Cold strongly interacting matter under intense magnetic fields

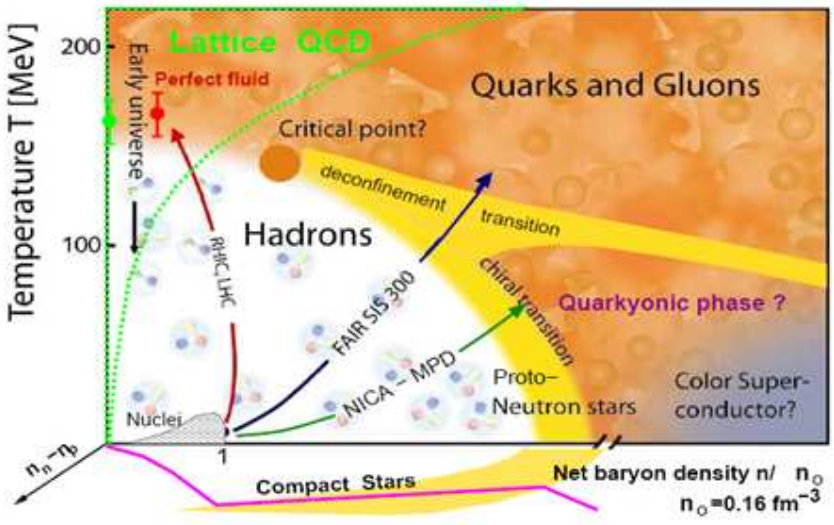
Valeria Paula Pagura
IFIC
In collaboration with:
Pablo Allen and **Norberto Scoccola**
Comisión Nacional de Energía Atómica - CONICET
Argentina

June 17, 2015

Introduction

Understanding the behavior of strongly interacting matter at finite temperature and/or density is of fundamental interest in **cosmology**, **neutron stars astrophysics**, **RHIC's physics**.

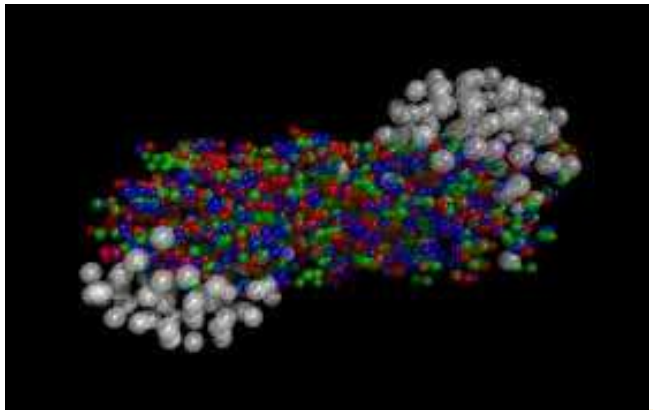
Low T and ρ: confinement + broken chiral symmetry
High T and/or ρ: free quarks + restaured chiral symmetry

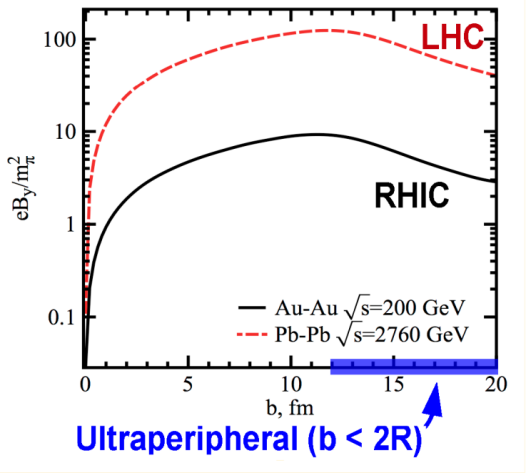


Lattice QCD:
$T_d \sim T_{ch} = 154 \pm 9$ MeV
Bazavov et al (2012)

Motivation: Intense magnetic fields

Non-central relativistic heavy ion collisions: $B \sim 10^{19}$ G

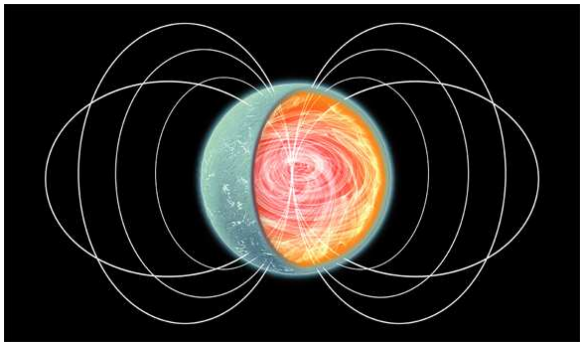




Magnetic field at $T = 0$

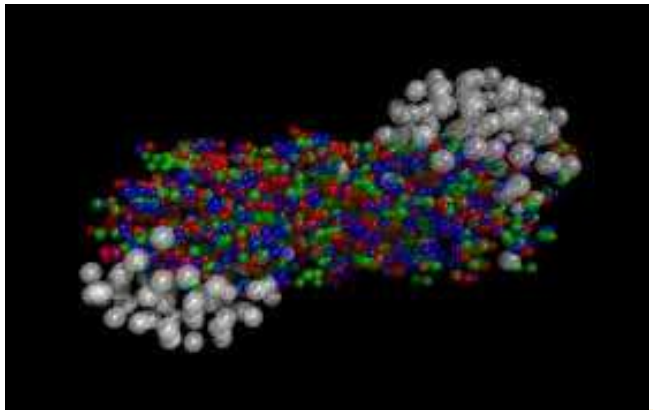
(A. Bzdak, V. Skokov (2012))

