

Baryon states with open/hidden charm in the extended local hidden gauge approach

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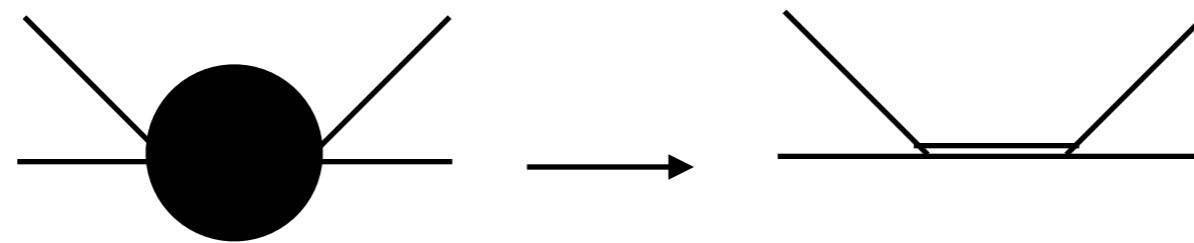
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W. H. Liang, T. Uchino, C. W. Xiao and E. Oset, Eur. Phys. J. A 51 (2015) 2, 16
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Baryon states with open/hidden charm

Negative parity open/hidden charm baryons are studied at the viewpoint of meson-baryon molecules.



Meson-Baryon:
with c (open charm)
with ccbar (hidden charm)

Baryon states:
 Λc^* , Σc^* (open charm)
 N^* with ccbar (hidden charm)

open charm baryon with $I=0, 1$
hidden charm baryon with $I=1/2$

As a negative parity open charm baryon, the two Λ_c states have been experimentally observed:

$\Lambda_c(2595)$

$J^P=1/2^-$
mass: ~2592 MeV
width: ~2.6 MeV

$\Lambda_c(2625)$

$J^P=3/2^-$
mass: ~2628 MeV
width: < 0.97 MeV

Generated states in several works (of hidden charm)

In addition to the two Lambda_c states, many states have been predicted in the open/hidden charm sector

Ref.	Model	M_R [MeV]	$N(1/2^-)$ g to main channels						$N(3/2^-)$ g to main channels				$N(5/2^-)$ g					
			$\Lambda_c \bar{D}$	$\Lambda_c \bar{D}^*$	$\Sigma_c \bar{D}$	$\Sigma_c \bar{D}^*$	$\Sigma_c^* \bar{D}^*$	M_R [MeV]	$\Lambda_c \bar{D}^*$	$\Sigma_c^* \bar{D}$	$\Sigma_c \bar{D}^*$	$\Sigma_c^* \bar{D}^*$	M_R [MeV]	$\Sigma_c^* \bar{D}^*$				
This work:	$(\mathbf{8}_2)_{2,0} \subset \mathbf{70}_{2,0}$	3918	3.1	0.5	0.2	2.6	2.6	3946	3.4	3.6	1.1	1.5	4027	5.6				
		3926	0.4	3.0	4.2	0.2	0.7											
	$(\mathbf{8}_4)_{2,0} \subset \mathbf{70}_{2,0}$	3974	0.4	2.2	2.1	3.4	3.1	3987	1.0	2.7	4.3	1.8	4006	1.0	1.6	3.2	4.2	4027
[65,66]	zero range vector exchange	3520			5.3			3430			5.6							
[75]	hidden gauge	4265	0.1		3.0			4415	0.1	2.8								
[76]	hidden gauge	4315	\times		\times													
[119]	quark model $uudc\bar{c}$	FS-CM 3933–4267						FS-CM 4013–4389					FS-CM 4119–4445					
		4013–4363						4136–4476					4156–4541	4236–4526	4236–4616			
		4119–4377																
		4136–4471																
		4156–4541																

Meson-baryon scattering

Within a unitary coupled channels approach, s-wave scattering amplitudes are given by

$$T = V + VGT = \frac{V}{1 - VG}$$

with the G function regularized in the cutoff method

$$G_l(\sqrt{s}) = \int_{|\vec{q}| \leq q_{\max}} \frac{d^3 q}{(2\pi)^3} \frac{\omega_l + E_l}{2\omega_l E_l} \frac{2M_l}{P^{02} - (\omega_l + E_l)^2 + i\epsilon}$$

The analysis of generated states

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_B} \longrightarrow \lim_{\sqrt{s} \rightarrow M_B} (\sqrt{s} - M_B) T_{ij}(\sqrt{s}) = g_i g_j \longrightarrow g_i G_i(M_B)$$

Wave function at the origin

*The value gG at the energy of generated states provides the wave function at the origin [1]

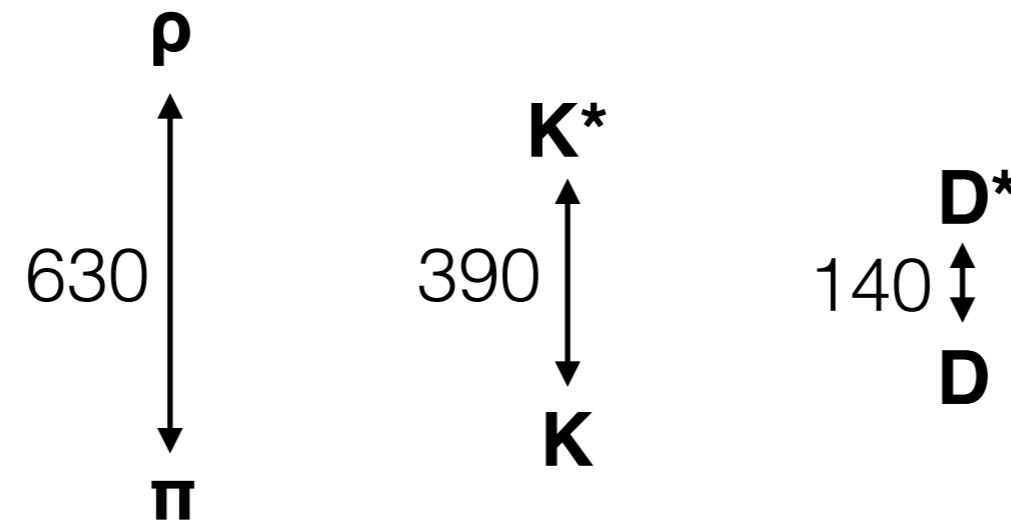
-> evaluate the component of the generated states in the coupled channel

To take into account pseudoscalar-baryon (PB) and vector-baryon (VB) sectors consistently
→Interaction V: based on the local hidden gauge approach [2,3,4]

- [1] D. Gamermann, J. Nieves, E. Oset and E. Ruiz Arriola, Phys. Rev. D 81, 014029 (2010)
- [2] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. 54, 1215 (1985)
- [3] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988)
- [4] U. G. Meissner, Phys. Rept. 161, 213 (1988)

PB-VB mixing

Both PB and VB sectors are considered →
PB-VB mixing: found to be important (even in the lighter sectors) [4,5]



mass difference between $P(D)$ and $V(D^*)$ is small in the charm sector
-> box diagram corrections
-> full coupled channels approach between PB and VB sectors

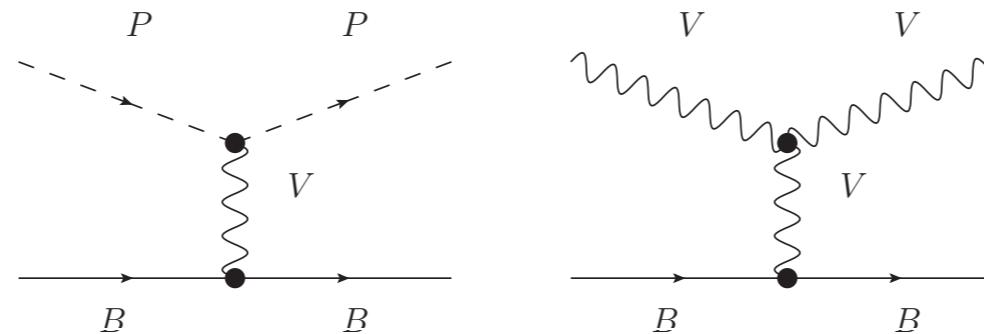
[4] K. P. Khemchandani, A. Martinez Torres, H. Kaneko, H. Nagahiro and A. Hosaka, Phys. Rev. D 84, 094018 (2011)
[5] E. J. Garzon and E. Oset, Phys. Rev. C 91, 025201 (2015)

