Higgs Physics

within and beyond the SM

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Exercises

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INSTITUCIÓ CATALANA DE RECERCA I ESTUDIS AVANÇATS

Ex. 1

Relativistic kinematics

- Two body decays: A→B+C
  - In the rest frame of the particle A, find the energy and momentum of the particles B and C

$$\left[E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2 \qquad p = \frac{\sqrt{\lambda(m_A, m_B, m_C)}}{2m_A} c \right]$$

$$\lambda(m_A, m_B, m_C) = (m_A + m_B + m_C)(m_A + m_B - m_C)(m_A - m_B + m_C)(m_A - m_B - m_C)$$

  - Application: compute the energies of the final particles in the following decays:
    - $\pi^- \to \mu^- \bar{\nu}_\mu$
    - $\pi^0 \to \gamma\gamma$
    - $\Omega^- \to \Lambda K^-$

$$\left[\begin{array}{cc} E_\mu \approx 0.11 \text{ GeV} & E_{\bar{\nu}_\mu} \approx 0.03 \text{ GeV} \\ \multicolumn{2}{c}{E_\gamma \approx 0.68 \text{ GeV}} \\ E_\Lambda \approx 1.13 \text{ GeV} & E_{K^-} \approx 0.53 \text{ GeV} \end{array} \right]$$

  - In the rest frame of the particle A, find the speed and the decay lengths (if they decay) of B and C

$$\left[v_B = \frac{pc^2}{E_B} \qquad d_B = \frac{p}{m_B} \tau_B \right]$$

  - Application: $\pi^- \to \mu^- \bar{\nu}_\mu$, what is the distance travelled by the muon before it decays?

$$\left[d_\mu \approx \frac{m_{\pi^-}^2 - m_\mu^2}{2 m_\mu m_{\pi^-}} c\tau_\mu \approx 186 \text{ m} \right]$$

Ex. 1

Relativistic kinematics

- N-body decays: $A \to B_1 + B_2 + \ldots + B_N$ ($B_1 \ldots B_N$ are a priori different particles)
  - In the rest frame of the particle A, find the minum and the maximum energy of the particle B_1
    - check first that the minimum energy of a system of two particles whose momenta add to a fixed value is obtained when there is no relative motion between the two particles.
    - check that two particles with no relative motion behave like a single particle whose mass is the sum of the masses of the two initial particles

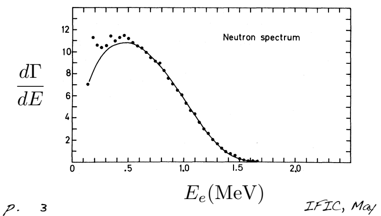
$$\left[E_{B_1}^{\min} = m_{B_1} c^2 \qquad E_{B_1}^{\max} = \frac{m_A^2 + m_{B_1}^2 - (m_{B_2} + \ldots + m_{B_N})^2}{2m_A} c^2 \right]$$

  - Application: compute the range of energy of an electron obtained from the decay of a muon at rest

$$\left[m_e c^2 \approx 511 \text{ keV} \leq E_e \leq 53 \text{ MeV} \approx \frac{m_\mu^2 + m_e^2}{2m_\mu} c^2 \right]$$

  - From the plot below showing the distribution of the electron energy in neutron β decay, compute the neutron-proton mass difference

$$\left[m_n - m_p \approx 1.3 \text{ MeV} \right]$$



C.J.Christensen et al, Phys. Rev. D5 (1972) 1628

Ex. 2

Higgs self-couplings

The Higgs potential is fully determined in terms of two parameters that can fixed by v, ie. m_Z, and m_H. Compute the self-couplings of the physical Higgs boson after EW symmetry breaking in terms of these two quantities.

