Entropy production in the early-cosmology pionic phase

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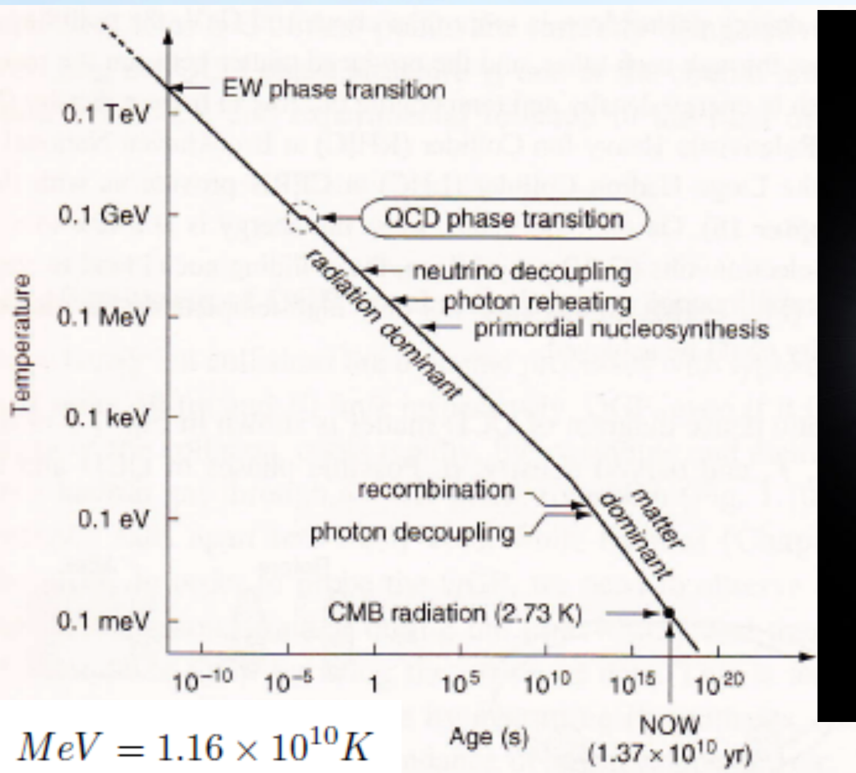
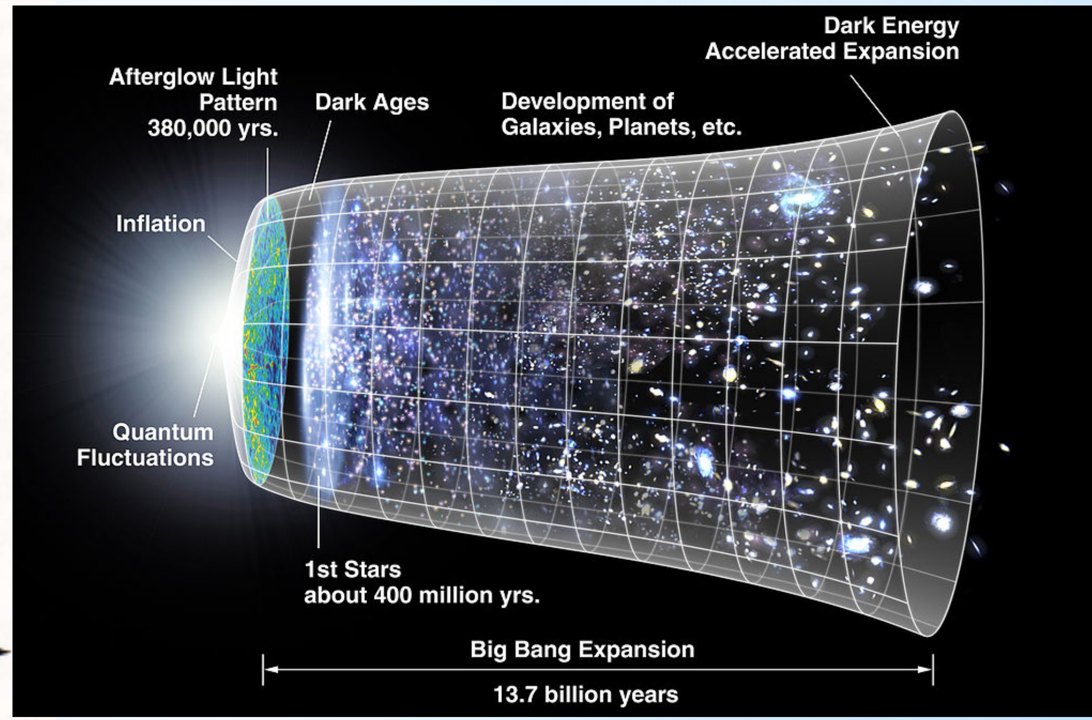
In the History of the Universe it is possible to distinguish different periods of time (epochs or eras) defined by important physical events:



$$MeV = 1.16 \times 10^{10} K$$



Early epoch:

From the Electroweak phase transition to T = 1 TeV to T = 1 GeV (t = 10^{-12} to 10^{-7} s)

After the Electroweak phase transition most of the particles have got masses

The Higgs H, the EW gauge bosons (W and Z), the top (t), button (b) and charm (c) quarks and the tau (τ) lepton decay very fast and don't survive this epoch.

Thus by the time T $\sim$ 1 GeV we are left just with the quarks up (u), dwon (d) and strange (s), the electron (e), the muon (μ), the three neutrinos (v) and their respective antiparticles and also photons (γ) and gluons (g)

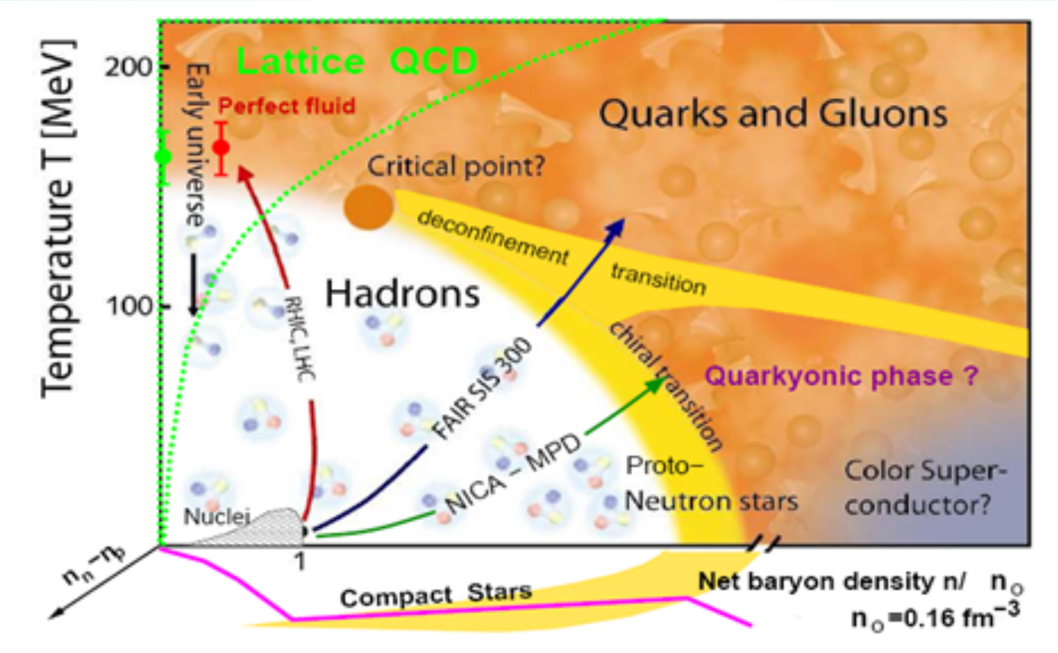
The quark-gluon plasma:

From T = 1 GeV to T = T_c = 175 MeV

Accesible to RICH and LHC

Major components are: Quarks: u, d, and s, gluons, leptons and photons.

This epoch ends at the QCD phase transition (confinement and spontaneous chiral symmetry breaking), most probably a cross-over.