Formalism

Vector exchange driving force

Relevant vertices are provided by the local hidden gauge approach extended to SU(4) [6]

$$\mathcal{L}_{PPV} = -ig \langle V^\mu [P, \partial_\mu P] \rangle$$



$$\mathcal{L}_{VVV} = ig \langle V^\mu [V^\nu, \partial_\mu V_\nu] \rangle$$

P, V, B: pseudoscalar, vector, baryon fields matrix

$$g = M_V / 2f_\pi$$

→Weinberg-Tomozawa (WT) interaction

$$V_{ij} = -\frac{C_{ij}}{4f^2} (2\sqrt{s} - M_{B_i} - M_{B_j}) \sqrt{\frac{M_{B_i} + E_i}{2M_{B_i}}} \sqrt{\frac{M_{B_j} + E_j}{2M_{B_j}}}$$

- **Heavy Quark Spin Symmetry (HQSS) is automatically fulfilled**
- **VB interaction has spin degeneracy [7]: spin degenerated states**

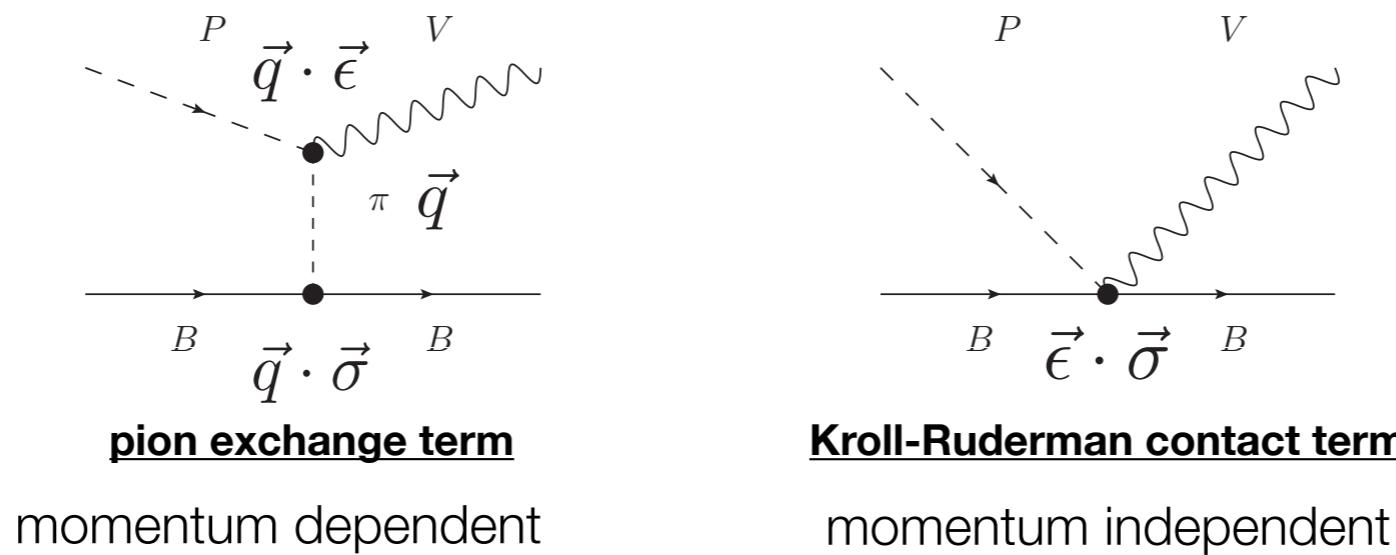
[6] J. -J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010)

[7] E. Oset and A. Ramos Eur. Phys. J. A 44, 445–454 (2010)

PB-VB mixing - pion exchange and contact term

Following the local hidden gauge approach, we can have PB-VB mixing terms [8]

VPP vertices are given by the local hidden gauge: pion exchange term
Constraint of gauge invariance: contact term



* We apply this mixing only to the D and D* mixing

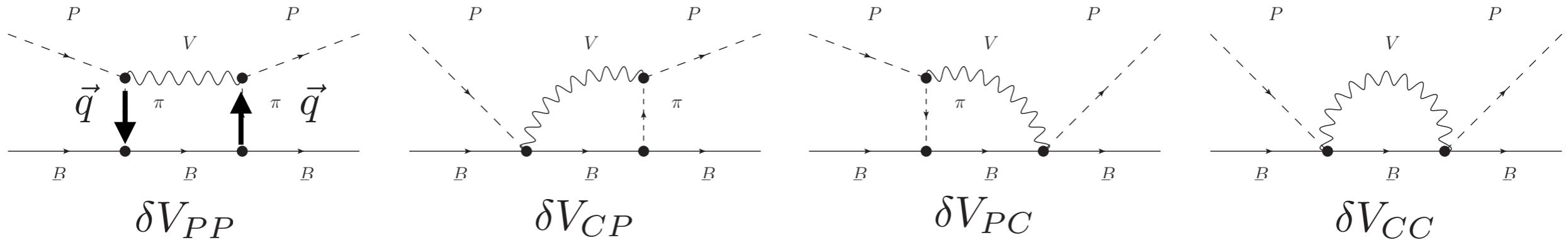
**PB-VB mixing intermediated by a pion depends on the momentum transfer
→ the box diagram**

Box potentials

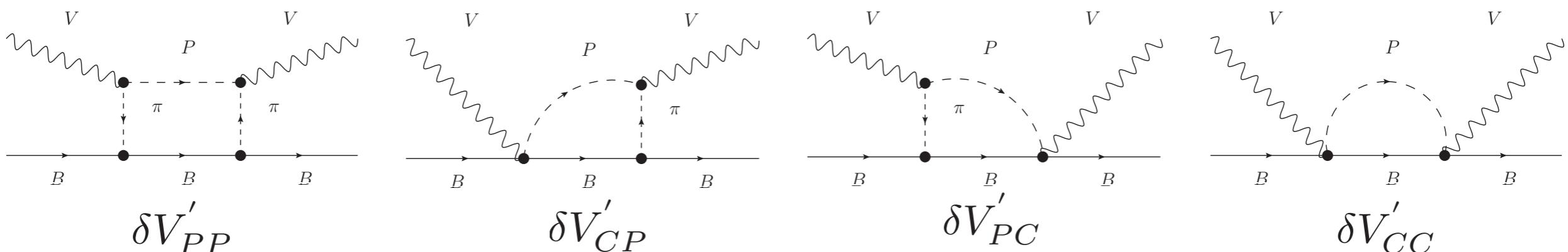
Momentum transfer q is integrated out in the box diagrams.

These potentials are now momentum independent and added to the Weinberg-Tomozawa terms.

