

# The molecular $K^* \bar{K}$ nature of the $f_1(1285)$ and its decays into $\pi a_0(980)$ ( $\pi f_0(980)$ ) and $\pi K \bar{K}$

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# Outline

F. A. Ju-Jun Xie, E. Oset, arXiv:1505.06134 [hep-ph]  
F. A. J. M. Dias, E. Oset, Eur. Phys. J. A51 (2015) 48

- introduction
- $f_1(1285) \rightarrow \pi a_0(980) (\pi f_0(980))$  and isospin breaking
  - formalism
  - results for the decay width
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- $f_1(1285) \rightarrow \pi K \bar{K}$  and isospin breaking
  - formalism
  - results for the decay width
  - invariant mass distributions
- conclusions

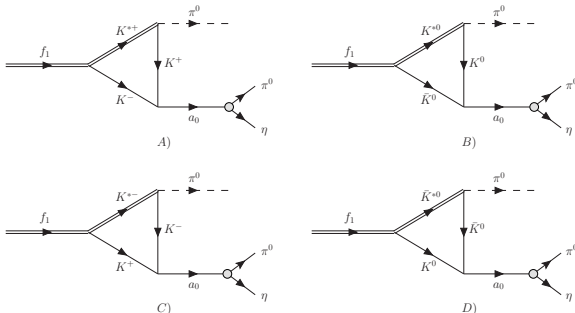
# Introduction

**starting point**  $\longrightarrow f_1(1285)$ ,  $a_0(980)$  and  $f_0(980)$  are dynamically generated by meson-meson interaction

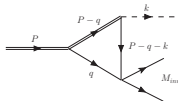
- $f_1(1285)$ 
  - $0^+(1^{++})$
  - appears in the pseudoscalar-vector interaction
  - it comes from the single channel  $K^* \bar{K} - c.c.$  [Roca et al., Phys. Rev. D 72, 014002 (2005)]
- $a_0(980)$ 
  - $1^-(0^{++})$
  - appears in the pseudoscalar-pseudoscalar interaction
  - building blocks:  $\pi\eta$ ,  $K\bar{K}$  [Oller et al., Nucl. Phys. A 620, 438 (1998)]
- $f_0(980)$ 
  - $0^+(0^{++})$
  - appears in the pseudoscalar-pseudoscalar interaction
  - building blocks:  $\pi\pi$ ,  $K\bar{K}$  [Oller et al., Nucl. Phys. A 620, 438 (1998)]

# $f_1(1285) \rightarrow a_0(980)\pi$ decay: formalism

- **B.R.** ( $f_1(1285) \rightarrow a_0(980)\pi$ ) =  $(36 \pm 7)\%$  excluding  $a_0(980) \rightarrow K\bar{K}$   
[Particle Data Group, Chin. Phys. C, 38, 090001 (2014)]
- dynamically generated picture  $\implies$  the process is described by the **triangular diagrams**:

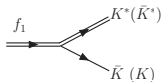


**Momenta assignment:**



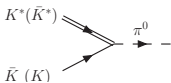
# $f_1(1285) \rightarrow a_0(980)\pi$ decay: formalism

## Vertices:



$$-it_1 = -ig_{f_1} C_1 \epsilon^\mu \epsilon'_\mu$$

- $g_{f_1} = 9687$  MeV, evaluated as the residue at the pole of  $T = [1 - VG]^{-1}V$  for  $K^* \bar{K} - c.c.$  in  $I = 0$  [the potential  $V$  is taken from [Roca et al., Phys. Rev. D 72, 014002 \(2005\)](#)]
- the loop function  $G$  is regularized by means of a cutoff  $q_{max} \simeq 1$  GeV
- the  $f_1$  couples to the  $I = 0$ ,  $C = +$  and  $G = +$  combination of  $K^* \bar{K}$ :  
 $-\frac{1}{2}(K^{*+}K^- + K^{*0}\bar{K} - K^{*-}K^+ - \bar{K}^{*0}K^0)$



$$-it_2 = -igC_2(2k - P + q)^\mu \epsilon'_\mu$$

- Hidden Gauge VPP Lagrangian  $\mathcal{L}_{VPP} = -ig\langle V^\mu [P, \partial_\mu P] \rangle$  [Phys. Rev. Lett. 54, 1215 (1985)]
- $g_V = \frac{m_V}{2f}$ ,  $m_V \simeq m_\rho$ ,  $f = 93$  MeV