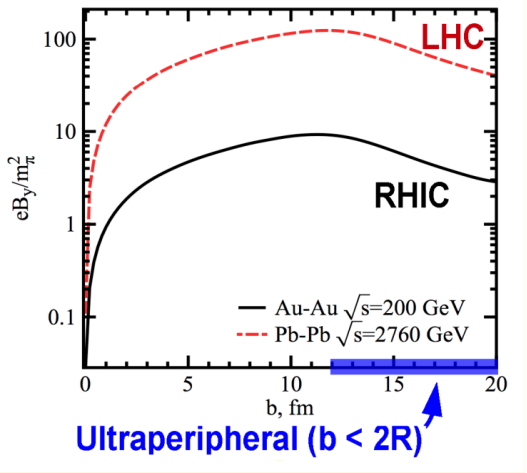
Magnetars: $B \sim 10^{13} - 10^{15}$ G (Duncan and Thompson (2012))



Motivation: Intense magnetic fields

Non-central relativistic heavy ion collisions: $B \sim 10^{19}$ G

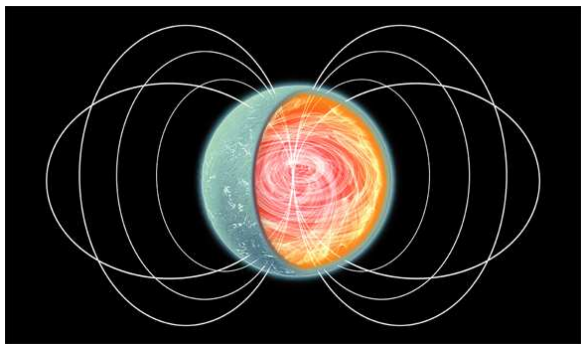




Magnetic field at $T = 0$

(A. Bzdak, V. Skokov (2012))

Magnetars: $B \sim 10^{13} - 10^{15}$ G (Duncan and Thompson (2012))



Including a non-zero uniform B

- How does QCD phase diagram looks like?
- Are there modifications in the nature of transitions?
- What is the fate of critical endpoints?

The subject has been studied from *ab initio* approach

- Lattice QCD, for $\mu = 0$ (D'Elia (10/11), Bali et al (11/12))

Effective models:

- NJL (Klevansky, Lemmer (1989); Klimenko et al. (1992,...); Gusynin, Miransky, Shokovy (1994/95); Ferrer, Incera et al (2003,...), Hiller, Osipov (2007/08); Menezes et al (2009);Fukushima, Ruggieri, Gatto (2010);...)
- χPT (Shushpanov, Smilga (1997); Agasian, Shushpanov (2000); Cohen, McGady, Werbos (07);...)
- Linear Sigma Model, MIT bag model ((Fraga, Mizher (2008), Fraga, Palhares (2012))

Nambu - Jona–Lasinio (NJL) Model

Model with chiral quark interaction (Nambu, Jona-Lasinio (1961))
Simplest version: **Scalar** and **pseudoscalar** interactions.

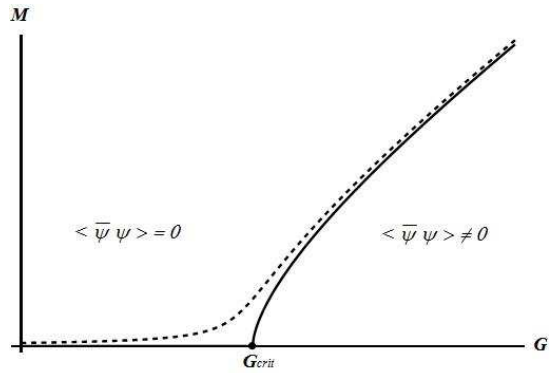
$$\mathcal{L}_{NJL} = \bar{\psi}(i\ \partial\!\!\!/ - m_c)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] \qquad \psi^\dagger = (u, d)$$

In the mean field approximation (MFA):

$$(\bar{\psi}\mathcal{O}\psi)^2 \to 2\langle\bar{\psi}\mathcal{O}\psi\rangle\bar{\psi}\mathcal{O}\psi$$

Dynamical fermion mass generation:

$$M = m_c - 2G\langle\bar{\psi}\psi\rangle \qquad G > G_{crit} \Rightarrow \langle\bar{\psi}\psi\rangle \neq 0$$