PB - VB - PB box



VB - PB - VB box



PP box: intermediate states are not necessarily only in the s-wave

CP, PC, PP boxes: intermediate states are in the s-wave

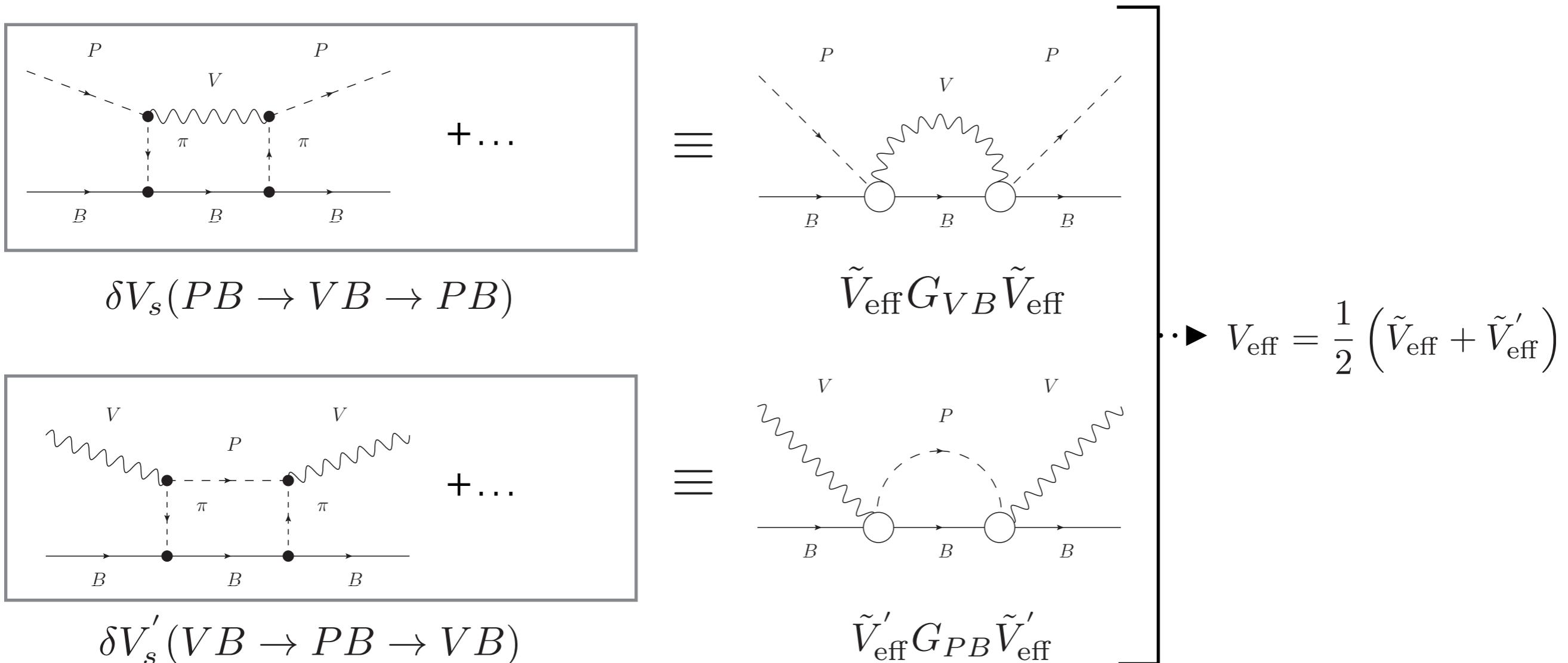
→ These box potentials resolve the spin degeneracy of the VB sectors

DN intermediate state in the D^*N ($J=1/2$) box is in the s-wave

DN intermediate state in the D^*N ($J=3/2$) box is in the d-wave

Effective transition potential

In order to fully take into account the PB-VB mixing, one step further, we implement the full coupled channels calculation by constructing **the effective PB-VB transition potential** [9]



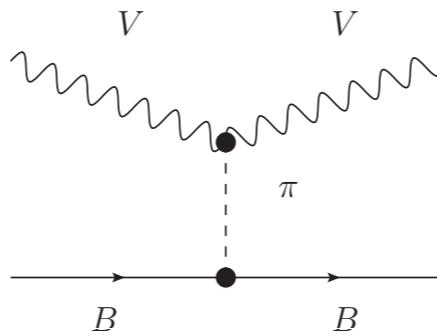
- **s-wave** components: used to construct **the effective potentials**
- **d-wave** (and other) components: still kept in **the box potentials**

Box diagram potential - Anomalous VVP term

Anomalous VVP vertex also provides the VB-VB potential via pion exchange [10,11]

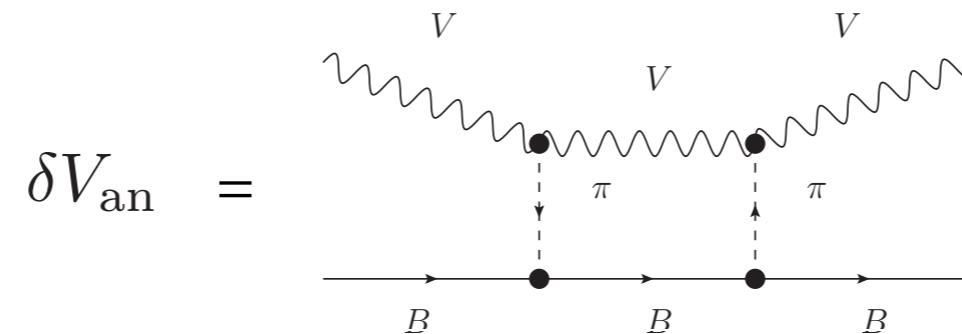
$$\mathcal{L}_{VVP} = \frac{G}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \partial_\mu V_\nu \partial_\alpha V_\beta \rangle$$

$$G = 3M_V^2/16\pi^2 f_\pi^2 \sim 14 \text{GeV}^{-1}$$



This VB - VB potential does not interfere with the WT interaction at tree level

→ These contributions are taken into account as box diagrams



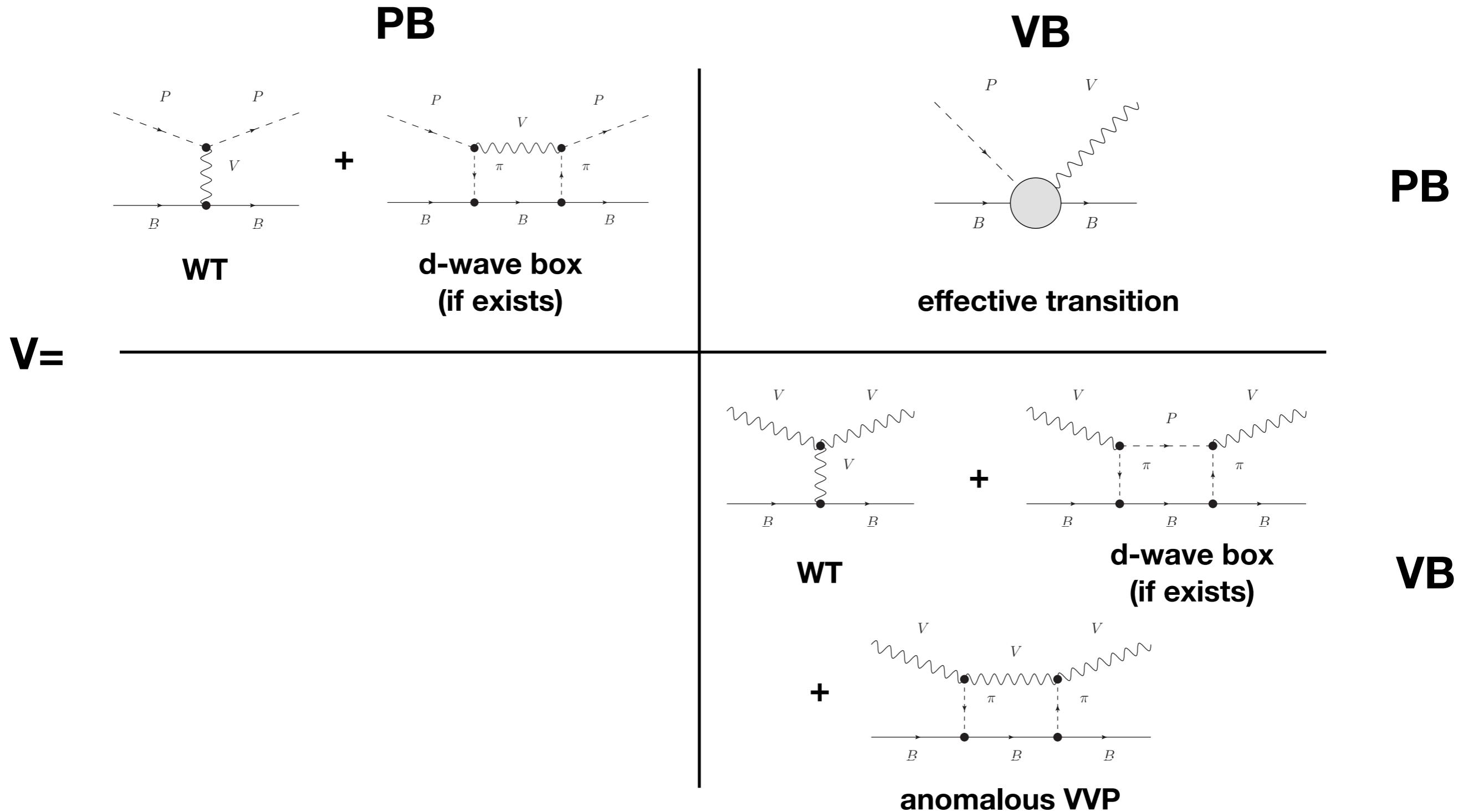
These box potentials just give extra contributions (attractions) only to VB potentials

*An anomalous process, like the V V P interaction, is one that does not conserve “natural” parity. The “natural” parity of a particle is defined as follows: it is +1 if the particle transforms as a true tensor of that rank, and -1 if it transforms as a pseudotensor, e.g. π , γ , ρ and a_1 have “natural” parity -1, +1, +1 and -1, respectively.

