The second class current decays $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ in the SM

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work in progress with Rafel Escribano and Pablo Roig

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Motivations

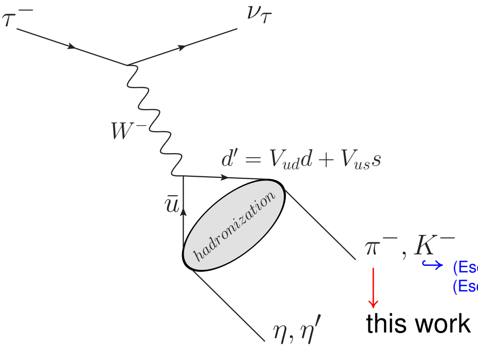
- $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ belong to the second-class current processes still unobserved in nature (Weinberg '58)
- It is an isospin violating process $(\underline{m_u \neq m_d}, e \neq 0)$
- Sensitive to the intermediate vector and scalar resonances $(\rho, \rho', a_0, a_0' ...)$ coupled to the $\bar{u}d$ operator

Purposes

- To describe the participating hadronic form factors
- To predict the decay spectra and to estimate the branching ratios
- To stimulate experimental collaborations to measure these decays

Outline

1. **Hadronic Matrix Element**
2. **Form Factors**
   - Vector Form Factor
   - Scalar Form Factor
     - Breit-Wigner
     - Final state interactions: Elastic case
     - Final state interactions: Coupled channels case
3. **Branching ratios predictions**
4. **Conclusions**



The decays proceed through the not yet evidenced second class current:

$$\mathrm{G-Parity}: G|X\rangle = e^{i\pi I_y} C|X\rangle = (-1)^I C|X\rangle$$

$$G|\bar{d}\gamma^\mu u\rangle = +|\bar{d}\gamma^\mu u\rangle \quad \neq \quad G|\pi^-\eta\rangle = -|\pi^-\eta\rangle$$

G-Parity violation

$\hookrightarrow$ (Escribano, González-Solís and Roig JHEP 1310 (2013) 039)
(Escribano, González-Solís, Jamin and Roig JHEP 1409 (2014) 042)

$$\mathcal{M} \overset{q^2<<M_W^2}{=} \frac{G_F}{\sqrt{2}} V_{ud}\bar{u}(p_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(p_\tau)\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu(1-\underbrace{\cancel{\gamma^5}}_{0^-,1^+\nrightarrow 0^+,1^-})u|0\rangle$$

The hadronic matrix element is generally parametrized as

$$\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu u|0\rangle = C^V_{\pi^-\eta^{(\prime)}}\left[(p_{\eta^{(\prime)}} - p_{\pi^-})^\mu F_+^{\pi^-\eta^{(\prime)}}(s) - (p_{\eta^{(\prime)}} + p_{\pi^-})^\mu F_-^{\pi^-\eta^{(\prime)}}(s)\right]$$

with $C^V_{\pi^-\eta^{(\prime)}} = \sqrt{2}$

Hadronic matrix element

Taking the divergence we obtain on the L.H.S

$$\langle 0|\partial_\mu(\bar{d}\gamma^\mu u)|\pi^+\eta^{(\prime)}\rangle = i\,\underbrace{(m_d - m_u)}_{\mathcal{O}(\varepsilon_{\pi\eta^{(\prime)}})\Rightarrow \text{suppresion}}\langle 0|\bar{d}u|\pi^+\eta^{(\prime)}\rangle = i\underbrace{\Delta^{QCD}_{K^0K^+}}_{\text{suppression}}\,C^S_{\pi^-\eta^{(\prime)}}F_0^{\pi^-\eta^{(\prime)}}(s) \qquad (1)$$

where $\Delta_{PQ} = M_P^2 - M_Q^2$, $C^S_{\pi^-\eta} = \sqrt{2/3}$, $C^S_{\pi^-\eta'} = 2/\sqrt{3}$

while on the R.H.S (vector current not conserved)

$$iq_\mu\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu u|0\rangle = iC^V_{\pi\eta^{(\prime)}}\left[(m^2_{\eta^{(\prime)}} - m^2_{\pi^-})F_+^{\pi\eta^{(\prime)}}(s) - sF_-^{\pi\eta^{(\prime)}}(s)\right] \qquad (2)$$

Equating eqs. (1,2) allows us to relate $F_-^{\pi^-\eta^{(\prime)}}(s)$ with $F_0^{\pi^-\eta^{(\prime)}}(s)$ as

$$F_-^{\pi\eta^{(\prime)}}(s) = -\frac{\Delta_{\pi^-\eta^{(\prime)}}}{s}\left[\frac{C^S_{\pi\eta^{(\prime)}}}{C^V_{\pi\eta^{(\prime)}}}\frac{\Delta^{QCD}_{K^0K^+}}{\Delta_{\pi^-\eta^{(\prime)}}}F_0^{\pi\eta^{(\prime)}}(s) + F_+^{\pi^-\eta^{(\prime)}}(s)\right] \qquad (3)$$