Parameters:

- Current quark mass m_c
- Coupling constant G (4 fermions)
- Cutoff Λ

Chosen to reproduce:

- $m_\pi = 139$ MeV
- $f_\pi = 92{,}4$ MeV
- Constiuent mass $M_0 \sim 300 - 400$ MeV

Generalized NJL model

$$\mathcal{L} = \bar{\psi}(x)(i\not{D} - m_c + \hat{\mu}\gamma^0)\psi + \mathcal{L}_{int}$$

scalar-pseudoscalar, **vector-axial-vector** 4-fermion couplings, t'Hooft determinant interaction

$$\begin{aligned}\mathcal{L}_{int} = & G_1 \sum_{a=0}^{3} \left[\left(\bar{\psi}\tau_a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda_a\psi\right)^2 \right] + G_2 \sum_{a=0}^{3} \left[\left(\bar{\psi}\gamma_\mu\tau_a\psi\right)^2 + \left(\bar{\psi}\gamma_\mu\gamma_5\tau_a\psi\right)^2 \right] \\ & G_3 \left[\left(\bar{\psi}\gamma_\mu\psi\right)^2 + \left(\bar{\psi}i\gamma_\mu\gamma_5\psi\right)^2 \right] + G_4 \left[\left(\bar{\psi}\gamma_\mu\psi\right)^2 - \left(\bar{\psi}\gamma_\mu\gamma_5\psi\right)^2 \right] \\ & +2G_D(d_+ + d_-), \qquad d_\pm = det\left[\bar{\psi}(1 \pm \gamma_5)\psi\right]\end{aligned}$$

Coupling to an homogeneous external magnetic field:

$$D_\mu = \partial_\mu - i\hat{q}\mathcal{A}_\mu, \quad \mathcal{A}_\mu = \delta_{\mu 2} x_1 B$$

$$\hat{q} = diag(q_u, q_d)$$

$$\hat{\mu} = diag(\mu_u, \mu_d)$$

Quark relation dispersions are modified: Landau levels

$$\Omega^{MFA}(\tilde{\mu}, B; \sigma_u, \sigma_d, \omega_u, \omega_d) = -\sum_{f=u,d} \theta_f + \Omega_{pot}$$

$$\Omega^{MFA}(\tilde{\mu}, B; \sigma_u, \sigma_d, \omega_u, \omega_d) = -\sum_{f=u,d} \theta_f + \Omega_{pot}$$

gas of quasi-particles

$$\theta_f = \theta_f^{\rm vac}(\sigma_f) + \theta_f^{\rm mag}(B; \sigma_f) + \theta_f^{\rm med}(\tilde{\mu}_f, B; \sigma_f)$$

$$M_f = m_c + \sigma_f \qquad \tilde{\mu}_f = \mu_f - \omega_f$$

$$\Omega^{MFA}(\tilde{\mu}, B; \sigma_u, \sigma_d, \omega_u, \omega_d) = -\sum_{f=u,d} \theta_f + \Omega_{pot}$$

gas of quasi-particles

$$\theta_f = \theta_f^{\rm vac}(\sigma_f) + \theta_f^{\rm mag}(B; \sigma_f) + \theta_f^{\rm med}(\tilde{\mu}_f, B; \sigma_f)$$

$$M_f = m_c + \sigma_f \qquad \tilde{\mu}_f = \mu_f - \omega_f$$

$$\Omega_{pot} = \frac{(1-c_s)(\sigma_u^2+\sigma_d^2) - 2c_s\,\sigma_u\,\sigma_d}{8g_s(1-2c_s)} - \frac{(1-2c_v)(\omega_u^2+\omega_d^2) + c_v\,\omega_u\,\omega_d}{8g_v(1-2c_v)}$$

$$\Omega^{MFA}(\tilde{\mu}, B; \sigma_u, \sigma_d, \omega_u, \omega_d) = -\sum_{f=u,d} \theta_f + \Omega_{pot}$$

gas of quasi-particles

$$\theta_f = \theta_f^{\rm vac}(\sigma_f) + \theta_f^{\rm mag}(B; \sigma_f) + \theta_f^{\rm med}(\tilde{\mu}_f, B; \sigma_f)$$

$$M_f = m_c + \sigma_f \qquad \tilde{\mu}_f = \mu_f - \omega_f$$

$$\Omega_{pot} = \frac{(1-c_s)(\sigma_u^2+\sigma_d^2) - 2c_s\,\sigma_u\,\sigma_d}{8g_s(1-2c_s)} - \frac{(1-2c_v)(\omega_u^2+\omega_d^2) + c_v\,\omega_u\,\omega_d}{8g_v(1-2c_v)}$$

- Λ, m_c
- g_v, g_s: effective scalar and vector couplings
- c_s: regulates flavor mixing $0 \leq c_s \leq 1/2$ (max. flavor mixing)
- c_v: regulates isoscalar-isovector mixing $0 \leq c_s \leq 1/2$ (pure isoscalar)
- g_v, c_s, c_v phenomenological (not well known) $\longrightarrow$ free parameters
  effective OGE: $g_v/g_s \sim 1/2$ and $\eta - \eta'$ mixing: $c_s \sim 0.2$

