Forward dispersion relations for πk scattering.

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Motivation

- πK scattering appears as final state in many hadronic processes.
- Good description in many previous works with UChPT: **Oller,Oset (1999).Dobado,Peláez (1997). Oller,Oset,Peláez (1999). Jamin,Oller,Pich (2000). Nicola,Peláez (2002).**
- Roy-Steiner analysis: Bttiker,Descotes-Genon,Moussallam (2004).
- $\kappa(800)$ appears in these works and in other papers: **D.Bugg. Zheng,Zhou. M. Ablikim et al (BES). E.M. Aitala et al (E791). G. Bonvicini et al (CLEO). J.M. Link et al (FOCUS).**
- However $\kappa(800)$ still needs confirmation according to PDG.
- We have been encouraged to perform a similar analysis for the $\kappa(800)$ as done for the $f_0(500)$ by our group.

Introduction

- Simple fits with unitarity and analyticity. There is no dynamical input, pure data analysis.
- Check of the FDR (valid range of energy).
- Impose FDR to the fits.
- Important for other purposes (Roy-Steiner input, poles...)
- Data obtained from LASS experiments (Aston et al., Estabrooks et al.).
- First step in a long term project.

- We form symmetric or antisymmetric amplitudes under $s \leftrightarrow u$ exchange.

$$T^+ = \frac{1}{3} T^{1/2} + \frac{2}{3} T^{3/2},$$
$$T^- = \frac{1}{3} T^{1/2} - \frac{1}{3} T^{3/2}. \tag{1}$$

- T^I is the amplitude of defined isospin I.

Forward dispersion relations

- We will take for our analysis $t = 0$, they are called FDR.
- For the symmetric $s \leftrightarrow u$ amplitude one subtraction is needed

$$Re(T^+(s)) = T^+(s_{th}) + \frac{s(s-s_{th})}{\pi} P \int_{s_{th}}^{\infty} \frac{Im T^+(s')(2s'-s_{th})}{s'(s'-s)(s'-s_{th})(s'+s-s_{th})} ds'. \quad (2)$$

- For the antisymmetric amplitude no subtraction is needed

$$Re(T^-(s)) = \frac{(2s-s_{th})}{\pi} P \int_{s_{th}}^{\infty} \frac{Im T^-(s')}{(s'-s)(s'+s-s_{th})} ds'. \quad (3)$$

Fits:Elastic region

- We use the unitary functional form for the partial waves

$$t_l^I(s) = \frac{1}{\sigma(s)} \frac{1}{cot\delta_l^I(s) - i} \tag{4}$$

- Where

$$cot\delta_l^I(s) = \frac{\sqrt{s}}{2q^{2l+1}} \sum B_n w(s)^n \tag{5}$$

- with $w(s) = \frac{\sqrt{y(s)} - \alpha\sqrt{y(s0) - y(s)}}{\sqrt{y(s)} + \alpha\sqrt{y(s0) - y(s)}}$ as our new variable (conformal mapping).
- Here $y(s) = (\frac{s - su}{s + su})^2$ defines the circular cut on the next figure.
- w used to maximize the analyticity domain.

Fits:Elastic region

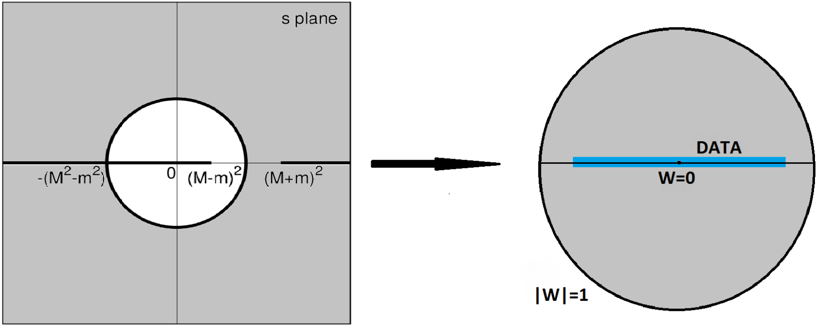

Figure: Structure of the PW.

- α is used to center the point of energy s_c for the expansion.