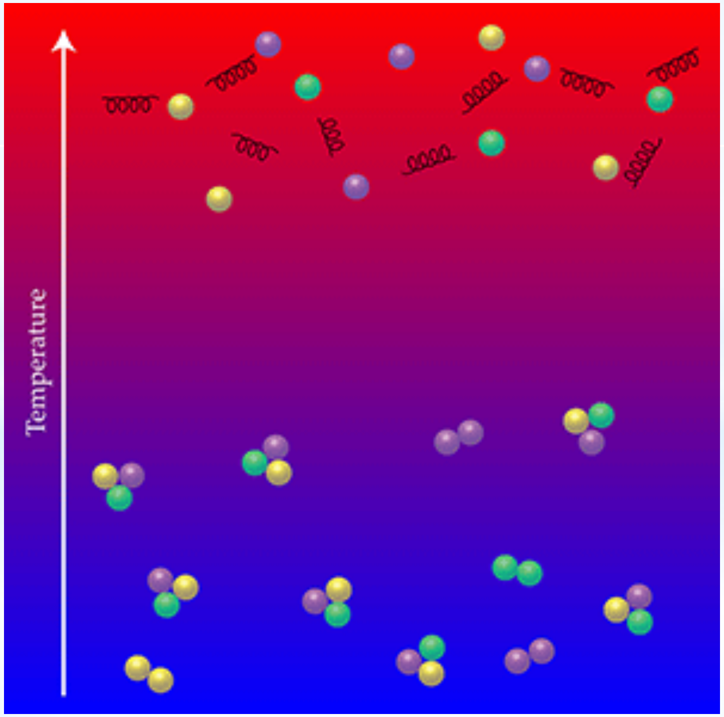


The QCD phase transition:

Around T_c = 175 MeV, t = 10^{-6} s

Hadronization takes place: $q\bar{q}$ ➔ M, q q q → B, gg ➔ G…

Confinement and chiral symmetry breaking (condensates <GG> and <u u + d d>)



The hadron era:

From $T=T_c$ = 175 MeV (t = 10^{-6} s) to T = 1 MeV (t=1s)

Most of hadrons decay very fast

Strong, electromagnetic, weak, ineractions still faster than cosmic expansion

The survival particles are pions, protons, neutrons, muons, electrons, neutrinos, photons and their corresponding antiparticles.

After some time even those hadrons annihilate

A small baryon asymmetry remains $\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$ $\sim$ 6 X 10^{-10}

About T = 80 MeV (t = 10^{-6} s) pions and muons decople from the photons and disappear (Beginning of the leptonic era) .

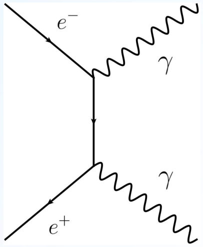
Some important events about T= 1 MeV (t = 1 s):

Neutrino decoupling when weak interction rate equals the cosmic expansion rate

$$\bar{\nu}\nu \leftrightarrow e^+e^-$$
$$\nu e \leftrightarrow \nu e$$

$$\frac{\Gamma_{int}}{H} \simeq \frac{G_F^2 T^5}{T^2/m_{\text{Planck}}} \simeq \left(\frac{T}{1\,\text{MeV}}\right)^3$$

Photon reheating at T = m_e = 0.5 MeV (t = 10 s) produced by electron-positron annihilation:



After that we are left just with many photons and a few protons, neutrons and electrons (remember baryonic asymmetry)

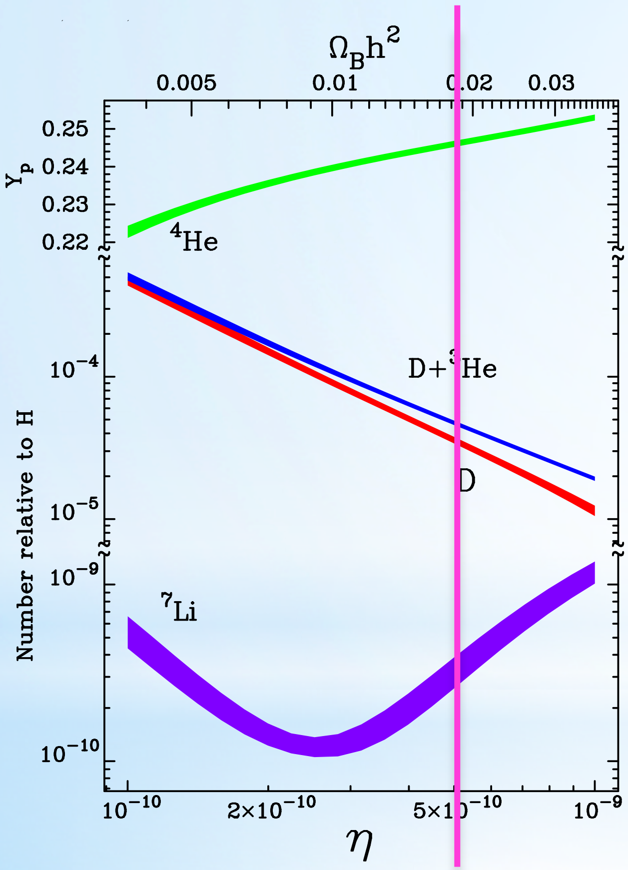
Neutrons decople from protons (as neutrinos from electrons) and start to decay

$$\frac{n_n}{n_p} = e^{-(m_n - m_p)/T} = e^{-\mathcal{Q}/T}. \qquad \mathcal{Q} = m_n - m_p = 1.293\ \text{MeV}$$

Primordial Nucleosinthesis

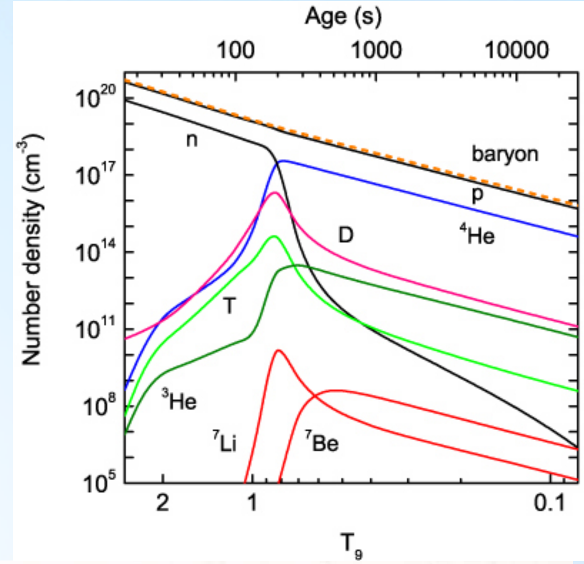
From T = 1 MeV to T = 0.1 MeV (t=1 s to 100 s)

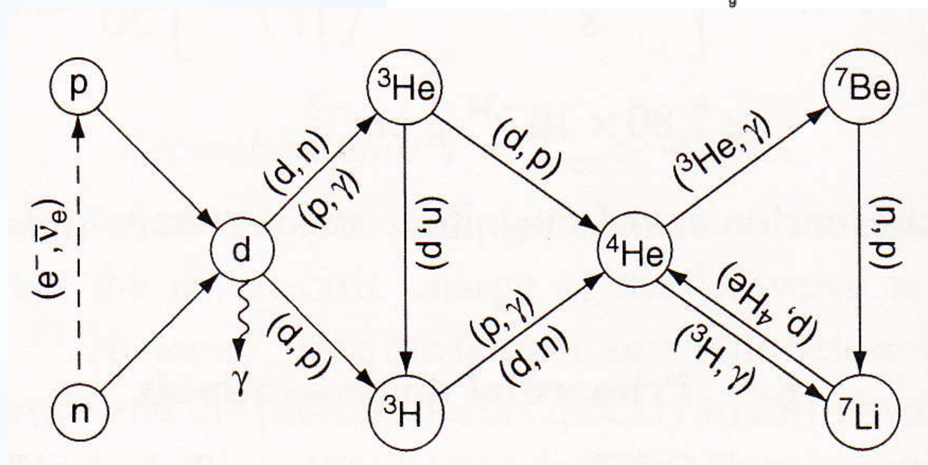
(Weinberg's famous three minutes)