# $f_1(1285) \rightarrow a_0(980)\pi$ decay: formalism



$$-it_3 = -it_{if}$$

- rescattering of the  $K\bar{K}$  pair
- $t_{if}$  is the element of the  $5 \times 5$  scattering matrix for the channels  $K^+K^-$  (1),  $K^0\bar{K}^0$  (2),  $\pi^0\eta$  (3),  $\pi^+\pi^-$  (4),  $\pi^0\pi^0$  (5) [Oller et al., Nucl. Phys. A 620, 438 (1998)]
- loop function regularized with a cutoff  $q'_{max} = 900$  MeV

due to the physical masses of the kaons, the rescattering of  $K\bar{K}$  dynamically generates the  $a_0(980)$  and  $f_0(980)$  leading to both  $\pi^0\eta$  and  $\pi^+\pi^-$  in the final state

isospin symmetry is broken

$\Rightarrow$  simultaneous study of  $f_1(1285) \rightarrow f_0(980)\pi$

# $f_1(1285) \rightarrow a_0(980)\pi$ decay: formalism

Putting everything together we get, for each diagram:

$$t = C g_{f1} g_{\vec{\epsilon} \cdot \vec{k}} (2\mathcal{I}_1 + \mathcal{I}_2) \quad t_{if} = \tilde{t} \vec{\epsilon} \cdot \vec{k} \quad t_{if}$$

|    | $C_1$  | $C_2$         | $C$            |
|----|--------|---------------|----------------|
| A) | $-1/2$ | $1/\sqrt{2}$  | $-1/2\sqrt{2}$ |
| B) | $-1/2$ | $-1/\sqrt{2}$ | $1/2\sqrt{2}$  |
| C) | $1/2$  | $-1/\sqrt{2}$ | $-1/2\sqrt{2}$ |
| D) | $1/2$  | $1/\sqrt{2}$  | $1/2\sqrt{2}$  |

$$\mathcal{I}_1 = -i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_K^2 + i\epsilon} \frac{1}{(P - k - q)^2 - m_K^2 + i\epsilon} \frac{1}{2\omega^*} \frac{1}{P^0 - q^0 - \omega^* + i\epsilon}$$

$$\mathcal{I}_2 = -i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_K^2 + i\epsilon} \frac{1}{(P - k - q)^2 - m_K^2 + i\epsilon} \frac{\vec{k} \cdot \vec{q}}{|\vec{k}|^2} \frac{1}{2\omega^*} \frac{1}{P^0 - q^0 - \omega^* + i\epsilon}$$

- integration in  $dq^0$  done analytically
- integration in  $d^3 q \rightarrow$  upper limit naturally provided by chiral unitary approach

the cutoff  $q_{max} \simeq 900$  MeV appears automatically as upper limit in  $\mathcal{I}_1$  and  $\mathcal{I}_2$  thanks to the  $K\bar{K} \rightarrow P\bar{P}$  potential used to dynamically generate the  $a_0$  and  $f_0$

$$V(\vec{q}, \vec{q}') = v \theta(q_{max} - |\vec{q}|) \theta(q_{max} - |\vec{q}'|)$$

$$t(\vec{q}, \vec{q}') = t \theta(q_{max} - |\vec{q}|) \theta(q_{max} - |\vec{q}'|)$$

# $f_1(1285) \rightarrow a_0(980)\pi$ decay: formalism

taking into account physical masses for the kaons, we can define  $\tilde{t}^+$  and  $\tilde{t}^0$  corresponding to  $\tilde{t}$  for the case of charged and neutral kaons respectively