Cold quark matter under intense B

Thermodynamic potential ($T \to 0$)

$$\Omega^{MFA}(\tilde{\mu}, B; \sigma_u, \sigma_d, \omega_u, \omega_d)$$

First order transitions

Solve the set of coupled "gap" equations for fixed μ, B

$$\frac{\partial \Omega}{\partial \phi} = 0 \qquad \phi = \sigma_u, \sigma_d, \omega_u, \omega_d$$

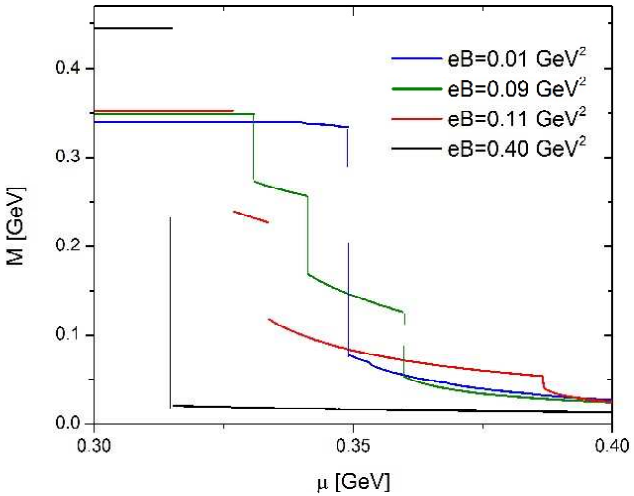
Crossovers

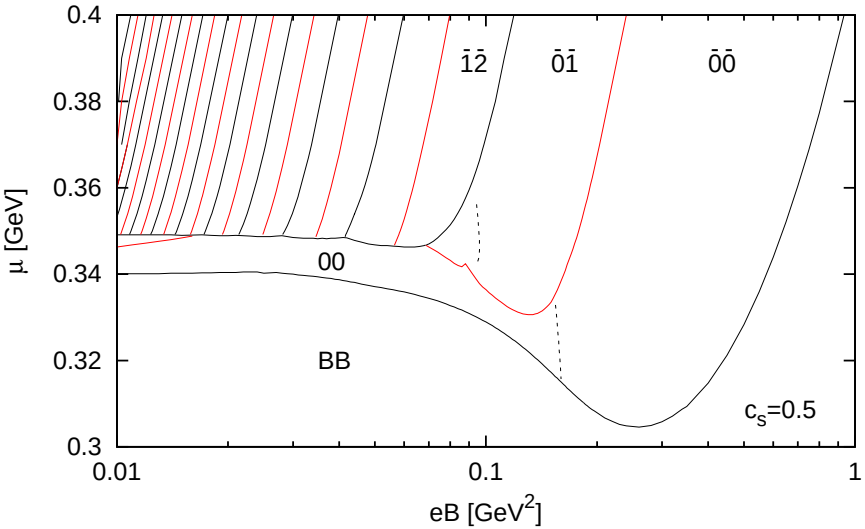
$$\chi_f = \partial \langle \bar{\psi}_f \psi_f \rangle / \partial m_c$$

Natural units: $B = 1.69 \times 10^{20}\mathrm{G} \to eB = 1\mathrm{GeV}^2$

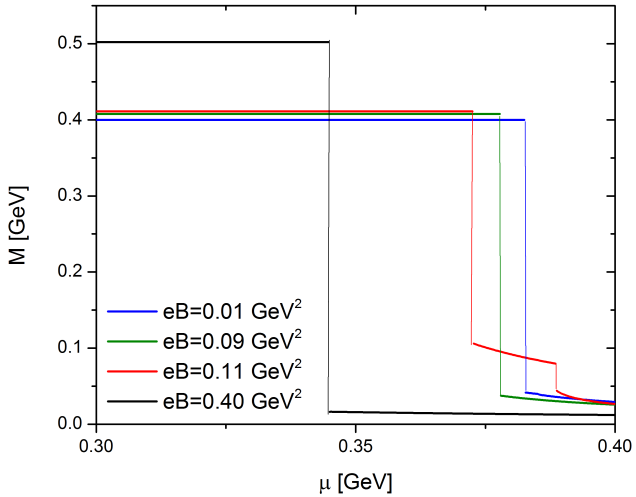
The "standard" NJL case ($c_s = 1/2,\ g_v/g_s = 0$)