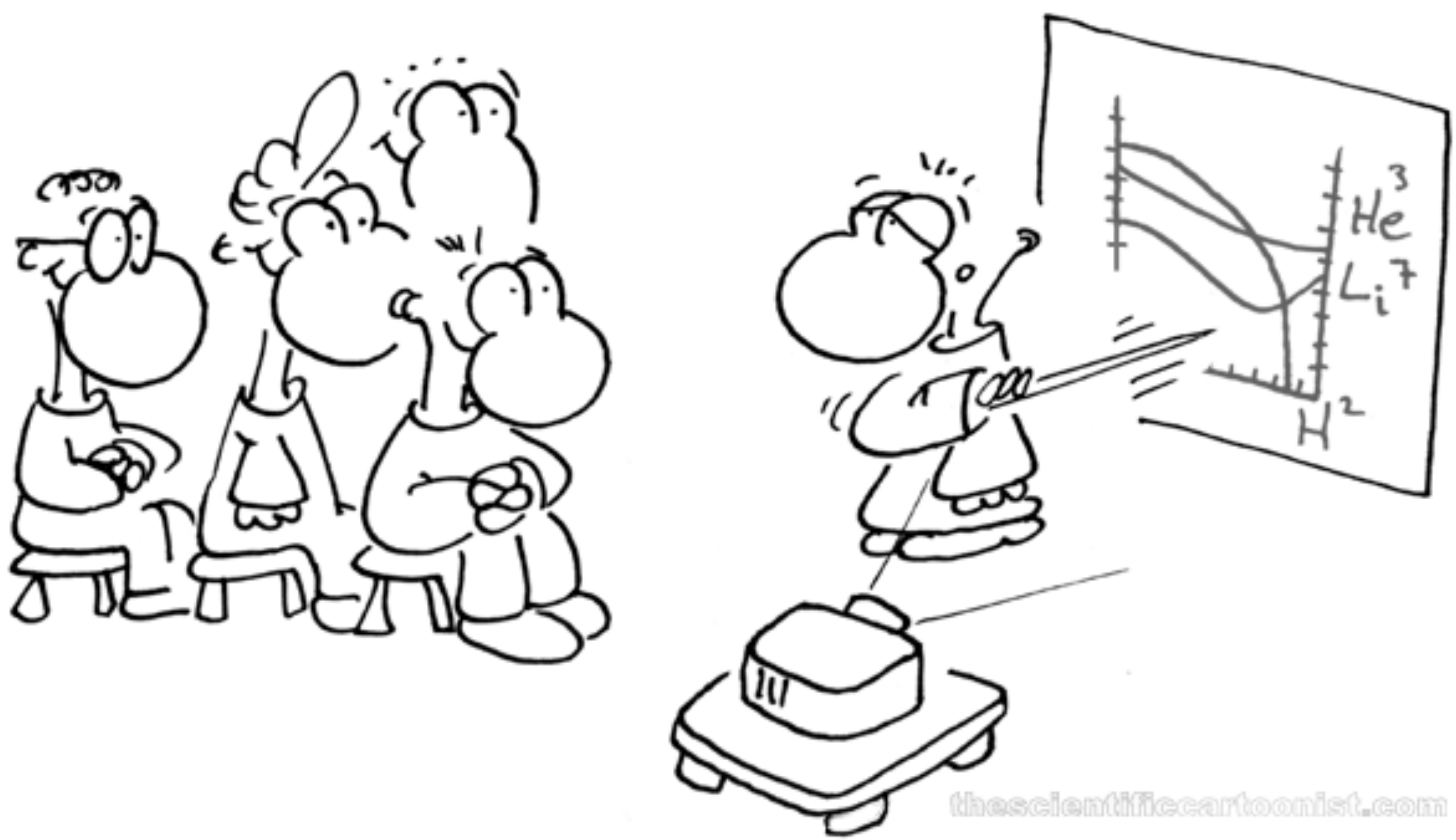
(Alpher, Bethe and Gamow)

$$Y_p \simeq \frac{4 \cdot n_n/2}{n_n + n_p} = \frac{2(n_n/n_p)}{1+(n_n/n_p)} = 0.25.$$





Nucleus	$g(^AZ) = 2I + 1$	$B(^AZ)$, MeV	Decay mode
^{2}H	3	2.22	stable
^{3}H	2	8.48	$\beta \rightarrow {}^3$He (12.6 yr)
^{3}He	2	7.72	stable
^{4}He	1	28.30	stable
^{6}Li	3	31.99	stable
^{7}Li	4	39.25	stable
^{7}Be	4	37.60	electron capture to ^{7}Li (53 days)
^{12}C	1	92.16	stable

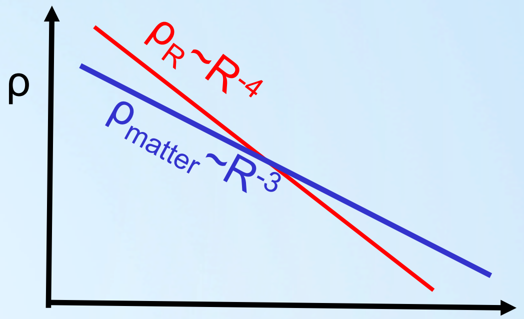


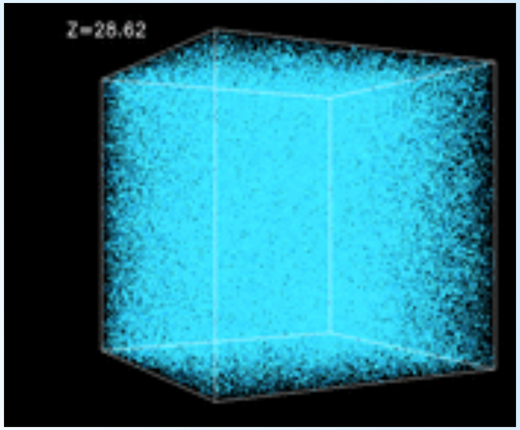
"The Big Bang produced Hydrogen, Helium, Deuterium and a small amount of Lithium. Unfortunately, it didn't generate enough of the last element for all our politicians."

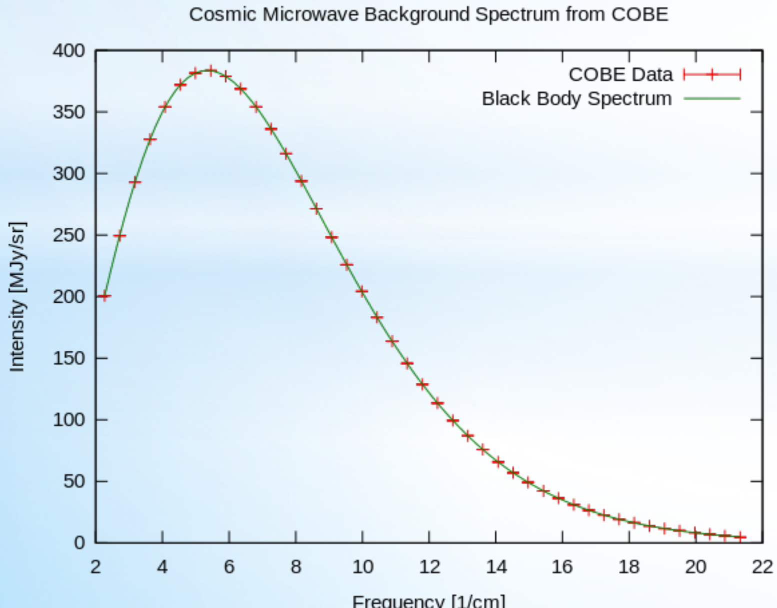
Further important events

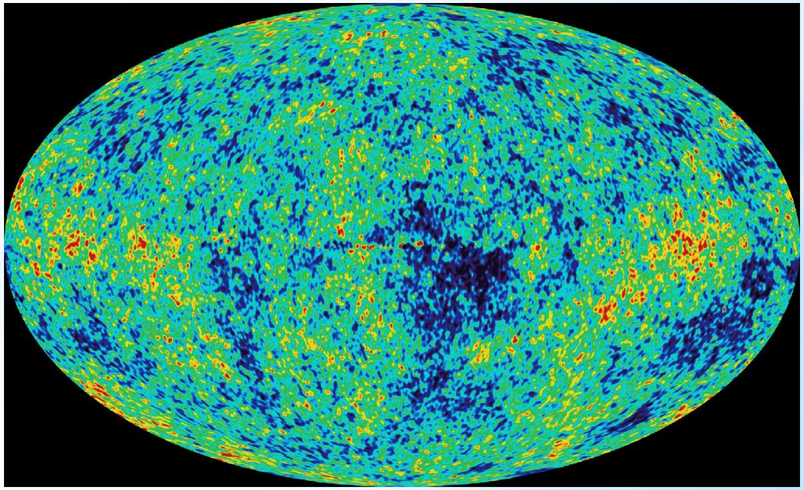
After T = 1 eV (1000 y) matter starts to dominate radiation.
End of the radiation era.
Begining of structure formation and BAO (t=7 10^4 y)
Recombination at T = 0.3 eV (t= 380.000 y)
The Universe becomes neutral also locally.
And finally photon decoupling
Origin of the current Cosmic Microwave Background CMB
Perfect black body radiation at T = 3000 K (T= 2.7 K today)
Temperature fluctuations are of the order of 10^{-5} or lesser
(bound on previous departure from homogeneity)