## TOTAL AMPLITUDE OF THE PROCESSES:

$$T_{\pi^0\eta} = (2\tilde{t}^{(+)} t_{K^+K^- \rightarrow \pi^0\eta} + 2\tilde{t}^{(0)} t_{K^0\bar{K}^0 \rightarrow \pi^0\eta}) \vec{\epsilon} \cdot \vec{k} = \tilde{T}_{\pi^0\eta} \vec{\epsilon} \cdot \vec{k}$$

$$T_{\pi^+\pi^-} = (2\tilde{t}^{(+)} t_{K^+K^- \rightarrow \pi^+\pi^-} + 2\tilde{t}^{(0)} t_{K^0\bar{K}^0 \rightarrow \pi^+\pi^-}) \vec{\epsilon} \cdot \vec{k} = \tilde{T}_{\pi^+\pi^-} \vec{\epsilon} \cdot \vec{k}$$

- $\tilde{t}^+$  and  $\tilde{t}^0$  have opposite signs

[see the table for the coefficients C]

- $t_{K^+K^- \rightarrow \pi^0\eta} = -t_{K^0\bar{K}^0 \rightarrow \pi^0\eta}$   
 $t_{K^+K^- \rightarrow \pi^+\pi^-} = t_{K^0\bar{K}^0 \rightarrow \pi^+\pi^-}$

[Oller et al., Nucl. Phys. A 620, 438 (1998)]

$\Rightarrow$

if isospin symmetry were exact ( $m_{K^0} = m_{K^\pm}$ )  
 $T_{\pi^+\pi^-} \equiv 0$  and the production of the  $f_0$  would be forbidden

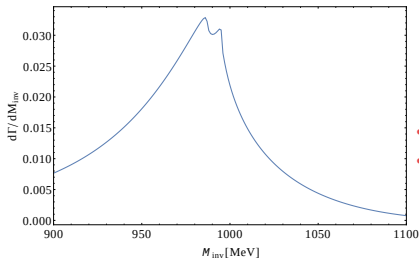
isospin symmetry is not exact  $\longrightarrow$  we can have both the  $\pi^0\eta$  and  $\pi^+\pi^-$  pairs in the final state



# $f_1(1285) \rightarrow a_0(980)\pi$ decay: results

We plot the invariant mass distribution for both cases:

$$\frac{d\Gamma}{dM_{inv}} = \frac{1}{(2\pi)^3} \frac{p_\pi |\vec{k}|}{4m_{f_1}^2} \frac{1}{3} |\tilde{T}|^2$$

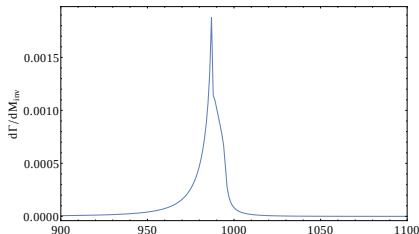


$f_1(1285) \rightarrow \pi^0 \pi^0 \eta$

- $a_0(980)$  produced with its normal width  $\simeq 50$  MeV
- high strength at the peak

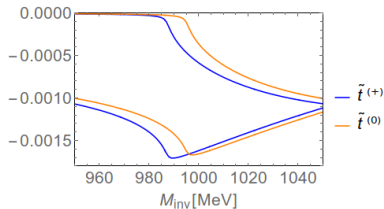
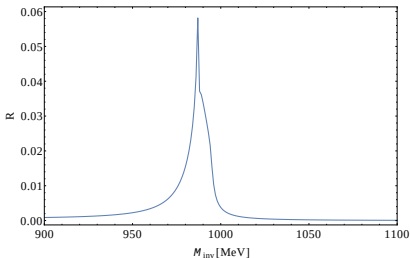
$f_1(1285) \rightarrow \pi^0 \pi^+ \pi^-$