The vectorial hadronic matrix element finally reads ($q^\mu = (p_{\eta^{(\prime)}} + p_{\pi^-})^\mu +$ and $q^2 = s$)

$$\langle\pi^-\eta^{(\prime)}|\bar{d}\gamma^\mu u|0\rangle =$$

$$\left[(p_{\eta^{(\prime)}} - p_\pi)^\mu + \frac{\Delta_{\pi^-\eta^{(\prime)}}}{s}q^\mu\right]C^V_{\pi\eta^{(\prime)}}F_+^{\pi\eta^{(\prime)}}(s) + \frac{\Delta^{QCD}_{K^0K^+}}{s}q^\mu C^S_{\pi^-\eta^{(\prime)}}F_0^{\pi^-\eta^{(\prime)}}(s) \qquad (4)$$

Differential decay width: $\tau^- \to \pi^-(\eta,\eta')\nu_\tau$

$$\frac{d\Gamma\left(\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau\right)}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{24\pi^3 s} S_{EW}|V_{ud}|^2 \underbrace{|F_+^{\pi^-\eta^{(\prime)}}(0)|^2}_{\mathcal{O}(\varepsilon_{\pi\eta})\Rightarrow \textit{suppression}} \left(1-\frac{s}{M_\tau^2}\right)^2$$

$$\left\{\left(1+\frac{2s}{M_\tau^2}\right) q^3_{\pi^-\eta^{(\prime)}}(s)|\widetilde{F}_+^{\pi^-\eta^{(\prime)}}(s)|^2 + \frac{3\Delta^2_{\pi^-\eta^{(\prime)}}}{4s} q_{\pi^-\eta^{(\prime)}}(s)|\widetilde{F}_0^{\pi^-\eta^{(\prime)}}(s)|^2\right\}$$

where

$$q_{PQ}(s) = \frac{\sqrt{s^2 - 2s\Sigma_{PQ} + \Delta^2_{PQ}}}{2\sqrt{s}}, \quad \Sigma_{PQ} = m_P^2 + m_Q^2, \quad \Delta_{PQ} = m_P^2 - m_Q^2$$

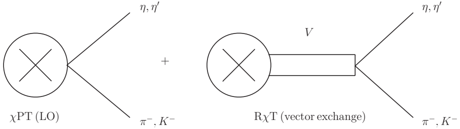
$$\widetilde{F}_{+,0}^{\pi^-\eta^{(\prime)}}(s) = \frac{F_{+,0}^{\pi^-\eta^{(\prime)}}(s)}{F_{+,0}^{\pi^-\eta^{(\prime)}}(0)}, \quad F_+^{\pi^-\eta^{(\prime)}}(0) = -\frac{C^S_{\pi^-\eta^{(\prime)}}}{C^V_{\pi^-\eta^{(\prime)}}} \frac{\Delta^{QCD}_{K^0K^+}}{\Delta_{\pi^-\eta^{(\prime)}}} F_0^{\pi^-\eta^{(\prime)}}(0)$$

Our next task:

$\Rightarrow$ to compute the vector and scalar Form Factors $F_+^{\pi^-\eta^{(\prime)}}(s)$ and $F_0^{\pi^-\eta^{(\prime)}}(s)$.

Vector Form Factor: RChT

The vector contribution current occurs via $\pi^0-\eta-\eta'$ mixing, so it is $\mathcal{O}(\varepsilon_{\pi\eta^{(\prime)}})$ and hence suppressed. The vector Form Factor is obtained by computing

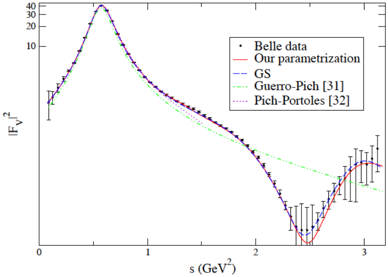


$$\begin{pmatrix} F_+^{\pi^-\eta}(s) \\ F_+^{\pi^-\eta'}(s) \end{pmatrix} = \underbrace{\begin{pmatrix} \varepsilon_{\pi\eta}\cos\theta_{\eta\eta'} - \varepsilon_{\pi\eta'}\sin\theta_{\eta\eta'} \\ \varepsilon_{\pi\eta'}\cos\theta_{\eta\eta'} + \varepsilon_{\pi\eta}\sin\theta_{\eta\eta'} \end{pmatrix}}_{\mathcal{O}(\varepsilon_{\pi\eta})\Rightarrow \textit{suppresion}} \times \underbrace{\left[1+\sum_{V=\rho,\rho',\rho''}\frac{F_V G_V}{F^2}\frac{s}{M_V^2-s}\right]}_{F_+^{\pi^-\pi^0}(s)}$$