[10] A. Bramon, A. Grau and G. Pancheri, Phys. Lett. B 344, 240 (1995)

[11] E. Oset, J. R. Pelaez and L. Roca, Phys. Rev. D 67, 073013 (2003)

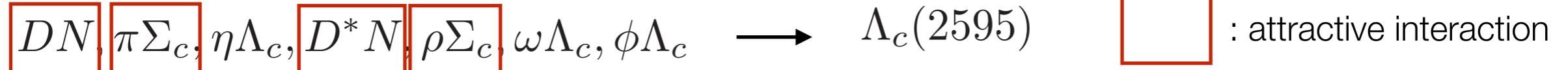
Summary of the full coupled channels approach



Open charm sector

The open charm baryons in the l=0 sector

J=1/2 (full coupled channels):



J=3/2 (VB sector with DN d-wave):



J=3/2 ($\pi\Sigma_c^*$):



* This channel is almost decoupled to other channels (D meson exchange)

Three cutoff parameters are used:

$q_{\max_P} \rightarrow$ PB G function

$q_{\max_V} \rightarrow$ VB G function

$q_{\max_B} \rightarrow$ all the box potentials

	set I	set II	set III
q_{\max}^B	600	800	1000
q_{\max}^V	771	737	715
q_{\max}^P	527	500	483

The three sets are used and all parameters are chosen to reproduce two Lambda_c states

Generated states in the l=0 sector

J=1/2 (full coupled):

	q_{max}^B	600	800	1000
	q_{max}^V	771	737	715
	q_{max}^P	527	500	483

●	$2592.26 + i0.56$	$2592.24 + i0.55$	$2592.14 + i0.52$
	$2610.44 + i48.68$	$2611.06 + i53.35$	$2611.32 + i56.28$
	$2757.25 + i1.20$	$2767.14 + i0.98$	$2772.41 + i0.86$
	$2970.01 + i0.45$	$2990.78 + i0.60$	$3004.54 + i0.64$

Lambda_c(2595) and 3 states

J=3/2 (VB sector with d-wave DN):

	q_{max}^B	600	800	1000
	q_{max}^V	771	737	715

●	2628.45	2628.35	2628.27
	$2969.64 + i0.91$	$2990.43 + i0.81$	$3004.21 + i0.77$

Lambda_c(2625) and 1 state

J=3/2 ($\pi\Sigma_c^*$):

q_{max}^P	527	500	483
	$2668.92 + i131.63$	$2665.81 + i136.10$	$2664.89 + i138.83$

All states except two Lambda_c states, have the theoretical uncertainty from cutoff, but within ± 20 MeV

Evaluation of generated states in the $|l=0$ sector

J=1/2 (full coupled):

2592.26 + $i0.56$	DN	$\pi\Sigma_c$	$\eta\Lambda_c$	
g_i	-8.18 + i0.61	0.54 + i0.00	-0.40 - $i0.03$	
$g_i G_i^{II}$	13.88 - i1.06	-10.30 - i0.69	1.76 - $i0.14$	
	D^*N	$\rho\Sigma_c$	$\omega\Lambda_c$	$\phi\Lambda_c$
g_i	9.81 + i0.77	-0.45 - $i0.04$	0.42 + $i0.03$	-0.59 - $i0.05$
$g_i G_i^{II}$	-26.51 - i2.10	2.07 + $i0.17$	-2.31 - $i0.19$	2.10 + $i0.17$
2611.06 + $i53.35$	DN	$\pi\Sigma_c$	$\eta\Lambda_c$	
g_i	0.08 - $i1.81$	1.78 + i1.40	0.03 - $i0.09$	
$g_i G_i^{II}$	-0.68 + $i3.13$	-55.22 - i18.22	-0.18 + $i0.39$	
	D^*N	$\rho\Sigma_c$	$\omega\Lambda_c$	$\phi\Lambda_c$
g_i	-1.56 + $i1.38$	0.09 - $i0.05$	-0.08 + $i0.05$	0.11 - $i0.07$
$g_i G_i^{II}$	4.66 - $i3.42$	-0.44 + $i0.20$	0.46 - $i0.25$	-0.42 + $i0.24$
2767.14 + $i0.98$	DN	$\pi\Sigma_c$	$\eta\Lambda_c$	
g_i	-3.70 + i0.04	0.02 - $i0.20$	-0.52 + $i0.00$	
$g_i G_i^{II}$	14.78 - i0.05	3.54 + $i2.76$	4.40 + $i0.02$	
	D^*N	$\rho\Sigma_c$	$\omega\Lambda_c$	$\phi\Lambda_c$
g_i	-3.97 + i0.05	0.47 - $i0.00$	-0.30 + $i0.00$	0.43 - $i0.00$
$g_i G_i^{II}$	15.47 - i0.16	-2.62 + $i0.01$	2.16 - $i0.02$	-1.82 + $i0.02$
2990.78 + $i0.60$	DN	$\pi\Sigma_c$	$\eta\Lambda_c$	
g_i	0.01 + $i0.00$	0.00 + $i0.00$	-0.00 - $i0.01$	
$g_i G_i^{II}$	0.09 + $i0.14$	0.01 + $i0.03$	0.16 - $i0.08$	
	D^*N	$\rho\Sigma_c$	$\omega\Lambda_c$	$\phi\Lambda_c$
g_i	-0.09 - $i0.11$	-5.44 - i0.02	-0.04 - $i0.01$	0.05 + $i0.02$
$g_i G_i^{II}$	1.57 + $i0.59$	44.54 + i0.20	0.50 + $i0.19$	-0.32 - $i0.12$

J=3/2 (VB sector):

2628.35	D^*N	$\rho\Sigma_c$	$\omega\Lambda_c$	$\phi\Lambda_c$
g_i	10.11	-0.55	0.49	-0.68
$g_i G_i^{II}$	-29.10	2.60	-2.78	2.50
2990.43 + $i0.81$	D^*N	$\rho\Sigma_c$	$\omega\Lambda_c$	$\phi\Lambda_c$
g_i	0.06 + $i0.11$	5.44 + i0.01	0.03 + $i0.02$	-0.04 - $i0.03$
$g_i G_i^{II}$	-1.23 - $i0.79$	-44.53 - i0.15	-0.39 - $i0.25$	0.25 + $i0.16$

- $\Lambda_c(2595)$ is dominated by DN and D^*N
- The state ($E=2767$ MeV, $J=1/2$) is orthogonal to $\Lambda_c(2595)$
- $\Lambda_c(2625)$ is dominated by D^*N
- The state of $\rho\Sigma_c$ is almost spin degenerated

The open charm baryons in the $l=1$ sector

The relevant box diagram is very small in the $l=1$ sector and thus we do not consider the PB-VB mixing. Neither anomalous VB term.

$$\delta V_{\text{box}}(I=1) = \frac{1}{9} \delta V_{\text{box}}(I=0)$$

J=1/2 (PB sector):

DN , $\pi\Sigma_c$, $\pi\Lambda_c$, $\eta\Sigma_c$

q_{max}^P	527	500	483
	$2673.14 + i51.55$	$2673.95 + i55.63$	$2674.34 + i58.25$

$2665.81 + i136.10$	DN	$\pi\Sigma_c$	$\pi\Lambda_c$	$\eta\Sigma_c$
g_i	$-1.07 - i0.58$	$1.55 + i1.55$	$0.02 + i0.04$	$0.02 + i0.04$
$g_i G_i^{II}$	$1.26 + i1.92$	$-72.39 - i15.30$	$-1.57 - i0.10$	$-0.02 - i0.13$