- narrow peak, width  $\simeq 10$  MeV  
in agreement with BESIII recent observations  
[arXiv:1505.06283 [hep-ex]]  
in agreement with what found in other reactions like  $\eta' \rightarrow \pi^0 \pi^+ \pi^-$   
[BES, Phys. Rev. Lett. 108,182001 (2012)]
- the peak is between the two  $K^+ K^-$  and  $K^0 \bar{K}^0$  thresholds



# $f_1(1285) \rightarrow a_0(980)\pi$ decay: results

due to the fact that the difference between  $|\tilde{t}^+|$  and  $|\tilde{t}^0|$  is significant only between the two thresholds



ratio of strength at the peak  $\simeq 6\%$   
 ten times smaller than what we got in the case of the  
 $\eta' \rightarrow \pi^0 \pi^0 \eta, \pi^0 \pi^+ \pi^-$   
 [Phys. Rev. D 86,114007 (2012)]

**the amount of isospin breaking depends on the reaction but can give important informations on the nature of the resonances involved**

# $f_1(1285) \rightarrow a_0(980)\pi$ decay: results

PARTIAL WIDTH FOR THE DECAY TO  $a_0\pi$ :

$$\Gamma_{a_0\pi} = 3 \int dM_{inv} \left( \frac{d\Gamma}{dM_{inv}} \right)_{\pi^0\eta} = 7.6 \text{ MeV}$$

$$BR(f_1 \rightarrow a_0\pi)|_{exp} = (36 \pm 7)\%$$

$$BR(f_1 \rightarrow a_0\pi)|_{th} \simeq 31\%$$

Two sources of **uncertainties**:

- cutoff used for the generation of the  $f_1(1285)$ : we change it of  $\pm 20$  MeV  $\rightarrow$  variation of  $\Gamma_{a_0\pi}$  of 2.5%
- cutoff used in  $\mathcal{I}_1$  and  $\mathcal{I}_2$ : we change it of  $\pm 20$  MeV  $\rightarrow$  variation of  $\Gamma_{a_0\pi}$  of 1.5%

$$BR(f_1 \rightarrow a_0\pi)|_{th} = (31.4 \pm 1.2)\%$$

$$\text{We also get: } \frac{\Gamma(\pi^0 f_0(980))}{\Gamma(\pi^0 a_0(980))} = 1.28 \times 10^{-2}$$

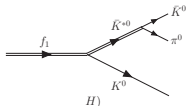
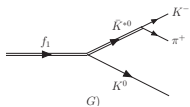
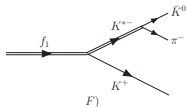
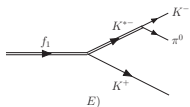
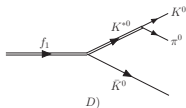
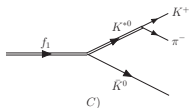
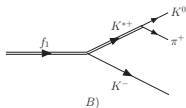
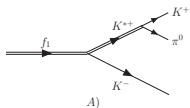
$$\text{much smaller than } \frac{\Gamma(\eta(1405) \rightarrow \pi^0 f_0(980))}{\Gamma(\eta(1405) \rightarrow \pi^0 a_0(980))} = 18 \times 10^{-2} \text{ [BESIII, Phys. Rev. Lett. 108,182001 (2012)]}$$

$$\text{and twice as big as } \frac{\Gamma(J/\psi \rightarrow \phi f_0(980))}{\Gamma(J/\psi \rightarrow \phi a_0(980))} = 0.6 \times 10^{-2} \text{ [BESIII, Phys. Rev D, 83, 032003 (2011)]}$$

# $f_1(1285) \rightarrow \pi K \bar{K}$ decay: tree level

$$BR(f_1 \rightarrow \pi K \bar{K})|_{exp} = (9.0 \pm 0.4)\%$$

[Particle Data Group, Chin. Phys. C, 38, 090001 (2014)]



## TREE LEVEL DIAGRAM:

Same vertices as before:

- $-it_1 = -ig_{f_1} C_1 \epsilon^\mu \epsilon'_\mu$
- $-it_2 = -ig C_2 (k - p)^\mu \epsilon'_\mu$

|       | $C_1$  |
|-------|--------|
| A) B) | $-1/2$ |
| C) D) | $1/2$  |
| E) F) | $-1/2$ |
| G) H) | $1/2$  |

|       | $C_2$         |
|-------|---------------|
| A) H) | $-1/\sqrt{2}$ |
| B) C) | $-1$          |
| D) E) | $1/\sqrt{2}$  |
| F) G) | $1$           |

- we sum the amplitudes for the diagrams with same final state:

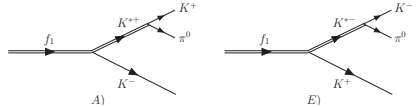
$$M_{tree}^{A+E} = M_{tree}^{D+H} = M_{tree}$$

$$M_{tree}^{B+G} = M_{tree}^{C+F} = \sqrt{2} M_{tree}$$

# $f_1(1285) \rightarrow \pi K \bar{K}$ decay: tree level

we take diagrams A) and E) as reference

$\rightarrow \pi^0 K^+ K^-$  final state



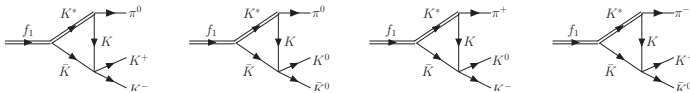
$$M_{\text{tree}} = \frac{gg_{f_1}}{2\sqrt{2}} \left( [(\vec{k} - \vec{p}) - \frac{m_\pi^2 - m_K^2}{m_{K^*}^2} (\vec{k} + \vec{p})] D_1 + [(\vec{k} - \vec{p}') + \frac{m_\pi^2 - m_K^2}{m_{K^*}^2} (\vec{k} + \vec{p}')] D_2 \right) \cdot \vec{\epsilon}$$

- $p$  and  $p'$  are the momenta of  $K^+$  and  $K^-$  respectively,  $k$  is the momentum of the  $\pi^0$
- $D_1 = \frac{1}{(k+p)^2 - m_{K^*}^2 + im_{K^*} \Gamma_{K^*}}$  and  $D_2 = \frac{1}{(k+p')^2 - m_{K^*}^2 + im_{K^*} \Gamma_{K^*}}$  are the propagators of the  $K^*$
- $\Gamma_{K^*} = \Gamma_{\text{on}} \left( \frac{q_{\text{off}}}{q_{\text{on}}} \right)^3$  is the total width of the  $K^*$

# $f_1(1285) \rightarrow \pi K \bar{K}$ decay: FSI

now we study the **final state interactions (FSI)** by means of the **triangular mechanism**

A)  $K \bar{K}$  interaction:



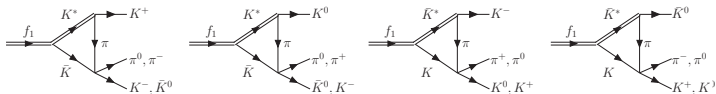
- rescattering  $\rightarrow$  generates the  $a_0(980)$  ( $I = 1$ )
- scattering amplitude  $t_{K\bar{K} \rightarrow K\bar{K}}^{I=1}(M_{K\bar{K}})$ : obtained using  $T = [1 - VG]^{-1} V$   
[with  $V$  taken from Oller et al., Nucl. Phys. A 620, 438 (1998)]  
 $\rightarrow$  good description of the  $a_0(980)$  using  $q_{\max} = 900$  MeV
- we sum the four diagrams with final state  $\pi^0 K^+ K^-$

$$M_{\text{FSI}}^{K\bar{K}} = -\frac{g g_{f_1}}{2\sqrt{2}} (2\mathcal{I}_1 + \mathcal{I}_2) 2 t_{K\bar{K} \rightarrow K\bar{K}}^{I=1}(M_{K\bar{K}}) \vec{\epsilon} \cdot \vec{k}$$

$$\begin{aligned} -M_{K\bar{K}}^2 &= (p + p')^2 = M_{f_1}^2 + m_\pi^2 - 2M_{f_1} \sqrt{|\vec{k}|^2 + m_\pi^2} \\ -\mathcal{I}_1, \mathcal{I}_2 &\text{ same as before} \end{aligned}$$