$p+e \rightarrow H+\gamma$









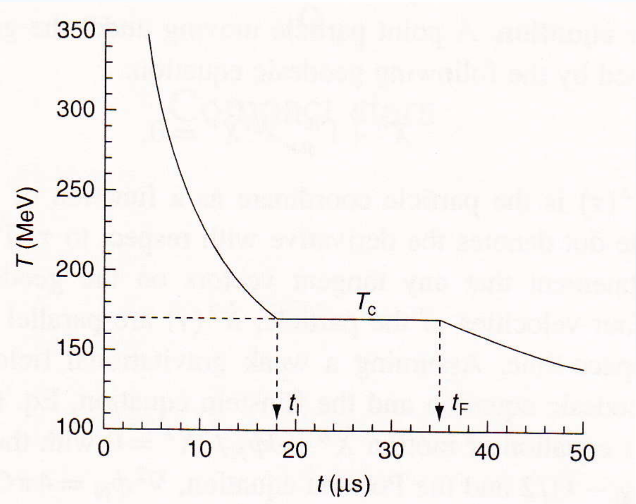
An important remark:

The QCD phase transition could produce big inhomogenities because of the fluctuations
(in addition to those produced by inflation or other previous phase transitions)

Those inhomogeneties must be washed up at the level of the CMB radiation or lower

Dissipative processes during the hadron era could do the job

Estudying how this works is the main point of this work



Matter dominated era

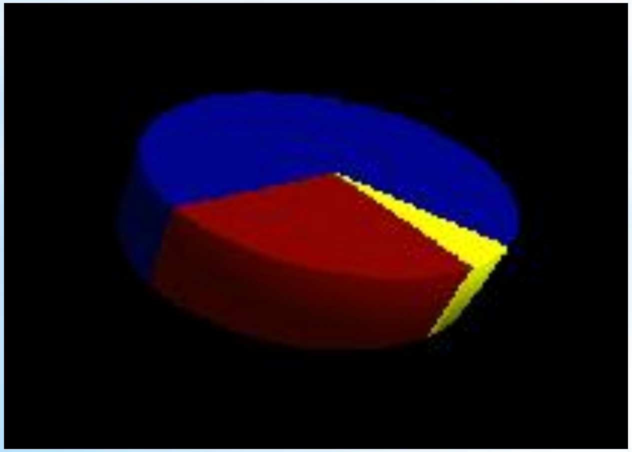
From T = 379.000 y to about 300 million years no brilliant single objects in the universe, just clouds of H and He (the Dark Eges)

At something about several hundred million years stars gallaxy and star formation

9×10^9 y Sun formation

10^{10} y Earliest forms of life

10^{10} y Dark energy dominated era



5% Ordinary Matter
25% Dark Matter
70% Dark Energy

This is the 1st Hadron Spanish Network Days meeting!

The Hadron era in more detail:

From T = 175 MeV, t = 10^{-6} s, R_H = 10 km, $M_H \sim M_\odot$
(just after the QCD phase transition)

Flat FRW metric $$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \quad (\kappa = 0)$$

Einstein Field Equations $$R_{ik} - \frac{1}{2}g_{ik}R = 8\pi\, G\, T_{ik}$$ One hundred years old!

Friedmann equation: $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$ and the matter equation: $$\frac{d\rho}{dt} = -\frac{3\dot{a}}{a}(\rho + P)$$

Total density: $$\rho = \rho_\gamma + \rho_{\nu,\bar{\nu}} + \rho_{e^\pm} + \rho_{\mu,\bar{\mu}} + \rho_{\pi^\pm,\pi^0} + \rho_{N,\bar{N}} + \dots$$

Strong, electromagnetic and weak interactions rates are much faster than the cosmic expansion rates:

Thus we assume local thermodynamics equilibrium:

$$f_i(\mathbf{r},\mathbf{p},t) = \frac{1}{e^{(p_\alpha U^\alpha(\mathbf{r},t)-\mu_i(\mathbf{r},t))/T(\mathbf{r},t)} \pm 1}$$

$\pi^0 \leftrightarrow \gamma\gamma$	$NN \leftrightarrow NN$	$e\pi \leftrightarrow e\pi$
$\pi\pi \leftrightarrow \pi\pi$	$N\bar{N} \leftrightarrow \gamma\gamma$	$e\,e \leftrightarrow e\,e$
$\pi^+ \leftrightarrow \mu^+\nu_\mu$	$\mu^+ \leftrightarrow e^+\nu_e\overline{\nu}_\mu$	$e\gamma \leftrightarrow e\gamma$
$\pi\pi \leftrightarrow \gamma\gamma$	$\mu\pi \leftrightarrow \mu\pi$	$\gamma\gamma \leftrightarrow e^-e^+$
$\nu_e\overline{\nu}_e \leftrightarrow e^+e^-$	$\mu^-\gamma \leftrightarrow \mu^-\gamma$	$\mu\mu \leftrightarrow \mu\mu$
$\nu_\mu\overline{\nu}_\mu \leftrightarrow \mu^+\mu^-$	$\gamma\gamma \leftrightarrow \mu^-\mu^+$	$\mu e \leftrightarrow \mu e$

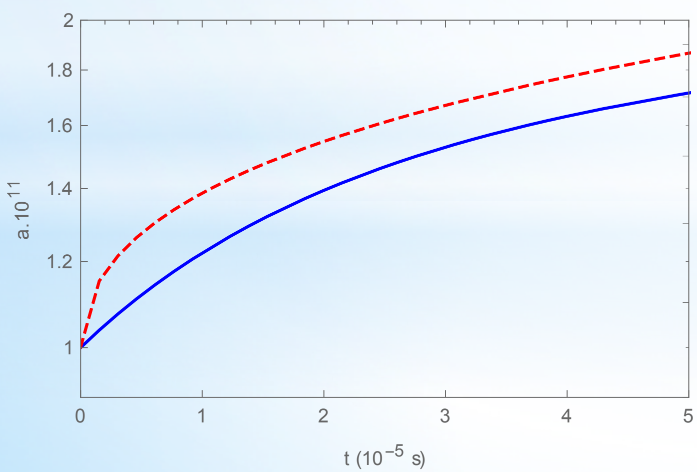
Chemical pottentials negligible

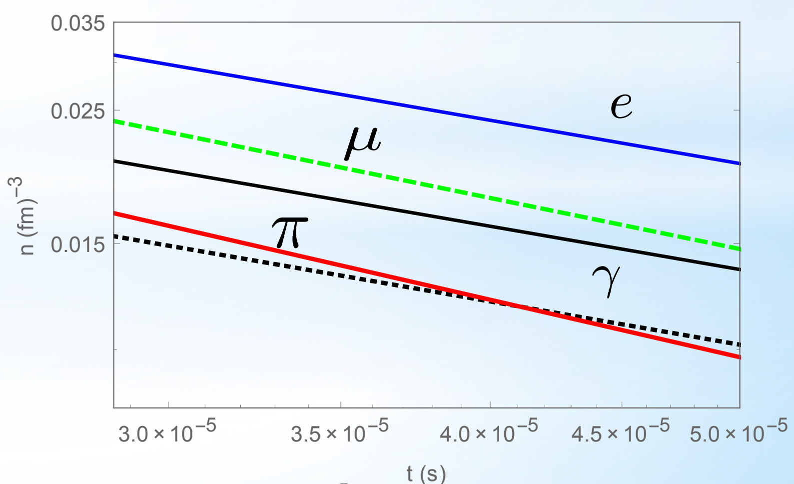
$$\rho = \Sigma_i \rho_i = \Sigma_i \frac{g_i}{(2\pi)^3}\int d^3p\, E\, f_i(\mathbf{r},\mathbf{p},t)$$

$$P = \Sigma_i P_i = \frac{1}{3}\Sigma_i \int d^3p\, f_i(\mathbf{r},\mathbf{p},t)\frac{|\mathbf{p}|^2}{E}$$

Solving Friedmann and matter equations:

$$\frac{dT}{dt} = -\frac{3\dot{a}}{a}(\rho+P)\frac{dT}{d\rho}$$





$t_{T=175MeV}$= 0 s $t_{T=100MeV}$ = 5 x10^{-5} s

$$n = \frac{3}{4\pi r_0{}^3} = 0.122\ \mathrm{fm}^{-3} = 1.22\cdot 10^{44}\ \mathrm{m}^{-3}$$

Thermodynamics:

Adiabatic, homogeneous expansion (entropy conservation)

