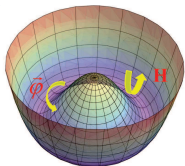
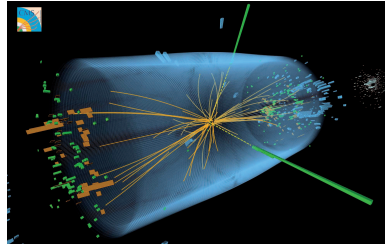
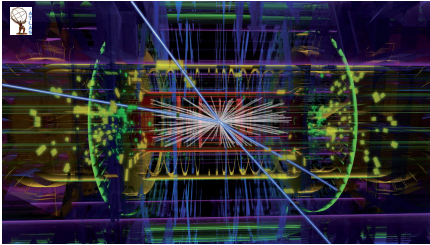


3. Electroweak Effective Theory

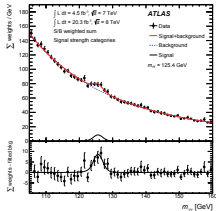


- Higgs Mechanism
- Custodial Symmetry
- Equivalence Theorem
- EW Effective Theory
- Linear Realization

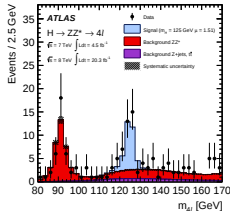
A New Higgs-Like Boson



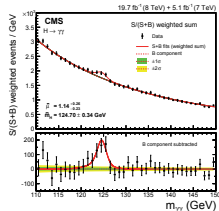
$H \rightarrow \gamma\gamma$



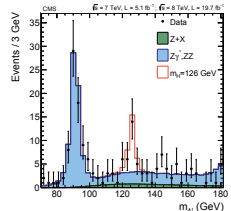
$H \rightarrow ZZ^* \rightarrow 4\ell$



$H \rightarrow \gamma\gamma$



$H \rightarrow ZZ^* \rightarrow 4\ell$



$$M_H = (125.09 \pm 0.21 \pm 0.11) \text{ GeV}$$

Great success of the Standard Model

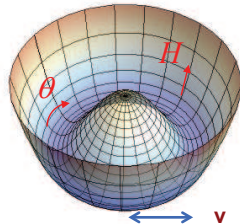
BEGHHK ($\equiv$ Higgs) Mechanism



Kibble Guralnik Hagen Englert Brout



Higgs 1964



Glashow



Weinberg



Salam

$$SU(2)_L \otimes U(1)_Y \quad v = 246 \text{ GeV}$$

$$M_Z \cos \theta_W = M_W = \frac{1}{2} v g$$



Fundación
Príncipe de Asturias



Beautiful Discovery

Boson, $J \neq 1$

Fermions = Matter ; Bosons = Forces

- **Fundamental Boson:** New interaction which is not gauge
- **Composite Boson:** New underlying dynamics



Beautiful Discovery

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If New Physics exists at Λ_{NP}

$$\delta M_H^2 \sim \frac{g^2}{(4\pi)^2} \Lambda_{\text{NP}}^2 \log \left(\frac{\Lambda_{\text{NP}}^2}{M_H^2} \right)$$

Which symmetry keeps M_H away from Λ_{NP} ?

- **Fermions:** Chiral Symmetry
- **Gauge Bosons:** Gauge Symmetry
- **Scalar Bosons:** Supersymmetry, Scale/Conformal Symmetry ... ?

Symmetries & Mass Scales

Fermions: $\psi_{L,R} \longrightarrow e^{i\alpha_{L,R}} \psi_{L,R}$ **Chiral symmetry**

$$\mathcal{L}_\psi = \bar{\psi} (i\not{\partial} - m_\psi) \psi = \bar{\psi}_L i\not{\partial} \psi_L + \bar{\psi}_R i\not{\partial} \psi_R - m_\psi (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

Symmetry recovered at $m_\psi = 0$



$$\delta m_\psi \propto m_\psi$$

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Symmetry recovered at $m_\psi = 0$ $\longrightarrow$ $\delta m_\psi \propto m_\psi$

Vectors: $A_\mu \longrightarrow A_\mu + \partial_\mu \theta$ **Gauge symmetry**

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu$$

Symmetry recovered at $m_A = 0$ $\longrightarrow$ $\delta m_A^2 \propto m_A^2$

Symmetries & Mass Scales

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Scalars: $\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2$ **Any symmetry?**

No additional symmetry at $m_\phi = 0$ $\longrightarrow \delta m_\phi^2 \propto M^2$ ($M = \text{any scale}$)

Symmetries & Mass Scales

Scalars:

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2$$

Any symmetry?

No additional symmetry at $m_\phi = 0$



$$\delta m_\phi^2 \propto M^2 \quad (M = \text{any scale})$$

- **Shift symmetry:** $\phi \longrightarrow \phi + c$

Pseudo-Goldstone Boson

Symmetries & Mass Scales

Scalars:

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2$$

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- **Shift symmetry:** $\phi \longrightarrow \phi + c$

Pseudo-Goldstone Boson

- **Scale symmetry:** $x \longrightarrow x/\lambda$, $\phi(x) \longrightarrow \lambda \phi(x/\lambda)$

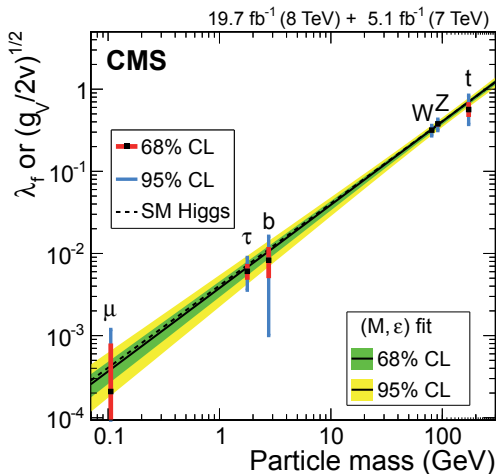
$$\mathbf{M} = 0 \quad , \quad \forall \mathbf{M}$$

Conformal Invariance. Dilaton

It is a Higgs Boson

$$\lambda_f = (m_f/M)^{1+\epsilon} \quad , \quad (g_V/2v)^{1/2} = (M_V/M)^{1+\epsilon}$$

Ellis-You, 1303.3879



SM: $\epsilon = 0$, $M = v = 246$ GeV

CMS: (95% CL)

$$\epsilon \in [-0.054, 0.100]$$

$$M \in [217, 279] \text{ GeV}$$



Possible Scenarios of EWSB

① **SM Higgs:** Favoured by EW precision tests

② **Alternative perturbative EWSB:**

Scalar Doublets and singlets

$$\rho_{\text{tree}} = \frac{M_W^2}{M_Z^2 c_W^2} = \frac{\sum_i v_i^2 [T_i(T_i + 1) - Y_i^2]}{2 \sum_i v_i^2 Y_i^2}$$

③ **Dynamical (non-perturbative) EWSB:**

Pseudo-Goldstone Higgs

Scalar Resonance



Possible Scenarios of EWSB

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③ **Dynamical (non-perturbative) EWSB:**