# $f_1(1285) \rightarrow \pi K \bar{K}$ decay: FSI

## B) $\pi K$ interaction:



- rescattering  $\rightarrow$  generates the  $\kappa(800)$  ( $I = 1/2$ )
- scattering amplitude:  $t_{\pi K \rightarrow \pi K}^{I=1/2}(M_{\pi K}) \rightarrow$  good description of the  $\kappa(800)$  using  $q_{\max} = 900$  MeV
- we sum the four diagrams with final state  $\pi^0 K^+ K^-$

$$M_{\text{FSI}}^{\pi K} = \frac{g}{2\sqrt{2}} (2\mathcal{I}'_1 + \mathcal{I}'_2) t_{\pi K \rightarrow \pi K}^{I=1/2}(M_{\pi K}^{(1)}) \vec{\epsilon} \cdot \vec{p} + \frac{g}{2\sqrt{2}} (2\mathcal{I}''_1 + \mathcal{I}''_2) t_{\pi K \rightarrow \pi K}^{I=1/2}(M_{\pi K}^{(2)}) \vec{\epsilon} \cdot \vec{p}'$$

- $\mathcal{I}'_1, \mathcal{I}'_2, \mathcal{I}''_1$  and  $\mathcal{I}''_2$  obtained from  $\mathcal{I}_1$  and  $\mathcal{I}_1$  with suitable substitution of kinematic variables
- $M_{\pi K}^{(1)} = \sqrt{(k+p')^2}$  and  $M_{\pi K}^{(2)} = \sqrt{(k+p)^2}$

$\Rightarrow$  full amplitude for the  $\pi^0 K^+ K^-$  final state:

$$M = M_{\text{tree}} + M_{\text{FSI}}^{K\bar{K}} + M_{\text{FSI}}^{\pi K}$$

# $f_1(1285) \rightarrow \pi K \bar{K}$ decay: results

## TOTAL DECAY WIDTH:

$$\Gamma_{K\bar{K}\pi} = 6 \frac{1}{64\pi^3 M_{f_1}} \int \int d\omega_{K^+} d\omega_{K^-} \sum |M|^2 \theta(1 - \cos^2 \theta_{K\bar{K}}) \theta(M_{f_1} - \omega_{K^+} - \omega_{K^-} - m_\pi)$$

- **factor 6**  $\longrightarrow$  accounts for all the final charges:  $\pi^0 K^+ K^-$ ,  $\pi^+ K^0 K^-$ ,  $\pi^- K^+ \bar{K}^0$  and  $\pi^0 K^0 \bar{K}^0$ , having weights 1, 2, 2, and 1
- $\omega_{K^+}$  and  $\omega_{K^-}$  energies of the  $K^+$  and  $K^-$
- $\cos \theta_{K\bar{K}}$  given by energy conservation

| $g_{f_1}$    | $\Gamma_{K\bar{K}\pi}$ | B.R.  |
|--------------|------------------------|-------|
| 9687 MeV (1) | 3.12                   | 12.9% |
| 7230 MeV (2) | 1.74                   | 7.2%  |

(1) Aceti et al., Eur. Phys. J. A 51, 4, 48 (2015)

(2) Roca et al., Phys. Rev. D 72, 014002 (2004)

$$BR(f_1 \rightarrow \pi K \bar{K})|_{exp} = (9.0 \pm 0.4)\%$$

$$B.R.(f_1 \rightarrow K \bar{K} \pi) = (7.2 - 12.9)\%$$

with  $g_{f_1} = 9687$  MeV and **only the tree level diagrams**:  $B.R.(f_1 \rightarrow K \bar{K} \pi) = 15\%$