Fits:Elastic region

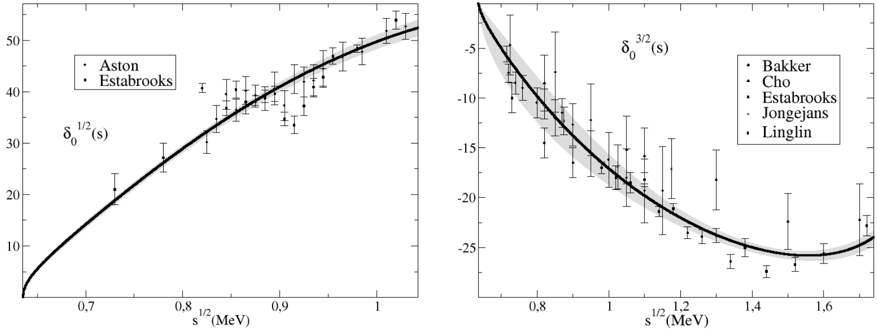

Figure: $S^{1/2}$ and $S^{3/2}$ phase shifts, the I=3/2 is elastic in the entire region.

- $S^{3/2}$ is elastic in the whole region.

Fits:Elastic region

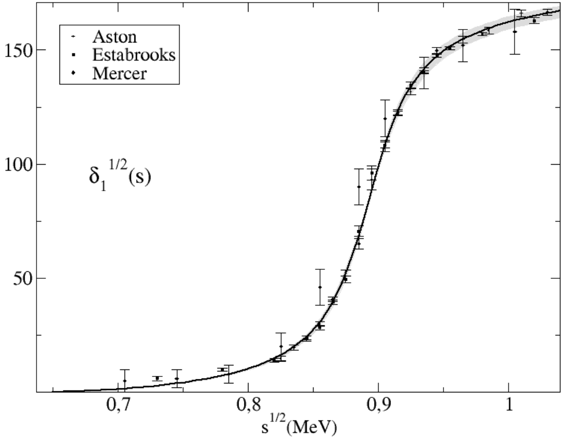

Figure: $P^{1/2}$ phase shifts.

- Good description of the $K^*(892)$.

Fits:Inelastic region

- In the inelastic region $t_l^I = \frac{\eta_l^I(s)e^{2i\delta_l^I(s)}-1}{2i} = |t_l^I|e^{i\phi_l^I}$.
- We use complex rational functions that near each resonance look like BW.
- We impose matching conditions on the inelastic ηk threshold.
- We neglect in this analysis the $P^{3/2}$ and $D^{3/2}$ PW, their phase shifts never reach 5 degrees.
- We use up to $F^{1/2}$ which is very small and neglect $G^{1/2}$ in the studied energy region.

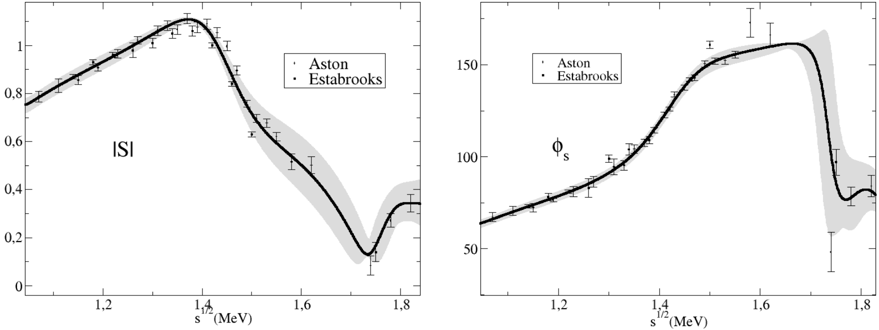

Figure: *S*-wave amplitude and total phase.

- Incompatibilities between Aston and Estabrooks sets of data.

Fits:Inelastic region

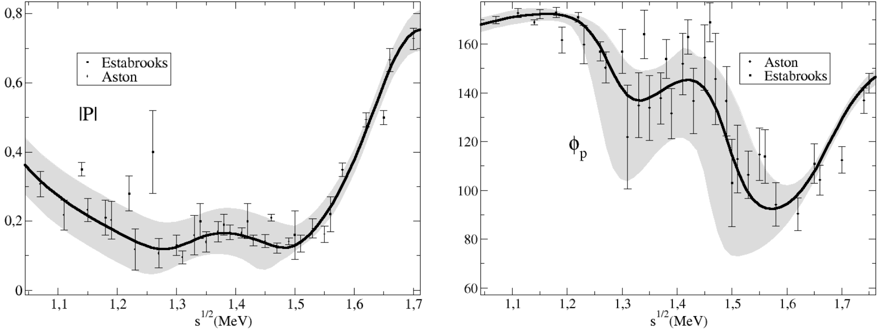

Figure: $P^{1/2}$-wave amplitude and total phase.