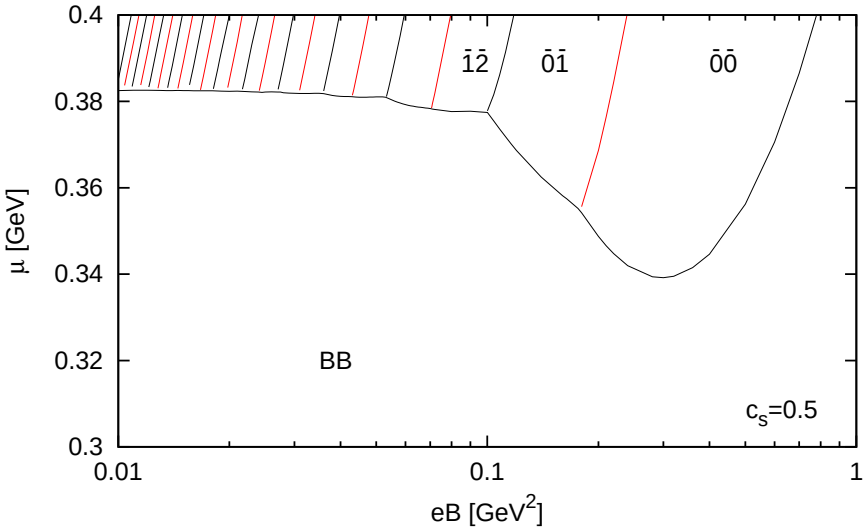
Set 1: $M_0 = 340$ MeV





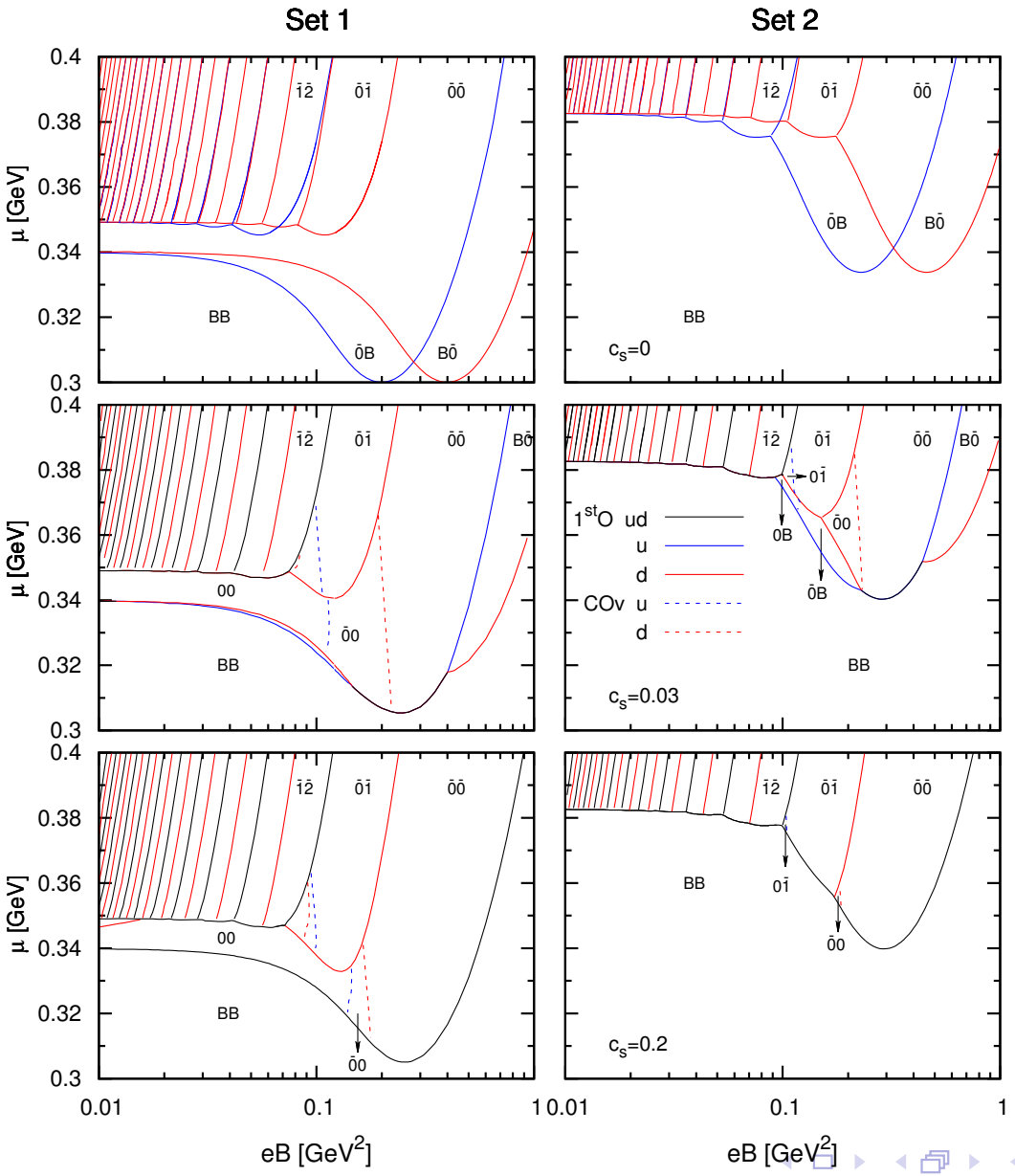
Set 2: $M_0 = 400$ MeV