General expression of the ρ parameter

If $SU(2)_L \times U(1)_Y$ is broken not only through a doublet, but also through a collection of scalar fields in the $2s_i+1$ representation of $SU(2)_L$, carrying a hypercharge y_i and acquiring a vev v_i, show that the ρ parameter is now given by

$$\rho = \frac{\sum_i (s_i(s_i+1) - y_i^2) v_i^2}{\sum_i 2 y_i^2 v_i^2}$$

What are the representations for which one obtains $\rho = 1$?

Ex. 3 Threshold behavior

Near threshold, the Higgs decay into a pair of fermions is highly suppressed by the third power of β ($\beta = \sqrt{1 - 4m_f^2/m_h^2}$). This is characteristic of a coupling of scalar field to fermions. Assuming that the Higgs were a pseudo-scalar, show that its decay rate into a fermion pair would be given instead by

$$\Gamma\left(h \to f\bar{f}\right) = \frac{N_c m_f^2 m_h}{8\pi v^2}\beta$$

Ex. 4 Unitarity bound

for a 2-to-2 process, the (angular) differential cross-section is related to the amplitude by

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{A}|^2$$

Partial wave amplitude decomposition: the partial waves are defined by

$$\mathcal{A} = 16\pi \sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)\, a_l$$

where Pl are the & polynomials ($P_0(x) = 1,\ P_1(x) = x, P_2(x) = 3x^2/2 - 1/2\ldots$ & $\int_{-1}^{1} dx P_l(x) P_{l'}(x) = \frac{2}{2l+1}\delta_{ll'}$)

Show that $a_l = \frac{1}{32\pi}\int_{-1}^{+1} d(\cos\theta) P_l(\cos\theta)\mathcal{A}$ and $\sigma = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)|a_l|^2$

Optical theorem: using the optical theorem: $\sigma = \mathrm{Im}\left(\mathcal{A}_{|\theta=0}\right)/s$

show that $|\mathrm{Re}\,(a_l)| \leq 1/2$

Consider: $W^+ W^- \to W^+ W^-$

Lee, Quigg, Thacker '77
Chanowitz, Gaillard '85

Compute a_0 for the SM without and with the Higgs. Compare the unitarity bounds with the NDA estimate of SM cutoff without a Higgs. What is the origin of the missing $\sqrt{\pi}$ factor?

Ex. 5 EW phase transition with H^6 potential

Show that the most general potential of degree-6 for the Higgs doublet that breaks EW symmetry can be always be written in the following form with λ positive

$$V(H) = \lambda\left(|H|^2 - \frac{v^2}{2}\right)^2 + \frac{1}{\Lambda^2}\left(|H|^2 - \frac{v^2}{2}\right)^3$$

1) Show that for this potential indeed breaks EW when $\Lambda^2 > v^2/(2\lambda)$

2) Compute the vev of the Higgs and the mass of the physical Higgs boson, compute the Higgs cubic self-interaction (and compare with the SM results)

We recall that the finite-temperature corrections generates a correction to the potential of the form

$$V = \frac{T^2}{24}\left(\sum_{\text{boson}} m^2 + \frac{1}{2}\sum_{\text{fermion}} m^2\right)$$

3) Compute the thermal mass of the Higgs boson for this model: $V = \frac{1}{2}cT^2h^2$

answer: $c = \frac{1}{16v^2}\left(8m_t^2 + 8m_W^2 + 4m_Z^2 + 4m_h^2 - 12\frac{v^4}{\Lambda^2}\right)$

4) Keeping only this thermal mass as the finite-temperature correction, show that the phase transition is 1st order if $\Lambda < \sqrt{3}v^2/m_h^2$

5) Compute the critical temperature and critical vev

6) Show that the phase transition is strong ($v_c/T_c > 1$) for $484\ \text{GeV} \leq \Lambda \leq 788\ \text{GeV}$

For the numerical application, you'll use

$$v = 246\ \text{GeV}, m_h = 125\ \text{GeV}, m_W = 80\ \text{GeV}, m_Z = 91\ \text{GeV}, m_t = 173\ \text{GeV}$$