Fits:Inelastic region

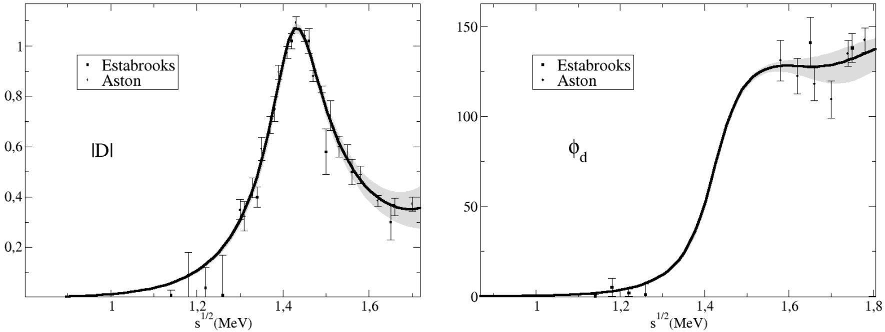

Figure: $D^{1/2}$-wave amplitude and total phase.

- No contribution below $1.2 - 1.4 GeV$.
- Above $1.7 GeV$ we use Regge parametrizations. **Peláez, Yndurain (**2004**)**

FDR check

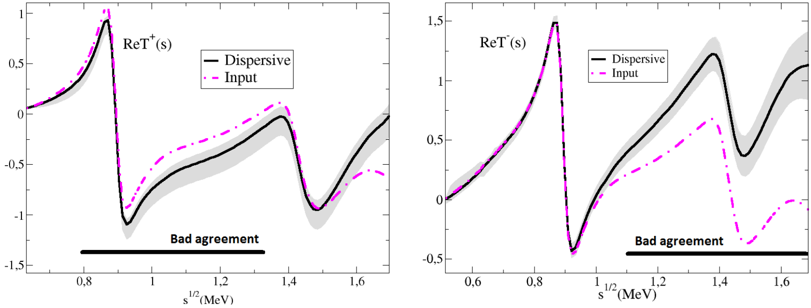

Figure: FDR unconstrained, symmetric and antisymmetric. It is clear the huge difference between the input and output

- Symmetric incompatibilities caused by the $P^{1/2}$ PW.
- Antisymmetric deviations due to Regge contribution.
- Room for improvement → Constrained fits.
- Above $1.7 GeV$ the discrepancies are too huge to impose FDR.

Constrained fits to data (CFD)

- We study the FDR up to $1.6 - 1.7 GeV$.
- We define a χ_1^2 between the input and output.
- There is a χ_2^2 between the UFD parameters and the new ones.
- After the minimization of $\chi_1^2 + \chi_2^2$ we obtain

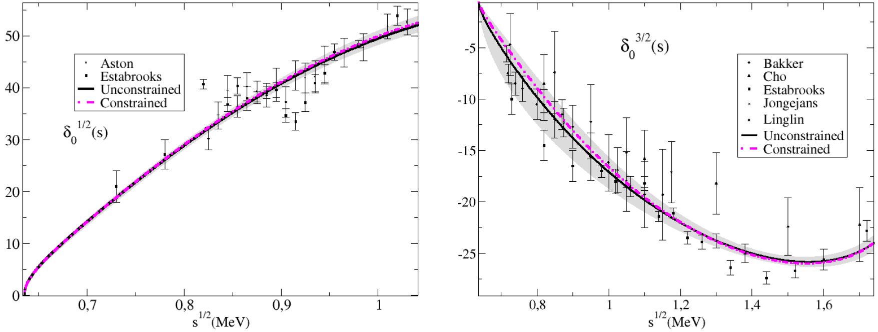

Figure: $S^{1/2}$ and $S^{3/2}$ phase shifts.