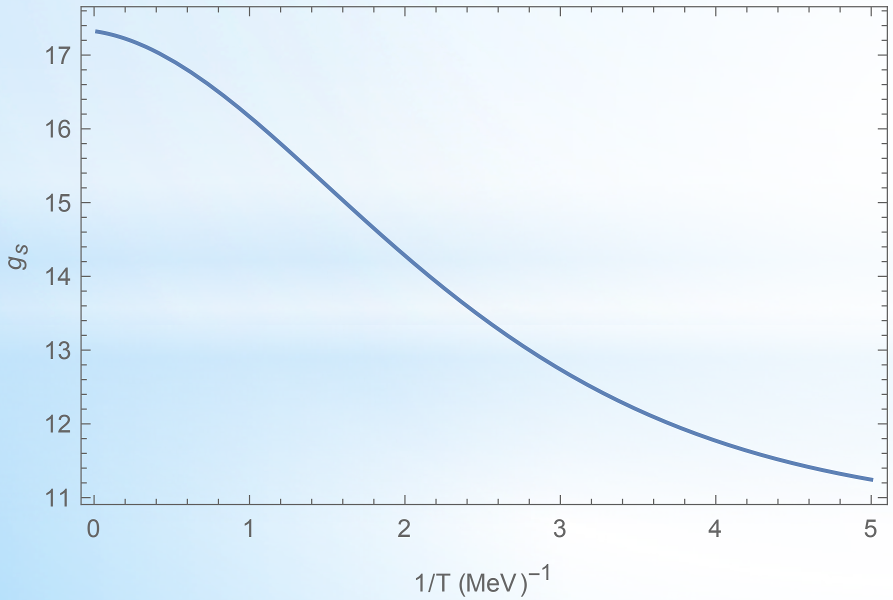
$$TdS = d(\rho V) + PdV.$$

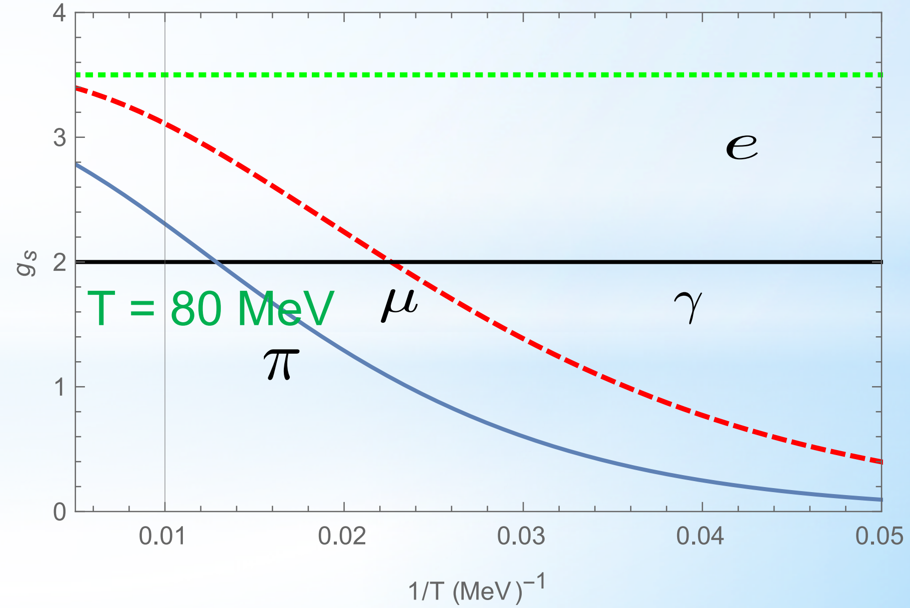
$$\pi^+\pi^- \leftrightarrow \pi^0\pi^0$$

$$\gamma\gamma \leftrightarrow \pi\pi$$

$$s = \frac{1}{T}(\rho + P) = \frac{dP}{dT} \qquad s = \frac{\rho_1 + \cdots + \rho_n + P_1 + \cdots + P_n}{T} \qquad sa^3 = s_0 a_0^3$$

Effective degrees of freedom $s(T) = g_s \frac{2\pi^2}{45} T^3 \rightarrow g_s(T)$





Why having a small pionic era (T= 175 to T = 80 Mev) is relevant?

↓

Because they are strongly interacting

↓

Large cross sections (not like photons)

↓

Small transport coefficients $\kappa \propto 1/\sigma$

↓

Small dissipation rates

↓

Largest dissipative inertia

↓

Still they have to erase any previous fluctuation to fit the CMB

Dissipation of thermal fluctuations

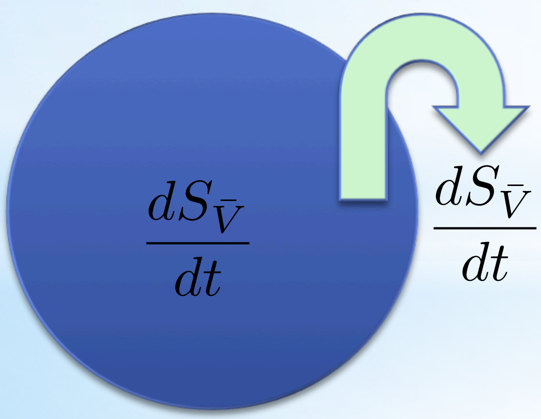
$$
\begin{aligned}
T(\mathbf{r},t) &= T_{back}(t) + \delta T(\mathbf{r},t) \\
s(\mathbf{r},t) &= s_{back}(t) + \delta s(\mathbf{r},t)\,.
\end{aligned}
$$

Heat equation $$\Delta\left(\delta T(\mathbf{r},t)\right) = \frac{\kappa(T)}{c_p(T)}\,\frac{\partial\left(\delta T(\mathbf{r},\mathbf{t})\right)}{\partial t}$$

Thermal conductivity $\kappa(T)$

specific heat at constant pressure $$c_p(T) = \left.\frac{\partial s_{back}(T)}{\partial T}\right|_P$$

$$\frac{dS_T}{dt} = \frac{dS_{\bar{V}}}{dt} + \frac{dS_V}{dt}$$



$$
\begin{aligned}
\frac{dS_{\bar{V}}}{dt} &= -\int_{\partial V} \mathbf{j}_s\cdot\mathbf{n}\,d\Sigma \\
\frac{dS_V}{dt} &= \int_V \sigma_s\,dV\,.
\end{aligned}
$$

σ_s entropy production per unit of volume and

$\mathbf{j}_s$ entropy current

$$\frac{dS_T}{dt} = \frac{d}{dt}\int_V s_T\,dV = -\int_{\partial V} d\Sigma\,\mathbf{j}_s\cdot\mathbf{n} + \int_V dV\,\sigma_s$$

$\mathbf{j}_e$ heat current

$$\frac{ds_T}{dt} = -\boldsymbol{\nabla}\cdot\mathbf{j}_s + \sigma_s$$

Gauss Law

$$\delta T(r,0) = \delta T_0\, e^{-\frac{r^2}{2R^2}}$$

$$\sigma_s = \mathbf{j}_e\cdot\boldsymbol{\nabla}\left(\frac{1}{T}\right)$$

Fourier Law

First Law $$dU = T\,dS_T = (\mathbf{j}_e \cdot \mathbf{n})\,d\Sigma\,dt$$

Integrating and using Gauss Theorem $$\Delta S_T = \int_{\partial V} d\Sigma\,dt\,\frac{\mathbf{j}_e \cdot \mathbf{n}}{T} = \int_V dV\,dt\,\boldsymbol{\nabla} \cdot \left(\frac{\mathbf{j}_e}{T}\right)$$

Using Fourier Law $$\Delta S_T = \int dV\,dt\,\boldsymbol{\nabla} \cdot \left(-\frac{1}{T}\kappa(T)\boldsymbol{\nabla} T\right)$$

Leibniz Rule $$\Delta S_T = \int dV\,dt\,\frac{\kappa(T)}{T^2}\left(|\boldsymbol{\nabla}\delta T|^2 - T\Delta\delta T\right) \qquad \Delta S_T \geq 0$$

Thus:

$$\sigma_s(r,t) = \frac{\kappa(T)}{T^2}|\boldsymbol{\nabla}\delta T(r,t)|^2\,,$$

$$-\boldsymbol{\nabla} \cdot \mathbf{j}_s(r,t) = \frac{\kappa(T)}{T}\Delta\delta T(r,t)\,.$$

Entropy pruduced inside V: $$\Delta S_V(\delta T_0) = \int dV\,dt\,\sigma_s(r,t)$$