Pseudo-Goldstone Higgs

Scalar Resonance



ATLAS SUSY Searches* - 95% CL Lower Limits

Status: Feb 2015

ATLAS Preliminary

$\sqrt{s} = 7, 8 \text{ TeV}$

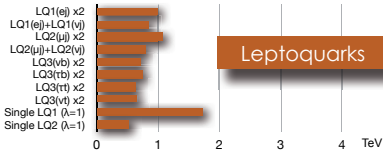
	Model	e, μ, τ, γ	Jets	E_T^{miss}	$\int \mathcal{L} d\Omega (\text{fb}^{-1})$	Mass limit	Reference
Inclusive Searches	MSUGRA/CMSSM	0	2-6 jets	Yes	20.3	$\tilde{L}, \tilde{R}$	$m(\tilde{g})=m(\tilde{t})$
	$\tilde{q}\tilde{q}, \tilde{q} \rightarrow q\tilde{g}$	0	2-6 jets	Yes	20.3	850 GeV	$m(\tilde{t}_1^0)=0 \text{ GeV}, m(\tilde{t}_1^{\pm} \text{ gen. q.})=m(2^{\text{nd}} \text{ gen. q.})$
	$\tilde{q}\tilde{q}, \tilde{q} \rightarrow q\tilde{g}$ (compressed)	1 γ	0-1 jet	Yes	20.3	250 GeV	$m(\tilde{g})-m(\tilde{t}_1^0) = m(\tilde{q})$
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow gg\tilde{t}_1^0$	0	2-6 jets	Yes	20.3		$m(\tilde{t}_1^0)=0 \text{ GeV}$
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow gg\tilde{t}_1^0$	1 e, μ	3-6 jets	Yes	20		$m(\tilde{t}_1^0)=300 \text{ GeV}, m(\tilde{t}^{\pm})=0.5(m(\tilde{t}_1^0)+m(\tilde{g}))$
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow gg\tilde{t}_1^0$	2 e, μ	0-3 jets	-	20		$m(\tilde{t}_1^0)=0 \text{ GeV}$
	GMSB ($\tilde{t}$ NLSP)	1-2 $\tau + 0-1 \ell$	0-2 jets	Yes	20.3		$\tan\beta > 20$
	GGM (bino NLSP)	2 γ	-	Yes	20.3		$m(\tilde{t}_1^0)=50 \text{ GeV}$
	GGM (wino NLSP)	1 $e, \mu + \gamma$	-	Yes	4.8	619 GeV	$m(\tilde{t}_1^0)=50 \text{ GeV}$
	GGM (higgsino-bino NLSP)	γ	1 b	Yes	4.8	900 GeV	$m(\tilde{t}_1^0)=220 \text{ GeV}$
3 rd gen. squarks direct production	GGM (higgsino NLSP)	2 e, μ	0-3 jets	Yes	5.8	690 GeV	$m(\text{NLSP}) > 200 \text{ GeV}$
	Gravitino LSP	0	mono-jet	Yes	20.3	$F^{1/2} \text{ scale}$	$m(\tilde{G}) > 1.8 \times 10^{-4} \text{ eV}, m(\tilde{g})=m(\tilde{g})=1.5 \text{ TeV}$
	$\tilde{g} \rightarrow b\tilde{b}\tilde{t}_1^0$	0	3 b	Yes	20.1		$m(\tilde{t}_1^0)=400 \text{ GeV}$
	$\tilde{g} \rightarrow t\tilde{t}\tilde{t}_1^0$	0	7-10 jets	Yes	20.3		$m(\tilde{t}_1^0) < 350 \text{ GeV}$
	$\tilde{g} \rightarrow t\tilde{t}\tilde{t}_1^0$	0-1 e, μ	3 b	Yes	20.1		$m(\tilde{t}_1^0)=400 \text{ GeV}$
	$\tilde{g} \rightarrow b\tilde{b}\tilde{t}_1^0$	0-1 e, μ	3 b	Yes	20.1		$m(\tilde{t}_1^0)=300 \text{ GeV}$
	$\tilde{b}_1\tilde{b}_1, \tilde{b}_1 \rightarrow b\tilde{t}_1^0$	0	2 b	Yes	20.1		$m(\tilde{t}_1^0)=90 \text{ GeV}$
	$\tilde{b}_1\tilde{b}_1, \tilde{b}_1 \rightarrow t\tilde{t}_1^0$	2 e, μ (SS)	0-3 b	Yes	20.3	100-620 GeV	$m(\tilde{t}_1^0)=2 m(\tilde{t}_1^0)$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow b\tilde{t}_1^0$	1-2 e, μ	1-2 b	Yes	4.7	110-167 GeV	$m(\tilde{t}_1^0) = 2m(\tilde{t}_1^0), m(\tilde{t}_1^0)=55 \text{ GeV}$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow W\tilde{b}_1^0$ or $\tilde{t}_1^0$	2 e, μ	0-2 jets	Yes	20.3	90-191 GeV	$m(\tilde{t}_1^0)=1 \text{ GeV}$
EW direct	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow t\tilde{t}_1^0$	0-1 e, μ	1-2 b	Yes	20		$m(\tilde{t}_1^0)=1 \text{ GeV}$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow c\tilde{t}_1^0$	0	mono-jet/ c -tag	Yes	20.3	90-240 GeV	$m(\tilde{t}_1)=m(\tilde{t}_1^0)=85 \text{ GeV}$
	$\tilde{t}_1\tilde{t}_1$ (natural GMSB)	2 e, μ	1 b	Yes	20.3		$m(\tilde{t}_1^0)=150 \text{ GeV}$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow t\tilde{t}_1^0$	3 e, μ (Z)	1 b	Yes	20.3		$m(\tilde{t}_1^0)=200 \text{ GeV}$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow t\tilde{t}_1^0$	2 e, μ	0	Yes	20.3	90-325 GeV	$m(\tilde{t}_1^0)=0 \text{ GeV}$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow t\tilde{t}_1^0$	2 e, μ	0	Yes	20.3	140-465 GeV	$m(\tilde{t}_1^0)=0 \text{ GeV}, m(\tilde{t}_1^0)=0.5(m(\tilde{t}_1^0)+m(\tilde{t}_1^0))$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow t\tilde{t}_1^0$	2 τ	-	Yes	20.3	100-350 GeV	$m(\tilde{t}_1^0)=0 \text{ GeV}, m(\tilde{t}_1^0)=0.5(m(\tilde{t}_1^0)+m(\tilde{t}_1^0))$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow t\tilde{t}_1^0$	3 e, μ	0	Yes	20.3	700 GeV	$m(\tilde{t}_1^0)=m(\tilde{t}_1^0), m(\tilde{t}_1^0)=0, m(\tilde{t}_1^0)=0.5(m(\tilde{t}_1^0)+m(\tilde{t}_1^0))$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow W\tilde{t}_1^0$	2-3 e, μ	0-2 jets	Yes	20.3	420 GeV	$m(\tilde{t}_1^0)=m(\tilde{t}_1^0), m(\tilde{t}_1^0)=0, \text{ sleptons decoupled}$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow W\tilde{t}_1^0$	e, μ, γ	0-2 b	Yes	20.3	250 GeV	$m(\tilde{t}_1^0)=m(\tilde{t}_1^0), m(\tilde{t}_1^0)=0, \text{ sleptons decoupled}$
Long-lived particles	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow t\tilde{t}_1^0$	4 e, μ	0	Yes	20.3	620 GeV	$m(\tilde{t}_1^0)=m(\tilde{t}_1^0), m(\tilde{t}_1^0)=0, m(\tilde{t}_1^0)=0.5(m(\tilde{t}_1^0)+m(\tilde{t}_1^0))$
	Direct $\tilde{t}_1\tilde{t}_1$ prod., long-lived $\tilde{t}_1^0$	Disapp. trk	1 jet	Yes	20.3	270 GeV	$m(\tilde{t}_1^0)=160 \text{ MeV}, \tau(\tilde{t}_1^0)=0.2 \text{ ns}$
	Stable, stopped $\tilde{g}$ R-hadron	0	1-5 jets	Yes	27.9	832 GeV	$m(\tilde{t}_1^0)=100 \text{ GeV}, 10 \mu\text{s} < \tau < 1000 \text{ s}$
	Stable $\tilde{g}$ R-hadron	trk	-	-	19.1	1.27 TeV	
	GMSB, stable $\tilde{t}, \tilde{t}_1^0 \rightarrow \tilde{t}(\tilde{t}_1^0)+\tau(e, \mu)$	1-2 μ	-	-	19.1	537 GeV	$10 < \tan\beta < 50$
	GMSB, $\tilde{t}_1^0 \rightarrow \gamma\tilde{G}$, long-lived $\tilde{t}_1^0$	2 γ	-	Yes	20.3	435 GeV	$2 < \tau(\tilde{t}_1^0) < 3 \text{ ns}, \text{SPS8 model}$
	$\tilde{q}\tilde{q}, \tilde{q} \rightarrow qq\tilde{g}$ (RPV)	1 μ , displ. vtx	-	-	20.3	1.0 TeV	$1.5 < \tau < 156 \text{ mm}, \text{BR}(\mu)=1, m(\tilde{t}_1^0)=108 \text{ GeV}$
	LFV $p\bar{p} \rightarrow \tilde{\nu}_e + X, \tilde{\nu}_e \rightarrow e + \mu$	2 e, μ	-	-	4.6	1.61 TeV	$A_{121}=0.10, A_{122}=0.05$
	LFV $p\bar{p} \rightarrow \tilde{\nu}_e + X, \tilde{\nu}_e \rightarrow e(\mu) + \tau$	1 $e, \mu + \tau$	-	-	4.6	1.1 TeV	$A_{121}=0.10, A_{1233}=0.05$
	Bilinear RPV CMSSM	2 e, μ (SS)	0-3 b	Yes	20.3	1.35 TeV	$m(\tilde{g})=m(\tilde{t}_1^0), \epsilon\tau_{\tilde{t}_1^0} < 1 \text{ mm}$
RPV	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow W\tilde{t}_1^0$	4 e, μ	-	Yes	20.3	750 GeV	$m(\tilde{t}_1^0) > 0.2 m(\tilde{t}_1^0), A_{121} \neq 0$
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow W\tilde{t}_1^0$	3 $e, \mu + \tau$	-	Yes	20.3	450 GeV	$m(\tilde{t}_1^0) > 0.2 m(\tilde{t}_1^0), A_{121} \neq 0$
	$\tilde{g} \rightarrow qq\tilde{g}$	0	6-7 jets	-	20.3	916 GeV	$\text{BR}(\mu)=\text{BR}(b)=\text{BR}(c)=0\%$
	$\tilde{g} \rightarrow t\tilde{t}_1, \tilde{t}_1 \rightarrow b\tilde{s}$	2 e, μ (SS)	0-3 b	Yes	20.3	850 GeV	
	Scalar charm, $\tilde{c} \rightarrow c\tilde{\chi}_1^0$	0	2 c	Yes	20.3	490 GeV	$m(\tilde{t}_1^0) < 200 \text{ GeV}$

*A selection of the available mass limits on new states or phenomena is shown. All limits quoted are observed minus 1 σ theoretical signal cross section uncertainty.

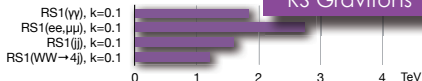
EFT

A. Pich – 2015

9

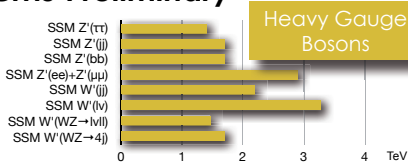


Leptoquarks

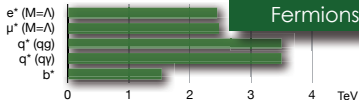


RS Gravitons

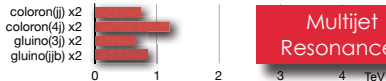
CMS Preliminary



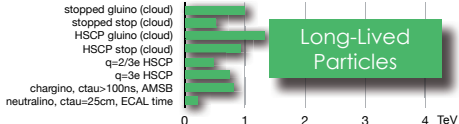
Heavy Gauge Bosons



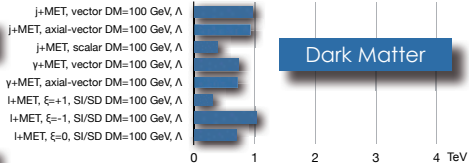
Excited Fermions



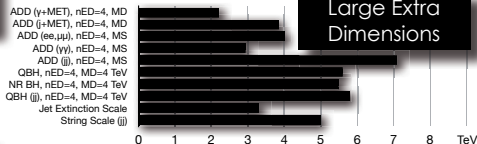
Multijet Resonances



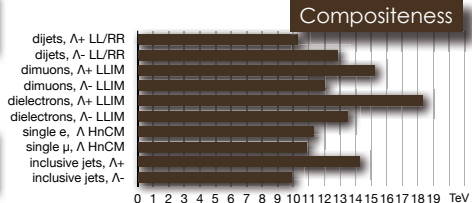
Long-Lived Particles



Dark Matter



Large Extra Dimensions



Compositeness

Energy Scale

Fields

Effective Theory

$\Lambda_{\text{NP}} \sim \text{TeV}$

S_n, P_n, V_n, A_n, F_n
 H, W, Z, γ, g
 τ, μ, e, ν_i
 t, b, c, s, d, u

Underlying Dynamics

..... Energy Gap



M_W

H, W, Z, γ, g
 τ, μ, e, ν_i
 t, b, c, s, d, u

Standard Model

Effective Field Theory

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(4)} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}} \mathcal{O}_i^{(D)}$$

- Most general Lagrangian with the SM gauge symmetries
- Light ($m \ll \Lambda_{\text{NP}}$) fields only
- The SM Lagrangian corresponds to $D = 4$
- $c_i^{(D)}$ contain information on the underlying dynamics:

$$\mathcal{L}_{\text{NP}} \doteq g_X (\bar{q}_L \gamma^\mu q_L) X_\mu \quad \rightarrow \quad \frac{g_X^2}{M_X^2} (\bar{q}_L \gamma^\mu q_L) (\bar{q}_L \gamma_\mu q_L)$$

- Options for $H(126)$:
 - $SU(2)_L$ doublet (SM)
 - Scalar singlet
 - Additional light scalars

Higgs Mechanism:

Gauge invariance

Massless $W^\pm, Z$ (spin 1)

3×2 polarizations = 6

Higgs Mechanism: 3 additional degrees of freedom $\varphi_i(x)$

Gauge invariance

Massless $W^\pm, Z$ (spin 1)