$\theta_{\eta\eta'} \sim \theta_P = (-13.3 \pm 1.0)^\circ$
(KLOE Coll. PLB 648 '07 267)

$\varepsilon_{\pi\eta} \sim 0.018(2)$
$\varepsilon_{\pi\eta'} \sim 0.005(1)$

(Kroll Mod.Phys.Lett. A20 (2005))

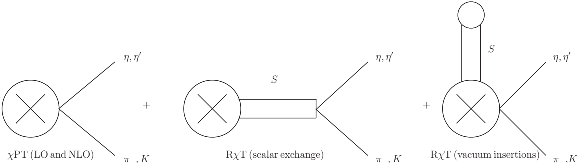


$F_+^{\pi^-\pi^0}$ analysis:
e.g. Roig, Gomez-Dumm Eur.Phys.J. C73 (2013)

We implement $F_+^{\pi^-\pi^0}$ from $\tau^- \to \pi^-\pi^0\nu_\tau$ Belle data to our work

Scalar Form Factor: Breit-Wigner

$$(m_d - m_u)\langle\pi^-\eta^{(\prime)}|\bar{d}u|0\rangle = \Delta^{QCD}_{K^0K^+} C^S_{\pi^-\eta^{(\prime)}} F_0^{\pi^-\eta^{(\prime)}}(s) =$$



Imposing $F_0^{\pi^-\eta^{(\prime)}}(s)$ to vanish for $s \to \infty$ we arrive at

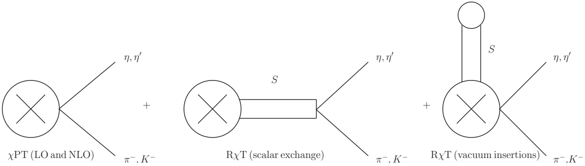
$$F_0^{\pi^-\eta^{(\prime)}}(s) = c_0^{\pi^-\eta^{(\prime)}} \frac{M_S^2 + \Delta_{\pi^-\eta^{(\prime)}}}{M_S^2 - s - iM_S\Gamma_S(s)}, \qquad \Gamma_S(s) = \Gamma_{a_0}(M_{a_0}^2)\left(\frac{s}{M_{a_0}^2}\right)^{3/2}\frac{h(s)}{h(M_{a_0}^2)}$$

$$h(s) = \sigma_{KK}(s) + \frac{2}{3}\sigma_{\pi\eta}(s)\left(c_0^{\pi^-\eta}\right)^2\left(1 + \frac{\Delta_{\pi\eta}}{s}\right)^2 + \frac{4}{3}\sigma_{\pi\eta'}(s)\left(c_0^{\pi^-\eta'}\right)^2\left(1 + \frac{\Delta_{\pi\eta'}}{s}\right)^2$$

$$c_0^{\pi^-\eta} = \cos\theta_{\eta\eta'} - \sqrt{2}\sin\theta_{\eta\eta'}, \qquad c_0^{\pi^-\eta'} = \cos\theta_{\eta\eta'} + \frac{1}{\sqrt{2}}\sin\theta_{\eta\eta'}$$

Scalar Form Factor: Breit-Wigner

$$(m_d - m_u)\langle \pi^- \eta^{(\prime)} | \bar{d}u | 0 \rangle = \Delta^{QCD}_{K^0K^+} C^S_{\pi^-\eta^{(\prime)}} F_0^{\pi^-\eta^{(\prime)}}(s) =$$



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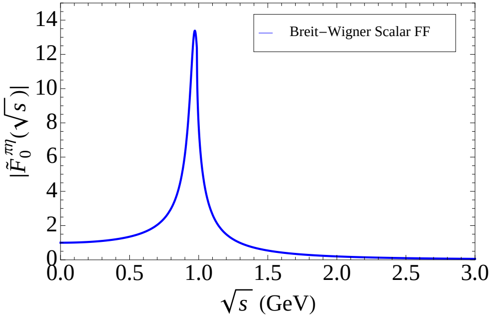
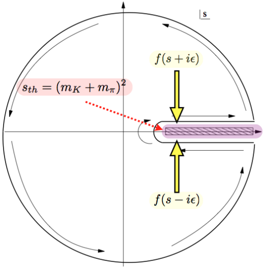
- Real part of the loop unconsidered