Background entropy density $$S_{back}(R, T_{back}) \simeq \frac{4}{3}\pi\left(\sqrt{2}R\right)^3 s_{back}(T_{back})$$

$$\Delta S_V / S_{back}$$

Viscous Hydrodynamics and transport coefficients:

$$T^{\mu\nu} = w u^\mu u^\nu - P\eta^{\mu\nu} + \tau^{\mu\nu}$$
$$n^\mu = n u^\mu + \nu^\mu \ .$$

$w = \epsilon + P$

Entalpy

$P = P(\epsilon)$

Equation of state

$\tau^{\mu\nu} u_\mu = 0$

$\nu^\mu u_\mu = 0$

Landau-Lifshitz frame

$$s^\mu = s u^\mu - \frac{\mu}{T}\nu^\mu$$

$\partial_\mu s^\mu \geq 0$ Second Law

Shear viscosity

Bulk viscosity

$$\tau^{\mu\nu} = 2\eta\nabla^{\{\mu}u^{\nu\}} + \left(\zeta - \frac{2}{3}\eta\right)\partial_\alpha u^\alpha \Delta^{\mu\nu}$$
$$\nu_\mu = -\kappa\left(\frac{nT}{w}\right)^2 \ \nabla_\mu\left(\frac{\mu}{T}\right) \ ,$$

Dissipative contribution to the energy-momentum tensor and current

Thermal conductivity

$\partial_\mu T^{\mu\nu} = 0$ Navier-Stokes Equations

Kinetic Theory:

Chapman-Enskog

$$n^\mu(t,\mathbf{x}) = g\int d^3p \frac{p^\mu}{(2\pi)^3 E_p} f_p(t,\mathbf{x})$$

$$f_p(t,\mathbf{x}) = n_p(t,\mathbf{x}) + f_p^{(1)}(t,\mathbf{x})$$

$$T^{\mu\nu}(t,\mathbf{x}) = g\int d^3p \frac{p^\mu p^\nu}{(2\pi)^3 E_p} f_p(t,\mathbf{x})$$

$$n_p(t,\mathbf{x}) = \frac{1}{e^{\beta(E_p-\mu)}-1}$$

Bose-Einstein

$$\tau^{\mu\nu}(t,\mathbf{x}) = g\int d^3p \frac{p^\mu p^\nu}{(2\pi)^3 E_p} f_p^{(1)}(t,\mathbf{x})$$

$$\nu^\mu(t,\mathbf{x}) = g\int d^3p \frac{p^\mu}{(2\pi)^3 E_p} f_p^{(1)}(t,\mathbf{x})$$

Viscous contribution to the energy-momentum tensor and current

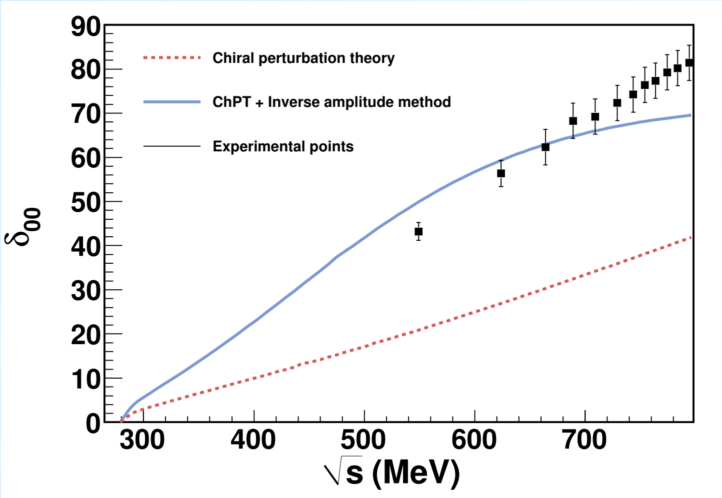
$$\frac{df_p}{dt} = C[f_3, f_p]$$

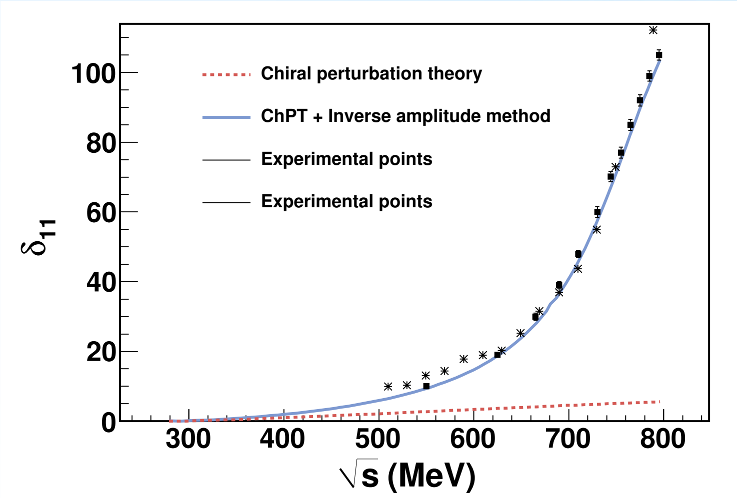
Boltzmann-Uehling-Uhlenbeck Equation

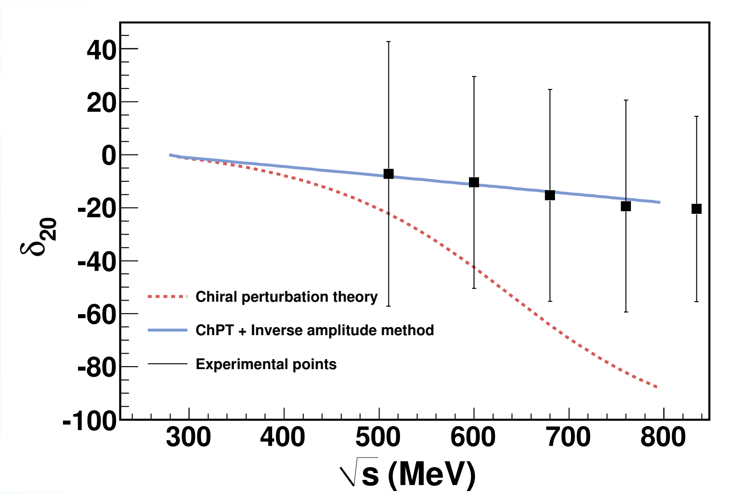
$$C[f_3,f_p] = \frac{g_3}{1+\delta_{3,p}}\int d\Gamma_{12,3p}\,\left[f_1 f_2(1+f_3)(1+f_p) - f_3 f_p(1+f_1)(1+f_2)\right]$$

$$d\Gamma_{12,3p} \equiv \frac{1}{2E_p}\overline{|T|^2}\prod_{i=1}^{3}\frac{d\mathbf{k}_i}{(2\pi)^3 2E_i}\,(2\pi)^4\delta^{(4)}(k_1+k_2-k_3-p)$$