Constrained fits to data (CFD)

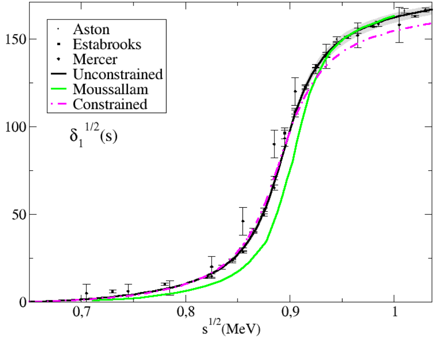

Figure: $P^{1/2}$ phase shifts.

- If the $K^*(892)$ is to be well described the phase shift must be lower at $1GeV$.
- Moussallam result→ solve Roy equations. They use the data at $0.935GeV$ as the matching point.

Constrained fits to data (CFD)

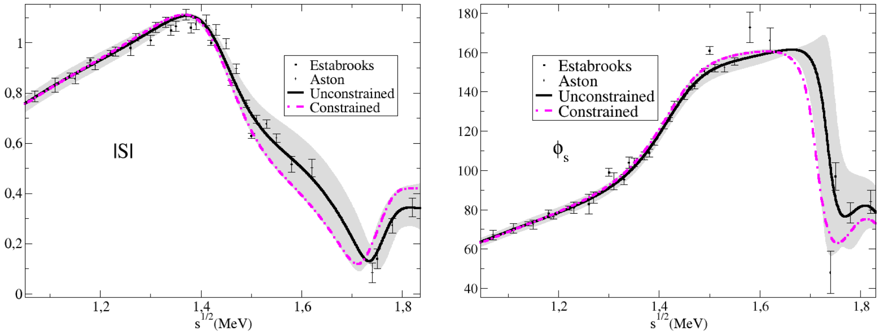

Figure: *S*-wave amplitude and total phase.

- Almost unchanged below 1.5 GeV.
- Constrained fit still compatible.

Constrained fits to data (CFD)

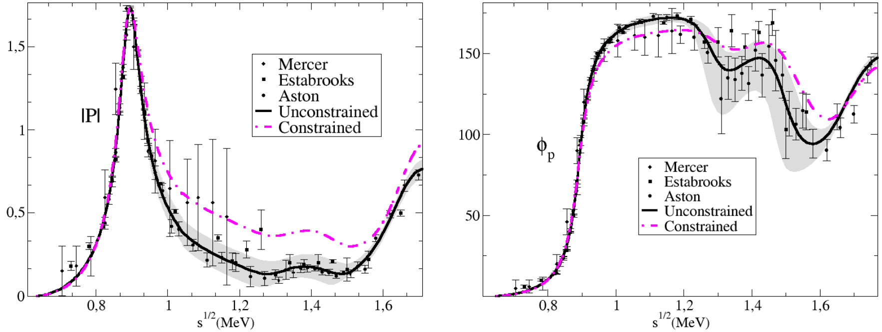

Figure: $P^{1/2}$-wave amplitude and total phase.

- We have tried to fit the data in the region $1 - 1.5 GeV$ but it spoils the T^+ and also the $K^*(892)$ after the minimization.
- The FDR demand a deviation from data in the P wave from 1 to 1.5 GeV.

Constrained fits to data (CFD)

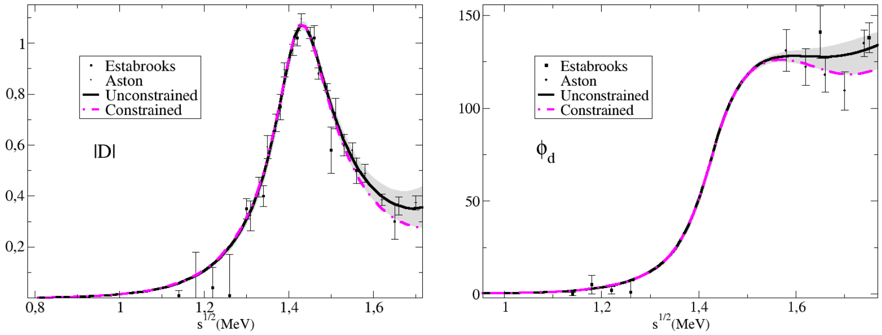

Figure: $D^{1/2}$ wave amplitude and total phase.

- Little changes around $1.6 GeV$.