$\Rightarrow$ violation of analyticity

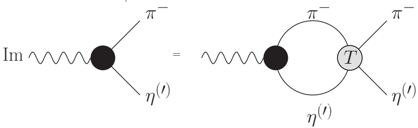


Scalar Form Factor: Elastic final state interactions

Analyticity and elastic unitarity ensured through a dispersion relation



$$F_0^{\pi^-\eta^{(\prime)}}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}F_0^{\pi^-\eta^{(\prime)}}(s')}{s'-s-i\varepsilon}$$



$$\mathrm{Im}F_0^{\pi^-\eta^{(\prime)}}(s) = \sigma_{\pi\eta^{(\prime)}}(s)F_0^{\pi^-\eta^{(\prime)}}T^*(s)$$
$$= F_0^{\pi^-\eta^{(\prime)}}\sin\delta_{1,0}^{\pi\eta^{(\prime)}}(s)e^{-i\delta_{1,0}^{\pi\eta^{(\prime)}}(s)}$$

§ (once subtracted) Omnès solution (Omnès '58)

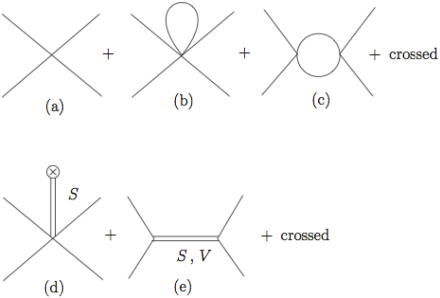
$$F_0^{\pi^-\eta^{(\prime)}}(s) = P(s)\exp\left[\frac{s-s_0}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s')}{(s'-s_0)(s'-s-i\varepsilon)}\right] = P(s)\Omega(s)$$

SFF: Omnès

Elastic unitarity: Form factor phase= $\delta_{\pi^-\eta^{(\prime)}}$ $2 \to 2$ elastic scattering

$$\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s) = \arctan \frac{\mathrm{Im}\, t_{1,0}(s)}{\mathrm{Re}\, t_{1,0}(s)}$$

$t_{1,0}$: unitarized S-waves of the $U(3) \times U(3)$ amplitudes in χPT at one-loop including resonances (Guo-Oller: Phys.Rev. D84 (2011) 034005) (Guo-Oller-Ruiz Elvira: 1206.4163)



$$\widetilde{c}_d = c_d/\sqrt{3}, \quad \widetilde{c}_m = c_m/\sqrt{3}$$

$$c_d = 19.8 \text{ MeV} , \quad c_m = 41.9 \text{ MeV}$$

$$M_{a_0,S_8} = 1397 \text{ MeV} , \quad M_{S_1} = 1100 \text{ MeV}$$

$$a_{SL}^{10,\pi\eta} = 2, \quad a_{SL}^{10,\pi\eta'} = -0.95$$

$$\Lambda_2 = -0.37$$

SFF: Omnès

Elastic unitarity: Form factor phase= $\delta_{1,0}^{\pi^-\eta^{(\prime)}}$ $2 \to 2$ elastic scattering

$$\delta_{1,0}^{\pi^-\eta^{(\prime)}}(s) = \arctan \frac{\mathrm{Im}\, t_{1,0}(s)}{\mathrm{Re}\, t_{1,0}(s)}, \quad t_{1,0}(s) = \frac{N_{1,0}(s)}{D_{1,0}(s)}$$

$$D(s) = D(s_0) + \frac{s - s_0}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im} D(s')}{(s' - s_0)(s' - s - i\varepsilon)}, \quad N(s) = \frac{s - s_0}{\pi} \int_{-\infty}^{s_L} ds' \frac{\mathrm{Im} N(s')}{(s' - s_0)(s' - s - i\varepsilon)}$$

Simplified perturbative solution

$$t_{1,0}(s) = \frac{N_{1,0}(s)}{1 + g(s) N_{1,0}(s)}, \quad N_{1,0}(s) = T_{1,0}^{\mathcal{O}(p^2)+res+loop} - g(s)(T_{1,0}^{\mathcal{O}(p^2)})^2$$