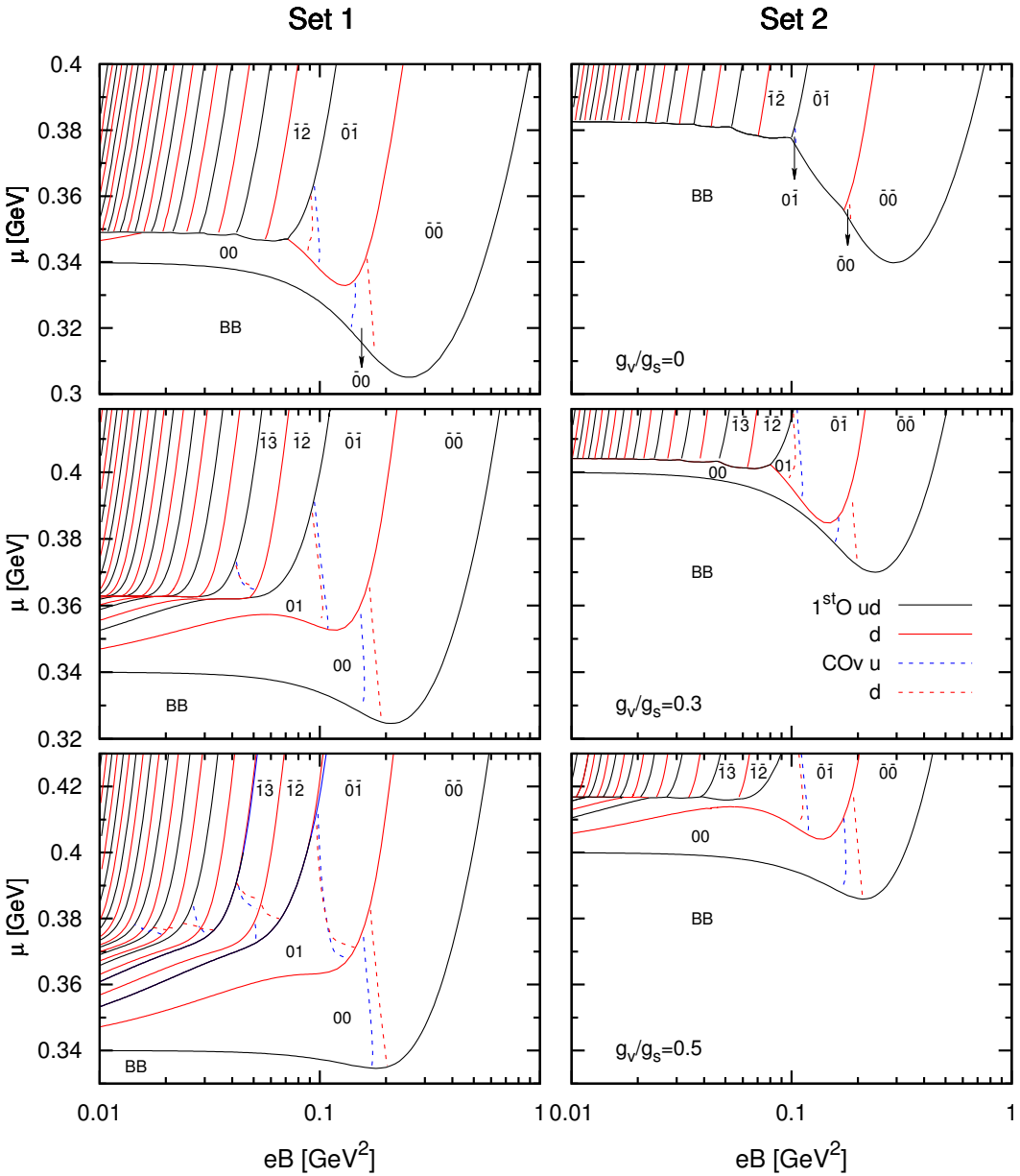


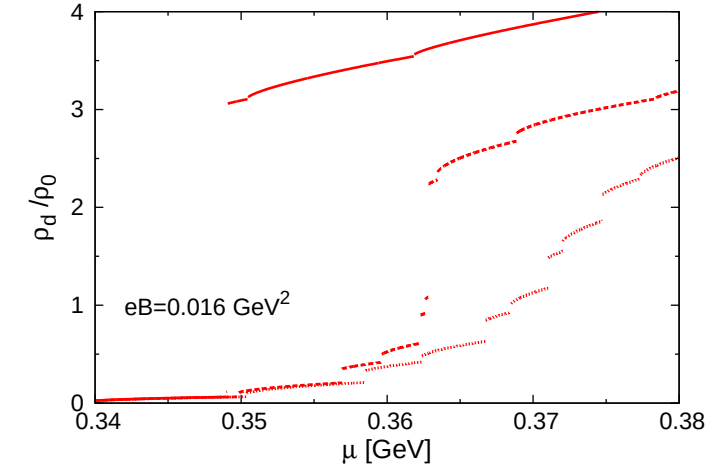
(P. Allen, N. Scoccola (2013))