* DN state is a virtual state

J=1/2, 3/2 (VB sector):

D^*N , $\rho\Sigma_c$, $\omega\Lambda_c$, $\phi\Lambda_c$

q_{max}^V	771	737	715
	$2922.09 + i0$	$2928.85 + i0$	$2932.66 + i0$
	$3133.27 + i4.07$	$3145.71 + i3.57$	$3153.51 + i3.22$

$2928.85 + i0$	D^*N	$\rho\Sigma_c$	$\rho\Lambda_c$	$\omega\Sigma_c$	$\phi\Sigma_c$
g_i	3.43	-0.79	-0.63	-0.34	0.48
$g_i G_i^{II}$	-27.18	5.69	6.72	2.40	-2.03
$3145.71 + i3.57$	D^*N	$\rho\Sigma_c$	$\rho\Lambda_c$	$\omega\Sigma_c$	$\phi\Sigma_c$
g_i	$-0.13 + i0.47$	$3.66 - i0.08$	$-0.15 - i0.10$	$-0.08 - i0.05$	$0.11 + i0.07$
$g_i G_i^{II}$	$-5.36 - i3.42$	$-47.40 + i0.34$	$4.89 - i0.81$	$1.02 + i0.67$	$-0.67 - i0.44$

Summary of the results of the open charm baryon

main channel	J	I	(E, Γ) [MeV]	Exp.	main decay channels
$\frac{1}{\sqrt{2}}(DN + D^*N)$, $\pi\Sigma_c$	1/2	0	2592, 1	$\Lambda_c(2595)$	$\pi\Sigma_c$
$\pi\Sigma_c$	1/2	0	2611, 106	-	$\pi\Sigma_c$
$\frac{1}{\sqrt{2}}(DN - D^*N)$	1/2	0	2767, 2	-	$\pi\Sigma_c$
D^*N	3/2	0	2628, 0	$\Lambda_c(2625)$	-
$\pi\Sigma_c^*$	3/2	0	2674, 111	-	$\pi\Sigma_c^*$
$\rho\Sigma_c$	1/2, 3/2	0	2990, 2	$\Lambda_c(2940)?$	D^*N
$\pi\Sigma_c$	1/2	1	2666, 272	-	$\pi\Sigma_c$, $\pi\Lambda_c$
D^*N	1/2, 3/2	1	2928, 0	-	-
$\rho\Sigma_c$	1/2, 3/2	1	3146, 7	-	D^*N , $\rho\Lambda_c$

- Six state ($I=0$) and three states ($I=1$) are observed
- $\Lambda_c(2595)$ and $\Lambda_c(2625)$ are reproduced dominated by DN and D^*N
- The state (2767 MeV, $J=1/2$) is an orthogonal state to $\Lambda_c(2592)$
- $\Lambda_c(2940)$ can be a negative parity state
- $\Sigma_c(2800)$ can be a positive parity state

Hidden charm sector

Channels of the hidden charm sector

In the previous work [6], the same sectors were investigated with the same WT potential. As a result, it was found that the coupling to the lighter sector, such as πN , $J/\psi N$ and so on, simply add an extra width ~ 30 MeV but do not modify the energy of states

→

We consider only channels which is formed with one anti-charmed meson and one charmed baryon

Σ_c and Σ_c^* mixing is found to be small with the quark model evaluation

J=1/2 (full coupled channels):

$\bar{D}\Sigma_c$, $(\bar{D}\Lambda_c)$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Lambda_c$

J=3/2 (VB sector with D^* bar-baryon d-wave):

$\bar{D}^*\Sigma_c$, $\bar{D}^*\Lambda_c$

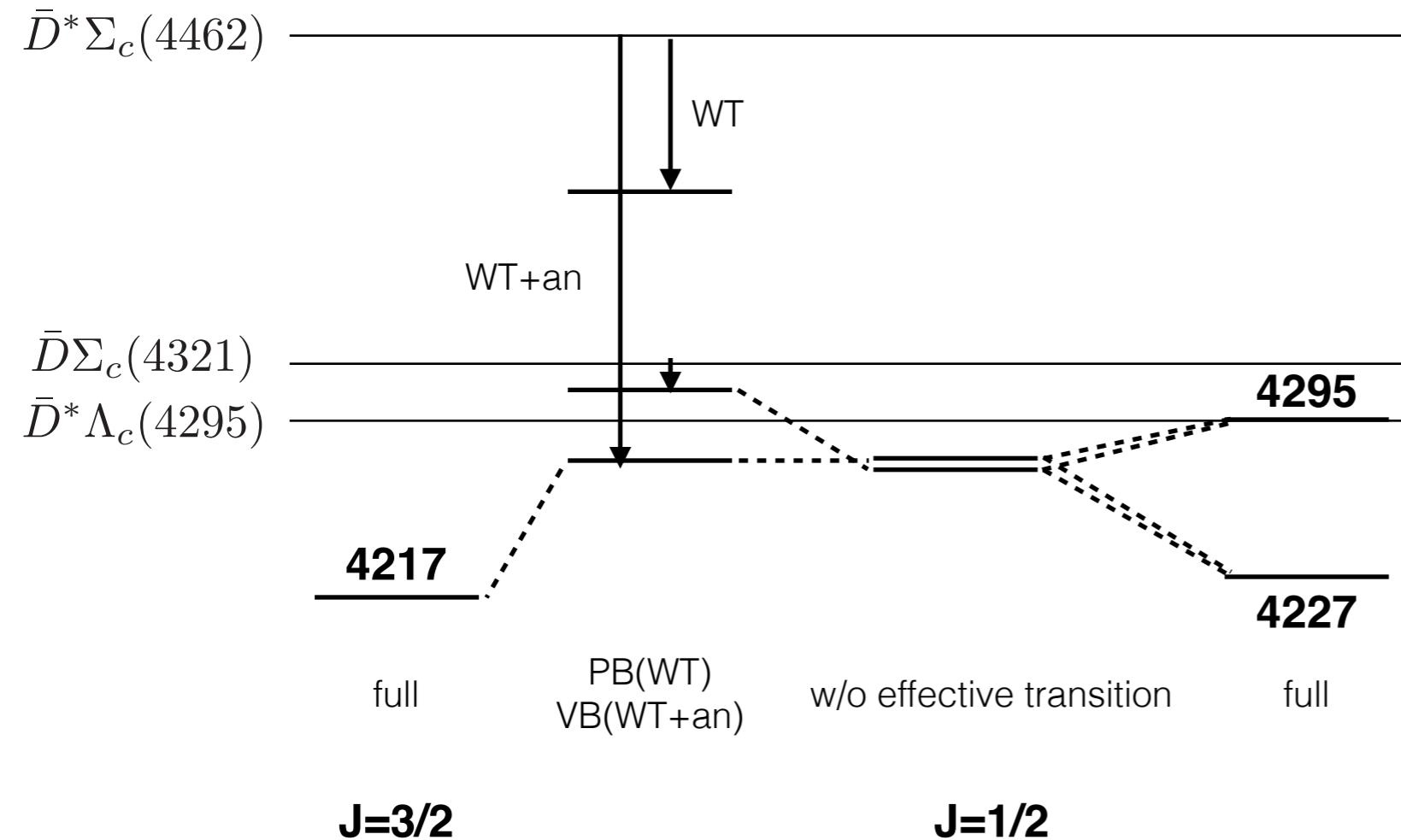
J=1/2, 5/2 (single channel):

$\bar{D}^*\Sigma_c^*$

J=3/2 (full coupled channels):