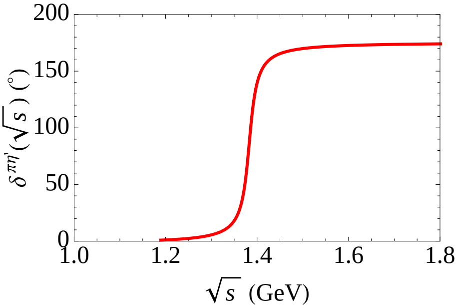
$g(s)$: meson one-loop scalar functions

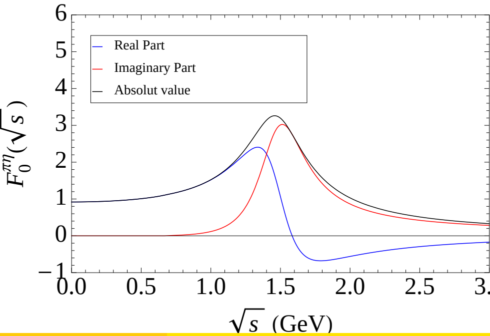
SFF: Omnès

Assuming $F_0^{\pi^-\eta^{(\prime)}}(s)$ to behave as s^{-1} : $F_0^{\pi\eta^{(\prime)}}(s) = P(s)\Omega(s)$,

$$\Omega(s) \sim s^{-\ell}, \quad \ell = \frac{1}{\pi}(\delta(\infty) - \delta(s_{sth})); \quad \delta(\infty) = n\pi \Rightarrow P(s) \;\; \text{constant} \;\; (n = 1)$$

our choice: $$P(s) = F_0^{Breit-Wigner}(0)$$





SFF: Closed expression

Once subtracted dispersion relation

$$F(s+i\varepsilon) = F(s_0) + \frac{s-s_0}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\sigma(s')t^*_{IJ}(s')F(s')}{(s'-s_0)(s'-s-i\varepsilon)} = F(s_0) + \widetilde{F}(s+i\varepsilon)$$

$$\begin{aligned}\widetilde{F}(s+i\varepsilon) - \widetilde{F}(s-i\varepsilon) &= 2i\sigma(s)t^*(s+i\varepsilon)F(s+i\varepsilon) \\ &= 2i\sigma(s)t^*(s+i\varepsilon)[F(s_0) + \widetilde{F}(s+i\varepsilon)]\end{aligned}$$

$$\left.\begin{aligned} t &= N/D \\ \mathrm{Im}\,t^{-1} &= -\sigma(s) \\ \mathrm{Im}D(s) &= -N\sigma(s) \end{aligned}\right\} \qquad \begin{aligned}&\widetilde{F}(s+i\varepsilon)D(s+i\varepsilon) - \widetilde{F}(s-i\varepsilon)D(s-i\varepsilon) \\ &= -2i\,\mathrm{Im}D(s)F(s_0),\end{aligned}$$

$$\begin{aligned}\widetilde{F}(s+i\varepsilon) &= \frac{1}{D(s+i\varepsilon)}\frac{-(s-s_0)}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}D(s')F(s_0)}{(s'-s_0)(s'-s)} \\ &= -D(s+i\varepsilon)^{-1}\left[D(s+i\varepsilon) - D(s_0)\right]F(s_0)\end{aligned}$$

SFF: Closed expression

$$F_0^{\pi\eta^{(\prime)}}(s) = \prod_{i=1} \frac{s - s_p^i}{s - s_z^i} D(s)^{-1} D(s_0) F_0(s_0)$$

s_p and s_z: poles and zeros of $D(s) = 1 - g(s)N(s)$

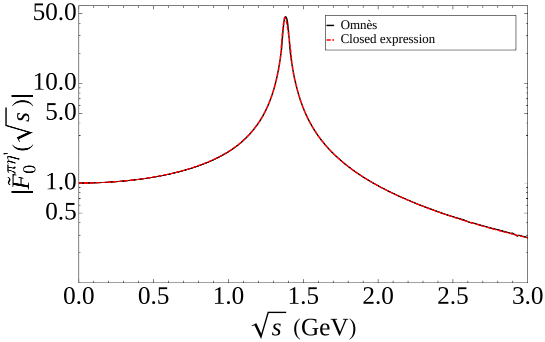
Iwamura, Kurihara, Takahashi '77
Kamal '79, Kamal, Cooper '80
Jamin, Oller, Pich '01

$s_0 = 0$

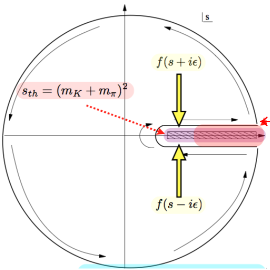
$F_0(s_0) = F_0^{BW}(0)$

$s_z = 1.390$ GeV

$N(s) = T_{1,0}^{\mathcal{O}(p^2)+res+\cancel{loop}}$

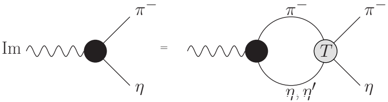


SFF: Coupled channels case

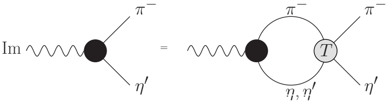


$$F_0^i(s) = \frac{1}{\pi}\sum_{j=1}^{2}\int_{s_i}^{\infty} ds' \frac{\Sigma_j(s')F_0^j(s')T_0^{i\to j}(s')^\star}{(s'-s-i\varepsilon)}$$