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+

3 Goldstones $\varphi_i(x)$

SSB



Massive $W^\pm, Z$

3×3 polarizations = 9

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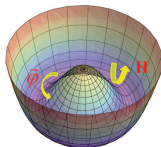


Massive $W^\pm, Z$

$$3 \times 3 \text{ polarizations} = 9$$

Spontaneous Symmetry Breaking

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$



$$\mu^2 < 0$$

$$\Phi(x) = \exp \left\{ i \vec{\sigma} \cdot \frac{\vec{\varphi}(x)}{v} \right\} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

Higgs Mechanism: 3 additional degrees of freedom $\varphi_i(x)$

Gauge invariance

Massless $W^\pm, Z$ (spin 1)

$$3 \times 2 \text{ polarizations} = 6$$

+

3 Goldstones $\varphi_i(x)$

SSB

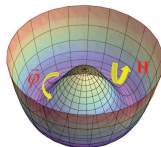


Massive $W^\pm, Z$

$$3 \times 3 \text{ polarizations} = 9$$

Spontaneous Symmetry Breaking

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$



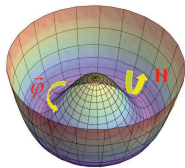
$$\mu^2 < 0$$

$$\Phi(x) = \exp \left\{ i \vec{\sigma} \cdot \frac{\vec{\varphi}(x)}{v} \right\} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

$$D_\mu \Phi = \left(\partial_\mu + \frac{i}{2} g \vec{\sigma} \cdot \vec{W}_\mu + \frac{i}{2} g' B_\mu \right) \Phi \quad ; \quad v^2 = -\mu^2 / \lambda$$

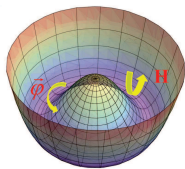
$$(D_\mu \Phi)^\dagger D^\mu \Phi \rightarrow M_W^2 W_\mu^\dagger W^\mu + \frac{M_Z^2}{2} Z_\mu Z^\mu$$

$$M_W = M_Z \cos \theta_W = \frac{1}{2} g v$$



$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$$

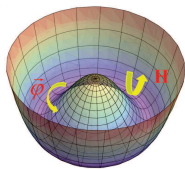
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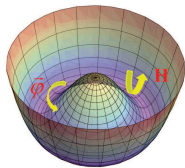
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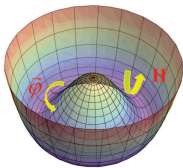
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Same Goldstone Lagrangian as QCD pions:

$$f_\pi \rightarrow v \quad , \quad \vec{\pi} \rightarrow \vec{\varphi} \rightarrow W_L^\pm, Z_L$$

EFFECTIVE LAGRANGIAN:

$$\mathcal{L}(U) = \sum_n \mathcal{L}_{2n}$$

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Derivative
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Goldstones become free at zero momenta

Goldstone Electroweak Effective Theory

$$\mathcal{L}_{\text{EW}}^{(2)} = -\frac{1}{2g^2} \langle \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \rangle - \frac{1}{2g'^2} \langle \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \rangle + \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle$$

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$$\mathcal{L}_2 = \frac{v^2}{4} \text{Tr} \left(D_\mu U^\dagger D^\mu U \right) \xrightarrow{U=1} \mathcal{L}_2 = M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$

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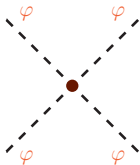
- QCD pions also generate small W, Z masses: $\delta_\pi M_W = \frac{1}{2} g f_\pi$

Goldstone interactions are determined by the underlying symmetry

$$\begin{aligned}\frac{v^2}{4} \langle \partial_\mu U^\dagger \partial^\mu U \rangle &= \partial_\mu \varphi^- \partial^\mu \varphi^+ + \frac{1}{2} \partial_\mu \varphi^0 \partial^\mu \varphi^0 \\ &+ \frac{1}{6v^2} \left\{ \left(\varphi^+ \overleftrightarrow{\partial}_\mu \varphi^- \right) \left(\varphi^+ \overleftrightarrow{\partial}^\mu \varphi^- \right) + 2 \left(\varphi^0 \overleftrightarrow{\partial}_\mu \varphi^+ \right) \left(\varphi^- \overleftrightarrow{\partial}^\mu \varphi^0 \right) \right\} \\ &+ O\left(\varphi^6/v^4\right)\end{aligned}$$

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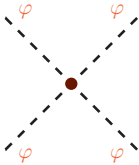
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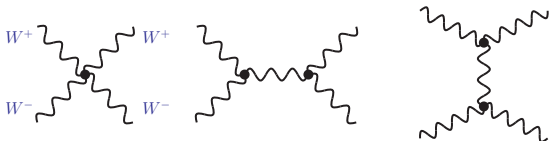


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Non-Linear Lagrangian:

$2\varphi \rightarrow 2\varphi, 4\varphi \dots$ related

Equivalence Theorem



Cornwall–Levin–Tiktopoulos

Vayonakis

Lee–Quigg–Thacker

$$\begin{aligned} T(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) &= \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right) \\ &= T(\varphi^+ \varphi^- \rightarrow \varphi^+ \varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right) \end{aligned}$$

The scattering amplitude grows with energy

Goldstone dynamics



derivative interactions

Tree-level violation of unitarity

Longitudinal Polarizations

$$k^\mu = (k^0, 0, 0, |\vec{k}|) \quad \rightarrow \quad \epsilon_L^\mu(\vec{k}) = \frac{1}{M_W} (|\vec{k}|, 0, 0, k^0) = \frac{k^\mu}{M_W} + \mathcal{O}\left(\frac{M_W}{|\vec{k}|}\right)$$

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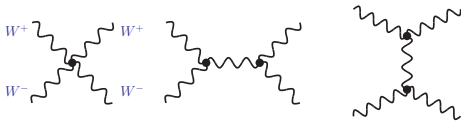
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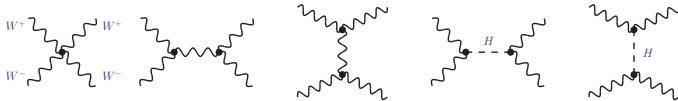
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**Gauge
Cancellation**

$$\begin{aligned} T(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) &= \frac{s+t}{v^2} + \mathcal{O}\left(\frac{M_W}{\sqrt{s}}\right) \\ &= T(\varphi^+ \varphi^- \rightarrow \varphi^+ \varphi^-) + \mathcal{O}\left(\frac{M_W}{\sqrt{s}}\right) \end{aligned}$$

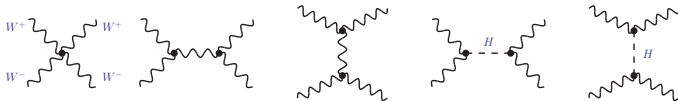
$$W_L^+ W_L^- \rightarrow W_L^+ W_L^-:$$



$$T_{\text{SM}} = \frac{1}{v^2} \left\{ s + t - \frac{s^2}{s - M_H^2} - \frac{t^2}{t - M_H^2} \right\} = -\frac{M_H^2}{v^2} \left\{ \frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right\}$$

Higgs-exchange exactly cancels the $O(s, t)$ terms in the SM

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Higgs-exchange exactly cancels the $O(s, t)$ terms in the SM

When $s \gg M_H^2$, $T_{\text{SM}} \approx -\frac{2M_H^2}{v^2}$, $a_0 \equiv \frac{1}{32\pi} \int_{-1}^1 d\cos\theta T_{\text{SM}} \approx -\frac{M_H^2}{8\pi v^2}$

Unitarity:

Lee-Quigg-Thacker

$$|a_0| \leq 1 \quad \Rightarrow \quad M_H < \sqrt{8\pi} v \underbrace{\sqrt{2/3}}_{W^+W^-, ZZ, HH} \approx 1 \text{ TeV}$$

What happens in QCD?

- QCD satisfies unitarity (it is a renormalizable theory)
- Pion scattering unitarized by exchanges of resonances (composite objects):
 - P-wave ($J = 1$) unitarized by ρ exchange
 - S-wave ($J = 0$) unitarized by σ exchange
- The σ meson is the QCD equivalent of the SM Higgs
- **BUT**, the σ is an 'effective' object generated through π rescattering (summation of pion loops)

Does not seem to work this way in the EW case, but ...

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\text{EW}}^{(4)} \Big|_{\text{CP-even}} = \sum_{i=0}^{14} a_i \mathcal{O}_i$$

(Appelquist, Longhitano)

$$\mathcal{O}_0 = v^2 \langle T_L V_\mu \rangle^2$$

$$\mathcal{O}_1 = \langle U \hat{B}_{\mu\nu} U^\dagger \hat{W}^{\mu\nu} \rangle$$

$$\mathcal{O}_3 = i \langle \hat{W}_{\mu\nu} [V^\mu, V^\nu] \rangle$$

$$\mathcal{O}_5 = \langle V_\mu V^\mu \rangle^2$$

$$\mathcal{O}_7 = 4 \langle V_\mu V^\mu \rangle \langle T_L V_\nu \rangle^2$$

$$\mathcal{O}_9 = -2 \langle T_L \hat{W}_{\mu\nu} \rangle \langle T_L [V^\mu, V^\nu] \rangle$$

$$\mathcal{O}_{11} = \langle (D_\mu V^\mu)^2 \rangle$$

$$\mathcal{O}_{13} = 2 \langle T_L D_\mu V_\nu \rangle^2$$

$$\mathcal{O}_2 = i \langle U \hat{B}_{\mu\nu} U^\dagger [V^\mu, V^\nu] \rangle$$

$$\mathcal{O}_4 = \langle V_\mu V_\nu \rangle \langle V^\mu V^\nu \rangle$$

$$\mathcal{O}_6 = 4 \langle V_\mu V_\nu \rangle \langle T_L V^\mu \rangle \langle T_L V^\nu \rangle$$

$$\mathcal{O}_8 = \langle T_L \hat{W}_{\mu\nu} \rangle^2$$

$$\mathcal{O}_{10} = 16 \{ \langle T_L V_\mu \rangle \langle T_L V_\nu \rangle \}^2$$

$$\mathcal{O}_{12} = 4 \langle T_L D_\mu D_\nu V^\nu \rangle \langle T_L V^\mu \rangle$$

$$\mathcal{O}_{14} = -2i \varepsilon^{\mu\nu\rho\sigma} \langle \hat{W}_{\mu\nu} V_\rho \rangle \langle T_L V_\sigma \rangle$$

$$V_\mu \equiv D_\mu U U^\dagger \quad , \quad D_\mu V_\nu \equiv \partial_\mu V_\nu - i [\hat{W}_\mu, V_\nu] \quad , \quad (V_\mu, D_\mu V_\nu, T_L) \rightarrow g_L (V_\mu, D_\mu V_\nu, T_L) g_L^\dagger$$