$\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c^*$

Results I: states generated by $\bar{D}\Sigma_c$ $\bar{D}^*\Lambda_c$ $\bar{D}^*\Sigma_c$



coupling constants and wave functions at the origin

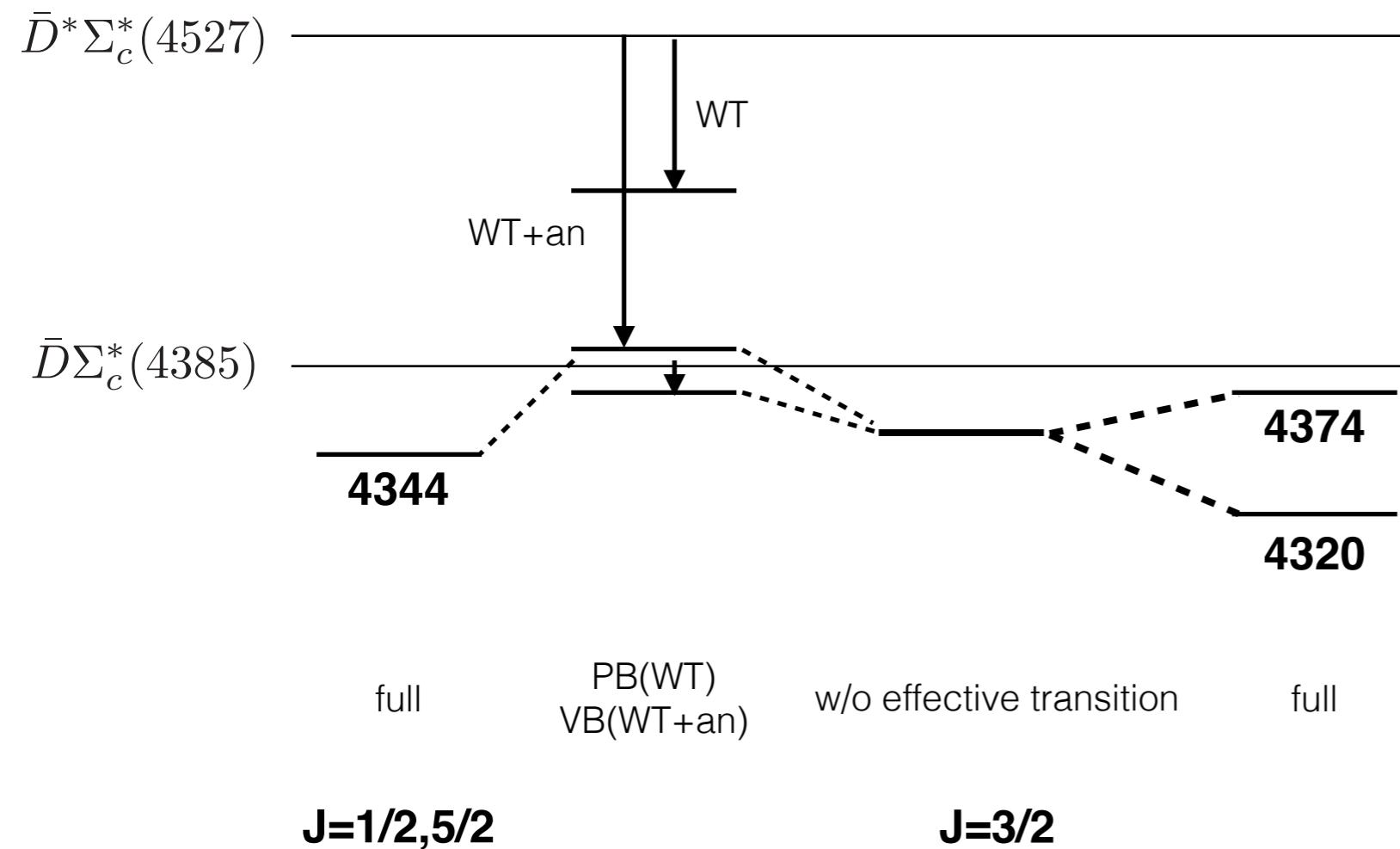
	$\bar{D}\Sigma_c$	$\bar{D}^*\Sigma_c$	$\bar{D}^*\Lambda_c$
$(4227.6, 21.1)$			
g_i	4.40	5.39	0.39
$g_i G_i$	-15.66	-24.17	-3.31
$(4295.1, 10.6)$	$\bar{D}\Sigma_c$	$\bar{D}^*\Sigma_c$	$\bar{D}^*\Lambda_c$
g_i	-1.27	2.28	-0.11
$g_i G_i$	8.46	-12.60	$2.09 + i0.04$

Two orthogonal states emerge

$$\sim \frac{1}{\sqrt{2}} (|\bar{D}^*\Sigma_c\rangle \pm |\bar{D}\Sigma_c\rangle)$$

- One VB bound state and two PB-VB admixture states appear
- In both admixture states, VB component is stronger than PB component
- Two admixture states are orthogonal: the relative sign of gG
- In the coupled channel, without the transition potential, energy of the two states are very close
- The mass difference between two orthogonal states is ~ 60 MeV

Results II: states generated by $\bar{D}\Sigma_c^*$ $\bar{D}^*\Sigma_c^*$



coupling constants and wave functions at the origin

	$\bar{D}\Sigma_c^*$	$\bar{D}^*\Sigma_c^*$
4324.85		
g_i	3.61	4.89
$g_i G_i^{II}$	-16.54	-24.22
4378.84	$\bar{D}\Sigma_c^*$	$\bar{D}^*\Sigma_c^*$
g_i	-1.25	3.00
$g_i G_i^{II}$	12.30	-17.57

Two orthogonal states emerge

$$\sim \frac{1}{\sqrt{2}} (|\bar{D}^*\Sigma_c^*\rangle \pm |\bar{D}\Sigma_c^*\rangle)$$

J=1/2,5/2

J=3/2

- One VB bound state and two PB-VB admixture states appear
- In both admixture states, VB component is stronger than PB component
- Two admixture states are orthogonal: the relative sign of gG
- In the coupled channel, without the transition potential, energy of the two states are very close
- The mass difference between two orthogonal states is ~ 60 MeV

Summary of the results of the hidden charm baryon

main channel	J	(E, Γ) [MeV]	main decay channels
$\frac{1}{\sqrt{2}}(\bar{D}^*\Sigma_c + \bar{D}\Sigma_c)$	1/2	4228, 21(51)	$\bar{D}\Lambda_c$
$\frac{1}{\sqrt{2}}(\bar{D}^*\Sigma_c - \bar{D}\Sigma_c)$	1/2	4295, 11(41)	$\bar{D}\Lambda_c$
$\bar{D}^*\Sigma_c$	3/2	4218, 103	$\bar{D}\Lambda_c$
$\bar{D}^*\Sigma_c^*$	1/2, 5/2	4344, 0	—
$\frac{1}{\sqrt{2}}(\bar{D}^*\Sigma_c^* + \bar{D}\Sigma_c^*)$	3/2	4325, 0	—
$\frac{1}{\sqrt{2}}(\bar{D}^*\Sigma_c^* - \bar{D}\Sigma_c^*)$	3/2	4378, 0	—

- Three state (with Σ_c) and three states (with Σ_c^*) are observed
- Two pair of the orthogonal states are observed
- The numbers in the parenthesis are obtained width with an extra 30 MeV width from the lighter sector

Summary of this open/hidden charm baryon study

- Negative parity open/hidden charm baryons are studied at the viewpoint of the meson-baryon molecule.
- Based on the local hidden gauge approach extended to SU(4), we have the WT term and the PB-VB mixing term. In addition, we also consider the extra VB potential from the anomalous WP interaction.
- The full coupled channels calculation is implemented with the effective transition potential from the relevant parts of the PB-VB mixing term.
- The two states corresponding to $\Lambda_c(2595)$ and $\Lambda_c(2625)$ are observed.
- From the evaluation of the wave function at the origin, it is found that several pairs of states emerge as orthogonal states.

Thank you!