Ex. 6 β function, gauge coupling running

The one-loop β function giving the running of the coupling constant of an SU(N) gauge symmetry is given by

$$\beta = \frac{dg}{d\log\mu} = -\frac{1}{16\pi^2}b_0 g^3 \quad \text{ie} \quad \frac{d\alpha}{d\log\mu} = -\frac{1}{2\pi}b_0\alpha^2$$

where the coefficient b_0 is computed to

$$b_0 = \frac{11}{3}T_2(\texttt{spin-1}) - \frac{2}{3}T_2(\texttt{chiral spin-1/2}) - \frac{1}{3}T_2(\texttt{complex spin-0})$$

$T_2(R)$ is defined from the traces of the product of two generators of SU(N) in the representation R

$$\mathrm{Tr}\left(T^a(R)T^b(R)\right) = T_2(R)\delta^{ab}$$

1) What should be the sign of b_0 to get an asymptotically free theory?
2) Compute $\alpha(\mu)$ from $\alpha(\mu_0)$
3) Compute b_0 for $U(1)_{em}$ with a single massive electron.
What is the value of the Landau pole of QED, ie the energy at which α_{em} blows up?
At which energy do we get a 1% departure from $\alpha(0)$?
4) Compute the coefficients b_0 for the 3 gauge groups of the SM
5) Compute the coefficients b_0 for the 3 gauge groups of the MSSM
6) In N=4 supersymmetric gauge theories, a supermultiplet contains 1 spin-1 field, 4 spin-1/2 chiral fields and 6 real spin-0 fields in the adjoint representation. Compute the coefficient b_0 for that theory? What do you conclude?

Ex. 7 EW oblique corrections

1 The oblique parameters are defined from physical observables:

$$\Delta m_W = -\frac{\alpha m_W}{4(c_0^2 - s_0^2)} S + \frac{\alpha c_0^2 m_W}{2(c_0^2 - s_0^2)} T + \frac{\alpha m_W}{8 s_0^2} U$$

$$\Delta s^2_{\rm eff} = \frac{\alpha}{4(c_0^2 - s_0^2)} S - \frac{\alpha c_0^2 s_0^2}{c_0^2 - s_0^2} T$$

$$\Delta \Gamma_{ll} = -\frac{2(1-4s_0^2)\alpha \Gamma^0_{ll}}{(1+(1-4s_0^2)^2)(c_0^2 - s_0^2)} S + \left(1 + \frac{8(1-4s_0^2)s_0^2 c_0^2}{(1+(1-4s_0^2)^2)(c_0^2 - s_0^2)}\right) \alpha \Gamma^0_{ll} T$$

We are using α_{em}, G_F and m_Z as input parameters

$s_{\rm eff}$ is defined via the LR asymmetry in Z-decay: $A_{LR} = \frac{(-1/2 + s^2_{\rm eff})^2 - s^4_{\rm eff}}{(-1/2 + s^2_{\rm eff})^2 + s^4_{\rm eff}}$.

c_0 and s_0 are SM tree-level values of the sin and cos of the weak mixing angle

2 Alternatively, the oblique parameters can be computed by looking at the gauge boson self-energies:

oblique parameters = modified propagators of $W^\pm$ and Z

$$\mathcal{L} = -\Pi_{+-}(p^2)\, W_+^\mu W_{-\,\mu} - \tfrac{1}{2}\Pi_{33}(p^2)\, W_3^\mu W_{3\,\mu} - \Pi_{3B}(p^2)\, W_3^\mu B_\mu - \tfrac{1}{2}\Pi_{BB}(p^2)\, B^\mu B_\mu$$

$$\widehat{S} = \frac{\alpha_{em}}{4 s_W^2} S = \frac{g}{g'} \Pi'_{3B}(0) \qquad \widehat{T} = \alpha_{em} T = \frac{(\Pi_{33}(0) - \Pi_{+-}(0))}{m_W^2} \qquad \widehat{U} = -\frac{\alpha_{em}}{4 s_W^2} U = \Pi'_{+-}(0) - \Pi'_{33}(0)$$