Other cuts $(K\bar{K}, \pi\eta' ...)$



$$F_0^{\pi\eta}(s) = \frac{1}{\pi}\int_{s_{th1}}^{\infty} ds' \frac{\sigma_{\pi\eta}(s')F_0^{\pi\eta}(s')T^*_{\pi\eta\to\pi\eta}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{th2}}^{\infty} ds' \frac{\sigma_{\pi\eta'}(s')F_0^{\pi\eta'}(s')T^*_{\pi\eta'\to\pi\eta}(s')}{s'-s-i\varepsilon}$$



$$F_0^{\pi\eta'}(s) = \frac{1}{\pi}\int_{s_{th1}}^{\infty} ds' \frac{\sigma_{\pi\eta}(s')F_0^{\pi\eta}(s')T^*_{\pi\eta\to\pi\eta'}(s')}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{s_{th2}}^{\infty} ds' \frac{\sigma_{\pi\eta'}(s')F_0^{\pi\eta'}(s')T^*_{\pi\eta'\to\pi\eta'}(s')}{s'-s-i\varepsilon}$$

SFF: Coupled channels case (closed expression)

$$F_0(s) = \prod_{i=1} \frac{s - s_p^i}{s - s_z^i} D(s)^{-1} D(s_0) F(s_0)$$

s_p and s_z: poles and zeros of $\det D(s)$

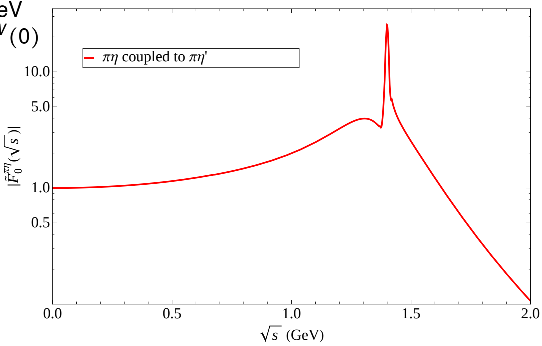
Iwamura, Kurihara, Takahashi PTF 58 (1977)
Kamal '79, Kamal, Cooper '80

$$F_0(s) = \begin{pmatrix} F_0^{\pi\eta}(s) \\ F_0^{\pi\eta'}(s) \end{pmatrix}, \quad s_z = 1.390\ \text{GeV}, \quad F_0(s_0) = F_0^{BW}(0)$$

$$D(s) = \mathbb{1} - g(s) N(s),$$

$$g(s) = \begin{pmatrix} g_{\pi\eta} & 0 \\ 0 & g_{\pi\eta'} \end{pmatrix},$$

$$N(s) = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta'\to\pi\eta} & N_{\pi\eta'\to\pi\eta'} \end{pmatrix},$$



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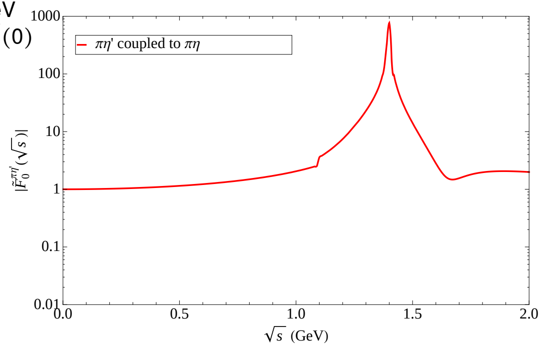

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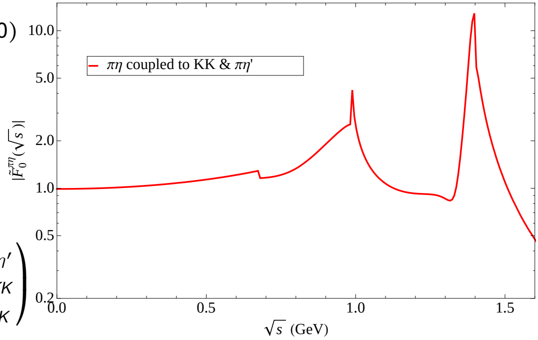
$$D(s) = \mathbb{1} - g(s) N(s),$$

$$g(s) = \begin{pmatrix} g_{\pi\eta} & 0 & 0 \\ 0 & g_{\pi\eta'} & 0 \\ 0 & 0 & g_{KK} \end{pmatrix},$$

$$N(s) = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to KK} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta'\to\pi\eta} & N_{\pi\eta'\to\pi\eta'} & N_{\pi\eta'\to KK} \\ N_{KK\to\pi\eta} & N_{KK\to\pi\eta'} & N_{KK\to KK} \end{pmatrix}$$