Symmetry breaking:

$$T_L \equiv U \frac{\sigma_3}{2} U^\dagger \quad , \quad \hat{B}_{\mu\nu} \equiv -g' \frac{\sigma_3}{2} B_{\mu\nu}$$

Low-Energy Effective Theory $\Rightarrow$ Power Counting

- Momentum expansion:

$$\Lambda \sim 4\pi v, M_X$$

$$\mathcal{A} = \sum_n \mathcal{A}_n \left(\frac{p}{\Lambda} \right)^n$$

- $U \sim \mathcal{O}(p^0)$, $D_\mu U, \hat{W}_\mu, \hat{B}_\mu \sim \mathcal{O}(p^1)$, $\hat{W}_{\mu\nu}, \hat{B}_{\mu\nu} \sim \mathcal{O}(p^2)$
- A general connected diagram with N_d vertices of $\mathcal{O}(p^d)$ and L Goldstone loops has a power dimension:

$$D = 2L + 2 + \sum_d N_d (d - 2)$$

$\Rightarrow$ Finite number of divergences / counterterms

NLO Predictions

- $\mathcal{L}_{EW}^{(2)}$ at one loop: Unitarity

Non-local (logarithmic) dependences unambiguously predicted

- $\mathcal{L}_{EW}^{(4)}$ at tree level: Local (polynomic) amplitude

Short-distance information encoded in the a_i couplings

Loop divergences reabsorbed through renormalized a_i

$$a_i = a_i^r(\mu) + \frac{\gamma_i}{16\pi^2} \left[\frac{2\mu^{D-4}}{4-D} + \log(4\pi) - \gamma_E \right]$$

$\hat{a}_i \equiv a_i/(16\pi)^2$ for different limits of the SM

	$M_H \rightarrow \infty$	$M_{t',b'} \rightarrow \infty$	$M_t \rightarrow \infty$
$\hat{a}_0$	$-\frac{3}{4}g'^2 \left[\log(M_H/\mu) - \frac{5}{12} \right]$	0	$\frac{3}{2} \frac{M_t^2}{v^2}$
$\hat{a}_1$	$-\frac{1}{6} \log(M_H/\mu) + \frac{5}{72}$	$-\frac{1}{2}$	$\frac{1}{3} \log(M_t/\mu) - \frac{1}{4}$
$\hat{a}_2$	$-\frac{1}{12} \log(M_H/\mu) + \frac{17}{144}$	$-\frac{1}{2}$	$\frac{1}{3} \log(M_t/\mu) - \frac{3}{4}$
$\hat{a}_3$	$\frac{1}{12} \log(M_H/\mu) - \frac{17}{144}$	$\frac{1}{2}$	$\frac{3}{8}$
$\hat{a}_4$	$\frac{1}{6} \log(M_H/\mu) - \frac{17}{72}$	$\frac{1}{4}$	$\log(M_t/\mu) - \frac{5}{6}$
$\hat{a}_5$	$\frac{2\pi^2 v^2}{M_H^2} + \frac{1}{12} \log(M_H/\mu) - \frac{79}{72} + \frac{9\pi}{16\sqrt{3}}$	$-\frac{1}{8}$	$-\log(M_t/\mu) + \frac{23}{24}$
$\hat{a}_6$	0	0	$-\log(M_t/\mu) + \frac{23}{24}$
$\hat{a}_7$	0	0	$\log(M_t/\mu) - \frac{23}{24}$
$\hat{a}_8$	0	0	$\log(M_t/\mu) - \frac{7}{12}$
$\hat{a}_9$	0	0	$\log(M_t/\mu) - \frac{23}{24}$
$\hat{a}_{10}$	0	0	$-\frac{1}{64}$
$\hat{a}_{11}$	—	$-\frac{1}{2}$	$-\frac{1}{2}$
$\hat{a}_{12}$	—	0	$-\frac{1}{8}$
$\hat{a}_{13}$	—	0	$-\frac{1}{4}$
$\hat{a}_{14}$	0	0	$\frac{3}{8}$

Decoupling:

Appelquist–Carazzone

The low-energy effects of heavy particles are either suppressed by inverse powers of the heavy masses, or they get absorbed into renormalizations of the couplings and fields of the EFT obtained by removing the heavy particles

SM: $M_W = M_Z \cos \theta_W = \frac{1}{2} g v$, $M_H = \sqrt{2\lambda} v$, $M_f = \frac{1}{\sqrt{2}} y_f v$

- Decoupling occurs when $v \rightarrow \infty$, keeping the couplings fixed

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2} \rightarrow 0$$

- There is no decoupling if some $M_i \rightarrow \infty$, keeping $v = 246 \text{ GeV}$
($g, \lambda, y_f \rightarrow \infty$)

Equations of Motion $\rightarrow$ Redundant Operators

Equivalent to field redefinitions which only affect higher-order terms

For massless fermions: $\partial_\mu \langle T_L V^\mu \rangle = 0$, $D_\mu V^\mu = 0$

$$\rightarrow \left\{ \begin{array}{l} \mathcal{O}_{11} = \mathcal{O}_{12} = 0 \\ \mathcal{O}_{13} = -\frac{g'^2}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{O}_1 - \mathcal{O}_4 + \mathcal{O}_5 - \mathcal{O}_6 + \mathcal{O}_7 + \mathcal{O}_8 \end{array} \right.$$

Equations of Motion $\rightarrow$ Redundant Operators

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$$\rightarrow \left\{ \begin{array}{l} \mathcal{O}_{11} = \mathcal{O}_{12} = 0 \\ \mathcal{O}_{13} = -\frac{g'^2}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{O}_1 - \mathcal{O}_4 + \mathcal{O}_5 - \mathcal{O}_6 + \mathcal{O}_7 + \mathcal{O}_8 \end{array} \right.$$

Heavy top:
$$\mathcal{O}_{11} \doteq \frac{g^4}{8M_W^4} m_t^2 \left\{ (\bar{t} \gamma_5 t)^2 - 4 \sum_{i,j} (\bar{d}_{iL} t_R) (\bar{t}_R d_{jL}) V_{tj} V_{ti}^* \right\}$$

Unitary Gauge: $U = 1$

All invariants reduce to polynomials of gauge fields

- Bilinear terms: $\mathcal{O}_0, \mathcal{O}_1, \mathcal{O}_8, \mathcal{O}_{11}, \mathcal{O}_{12}, \mathcal{O}_{13}$

→ Oblique corrections $(\Delta r, \Delta \rho, \Delta k \longleftrightarrow S, T, U)$

- Trilinear terms: $\mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_9, \mathcal{O}_{14}$

- Quartic terms: $\mathcal{O}_4, \mathcal{O}_5, \mathcal{O}_6, \mathcal{O}_7, \mathcal{O}_{10}$

- $\mathcal{O}_{11} \sim m_t^2 (\bar{\psi}\psi)(\bar{\psi}\psi)$: $Z\bar{b}b, B^0-\bar{B}^0, \varepsilon_K \dots$

$$\varphi^a \varphi^b \rightarrow \varphi^c \varphi^d:$$



$$\mathcal{A}(\varphi^a \varphi^b \rightarrow \varphi^c \varphi^d) = A(s, t, u) \delta_{ab} \delta_{cd} + A(t, s, u) \delta_{ac} \delta_{bd} + A(u, t, s) \delta_{ad} \delta_{bc}$$

$$\begin{aligned} A(s, t, u) = & \frac{s}{v^2} + \frac{4}{v^2} \left[a_4^r(\mu) (t^2 + u^2) + 2 a_5^r(\mu) s^2 \right] \\ & + \frac{1}{16\pi^2 v^2} \left\{ \frac{5}{9} s^2 + \frac{13}{18} (t^2 + u^2) + \frac{1}{12} (s^2 - 3t^2 - u^2) \log \left(\frac{-t}{\mu^2} \right) \right. \\ & \left. + \frac{1}{12} (s^2 - t^2 - 3u^2) \log \left(\frac{-u}{\mu^2} \right) - \frac{1}{2} s^2 \log \left(\frac{-s}{\mu^2} \right) \right\} \end{aligned}$$

$$a_i = a_i^r(\mu) + \frac{\gamma_i}{16\pi^2} \left[\frac{2\mu^{D-4}}{4-D} + \log(4\pi) - \gamma_E \right] \quad , \quad \gamma_4 = -\frac{1}{12} \quad , \quad \gamma_5 = -\frac{1}{24}$$

$$\varphi^a \varphi^b \rightarrow \varphi^c \varphi^d$$



+ Higgs (tree + 1-loop) contributions

$$\mathcal{L} = \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle \left[1 + 2a \frac{H}{v} + b \frac{H^2}{v^2} \right]$$

Espriu–Mescia–Yencho, Delgado–Dobado–Llanes-Estrada

$$\begin{aligned} A(s, t, u) = & \frac{s}{v^2} (1 - a^2) + \frac{4}{v^2} \left[a_4^r(\mu) (t^2 + u^2) + 2 a_5^r(\mu) s^2 \right] \\ & + \frac{1}{16\pi^2 v^2} \left\{ \frac{1}{9} (14 a^4 - 10 a^2 - 18 a^2 b + 9 b^2 + 5) s^2 + \frac{13}{18} (1 - a^2)^2 (t^2 + u^2) \right. \\ & - \frac{1}{2} (2 a^4 - 2 a^2 - 2 a^2 b + b^2 + 1) s^2 \log \left(\frac{-s}{\mu^2} \right) \\ & \left. + \frac{1}{12} (1 - a^2)^2 \left[(s^2 - 3t^2 - u^2) \log \left(\frac{-t}{\mu^2} \right) + (s^2 - t^2 - 3u^2) \log \left(\frac{-u}{\mu^2} \right) \right] \right\} \end{aligned}$$

$$\gamma_4 = -\frac{1}{12} (1 - a^2)^2, \quad \gamma_5 = -\frac{1}{48} (2 + 5 a^4 - 4 a^2 - 6 a^2 b + 3 b^2)$$

$$\varphi^a \varphi^b \rightarrow \varphi^c \varphi^d$$



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$$\begin{aligned} A(s, t, u) = & \frac{s}{v^2} (1 - a^2) + \frac{4}{v^2} \left[a_4^r(\mu) (t^2 + u^2) + 2 a_5^r(\mu) s^2 \right] \\ & + \frac{1}{16\pi^2 v^2} \left\{ \frac{1}{9} (14 a^4 - 10 a^2 - 18 a^2 b + 9 b^2 + 5) s^2 + \frac{13}{18} (1 - a^2)^2 (t^2 + u^2) \right. \\ & - \frac{1}{2} (2 a^4 - 2 a^2 - 2 a^2 b + b^2 + 1) s^2 \log \left(\frac{-s}{\mu^2} \right) \\ & \left. + \frac{1}{12} (1 - a^2)^2 \left[(s^2 - 3t^2 - u^2) \log \left(\frac{-t}{\mu^2} \right) + (s^2 - t^2 - 3u^2) \log \left(\frac{-u}{\mu^2} \right) \right] \right\} \end{aligned}$$

$$\gamma_4 = -\frac{1}{12} (1 - a^2)^2, \quad \gamma_5 = -\frac{1}{48} (2 + 5 a^4 - 4 a^2 - 6 a^2 b + 3 b^2)$$