Ex. 7 T from higher dimensional operator

We want to compute the oblique parameters when the following dimension-6 operator is added to the SM

$$\mathcal{L} = \frac{1}{\Lambda^2} \left| H^\dagger D_\mu H \right|^2$$

1 In the unitary gauge, show that this operator gives only a correction to Π_{33} equal to

$$\Delta \Pi_{33} = -\frac{g^2 v^4}{8 \Lambda^2}$$

2 Conclude that $T = -\frac{1}{2\sqrt{2}\alpha G_F \Lambda^2}$

3 Write m_W, and $s_{\rm eff}$ in terms of the input observables: α, G_F and m_Z.
And rederive the expression of T

$$\Delta m_W = \frac{c_0^2 m_W}{4\sqrt{2}(c_0^2 - s_0^2) G_F \Lambda^2} \qquad \Delta s^2_{\rm eff} = -\frac{-s_0^2 c_0^2}{2\sqrt{2}(s_0^2 - c_0^2) G_F \Lambda^2}$$

4 Check that T measures the deviation to ρ=1 $\quad \rho = \frac{m_W^2}{c_W^2 m_Z^2} \approx 1 + \alpha T$

Ex. 7 S from higher dimensional operator

We want to compute the oblique parameters when the following dimension-6 operator is added to the SM

$$\mathcal{L} = \frac{1}{\Lambda^2} H^\dagger W_{\mu\nu} H B_{\mu\nu}$$

1 In the unitary gauge, show that this operator gives only a correction to Π_{30} equal to

$$\Delta \Pi'_{3B} = \frac{v^2}{\Lambda^2}$$

2 Conclude that $S = \frac{4 s_W c_W}{\sqrt{2}\alpha_{em} G_F \Lambda^2}$

3 Because of the kinetic mixing, the Z and the γ are not obtained from the usual weak rotation from W_3 and B. Find the correct expressions of Z and γ.

4 The expression of e in terms of g and g' receives some corrections compared to its SM expression. Derive these corrections.

5 Write m_W in terms of the input observables: α, G_F and m_Z.
And rederive the expression of S

Ex. 8 Composite Higgs anomalous couplings

The Higgs boson doesn't have to be an elementary particle. It could be a bound state emerging from a strongly coupled sector. Below the compositeness scale of the Higgs, f, the dynamics of such a composite Higgs boson is well captured by the SM Lagrangian supplemented by a few dimension-six operators:

$$\mathcal{L} = |D_\mu H|^2 + \frac{c_H}{2f^2}\left(\partial_\mu |H|^2\right)^2 + \mu^2 |H|^2 - \lambda |H|^4 - \frac{c_6 \lambda}{3 f^2}|H|^6 - y_f \left(H \bar{f}_L f_R \left(1 + \frac{c_y}{f^2}|H|^2\right) + \text{h.c.}\right)$$

1) Compute the corrections to the Higgs self-couplings to the lowest order in $\xi = v^2/f^2$

(check that $\hat{h} = \left(1 + \frac{c_H \xi}{2}\right) h + \frac{c_H \xi}{2}\frac{h^2}{v} + \frac{c_H \xi}{6}\frac{h^3}{v^2}$ is canonically normalized)

2) Compute the coefficients a,b,c for the effective Lagrangian:

$$\mathcal{L}_{\rm EWSB} = \frac{v^2}{4} \text{Tr}\left(D_\mu \Sigma^\dagger D_\mu \Sigma\right) \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - \lambda \bar{\psi}_L \Sigma \psi_R \left(1 + c\frac{h}{v}\right)$$

3) What is the high-energy behavior of the amplitudes for WW→WW and WW→ hh?
4) Compute the corrections to the decay width of the Higgs into a pair of fermions
5) Compute the corrections to the decay width of the Higgs into a pair of bosons