ChPT plus the IAM method fit very well pion scattering at the required energies

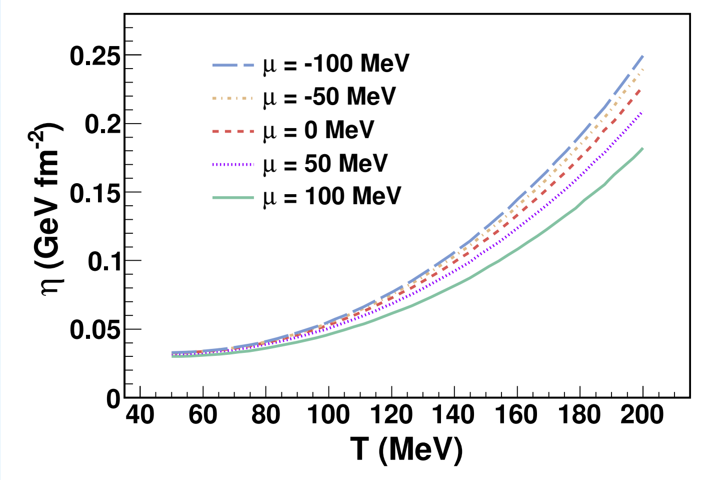


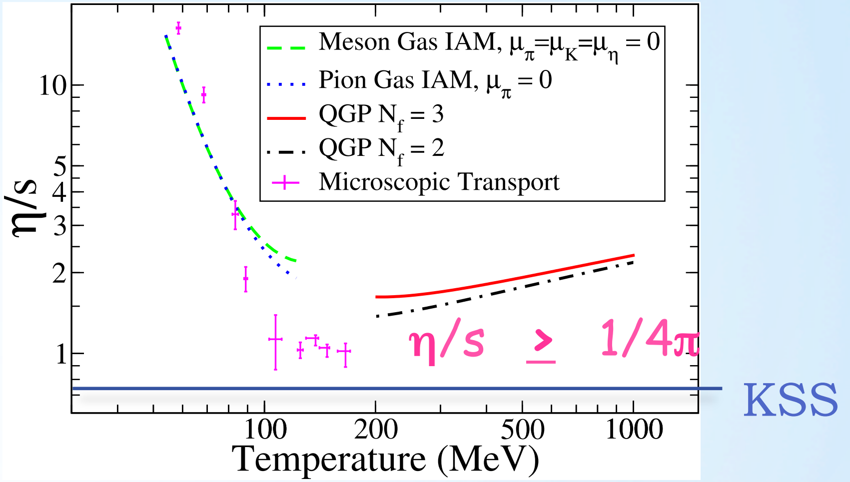


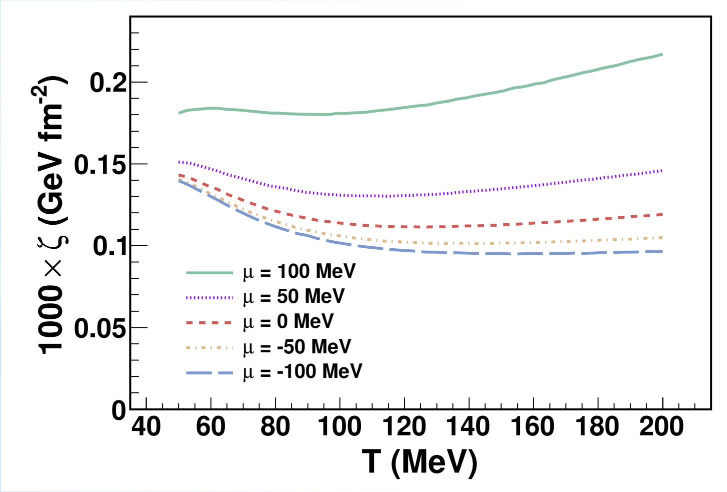


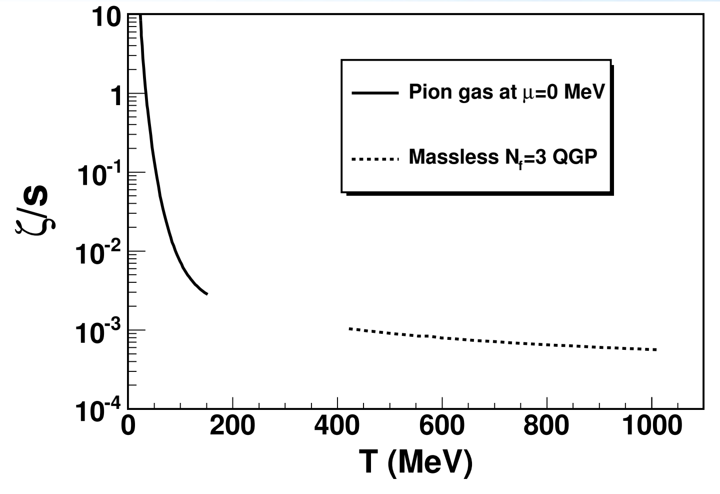
Tranport coefficients as function of the temperature

Kovtun, Son and Starinets bound



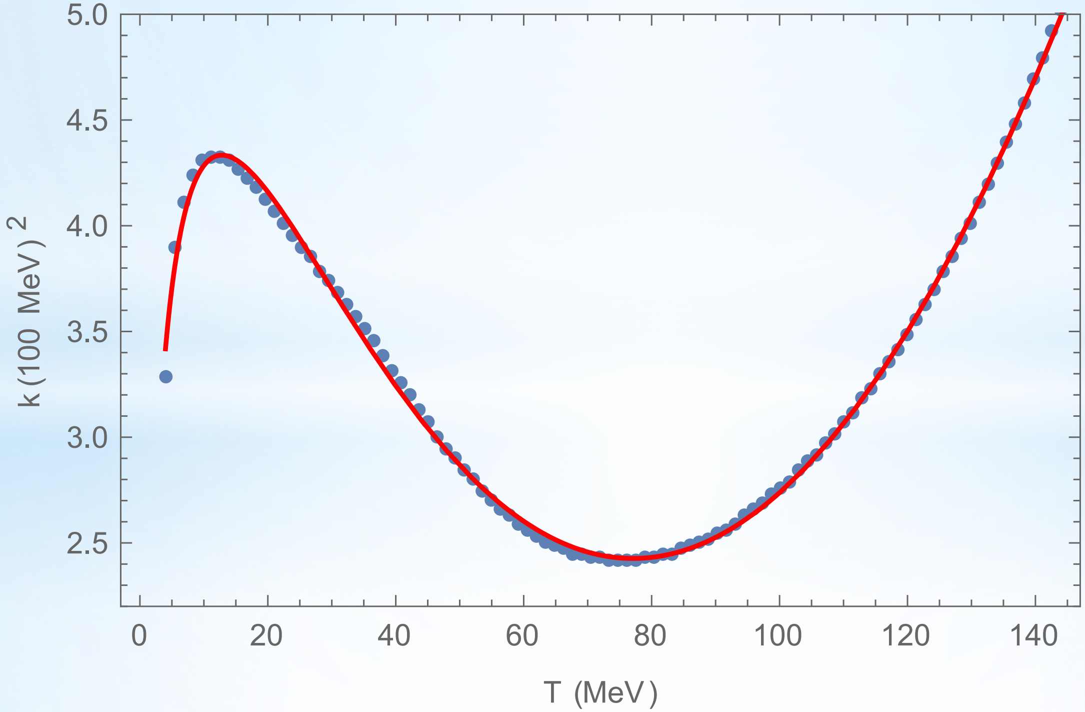






AD, F. Llanes-Estrada and Torres-Rincon
Gomez-Nicola and Fernandez-Fraile

Heat conductivity



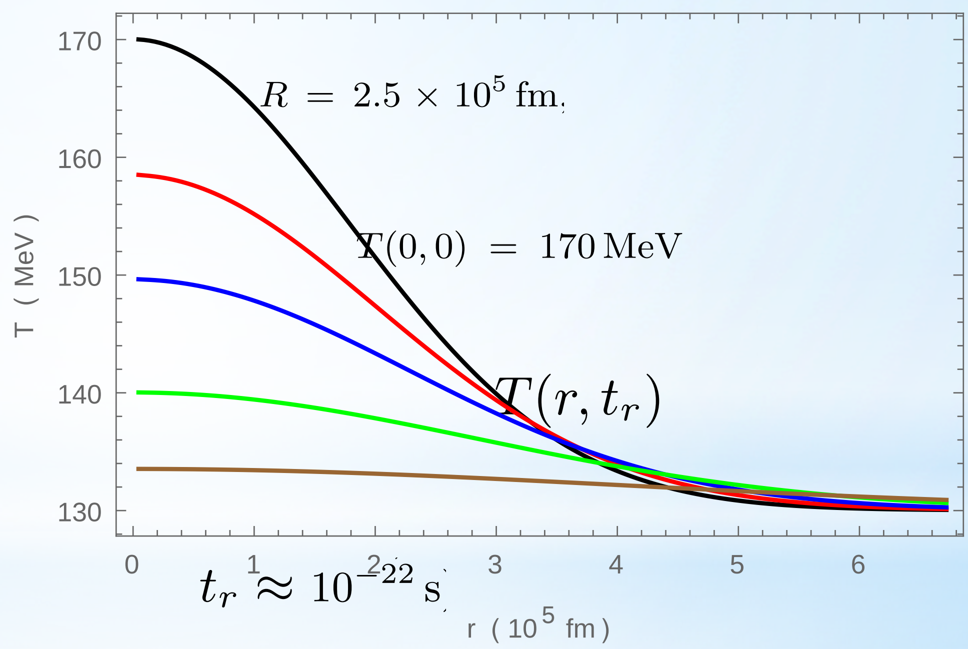
$$m_\pi \simeq f_\pi$$

$\Delta S_V / S_{back}$

$\Delta S_V / S_{back}$ (in units of 10^{-3})