SM: $a = b = 1$, $a_4 = a_5 = 0$



$$A(s, t, u) \sim \mathcal{O}(M_H^2/v^2)$$

Yukawa Couplings

$$\mathcal{L}_Y = -v \left\{ \bar{Q}_L U(\varphi) \left[\hat{Y}_u \mathcal{P}_+ + \hat{Y}_d \mathcal{P}_- \right] Q_R + \bar{L}_L U(\varphi) \hat{Y}_\ell \mathcal{P}_+ L_R + \text{h.c.} \right\}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \rightarrow g_L U(\varphi) g_R^\dagger \quad , \quad Q_L \rightarrow g_L Q_L \quad , \quad Q_R \rightarrow g_R Q_R \quad , \quad \mathcal{P}_\pm \rightarrow g_R \mathcal{P}_\pm g_R^\dagger$$

Yukawa Couplings

$$\mathcal{L}_Y = -v \left\{ \bar{Q}_L U(\varphi) \left[\hat{Y}_u \mathcal{P}_+ + \hat{Y}_d \mathcal{P}_- \right] Q_R + \bar{L}_L U(\varphi) \hat{Y}_\ell \mathcal{P}_+ L_R + \text{h.c.} \right\}$$

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Symmetry Breaking:

$$\mathcal{P}_\pm = \frac{1}{2} (I_2 \pm \sigma_3)$$

Yukawa Couplings

$$\mathcal{L}_Y = -v \left\{ \bar{Q}_L U(\varphi) \left[\hat{Y}_u \mathcal{P}_+ + \hat{Y}_d \mathcal{P}_- \right] Q_R + \bar{L}_L U(\varphi) \hat{Y}_\ell \mathcal{P}_+ L_R + \text{h.c.} \right\}$$

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Symmetry Breaking:

$$\mathcal{P}_\pm = \frac{1}{2} (I_2 \pm \sigma_3)$$

Flavour Structure:

$$\hat{Y}_{u,d,\ell}$$

3 × 3 matrices in flavour space

$$\mathcal{O}_{\psi V1} = i \bar{Q}_L \gamma^\mu Q_L \langle V_\mu T_L \rangle$$

$$\mathcal{O}_{\psi V2} = i \bar{Q}_L \gamma^\mu T_L Q_L \langle V_\mu T_L \rangle$$

$$\mathcal{O}_{\psi V3} = i \bar{Q}_L \gamma^\mu \tilde{P}_{12} Q_L \langle V_\mu \tilde{P}_{21} \rangle$$

$$\mathcal{O}_{\psi V4} = i \bar{u}_R \gamma^\mu u_R \langle V_\mu T_L \rangle$$

$$\mathcal{O}_{\psi V5} = i \bar{d}_R \gamma^\mu d_R \langle V_\mu T_L \rangle$$

$$\mathcal{O}_{\psi V6} = i \bar{u}_R \gamma^\mu d_R \langle V_\mu \tilde{P}_{21} \rangle$$

$$\mathcal{O}_{\psi V7} = i \bar{L}_L \gamma^\mu L_L \langle V_\mu T_L \rangle$$

$$\mathcal{O}_{\psi V8} = i \bar{L}_L \gamma^\mu T_L L_L \langle V_\mu T_L \rangle$$

$$\mathcal{O}_{\psi V9} = \bar{L}_L \gamma^\mu \tilde{P}_{12} L_L \langle V_\mu \tilde{P}_{21} \rangle$$

$$\mathcal{O}_{\psi V10} = \bar{\ell}_R \gamma^\mu \ell_R \langle V_\mu T_L \rangle$$

$$\mathcal{O}_{\psi V3}^\dagger$$

$$\mathcal{O}_{\psi V6}^\dagger$$

$$\mathcal{O}_{\psi S1,2} = \bar{Q}_L \tilde{P}_\pm U Q_R \langle D_\mu U^\dagger D^\mu U \rangle$$

$$\mathcal{O}_{\psi S3,4} = \bar{Q}_L \tilde{P}_\pm U Q_R \langle V_\mu T_L \rangle^2$$

$$\mathcal{O}_{\psi S5} = \bar{Q}_L \tilde{P}_{12} U Q_R \langle V_\mu \tilde{P}_{21} \rangle \langle V^\mu T_L \rangle$$

$$\mathcal{O}_{\psi S6} = \bar{Q}_L \tilde{P}_{12} Q_R \langle V_\mu \tilde{P}_{12} \rangle \langle V^\mu T_L \rangle$$

$$\mathcal{O}_{\psi S7} = \bar{L}_L \tilde{P}_- U L_R \langle D_\mu U^\dagger D^\mu U \rangle$$

$$\mathcal{O}_{\psi S8} = \bar{L}_L \tilde{P}_- U L_R \langle V^\mu T_L \rangle^2$$

$$\mathcal{O}_{\psi S9} = \bar{L}_L \tilde{P}_{12} U L_R \langle V_\mu \tilde{P}_{12} \rangle \langle V^\mu T_L \rangle$$

$$\mathcal{O}_{\psi T1} = \bar{Q}_L \sigma^{\mu\nu} \tilde{P}_{12} U Q_R \langle V_\mu \tilde{P}_{21} \rangle \langle V^\nu T_L \rangle$$

$$\mathcal{O}_{\psi T2} = \bar{Q}_L \sigma^{\mu\nu} \tilde{P}_{21} U Q_R \langle V_\mu \tilde{P}_{12} \rangle \langle V^\nu T_L \rangle$$

$$\mathcal{O}_{\psi T3,4} = \bar{Q}_L \sigma^{\mu\nu} \tilde{P}_\pm U Q_R \langle V_\mu \tilde{P}_{12} \rangle \langle V_\nu \tilde{P}_{21} \rangle$$

$$\mathcal{O}_{\psi T5} = \bar{L}_L \sigma^{\mu\nu} \tilde{P}_{12} U L_R \langle V_\mu \tilde{P}_{21} \rangle \langle V^\nu T_L \rangle$$

$$\mathcal{O}_{\psi T6} = \bar{L}_L \sigma^{\mu\nu} \tilde{P}_- U L_R \langle V_\mu \tilde{P}_{12} \rangle \langle V_\nu \tilde{P}_{21} \rangle$$

$$V_\mu = D_\mu U U^\dagger, \quad T_L = U \frac{\sigma_3}{2} U^\dagger, \quad \tilde{P}_{12} = U \frac{\sigma_{1+i2}}{2} U^\dagger, \quad \tilde{P}_{21} = U \frac{\sigma_{1-i2}}{2} U^\dagger, \quad \tilde{P}_\pm = U P_\pm U^\dagger$$

NLO Operators (cont.)

Buchalla–Catá

$$\begin{aligned}
 \mathcal{O}_{LL6} &= \bar{Q}_L \gamma^\mu T_L Q_L \bar{Q}_L \gamma_\mu T_L Q_L & \mathcal{O}_{LL7} &= \bar{Q}_L \gamma^\mu T_L Q_L \bar{Q}_L \gamma_\mu Q_L & \mathcal{O}_{LL8} &= \bar{q}_{L\alpha} \gamma^\mu T_L Q_{L\beta} \bar{Q}_{L\beta} \gamma_\mu T_L Q_{L\alpha} \\
 \mathcal{O}_{LL10} &= \bar{Q}_L \gamma^\mu T_L Q_L \bar{L}_L \gamma_\mu T_L L_L & \mathcal{O}_{LL11} &= \bar{Q}_L \gamma^\mu T_L Q_L \bar{L}_L \gamma_\mu L_L & \mathcal{O}_{LL9} &= \bar{Q}_{L\alpha} \gamma^\mu T_L Q_{L\beta} \bar{Q}_{L\beta} \gamma_\mu Q_{L\alpha} \\
 \mathcal{O}_{LL12} &= \bar{Q}_L \gamma^\mu Q_L \bar{L}_L \gamma_\mu T_L L_L & \mathcal{O}_{LL13} &= \bar{Q}_L \gamma^\mu T_L L_L \bar{L}_L \gamma_\mu T_L Q_L & \mathcal{O}_{LL14} &= \bar{Q}_L \gamma^\mu T_L L_L \bar{L}_L \gamma_\mu Q_L \\
 \mathcal{O}_{LL15} &= \bar{L}_L \gamma^\mu T_L L_L \bar{L}_L \gamma_\mu T_L L_L & \mathcal{O}_{LL16} &= \bar{L}_L \gamma^\mu T_L L_L \bar{L}_L \gamma_\mu L_L & & \\
 \mathcal{O}_{LR10} &= \bar{Q}_L \gamma^\mu T_L Q_L \bar{u}_R \gamma_\mu u_R & \mathcal{O}_{LR12} &= \bar{Q}_L \gamma^\mu T_L Q_L \bar{d}_R \gamma_\mu d_R & \mathcal{O}_{LR11} &= \bar{Q}_L \gamma^\mu t^a T_L Q_L \bar{u}_R \gamma_\mu t^a u_R \\
 \mathcal{O}_{LR14} &= \bar{u}_R \gamma^\mu u_R \bar{L}_L \gamma_\mu T_L L_L & \mathcal{O}_{LR15} &= \bar{d}_R \gamma^\mu d_R \bar{L}_L \gamma_\mu T_L L_L & \mathcal{O}_{LR13} &= \bar{Q}_L \gamma^\mu t^a T_L Q_L \bar{d}_R \gamma_\mu t^a d_R \\
 \mathcal{O}_{LR16} &= \bar{Q}_L \gamma^\mu T_L Q_L \bar{\ell}_R \gamma_\mu \ell_R & \mathcal{O}_{LR17} &= \bar{L}_L \gamma^\mu T_L L_L \bar{\ell}_R \gamma_\mu \ell_R & \mathcal{O}_{LR18} &= \bar{Q}_L \gamma^\mu T_L L_L \bar{\ell}_R \gamma_\mu d_R \\
 \mathcal{O}_{ST5} &= \bar{Q}_L \tilde{P}_+ U_{QR} \bar{Q}_L \tilde{P}_- U_{QR} & \mathcal{O}_{ST6} &= \bar{Q}_L \tilde{P}_{21} U_{QR} \bar{Q}_L \tilde{P}_{12} U_{QR} & \mathcal{O}_{ST7} &= \bar{Q}_L t^a \tilde{P}_+ U_{QR} \bar{Q}_L t^a \tilde{P}_- U_{QR} \\
 \mathcal{O}_{ST9} &= \bar{Q}_L \tilde{P}_+ U_{QR} \bar{L}_L \tilde{P}_- U_{LR} & \mathcal{O}_{ST10} &= \bar{Q}_L \tilde{P}_{21} U_{QR} \bar{L}_L \tilde{P}_{12} U_{LR} & \mathcal{O}_{ST8} &= \bar{Q}_L t^a \tilde{P}_{21} U_{QR} \bar{Q}_L t^a \tilde{P}_{12} U_{QR} \\
 \mathcal{O}_{ST11} &= \bar{Q}_L \sigma^{\mu\nu} \tilde{P}_+ U_{QR} \bar{L}_L \sigma_{\mu\nu} \tilde{P}_- U_{LR} & \mathcal{O}_{ST12} &= \bar{Q}_L \sigma^{\mu\nu} \tilde{P}_{21} U_{QR} \bar{L}_L \sigma_{\mu\nu} \tilde{P}_{12} U_{LR} & & \\
 \mathcal{O}_{FY4} &= \bar{Q}_L t^a \tilde{P}_- U_{QR} \bar{Q}_L t^a \tilde{P}_- U_{QR} & \mathcal{O}_{FY8} &= \bar{Q}_L \sigma^{\mu\nu} \tilde{P}_- U_{QR} \bar{L}_L \sigma_{\mu\nu} \tilde{P}_- U_{LR} & & \\
 \mathcal{O}_{FY1} &= \bar{Q}_L \tilde{P}_+ U_{QR} \bar{Q}_L \tilde{P}_+ U_{QR} & \mathcal{O}_{FY3} &= \bar{Q}_L \tilde{P}_- U_{QR} \bar{Q}_L \tilde{P}_- U_{QR} & \mathcal{O}_{FY2} &= \bar{Q}_L t^a \tilde{P}_+ U_{QR} \bar{Q}_L t^a \tilde{P}_+ U_{QR} \\
 \mathcal{O}_{FY5} &= \bar{Q}_L \tilde{P}_- U_{QR} \bar{Q}_R U^\dagger \tilde{P}_+ Q_L & \mathcal{O}_{FY7} &= \bar{Q}_L \tilde{P}_- U_{QR} \bar{L}_L \tilde{P}_- U_{LR} & \mathcal{O}_{FY6} &= \bar{Q}_L t^a \tilde{P}_- U_{QR} \bar{Q}_R t^a U^\dagger \tilde{P}_+ Q_L \\
 \mathcal{O}_{FY9} &= \bar{L}_L \tilde{P}_- U_{LR} \bar{Q}_R U^\dagger \tilde{P}_+ Q_R & \mathcal{O}_{FY10} &= \bar{L}_L \tilde{P}_- U_{LR} \bar{L}_L \tilde{P}_- U_{LR} & \mathcal{O}_{FY11} &= \bar{L}_L \tilde{P}_- U_{QR} \bar{Q}_R U^\dagger \tilde{P}_+ L_L
 \end{aligned}$$