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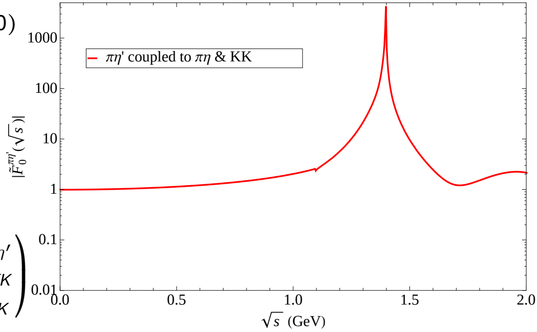
Iwamura, Kurihara, Takahashi PTF 58 (1977)
Kamal '79, Kamal, Cooper '80

$$F_0(s) = \begin{pmatrix} F_0^{\pi\eta}(s) \\ F_0^{\pi\eta'}(s) \end{pmatrix}, \quad \begin{array}{l} s_z = 1.390 \text{ GeV} \\ F_0(s_0) = F_0^{BW}(0) \end{array}$$

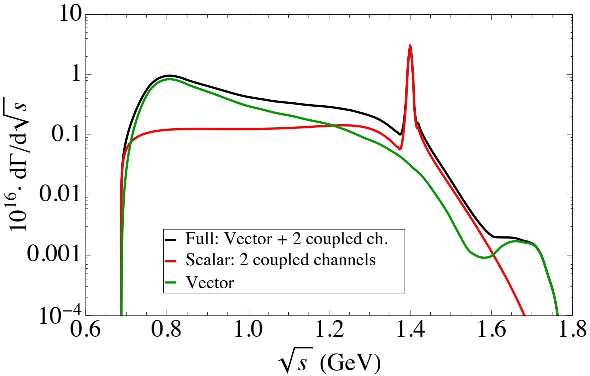
$$D(s) = \mathbb{1} - g(s) N(s),$$

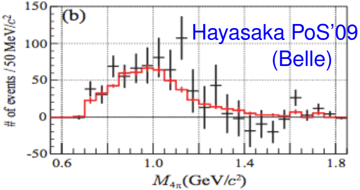
$$g(s) = \begin{pmatrix} g_{\pi\eta} & 0 & 0 \\ 0 & g_{\pi\eta'} & 0 \\ 0 & 0 & g_{KK} \end{pmatrix},$$

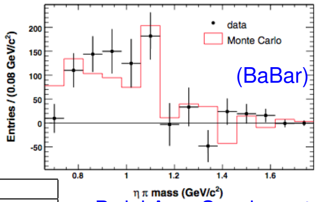
$$N(s) = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to KK} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta'\to\pi\eta} & N_{\pi\eta'\to\pi\eta'} & N_{\pi\eta'\to KK} \\ N_{KK\to\pi\eta} & N_{KK\to\pi\eta'} & N_{KK\to KK} \end{pmatrix}$$



Branching Ratio of $\tau^- \to \pi^- \eta \nu_\tau$







P. del Amo Sanchez *et.al* (PRD 83 032002 '11)

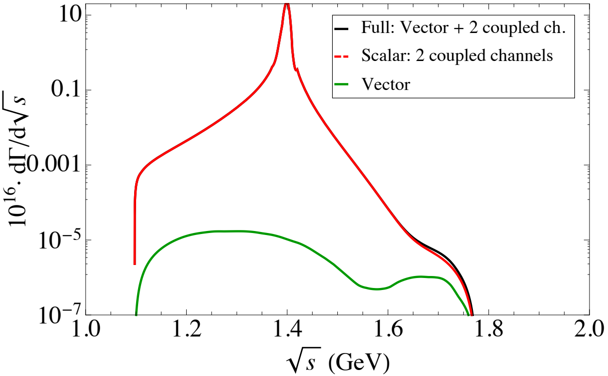
BR_V	BR_S	BR	Reference
0.25	1.60	1.85	Tisserant, Truong'82
0.12	1.38	1.50	Pich'87
0.15	1.06	1.21	Neufeld, Rupertsberger'94
0.36	1.00	1.36	Nussinov, Soffer'08
$[0.2, 0.6]$	$[0.2, 2.3]$	$[0.4, 2.9]$	Paver, Riazuddin'10
0.44	0.04	0.48	Volkov, Kostunin'12
0.13	0.20	0.33	Descotes-Genon, Moussallam'14
0.9 ± 0.2	2.70 ± 1.10	3.60 ± 1.12	Our prediction: Breit-Wigner (prel.)
0.9 ± 0.2	0.28 ± 0.07	1.18 ± 0.21	Our prediction: Elastic rescattering (prel.)
0.9 ± 0.2	0.54 ± 0.13	1.44 ± 0.23	Our prediction: 2 coupled channels (prel.)
0.9 ± 0.2	0.31 ± 0.08	1.21 ± 0.22	Our prediction: 3 coupled channels (prel.)