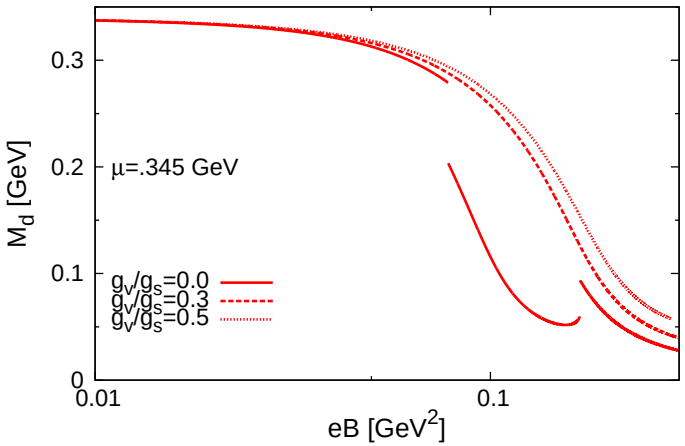
The rôle of Flavor Mixing



The rôle of vector interaction ($c_s = 0.2$, $c_v = 0$)







reduction of magnetic anticatalysis

The rôle of stellar matter conditions

- β equilibrium

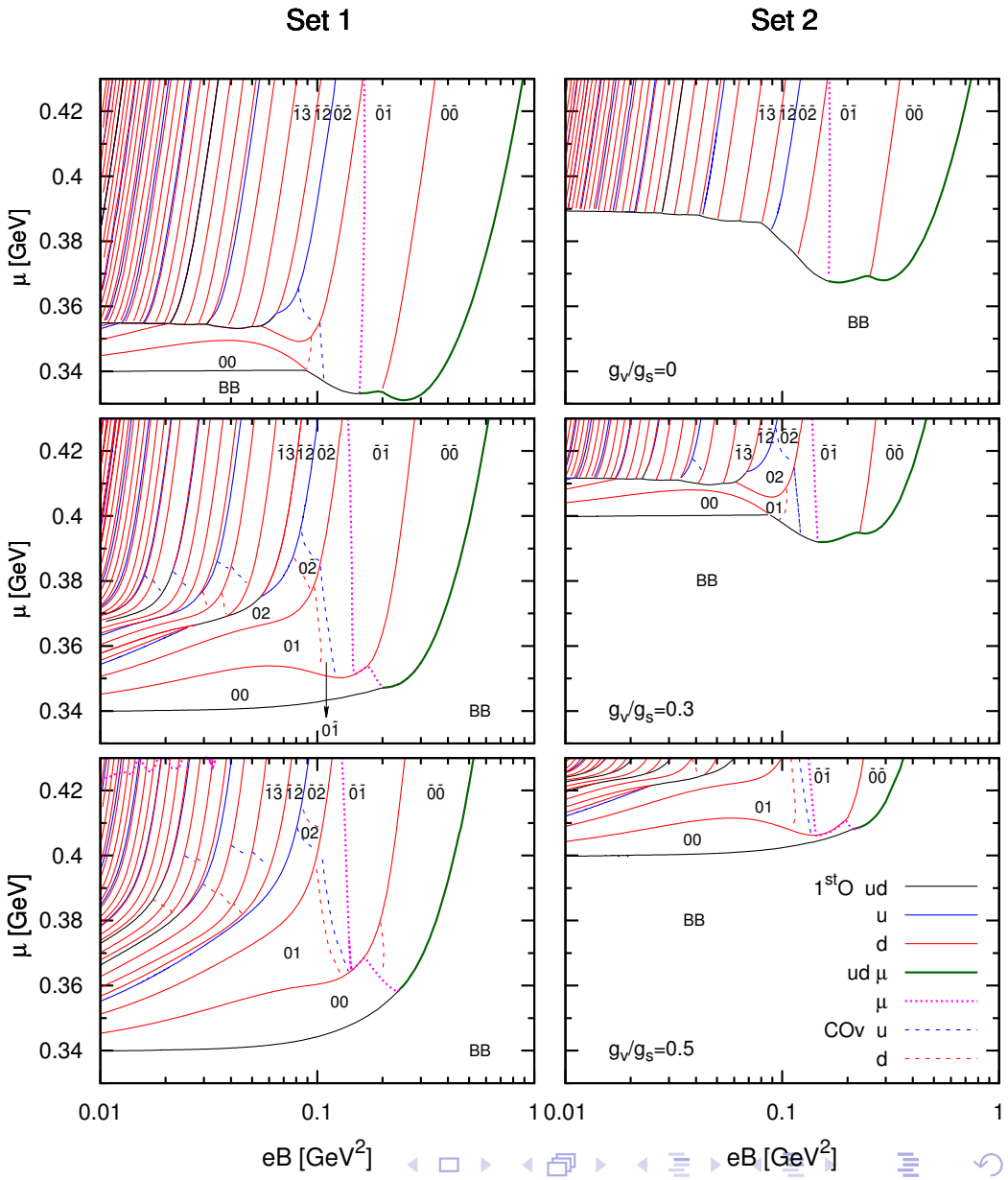
$$\mu_u = \mu - 2/3\mu_e$$

$$\mu_d = \mu + \mu_e/3$$

- charge neutrality

$$\rho_e + \rho_\mu = (2\rho_u - \rho_d)/3$$

- e and q populate their LLL simultaneously
- μ require higher chemical potential