Including a Light (singlet) Higgs in the EWET

Let us assume that $\mathbf{h(126)}$ is an $\mathrm{SU(2)_{L+R}}$ scalar singlet

All Higgsless operators can be multiplied by an arbitrary function of $\mathbf{h}$:

$$\begin{aligned}\mathcal{O}_X &\longrightarrow \tilde{\mathcal{O}}_X \equiv \mathcal{F}_X(h) \mathcal{O}_X \\ \mathcal{F}_X(h) &= \sum_{n=0} c_X^{(n)} \left(\frac{h}{v}\right)^n\end{aligned}$$

In addition, the LO Lagrangian should include the **scalar potential**:

$$V(h) = v^4 \sum_{n=2} c_V^{(n)} \left(\frac{h}{v}\right)^n$$

Plus operators with derivatives $(\partial_\mu h)$:

$$F_X \equiv F_X(h)$$

$$\begin{aligned}
 \mathcal{O}_{D7} &= -\langle V_\mu V^\mu \rangle \frac{\partial_\nu h \partial^\nu h}{v^2} F_{D7} & \mathcal{O}_{D8} &= -\langle V_\mu V_\nu \rangle \frac{\partial^\mu h \partial^\nu h}{v^2} F_{D8} & \mathcal{O}_{D11} &= \frac{(\partial_\mu h \partial^\mu h)^2}{v^4} F_{D11} \\
 \mathcal{O}_{D6} &= -\langle \tilde{T}_L V_\mu V_\nu \rangle \langle \tilde{T}_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{D6} & \mathcal{O}_{D9} &= -\langle \tilde{T}_L V_\mu \rangle \langle \tilde{T}_L V^\mu \rangle \frac{\partial_\nu h \partial^\nu h}{v^2} F_{D9} \\
 \mathcal{O}_{D10} &= -\langle \tilde{T}_L V_\mu \rangle \langle \tilde{T}_L V_\nu \rangle \frac{\partial^\mu h \partial^\nu h}{v^2} F_{D10} & \mathcal{O}_{\psi S18} &= \bar{L}_L \tilde{P}_- U_{LR} \frac{\partial_\mu h \partial^\mu h}{v^2} F_{\psi S18} \\
 \mathcal{O}_{\psi S10} &= -i \bar{Q}_L \tilde{P}_+ U_{QR} \langle \tilde{T}_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S10} & \mathcal{O}_{\psi S11} &= -i \bar{Q}_L \tilde{P}_- U_{QR} \langle \tilde{T}_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S11} \\
 \mathcal{O}_{\psi S12} &= -i \bar{Q}_L \tilde{P}_{12} U_{QR} \langle \tilde{P}_{21} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S12} & \mathcal{O}_{\psi S13} &= -i \bar{Q}_L \tilde{P}_{21} U_{QR} \langle \tilde{P}_{12} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S13} \\
 \mathcal{O}_{\psi S14} &= \bar{Q}_L \tilde{P}_+ U_{QR} \frac{\partial_\mu h \partial^\mu h}{v^2} F_{\psi S14} & \mathcal{O}_{\psi S15} &= \bar{Q}_L \tilde{P}_- U_{QR} \frac{\partial_\mu h \partial^\mu h}{v^2} F_{\psi S15} \\
 \mathcal{O}_{\psi S16} &= -i \bar{L}_L \tilde{P}_- U_{LR} \langle \tilde{T}_L V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S16} & \mathcal{O}_{\psi S17} &= -i \bar{L}_L \tilde{P}_{12} U_{LR} \langle \tilde{P}_{21} V_\mu \rangle \frac{\partial^\mu h}{v} F_{\psi S17} \\
 \mathcal{O}_{\psi T7} &= -i \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_+ U_{QR} \langle \tilde{T}_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T7} & \mathcal{O}_{\psi T8} &= -i \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_- U_{QR} \langle \tilde{T}_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T8} \\
 \mathcal{O}_{\psi T9} &= -i \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_{21} U_{QR} \langle \tilde{P}_{12} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T9} & \mathcal{O}_{\psi T10} &= -i \bar{Q}_L \sigma_{\mu\nu} \tilde{P}_{12} U_{QR} \langle \tilde{P}_{21} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T10} \\
 \mathcal{O}_{\psi T11} &= -i \bar{L}_L \sigma_{\mu\nu} \tilde{P}_- U_{LR} \langle \tilde{T}_L V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T11} & \mathcal{O}_{\psi T12} &= -i \bar{L}_L \sigma_{\mu\nu} \tilde{P}_{12} U_{LR} \langle \tilde{P}_{21} V^\mu \rangle \frac{\partial^\nu h}{v} F_{\psi T12}
 \end{aligned}$$

Assumes that $H(126)$ and $\vec{\varphi}$ combine into an $SU(2)_L$ doublet:

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \frac{1}{2} (v + H) U(\vec{\varphi}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The SM Lagrangian is the low-energy effective theory with $D = 4$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}} \mathcal{O}_i^{(D)}$$

- **1 operator with $D = 5$:** $\mathcal{O}^{(5)} = \bar{L}_L \tilde{\Phi} \tilde{\Phi}^T L_L^c$ (violates L by 2 units)
Weinberg
- **59 independent $\mathcal{O}_i^{(6)}$ preserving B and L** (for 1 generation)
Buchmuller–Wyler, Grzadkowski–Iskrzynski–Misiak–Rosiek
- **5 independent $\mathcal{O}_i^{(6)}$ violating B and L** (for 1 generation)
Weinberg, Wilczek–Zee, Abbott–Wise,

D = 6 Operators (other than 4-fermion ones)

Grzadkowski–Iskrzynski–Misiak–Rosiek

χ^3		ϕ^6 and $\phi^4 D^2$		$\psi^2 \phi^3$	
$\mathcal{O}_G$	$f^{ABC} G_{\mu}^{A\nu} G_{\nu}^{B\rho} G_{\rho}^{C\mu}$	$\mathcal{O}_{\Phi}$	$(\Phi^\dagger \Phi)^3$	$\mathcal{O}_{e\Phi}$	$(\Phi^\dagger \Phi) (\bar{l}_p e_r \Phi)$
$\mathcal{O}_{\tilde{G}}$	$f^{ABC} \tilde{G}_{\mu}^{A\nu} G_{\nu}^{B\rho} G_{\rho}^{C\mu}$	$\mathcal{O}_{\Phi\Box}$	$(\Phi^\dagger \Phi) \Box (\Phi^\dagger \Phi)$	$\mathcal{O}_{u\Phi}$	$(\Phi^\dagger \Phi) (\bar{q}_p u_r \tilde{\Phi})$
$\mathcal{O}_W$	$\varepsilon^{IJK} W_{\mu}^{I\nu} W_{\nu}^{J\rho} W_{\rho}^{K\mu}$	$\mathcal{O}_{\Phi D}$	$(\Phi^\dagger D^\mu \Phi)^* (\Phi^\dagger D_\mu \Phi)$	$\mathcal{O}_{d\Phi}$	$(\Phi^\dagger \Phi) (\bar{q}_p d_r \Phi)$
$\mathcal{O}_{\tilde{W}}$	$\varepsilon^{IJK} \tilde{W}_{\mu}^{I\nu} W_{\nu}^{J\rho} W_{\rho}^{K\mu}$				
$\chi^2 \phi^2$		$\psi^2 \chi \phi$		$\psi^2 \phi^2 D$	
$\mathcal{O}_{\Phi G}$	$\Phi^\dagger \Phi G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \Phi W_{\mu\nu}^I$	$\mathcal{O}_{\Phi l}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi) (\bar{l}_p \gamma^\mu l_r)$
$\mathcal{O}_{\Phi \tilde{G}}$	$\Phi^\dagger \Phi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \Phi B_{\mu\nu}$	$\mathcal{O}_{\Phi l}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu^I \Phi) (\bar{l}_p \tau^I \gamma^\mu l_r)$
$\mathcal{O}_{\Phi W}$	$\Phi^\dagger \Phi W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\Phi} G_{\mu\nu}^A$	$\mathcal{O}_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi) (\bar{e}_p \gamma^\mu e_r)$
$\mathcal{O}_{\Phi \tilde{W}}$	$\Phi^\dagger \Phi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\Phi} W_{\mu\nu}^I$	$\mathcal{O}_{\Phi q}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi) (\bar{q}_p \gamma^\mu q_r)$
$\mathcal{O}_{\Phi B}$	$\Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\Phi} B_{\mu\nu}$	$\mathcal{O}_{\Phi q}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu^I \Phi) (\bar{q}_p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{\Phi \tilde{B}}$	$\Phi^\dagger \Phi \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \Phi G_{\mu\nu}^A$	$\mathcal{O}_{\Phi u}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi) (\bar{u}_p \gamma^\mu u_r)$
$\mathcal{O}_{\Phi WB}$	$\Phi^\dagger \tau^I \Phi W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \Phi W_{\mu\nu}^I$	$\mathcal{O}_{\Phi d}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi) (\bar{d}_p \gamma^\mu d_r)$
$\mathcal{O}_{\Phi \tilde{W}B}$	$\Phi^\dagger \tau^I \Phi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \Phi B_{\mu\nu}$	$\mathcal{O}_{\Phi ud}$	$i(\tilde{\Phi}^\dagger D_\mu \Phi) (\bar{u}_p \gamma^\mu d_r)$