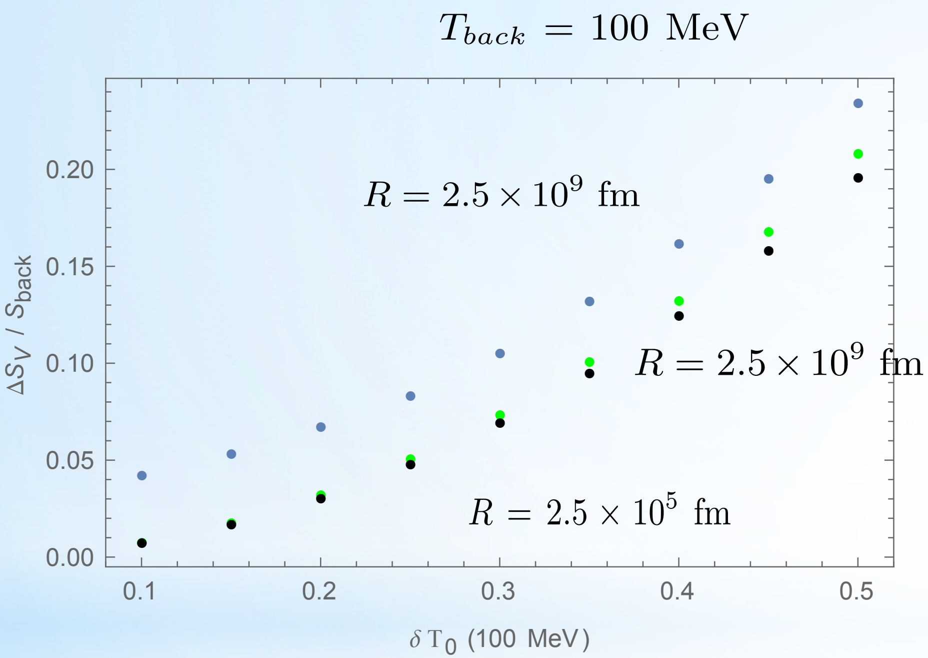
T_{back}	δT_0	R_1	R_2	R_3	R_4
138	35	32.6	35.1	35.1	46.1
	30	24.0	25.8	25.8	35.3
	25	16.6	17.9	17.9	28.5
	20	10.6	11.5	11.5	22.4
	15	5.98	6.44	6.44	17.7
	10	2.66	2.86	2.86	19.8
	5	0.7	0.7	0.7	11.4
100	50	196.0	208.4	208.4	234.4
	45	158.4	168.3	168.3	195.5
	40	124.7	132.4	132.4	162.0
	35	95.1	100.9	100.9	132.4
	30	69.6	73.7	73.7	105.5
	25	48.0	50.9	50.9	83.4
	20	30.5	32.3	32.3	67.3
	15	17.0	18.0	18.0	53.5
	10	7.5	7.9	7.9	42.4
	5	1.9	2.0	2.0	49.7

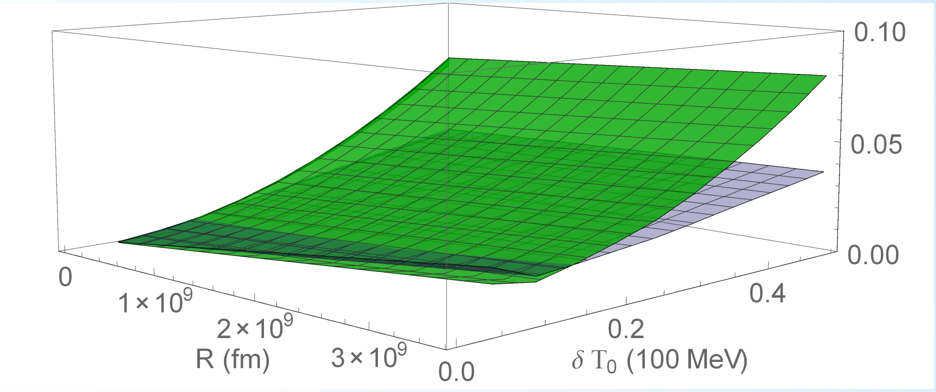
fm



MeV

$\Delta S_V / S_{back}$





Summary and open questions

After the QCD phase transition there was a small period of time where pions carried an important fraction of the entropy of the universe (more than photons!)

That era lasted from T = 175 Mev to T = 80 MeV ($t = 10^{-6}$ s to 5×10^{-5} s)

In that period of time pions interactions dominated dissipative processes

The physics of this era is well known but not very well developed

As the density is relatively small kinetic theory can be applied

It could be used to study entropy production by dissipation and to set bounds on the fluctuation spectrum at the end of the QCD phase transition to make them compatible with CMB

More work is needed in that interesting direction

Thank you very much
for your attention

Brief thermal history of the Universe

event	time	z	T
Planck time graviton decoupling	10^{-43} s	10^{37}	10^{19} GeV (10^{31} K)
GUT/Inflation/baryogenesis	10^{-35} s	10^{32}	10^{14} GeV (10^{26} K)
EW unification	10^{-12} s	10^{21}	10^{3} GeV (10^{15} K)
Quark-hadron transition	10^{-6} s	10^{18}	1 GeV (10^{12} K)
Neutrino decoupling	1 s	10^{15}	1 MeV (10^{9} K)
e^+e^- annihilation	1 s	10^{15}	1 MeV (10^{9} K)
nucleosinthesis	1-100 s	10^{14}	0.1-1 MeV (10^{8}-10^{9} K)
Matter-radiation equality	10^{3} years	10^{4}	1 eV (10^{31} K)
recombination	10^{5} years	10^{3}	10^{-1} eV (10^{3} K)
photon decoupling	10^{5} years	10^{3}	10^{-1} eV (10^{3} K)

Quiral Perturbation Theory (Momentum and mass expansion)

$$t_{IJ} \simeq t_{IJ}^{(0)} + t_{IJ}^{(1)} + t_{IJ}^{(2)} + \ldots$$

$$\begin{aligned} \mathrm{Im} t_{IJ}^{(0)} &= 0 \\ \mathrm{Im} t_{IJ}^{(1)} &= \sigma_{\alpha\beta} t_{IJ}^{(0)2} \\ \mathrm{Im}(t_{IJ}^{(2)} + t_{IJ}^{(1)}) &= \sigma_{\alpha\beta}\left(t_{IJ}^{(0)2} + 2t_{IJ}^{(0)}\mathrm{Re}t_{IJ}^{(1)}\right) \simeq \sigma_{\alpha\beta} \mid t_{IJ}^{(0)} + t_{IJ}^{(1)} \mid^2 \end{aligned}$$

The Inverse Amplitude Method

$$t_{IJ}(s) = C_0 + C_1 s + C_2 s^2 + \frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty} \frac{\mathrm{Im} t_{IJ}(s')ds'}{s'^3(s'-s-i\epsilon)} + LC(t_{IJ})$$

$$\begin{aligned} t_{IJ}^{(0)} &= a_0 + a_1 s \\ t_{IJ}^{(1)} &= b_0 + b_1 s + b_2 s^2 + \frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty} \frac{\mathrm{Im} t_{IJ}^{(1)}(s')ds'}{s'^3(s'-s-i\epsilon)} + LC(t_{IJ}^{(1)}) \end{aligned}$$

Dobado and Peláez

$$G(s) = t_{IJ}^{(0)2}/t_{IJ}$$

$$\mathrm{Im}G = -t_{IJ}^{(0)2}\frac{\mathrm{Im}t_{IJ}}{\mid t_{IJ}\mid^2} = -t_{IJ}^{(0)2}\sigma = -\mathrm{Im}t_{IJ}^{(1)}$$

$$G(s) = G_0 + G_1 s + G_2 s^2 + \frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty} \frac{\mathrm{Im}G(s')ds'}{s'^3(s'-s-i\epsilon)} + LC(G) + PC$$

$$\begin{aligned} \frac{t_{IJ}^{(0)2}}{t_{IJ}} &\simeq a_0 + a_1 s - b_0 - b_1 s - b_2 s^2 \\ &- \frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty} \frac{\mathrm{Im} t_{IJ}^{(1)}(s')ds'}{s'^3(s'-s-i\epsilon)} - LC(t_{IJ}^{(1)}) + PC \simeq t_{IJ}^{(0)} - t_{IJ}^{(1)} \end{aligned}$$

$$t_{IJ} \simeq \frac{t_{IJ}^{(0)2}}{t_{IJ}^{(0)} - t_{IJ}^{(1)}}$$

$$\mathrm{Im}t_{IJ} = \sigma_{\alpha\beta} \mid t_{IJ}\mid^2$$

The Inverse Amplitude Method

Lowest order ChPT (Weinberg Theorems) is only valid at very low energies.

However second order ChPT suplemented with Dispersion Relations (the Inverse amplitude method) makes it possible a simultaneous description of ππ→ππ and πK→πK up to 800-1000 MeV including resonances