Summary and outlook

- We have analyzed the effect of an intense magnetic field on quark matter in the context of NJL-type models.
- The detailed form of the phase diagram at low T is rather different depending on the parameterization used. In fact, there can be a quite rich structure due to the subsequent population of the Landau levels as μ increases.
- For arbitrary (small) flavor mixing (FM) the phase diagram can be rather complex. For phenomenological acceptable values of FM ($c_s \sim 0{,}2$) results similar to maximum flavor mixing.
- Vector mesons as well as charge neutrality and beta equilibrium conditions tend to make the IMC effect weaker (or even disappear)

thank you for your attention

Thermodynamic potential

$$\Omega^{MFA} = -\sum_{f=u,d} \theta_f + \Omega_{pot} \qquad \theta_f : \text{gas of quasi-particles } f$$

$$\theta_f^{\rm vac} = 4N_c \int^{\Lambda} \frac{d^3p}{(2\pi)^3} \sqrt{p^2+M^2} \quad \text{(needs to be regularized)}$$

$$\theta_f^{\rm mag} = \frac{N_c}{2\pi^2} (|q_f|B)^2 \left[\zeta^{(1,0)}(-1,x_f) - \frac{1}{2}(x_f^2 - x_f)\ln x_f + \frac{x_f^2}{4} \right]$$

$$\theta_f^{\rm med} = \frac{N_c}{4\pi^2} |q_f|B \sum_{\nu=0}^{\nu_f^{max}} \alpha_\nu \left[\tilde{\mu}_f \sqrt{\tilde{\mu}_f^2 - s_f(\nu,B)^2} - s_f(\nu,B)^2 \ln \left(\frac{\tilde{\mu}_f + \sqrt{\tilde{\mu}_f^2 - s_f(\nu,B)^2}}{s_f(\nu,B)} \right) \right] \text{(Landau levels)}$$

$$M_f = m_c + \sigma_f \qquad \tilde{\mu}_f = \mu_f - \omega_f$$

$\zeta(z,x_f)$: Riemann-Hurwitz zeta function $\qquad x_f = M_f^2/(2|q_f|B)$

$s_f(\nu,B) = \sqrt{M_f^2 + 2|q_f|B\nu} \qquad \nu_f^{max} < (\tilde{\mu}_f^2 - M_f^2)/(2|q_f|B)$

gap equations I

The explicit form of the gap equations is

$$\phi_u + \frac{(1-c_s)\sigma_u - c_s\sigma_d}{4g_s(1-2c_s)} = 0 \quad ; \quad \rho_u - \frac{(1-2c_v)\omega_u - c_v\omega_d}{4g_v(1-2c_v)} = 0$$

$$\phi_d + \frac{(1-c_s)\sigma_d - c_s\sigma_u}{4g_s(1-2c_s)} = 0 \quad ; \quad \rho_d - \frac{(1-2c_v)\omega_d - c_v\omega_u}{4g_v(1-2c_v)} = 0$$

where ρ_f in the quark density for each flavor

$$\rho_f = \frac{N_c}{2\pi^2}|q_f|B\sum_{\nu=0}^{\nu_f^{max}}\alpha_\nu\sqrt{\tilde{\mu}_f^2 - s_f(\nu,B)^2}$$

gap equations II

and ϕ_f can be written as the sum of three terms

$$\phi_f^{\rm vac} = -\frac{N_c M_f}{2\pi^2}\left[\Lambda\sqrt{\Lambda^2+M_f^2} - M_f^2 \ln\left(\frac{\Lambda+\sqrt{\Lambda^2+M_f^2}}{M_f}\right)\right],$$

$$\phi_f^{\rm mag} = -\frac{N_c M_f |q_f| B}{2\pi^2}\left[\ln\Gamma(x_f) - \frac{1}{2}\ln(2\pi) + x_f - \frac{1}{2}(2x_f-1)\ln(x_f\right.$$

$$\phi_f^{\rm med} = -\frac{N_c}{2\pi^2}\, M_f\, |q_f| B \sum_{\nu=0}^{\nu_f^{max}} \alpha_\nu\, \ln\left[\frac{\tilde{\mu}_f + \sqrt{\tilde{\mu}_f^2 - s_f(\nu,B)^2}}{s_f(\nu,B)}\right].$$