$$q = q_L, \quad l = l_L, \quad u = u_R, \quad d = d_R, \quad e = e_R, \quad \overleftrightarrow{D}_\mu^I \equiv \tau^I \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \tau^I, \quad p, r = \text{generation indices}$$

D = 6 Four-Fermion Operators

Grzadkowski–Iskrzynski–Misiak–Rosiek

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating			
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s^j q_t^i)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_p^\alpha)^T C u_r^\beta] [(q_s^j)^T C l_t^k]$		
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(u_s^\gamma)^T C e_t]$		
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \varepsilon_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^m)^T C l_t^n]$		
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	$\mathcal{O}_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma} (\tau^I \varepsilon)_{jk} (\tau^I \varepsilon)_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^m)^T C l_t^n]$		
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^\gamma)^T C e_t]$		

$q = q_L, l = l_L, u = u_R, d = d_R, e = e_R, p, r, s, t = \text{generation indices}$

Equivalent Bases of CP-even Operators (EW bosons only)

Willenbrock–Zhang

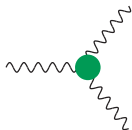
BW	HISZ	GGPR
$\mathcal{O}_W = \epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{WWW} = \text{Tr} [\hat{W}_{\mu\nu} \hat{W}^{\nu\rho} \hat{W}_\rho^\mu]$	$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon_{abc} W_\mu^{a\nu} W_\nu^b W^{c\rho\mu}$
$\mathcal{O}_{\Phi W} = \Phi^\dagger \Phi W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi$	—
$\mathcal{O}_{\Phi B} = \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi$	$\mathcal{O}_{BB} = g'^2 H ^2 B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\Phi WB} = \Phi^\dagger \sigma^I \Phi W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi$	—
—	$\mathcal{O}_W = (D_\mu \Phi)^\dagger \hat{W}^{\mu\nu} (D_\nu \Phi)$	$\mathcal{O}_{HW} = ig (D^\mu H)^\dagger \sigma^a (D^\nu H) W_{\mu\nu}^a$
—	$\mathcal{O}_B = (D_\mu \Phi)^\dagger \hat{B}^{\mu\nu} (D_\nu \Phi)$	$\mathcal{O}_{HB} = ig' (D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$
—	$\mathcal{O}_{DW} = \text{Tr} ([D_\mu, \hat{W}_{\nu\rho}] [D^\mu, \hat{W}^{\nu\rho}])$	$\mathcal{O}_{2W} = -\frac{1}{2} (D^\mu W_{\mu\nu}^a)^2$
—	$\mathcal{O}_{DB} = -\frac{g'^2}{2} (\partial_\mu B_{\nu\rho}) (\partial^\mu B^{\nu\rho})$	$\mathcal{O}_{2B} = -\frac{1}{2} (\partial^\mu B_{\mu\nu})^2$
—	—	$\mathcal{O}_W = \frac{ig}{2} (H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W_{\mu\nu}^a$
—	—	$\mathcal{O}_B = \frac{ig'}{2} (H^\dagger \overleftrightarrow{D}^\mu H) \partial^\nu B_{\mu\nu}$
$\mathcal{O}_{\Phi D} = (\Phi^\dagger D^\mu \Phi)^* (\Phi^\dagger D_\mu \Phi)$	$\mathcal{O}_{\Phi,1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi)$	$\mathcal{O}_T = \frac{1}{2} (H^\dagger \overleftrightarrow{D}^\mu H)^2$
$\mathcal{O}_{\Phi \square} = (\Phi^\dagger \Phi) \square (\Phi^\dagger \Phi)$	$\mathcal{O}_{\Phi,2} = \frac{1}{2} \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi)$	$\mathcal{O}_H = \frac{1}{2} (\partial^\mu H ^2)^2$
$\mathcal{O}_\Phi = (\Phi^\dagger \Phi)^3$	$\mathcal{O}_{\Phi,3} = \frac{1}{3} (\Phi^\dagger \Phi)^3$	$\mathcal{O}_6 = \lambda H ^6$
—	$\mathcal{O}_{\Phi,4} = (D_\mu \Phi)^\dagger (D^\mu \Phi) (\Phi^\dagger \Phi)$	—

BW = Buchmuller–Wyler, Grzadkowski et al.

HISZ = Hagiwara et al.

GGPR = Giudice et al.

Anomalous Triple Gauge Couplings



$$\mathcal{L}_{WWV} = -i g_{WWV} \left\{ g_1^V (W_{\mu\nu}^+ W^{-\mu} V^\nu - W_\mu^+ W^{-\mu\nu} V_\nu) \right. \\ \left. + \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{M_W^2} W_{\mu\nu}^+ W^{-\nu\rho} V_\rho^\mu \right\}$$

$$g_{WW\gamma} = e = g \cos \theta_W \quad , \quad g_{WWZ} = g \cos \theta_W \quad , \quad \kappa_V = 1 + \Delta\kappa_V \quad , \quad g_1^V = 1 + \Delta g_1^V$$

Relevant operators:

$$\mathcal{O}_W, \mathcal{O}_{\Phi WB}, \mathcal{O}_{\phi I}^{(3)} \quad (\text{BW})$$

$$\mathcal{O}_{WWW}, \mathcal{O}_W, \mathcal{O}_B, \mathcal{O}_{BW}, \mathcal{O}_{DW} \quad (\text{HISZ})$$

$$\mathcal{O}_{3W}, \mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{2W}, \mathcal{O}_W, \mathcal{O}_B \quad (\text{GGPR})$$

D = 6 Relations:

$$\lambda_\gamma = \lambda_Z \quad , \quad \Delta g_1^Z = \Delta\kappa_Z + \tan^2 \theta_W \Delta\kappa_\gamma$$

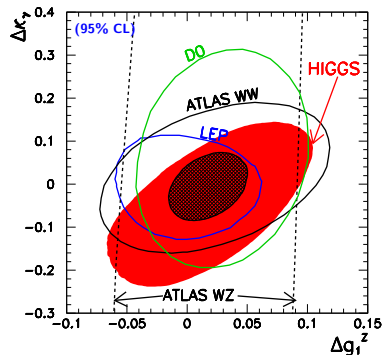
Anomalous Triple Gauge Couplings

HISZ basis: $\mathcal{L}_{\text{eff}} = \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n$

Corbett–Eboli–González-Fraile–González-García

$$\lambda_\gamma = \lambda_z = \frac{3g^2 M_W^2}{2\Lambda^2} f_{WWW} \quad , \quad \Delta\kappa_\gamma = \frac{g^2 v^2}{8\Lambda^2} (f_W + f_B)$$

$$\Delta\kappa_z = \frac{g^2 v^2}{8\Lambda^2} (f_W - \tan^2 \theta_W f_B) \quad , \quad \Delta g_1^Z = \frac{g^2 v^2}{8 \cos^2 \theta_W \Lambda^2} f_W$$



Higgs data: (90% CL)

$$-0.047 \leq \Delta g_1^Z \leq 0.089 \quad , \quad -0.19 \leq \Delta\kappa_\gamma \leq 0.099$$

$$\Rightarrow -0.019 \leq \Delta\kappa_z \leq 0.083$$

Combined: (90% CL)

$$-0.005 \leq \Delta g_1^Z \leq 0.040 \quad , \quad -0.058 \leq \Delta\kappa_\gamma \leq 0.047$$

$$\Rightarrow -0.004 \leq \Delta\kappa_z \leq 0.040$$

LEP, D0, ATLAS assume $\lambda_\gamma = \lambda_z = 0$

Global Fit to $\mathcal{L}_{\text{eff}}^{(D=6)}$

Pomarol–Riva

GGPR basis:

$$\mathcal{O}_H = \frac{1}{2} (\partial^\mu |H|^2)^2$$

$$\mathcal{O}_T = \frac{1}{2} (H^\dagger \overleftrightarrow{D}_\mu H)^2$$

$$\mathcal{O}_6 = \lambda |H|^6$$

$$\mathcal{O}_W = \frac{ig}{2} (H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) D_\nu W^{a\mu\nu}$$

$$\mathcal{O}_B = \frac{ig'}{2} (H^\dagger \overleftrightarrow{D}_\mu H) \partial_\nu B^{\mu\nu}$$

$$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{O}_{GG} = g_s^2 |H|^2 G_{\mu\nu}^A G^{A\mu\nu}$$

$$\mathcal{O}_{HW} = ig (D^\mu H)^\dagger \sigma^a (D^\nu H) W_{\mu\nu}^a$$

$$\mathcal{O}_{HB} = ig' (D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$$

$$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon_{abc} W_\mu^{a\nu} W_{\nu\rho}^b W^{c\rho\mu}$$

$$\mathcal{O}_{y_u} = y_u |H|^2 \bar{Q}_L \tilde{H} u_R + \text{h.c.}$$

$$\mathcal{O}_R^u = i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{u}_R \gamma^\mu u_R)$$

$$\mathcal{O}_L^q = i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{Q}_L \gamma^\mu Q_L)$$

$$\mathcal{O}_L^{(3)q} = i (H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) (\bar{Q}_L \sigma^a \gamma^\mu Q_L)$$

$$\mathcal{O}_{LL}^{(3)ql} = (\bar{Q}_L \sigma^a \gamma_\mu Q_L) (\bar{L}_L \sigma^a \gamma^\mu L_L)$$

