36.6	35.5	35.5	35.5 ± 0.8
$\mid g \mid$ (GeV)	10.85	10.60	10.37	10.41	10.6 ± 0.20
P (%)	86.68	82.15	84.09	87.16	85 ± 2
a_0 (fm)	−1.32	−1.24	−1.25	−1.25	-1.27 ± 0.03
r_0 (fm)	0.30	0.22	0.19	0.19	0.23 ± 0.05

Table 4. Dependence of the properties of the KD bound state on q_{max}.

q_{max} (MeV)	770	875	1075	1275	Average
B (MeV)	45.8	45.6	44.9	44.2	45.0 ± 0.7
$\mid g \mid$ (GeV)	10.67	10.15	10.32	10.31	10.4 ± 0.2
P (%)	60.30	57.42	63.33	66.10	62 ± 3
a_0 (fm)	−1.010	−0.967	−0.980	−0.986	-0.99 ± 0.02
r_0 (fm)	0.07	−0.03	−0.04	−0.06	-0.02 ± 0.05

Table 5. Dependence of the properties of the KD^* bound state on q_{max}.

- ✓ Use of physical D and D* meson masses ➔ Tiny effect
  (note that m_π already small, 156 MeV vs the physical 140 MeV value)

Summary of results

Summing all the systematic errors in quadrature, our final results are:

$$B(KD) = 38 \pm 18 \pm 9 \text{ MeV}$$
$$P(KD) = 72 \pm 13 \pm 5 \ \%$$
$$a(KD) = -1.3 \pm 0.5 \pm 0.1 \ \text{fm}$$
$$r_0(KD) = -0.1 \pm 0.3 \pm 0.1 \ \text{fm}$$

$$B(KD^*) = 44 \pm 22 \pm 26 \text{ MeV}$$
$$P(KD^*) = (57 \pm 21 \pm 6) \ \%$$
$$a(KD^*) = -1.1 \pm 0.5 \pm 0.2 \ \text{fm}$$
$$r_0(KD^*) = -0.2 \pm 0.3 \pm 0.1 \ \text{fm}$$

where the first error is statistical and the second systematic.

Conclusions

We have performed a reanalysis of the lattice data for s-wave scattering KD and KD^* channels that takes into account the three levels obtained in the spectra.

The essence of the new analysis is the use of an auxiliary potential which is allowed to be energy dependent. We tried two forms of this energy dependence: one inspired from chiral unitary models, and the other containing a CDD pole to account for possible genuine states. The results are compatible within errors.

We have evaluated other sources of systematic errors and found them to be smaller than the statistical ones.

Our results indicate that the $D^*_{s0}(2317)$ and the $D_{s1}(2560)$ states are mostly of meson-meson nature and we established a probability to find KD and KD^* in those states in an amount of about $(72\pm13\pm5)$ % and $(57\pm21\pm6)$ %, respectively.

In order to improve the analysis we need:

- more accuracy in the lattice spectra
- additional levels, or spectra calculated at other lattice sizes
- simulations with ηD_s and ηD^*_s interpolators ➔ simulation is underway