$BR^{BaBar}_{exp} < 9.9 \cdot 10^{-5}$ 95%CL

$BR^{Belle}_{exp} < 7.3 \cdot 10^{-5}$ 90%CL

in units of 10^{-5}

Branching Ratio of $\tau^- \to \pi^- \eta' \nu_\tau$



BR_V	BR_S	BR	Reference
$< 10^{-7}$	$[0.2, 1.3] \cdot 10^{-6}$	$[0.2, 1.4] \cdot 10^{-6}$	Nussinov, Soffer'09
$\left[1.4 \cdot 10^{-9}, 3.4 \cdot 10^{-8}\right]$	$\left[6.0 \cdot 10^{-8}, 1.8 \cdot 10^{-7}\right]$	$\left[6.1 \cdot 10^{-8}, 2.1 \cdot 10^{-7}\right]$	Paver, Riazuddin'11
$1.1 \cdot 10^{-8}$	$2.6 \cdot 10^{-8}$	$3.7 \cdot 10^{-8}$	Volkov, Kostunin'12
$\left[10^{-11}, 10^{-9}\right]$	$\left[0.2 \cdot 10^{-8}, 4.4 \cdot 10^{-8}\right]$	$\left[0.2 \cdot 10^{-8}, 4.4 \cdot 10^{-8}\right]$	Our prediction: Breit-Wigner (prel.)
$\left[10^{-11}, 10^{-9}\right]$	$\sim 0.15 \cdot 10^{-6}$	$\sim 0.15 \cdot 10^{-6}$	Our prediction: Elastic rescattering (prel.)
$\left[10^{-11}, 10^{-9}\right]$	$\sim 10^{-6}$	$\sim 10^{-6}$	Our estimate: 2 coupled channels (prel.)
$\left[10^{-11}, 10^{-9}\right]$	$\sim 10^{-6}$	$\sim 10^{-6}$	Our estimate: 3 coupled channels (prel.)

$BR_{exp}^{BaBar} < 4 \cdot 10^{-6}$, $BR_{exp}^{BaBar} < 7.2 \cdot 10^{-6}$, $BR_{exp}^{CLEO} < 7.4 \cdot 10^{-5}$ $90\% CL$

Branching Ratio estimates: $\eta^{(\prime)} \to \pi^+ \ell^- \nu_\ell \quad (\ell = e, \mu)$

$$\frac{d\Gamma}{d\sqrt{s}} = \frac{G_F^2 |V_{ud} F_+(0)|^2 (C_V^{\pi\eta})^2 (s - m_l^2)^2}{24\pi^3 M_\eta^3 s}$$

$$\left\{ (2s + m_\ell^2) q_{\pi\eta}^3 |\widetilde{F}_+(s)|^2 + \frac{3}{4s} \Delta_{\pi\eta}^2 {m_\ell}^2 q_{\pi\eta} |\widetilde{F}_0(s)|^2 \right\}$$

Decay	Descotes-Genon, Moussallam '14	Our estimate
$\eta \to \pi^+ e^- \nu_e + c.c.$	$\sim 1.40 \cdot 10^{-13}$	$1.9 \cdot 10^{-13}$
$\eta \to \pi^+ \mu^- \nu_\mu + c.c.$	$1.02 \cdot 10^{-13}$	$1.4 \cdot 10^{-13}$
$\eta' \to \pi^+ e^- \nu_\mu + c.c.$		$1.3 \cdot 10^{-16}$
$\eta' \to \pi^+ \mu^- \nu_\mu + c.c.$		$1.2 \cdot 10^{-16}$

Conclusions

- $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$ are suppressed because of both G-parity and isospin violation
- Vector Form Factor: related to the very well-known vector form factor of the $\pi^-\pi^0$ mode and, therefore, our predictions are robust
- Scalar Form Factor: i) Simple Breit-Wigner non-analytic ii) Analytic and elastic unitary (Omnès). iii) Proposal for coupled channels: resonances generated dynamically through final state interactions
- I would like to encourage experimental groups to measure these interesting physical processes

Extra slides

Possible new physics contributions: Charged Higgs