**the contribution of the FSI is very small: cancellation due to their opposite sign**

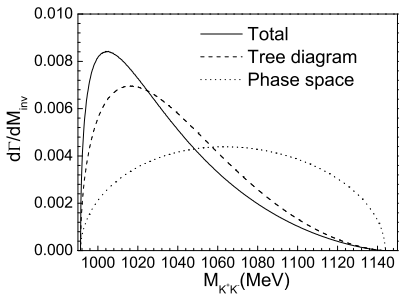


# $f_1(1285) \rightarrow \pi K \bar{K}$ decay: results

INVARIANT MASS DISTRIBUTION  $f_1(1285) \rightarrow \pi^0 K^+ K^-$  as a function of  $M_{K^+ K^-}$ :

$$\frac{d\Gamma}{dM_{K^+ K^-}} = \frac{M_{K^+ K^-}}{64\pi^3 M_{f_1}^2} \int d\omega_{K^+} \overline{\sum} |M|^2 \theta(1 - \cos^2 \theta_{K\bar{K}}) \theta(M_{f_1} - \omega_{K^+} - \omega_{K^-} - m_\pi) \theta(\omega_{K^-} - m_K)$$

$$\text{with } \omega_{K^-} = \frac{1}{2M_{f_1}} (M_{K^+ K^-}^2 + M_{f_1}^2 - m_\pi^2) - \omega_{K^+}$$



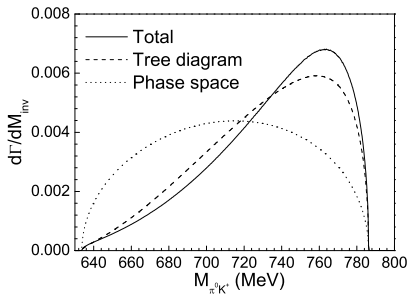
- shape of the tree level very different
- we can see a peak at low  $M_{K^+ K^-}$
- the peak is shifted closer to the mass of the  $a_0(980)$  by the FSI

# $f_1(1285) \rightarrow \pi K \bar{K}$ decay: results

INVARIANT MASS DISTRIBUTION  $f_1(1285) \rightarrow \pi^0 K^+ K^-$  as a function of  $M_{\pi^0 K^+}$ :

$$\frac{d\Gamma}{dM_{\pi^0 K^+}} = \frac{M_{\pi^0 K^+}}{64\pi^3 M_{f_1}^2} \int d\omega_{K^+} \overline{\sum} |M|^2 \theta(1 - \cos^2 \theta_{K\bar{K}}) \theta(M_{f_1} - \omega_{K^+} - \omega_{K^-} - m_\pi) \theta(\omega_{K^-} - m_K)$$

$$\text{with } \omega_{K^-} = \frac{1}{2M_{f_1}} (M_{f_1}^2 + m_K^2 - M_{\pi^0 K^+}^2)$$



- now the peak is at high  $M_{\pi^0 K^+}$
- the peak is shifted closer to the mass of the  $\kappa(800)$  by the FSI

presence of the peak  
 $\Rightarrow$  tied to the  $K^*$  propagator  
 $\Rightarrow$  tied to the  $\bar{K}^* K - c.c.$  nature  
 of the  $f_1(1285)$

# Conclusions

The assumption that the  $f_1(1285)$ ,  $a_0(980)$  and  $f_0(980)$  are dynamically generated resonances is supported by our findings:

- a branching ratio for  $f_1(1285) \rightarrow \pi\eta$  in good agreement with experiment within errors
- a branching ratio for  $f_1(1285) \rightarrow \pi K\bar{K}$  in good agreement with experiment within errors

We also observed:

- an amount of isospin breaking in  $f_1(1285) \rightarrow \pi^0 f_0$  different from other reaction  
 $\implies$  it appears to be strongly dependent on the process
- shape of the invariant mass distributions of  $f_1(1285) \rightarrow \pi K\bar{K}$  tied to the nature of the  $f_1$   
 $\implies$  **a measurement of these quantities would help understanding the nature of the  $f_1(1285)$**

# The molecular $K^* \bar{K}$ nature of the $f_1(1285)$ and its decays into $\pi a_0(980)$ ( $\pi f_0(980)$ ) and $\pi K \bar{K}$

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