Backup slides

Coefficients of WT term in the open charm sector

|=0

C_{ij}	DN	$\pi\Sigma_c$	$\eta\Lambda_c$	C_{ij}	D^*N	$\rho\Sigma_c$	$\omega\Lambda_c$	$\phi\Lambda_c$	C_{ij}	$\pi\Sigma_c^*$
DN	3	$-\sqrt{\frac{3}{2}}$	$\frac{3}{\sqrt{2}}$	D^*N	3	$-\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}$	$-\sqrt{3}$	$\pi\Sigma_c^*$	4
$\pi\Sigma_c$		4	0	$\rho\Sigma_c$	4	0	0	0		
$\eta\Lambda_c$			0	$\omega\Lambda_c$		0	0	0		

|=1

C_{ij}	DN	$\pi\Sigma_c$	$\pi\Lambda_c$	$\eta\Sigma_c$	C_{ij}	D^*N	$\rho\Sigma_c$	$\rho\Lambda_c$	$\omega\Sigma_c$	$\phi\Sigma_c$
DN	1	-1	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{3}{2}}$	D^*N	1	-1	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{1}{2}}$	1
$\pi\Sigma_c$		2	0	0	$\rho\Sigma_c$	2	0	0	0	0
$\pi\Lambda_c$			0	0	$\rho\Lambda_c$		0	0	0	0
$\eta\Sigma_c$				0	$\omega\Sigma_c$			0	0	0
					$\phi\Sigma_c$					0

Full coupled channels I: $\bar{D}\Sigma_c$ $\bar{D}^*\Lambda_c$ $\bar{D}^*\Sigma_c$

J=1/2: coupled channels

$$V = \left(\begin{array}{c|ccc} & \bar{D}\Sigma_c & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline \bar{D}\Sigma_c & V_{WT} & 0 & 0 \\ \bar{D}^*\Sigma_c & & V_{WT} & 0 \\ \bar{D}^*\Lambda_c & & & V_{WT} \end{array} \right) + \left(\begin{array}{c|cccc} & \bar{D}\Sigma_c & & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline \bar{D}\Sigma_c & \delta V_d(\bar{D}^*\Sigma_c) + \delta V_d(\bar{D}^*\Lambda_c) & V_{\text{eff}} & V_{\text{eff}} \\ \bar{D}^*\Sigma_c & & 0 & 0 \\ \bar{D}^*\Lambda_c & & & 0 \end{array} \right)$$

$$+ \left(\begin{array}{c|ccc} & \bar{D}\Sigma_c & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline \bar{D}\Sigma_c & 0 & 0 & 0 \\ \bar{D}^*\Sigma_c & & \delta V_{\text{an}}(\bar{D}^*\Sigma_c) + \delta V_{\text{an}}(\bar{D}^*\Lambda_c) & 0 \\ \bar{D}^*\Lambda_c & & & \delta V_{\text{an}}(\bar{D}^*\Sigma_c) \end{array} \right)$$

d-wave box potentials are added only to PB sector

J=3/2: two single channels

$$V = \left(\begin{array}{c|cc} & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline \bar{D}^*\Sigma_c & V_{WT} & 0 \\ \bar{D}^*\Lambda_c & & V_{WT} \end{array} \right) + \left(\begin{array}{c|cc} & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline \bar{D}^*\Sigma_c & \delta V(\bar{D}\Sigma_c) + \delta V(\bar{D}\Lambda_c) & 0 \\ \bar{D}^*\Lambda_c & & \delta V(\bar{D}\Sigma_c) \end{array} \right)$$

$$+ \left(\begin{array}{c|cc} & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline \bar{D}^*\Sigma_c & \delta V_{\text{an}}(\bar{D}^*\Sigma_c) + \delta V_{\text{an}}(\bar{D}^*\Lambda_c) & 0 \\ \bar{D}^*\Lambda_c & & \delta V_{\text{an}}(\bar{D}^*\Sigma_c) \end{array} \right)$$

There is no transition between two channels \rightarrow two single channels

Full coupled channels II: $\bar{D}\Sigma_c^*$ $\bar{D}^*\Sigma_c^*$

J=1/2, 5/2: single channel

$$V = \left(\begin{array}{c|c} \bar{D}^*\Sigma_c^* & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}^*\Sigma_c^* & V_{WT} \end{array} \right) + \left(\begin{array}{c|c} \bar{D}^*\Sigma_c^* & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}^*\Sigma_c^* & \delta V(\bar{D}\Sigma_c^*) \end{array} \right) + \left(\begin{array}{c|c} \bar{D}^*\Sigma_c^* & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}^*\Sigma_c^* & \delta V_{\text{an}}(\bar{D}^*\Sigma_c^*) \end{array} \right)$$

J=3/2: coupled channels

$$V = \left(\begin{array}{c|cc} \bar{D}\Sigma_c^* & \bar{D}\Sigma_c^* & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}\Sigma_c^* & V_{WT} & 0 \\ \bar{D}^*\Sigma_c^* & V_{WT} & V_{WT} \end{array} \right) + \left(\begin{array}{c|cc} \bar{D}\Sigma_c^* & \bar{D}\Sigma_c^* & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}\Sigma_c^* & \delta V_{\neq s}(\bar{D}^*\Sigma_c^*) & V_{\text{eff}} \\ \bar{D}^*\Sigma_c^* & & \delta V_d(\bar{D}\Sigma_c^*) \end{array} \right) \\ + \left(\begin{array}{c|cc} \bar{D}\Sigma_c^* & \bar{D}\Sigma_c^* & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}\Sigma_c^* & 0 & 0 \\ \bar{D}^*\Sigma_c^* & & \delta V_{\text{an}}(\bar{D}^*\Sigma_c^*) \end{array} \right)$$

It is not simple to extract the s-wave component of the VB intermediate state

$$V_{\text{eff}} = \tilde{V}'_{\text{eff}} = \sqrt{\frac{\delta V'_s(\bar{D}\Sigma_c^*)}{G_{\bar{D}\Sigma_c^*}}} \longrightarrow \delta V_{\neq s}(\bar{D}^*\Sigma_c^*) = \delta V_{\text{total}}(\bar{D}^*\Sigma_c^*) - V_{\text{eff}} G_{\bar{D}^*\Sigma_c^*} V_{\text{eff}}$$

s-wave

Uncertainty from the cutoff

	set I	set II	set III
q_{\max}^B	600	800	1000
q_{\max}^V	771	737	715
q_{\max}^P	527	500	483

	set I	set II	set III
peak 1	4241.7	4227.6	4218.6
width 1	19.5	21.1	21.5
peak 2	4296.8	4295.1	4294.5
width 2	13.1	10.6	9.6

J=1/2 $\bar{D}\Sigma_c \bar{D}^*\Sigma_c \bar{D}^*\Lambda_c$

	set I	set II	set III
Pole	4354.5	4344.1	4337.5

J=1/2, 5/2 $\bar{D}^*\Sigma_c^*$

	set I	set II	set III
peak	4250.5	4217.7	4205.8
width	140.8	103.2	82.0

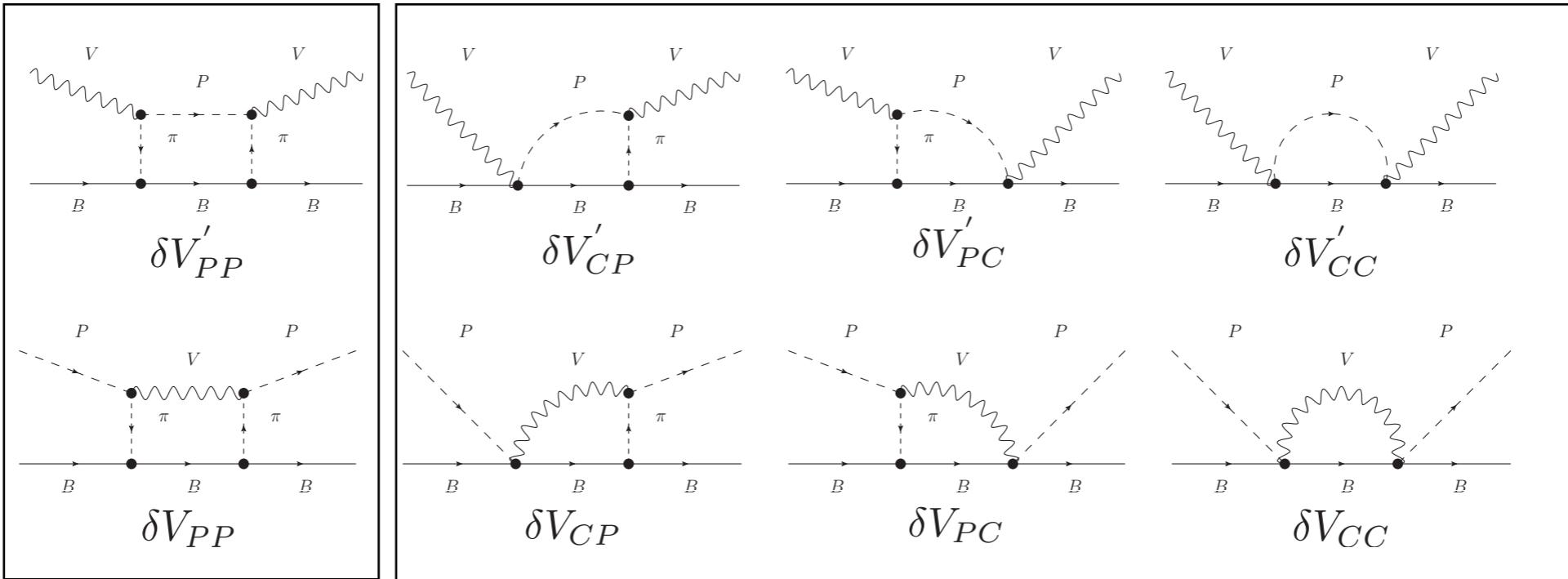
***J=3/2** $\bar{D}^*\Sigma_c \bar{D}^*\Lambda_c$