Constrained fits to data (CFD)

- The change in the symmetric amplitude around $1 - 1.2 GeV$ is caused by the change of the P-wave. The Regge contribution in this region is small.
- The huge change of the antisymmetric one is caused by the Regge πk factorization constant.

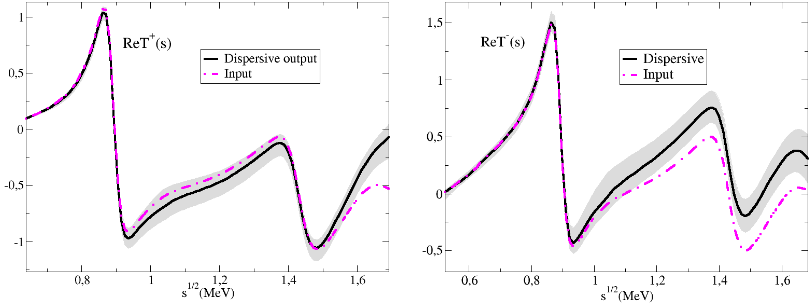

Figure: FDR constrained, symmetric and antisymmetric. They fairly compatible up to $1.6 GeV$

Constrained fits to data (CFD)

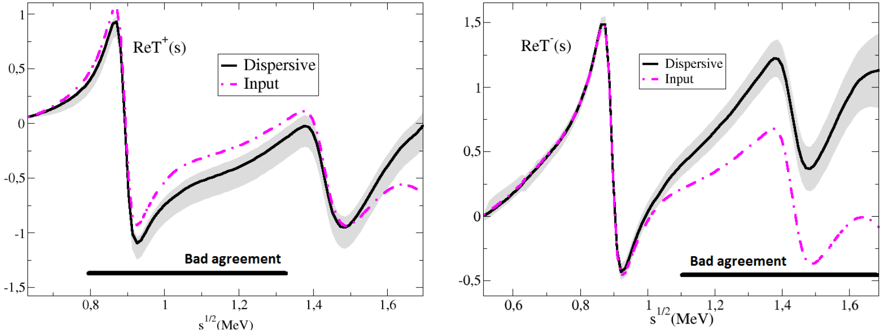

Figure: FDR unconstrained, symmetric and antisymmetric. It is clear the huge difference between the input and output

Scattering lengths

- Now we can obtain the threshold parameters for the most important partial waves

Table: Scattering lengths.

SL	UFD	CFD	Moussallam result
$m_\pi a_0^{1/2}$	0.217	0.218±0.014	0.224± 0.022
$m_\pi a_0^{3/2}$	-0.0790±0.03	-0.0614±0.025	-0.0448± 0.0077
$m_\pi^3 a_1^{1/2}$	0.0259±0.008	0.0242±0.007	0.019± 0.001

Pole parameters

- For the kappa resonance $K_0(800)$ we obtain

Table: $\kappa(800)$ parameters.

Group	Mass	Width
UFD	680 ± 19	677 ± 24
CFD	681 ± 19	674 ± 23
Moussallam et al.	658 ± 13	557 ± 24
D.Bugg	663 ± 34	658 ± 44
Zheng,Zhou	694 ± 53	606 ± 59

- The values of the masses are compatible. However we expect to obtain a lower value of the partial width in a future analysis with the help of πK Roy equations.

Conclusions

- We have studied FDR in πk up to $1.7 GeV$.
- Imposing FDR gields significant changes in $|P^{1/2}|$ above $1 GeV$.
- Our solution still describe the data of the experiments in the energy region for the rest of the PW.
- $\kappa(800)$ mass compatible with PDG and Roy-Steiner equations but width comes wider.
- This is first step on our program to obtain fits consistent with Roy-Steiner equations and rigorous determination of $\kappa(800)$.

¡Thank you for your atention!

FDR deviation over 1.7 GeV

- Too huge differences in both T^+ and T^-.
- We can't correct only the PW or the Regge, too much deviation from the old parameters in the PW.

Regge parametrization

- The problem arises with the factorization constants $f_{k/\pi}$ (Pomeron) and $g_{k/\pi}$ (Rho).
- The problem in the antisymmetric amplitude is caused by $g_{k/\pi}$.
- The previous works in $\pi\pi$ scattering together with the paper of the Regge parametrizations suggest a lower value (30 percent) of this constant.