	set I	set II	set III
Pole 1	4330.6	4324.9	4319.9
Pole 2	4384.1	4377.8	4374.4

J=3/2 $\bar{D}\Sigma_c^* \bar{D}^*\Sigma_c^*$

Except the * sector, the uncertainty is $\sim \pm 10$ MeV

Extract the s-wave component from the box



intermediate states are
not necessarily in the s-wave

intermediate states are
always in the s-wave

When an intermediate state of a PP box has s- wave and d-wave components...

$$q_i q_j \rightarrow \frac{1}{3} q^2 \delta_{ij} \quad (\vec{\sigma} \cdot \vec{q}) (\vec{\epsilon} \cdot \vec{q}) \rightarrow \epsilon_i q_i \sigma_j q_j = \epsilon_i \sigma_j \left\{ \frac{1}{3} q^2 \delta_{ij} + \left(q_i q_j - \frac{1}{3} q^2 \delta_{ij} \right) \right\}$$

s-wave

$$\delta V_{PP} \rightarrow \left\{ \begin{array}{l} \delta V_{PP}^s = \frac{1}{3} \delta V_{PP} \\ \delta V_{PP}^d = \frac{2}{3} \delta V_{PP} \end{array} \right.$$

In the same manner, one can also decompose the the VB-PB-VB box potentials with the same factor, 1/3 and 3/2

Sign of the effective transition potential

From the definition, each ingredient of the effective potential is found to be a doubled-value function.

$$\tilde{V}_{\text{eff}} = \pm \sqrt{\frac{\delta V(PB \rightarrow VB \rightarrow PB)}{G_{VB}}} \quad \tilde{V}'_{\text{eff}} = \pm \sqrt{\frac{\delta V'(VB \rightarrow PB \rightarrow VB)}{G_{PB}}}$$

Not to be cancelled out, the two signs should be taken as the same

$$V_{\text{eff}} = \pm \frac{1}{2} \left(\tilde{V}_{\text{eff}} + \tilde{V}'_{\text{eff}} \right)$$

In the case of the two coupled channels, the change of sign of the transition potentials

$$V = \begin{pmatrix} V_{11} & \pm V_{12} \\ & V_{22} \end{pmatrix} \quad \xrightarrow{T=V+VGT} \quad T = \begin{pmatrix} T_{11} & \pm T_{12} \\ & T_{22} \end{pmatrix} \quad T_{12}(\sqrt{s}) \sim \pm \frac{g_1 g_2}{\sqrt{s} - M_R + i\Gamma_R}$$

close to the pole

Energies of generated states: not depend

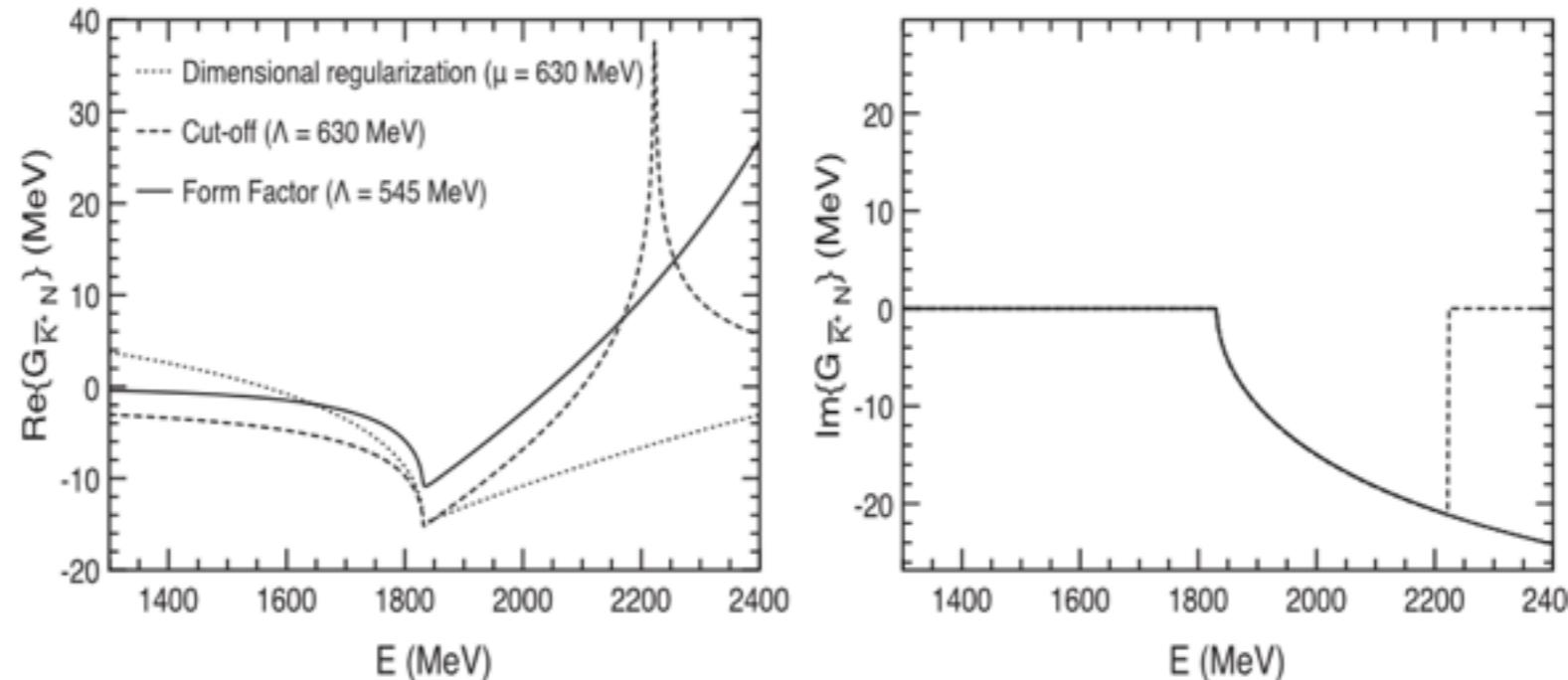
Relative sign of coupling constants (and wave function at the origin): depend



In the present work, we utilize the effective potentials with negative real part
(as virtual pion is exchanged)

G function: dimensional regularisation vs cutoff

Real and imaginary part of the G function regularised in several ways



K. P. Khemchandani, A. Martinez Torres, H. Kaneko, H. Nagahiro and A. Hosaka, Phys. Rev. D 84, 094018 (2011)

Recalling the form of the scattering amplitude $T = V + VGT = \frac{V}{1 - VG}$

Even with repulsive interactions ($\text{Re}V > 0$), the denominator of the amplitude can be 0

**Amplitude can have unphysical poles
below the threshold: dimensional regularisation
above the threshold: cut off**

$\bar{D}\Lambda_c$ has the repulsive interaction and its threshold is below the generated states