$$\mathcal{O}_{y_d} = y_d |H|^2 \bar{Q}_L H d_R + \text{h.c.}$$

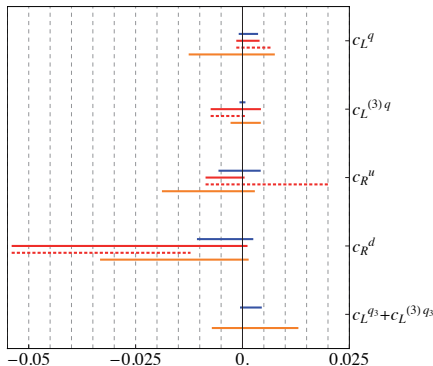
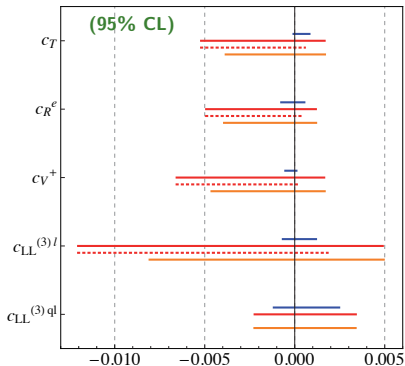
$$\mathcal{O}_R^d = i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{d}_R \gamma^\mu d_R)$$

$$\mathcal{O}_{y_e} = y_e |H|^2 \bar{L}_L H e_R + \text{h.c.}$$

$$\mathcal{O}_R^e = i (H^\dagger \overleftrightarrow{D}_\mu H) (\bar{e}_R \gamma^\mu e_R)$$

$$\mathcal{O}_{LL}^{(3)l} = (\bar{L}_L \sigma^a \gamma^\mu L_L) (\bar{L}_L \sigma^a \gamma_\mu L_L)$$

LEP-I + SLC + KLOE (CKM univ) + $pp \rightarrow l\bar{\nu}$ (LHC)

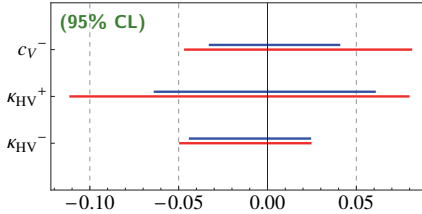
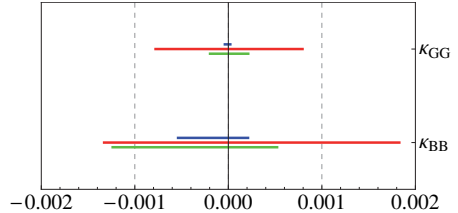


— All ; - - - without KLOE ; — include $\mathcal{O}_L^t + \mathcal{O}_L^{(3)t}$; — All other $\mathcal{O}_i$ set to zero

$$\Delta\mathcal{L} = \frac{c_T}{v^2} \mathcal{O}_T + \frac{c_V^+}{M_W^2} (\mathcal{O}_W + \mathcal{O}_B) + \frac{c_{LL}^{(3)l}}{v^2} \mathcal{O}_{LL}^{(3)l} + \frac{c_R^e}{v^2} \mathcal{O}_R^e$$

$$+ \frac{c_L^q}{v^2} \mathcal{O}_L^q + \frac{c_L^{(3)q}}{v^2} \mathcal{O}_L^{(3)q} + \frac{c_R^u}{v^2} \mathcal{O}_R^u + \frac{c_R^d}{v^2} \mathcal{O}_R^d + \frac{c_{LL}^{(3)ql}}{v^2} \mathcal{O}_{LL}^{(3)ql}$$

$$c_V^\pm = \frac{1}{2}(c_W \pm c_B)$$

LEP-II + $H \rightarrow Z\gamma$ (LHC) $H \rightarrow VV, f\bar{f}$ (LHC)

— All ; — All other $\mathcal{O}_i$ set to zero ; — c_H and c_{Yf} effects constrained to be below 50% of SM

$$\Delta\mathcal{L}_{\text{TGC}} = \frac{\kappa_{HV}^+}{m_W^2} (\mathcal{O}_{HW} + \mathcal{O}_{HB}) + \frac{c_V^- + \kappa_{HV}^-}{2m_W^2} \mathcal{O}_+ + \frac{\kappa_{3W}}{m_W^2} \mathcal{O}_{3W}$$

$$\Delta\mathcal{L}_{\text{Higgs}} = \frac{c_H}{v^2} \mathcal{O}_H + \sum_{f=t,b,\tau} \frac{c_{Yf}}{v^2} \mathcal{O}_{Yf} + \frac{c_6}{v^2} \mathcal{O}_6 + \frac{\kappa_{BB}}{M_W^2} \mathcal{O}_{BB} + \frac{\kappa_{GG}}{M_W^2} \mathcal{O}_{GG} + \frac{c_V^- - \kappa_{HV}^-}{2M_W^2} \mathcal{O}_-$$

$$\mathcal{O}_{\pm} \equiv (\mathcal{O}_W - \mathcal{O}_B) \pm (\mathcal{O}_{HW} - \mathcal{O}_{HB}) \quad , \quad \mathcal{O}_{WW} = 4\mathcal{O}_- + \mathcal{O}_{BB} \quad , \quad \kappa_{HV}^{\pm} = \frac{1}{2}(\kappa_{HW} \pm \kappa_{HB})$$

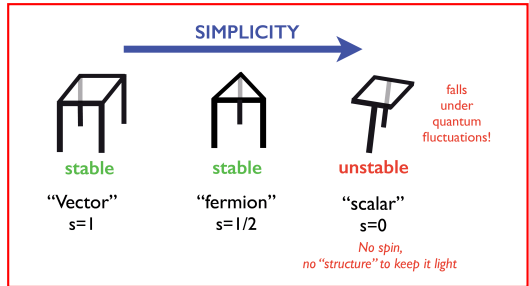
Backup Slides

Which symmetry keeps M_H away from Λ_{NP} ?

A. Pomarol

Quantum Stability

Vectors/fermions
protected by
gauge/chiral symmetries



Proposed Solutions

SUSY / Composite Higgs

1) Keep the Higgs elementary, but protect it by symmetries: **Supersymmetry**

Higgs (boson)  Higgsino (fermion)



2) The Higgs is not elementary: **Composite Higgs**

Higgs made of fermions
(as a pion in strong interactions)



Spin, Mass & Degrees of Freedom

	$J = 1$	$J = \frac{1}{2}$	$J = 0$
$M = 0$	2 d.o.f. Trans. Pol.	2 d.o.f. ψ_L	1 d.o.f.
$M \neq 0$	3 d.o.f. Trans & Long.	4 d.o.f. ψ_L, ψ_R	1 d.o.f.

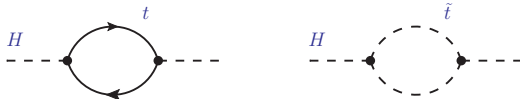
Vector ($2 \neq 3$) and fermion masses are safe ($2 \neq 4$)

Scalar masses not protected (continuous $m \rightarrow 0$ limit)

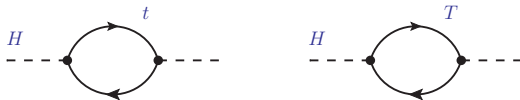
Higgs Self-energy: The top is the largest SM contribution

δM_H^2 stabilized through new-physics contributions

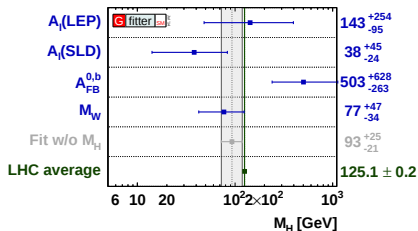
- **SUSY:** stop loops



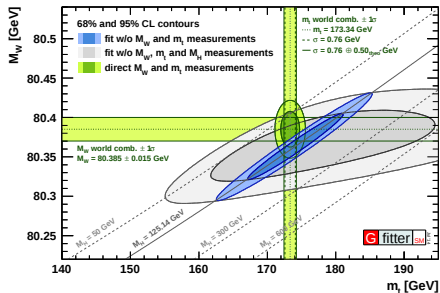
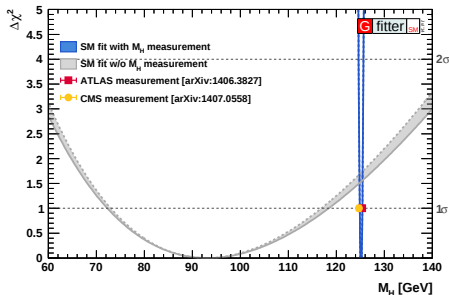
- **Composite Higgs:** fermionic top partners



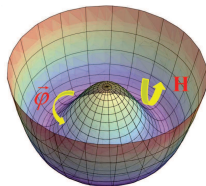
SM Higgs



**Favoured by
EW precision tests**



SM Higgs Potential



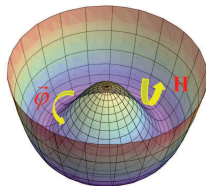
$$\Phi(x) = \exp \left\{ \frac{i}{v} \vec{\sigma} \vec{\varphi}(x) \right\} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

$$V(\Phi) + \frac{\lambda}{4} v^4 = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 = \frac{1}{2} M_H^2 H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4$$

$$v = (\sqrt{2} G_F)^{-1/2} = 246 \text{ GeV}$$

$$M_H = (125.09 \pm 0.24) \text{ GeV} \quad \Rightarrow \quad \lambda = \frac{M_H^2}{2v^2} = 0.13$$

SM Higgs Potential



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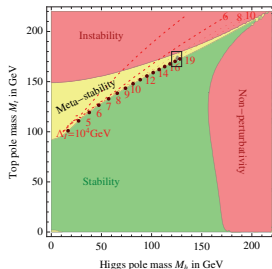
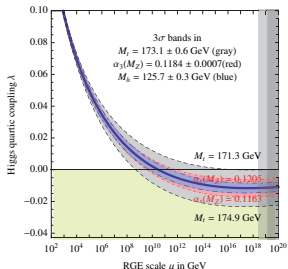
$$M_H = (125.09 \pm 0.24) \text{ GeV} \quad \Rightarrow \quad \lambda = \frac{M_H^2}{2v^2} = 0.13$$

$$M_H^2 = 2v^2 \lambda(\mu) + \frac{2v^2 y_t^2}{(4\pi)^2} [2\lambda + 3(\lambda - y_t^2) \log(m_t^2/\mu^2)] + \dots$$

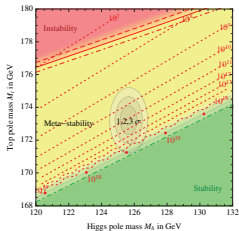
$$y_t = \sqrt{2} m_t / v \approx 1$$

Vacuum Stability: $\lambda(\Lambda) \geq 0$

Degrassi et al, 1205.6497, 1307.3536



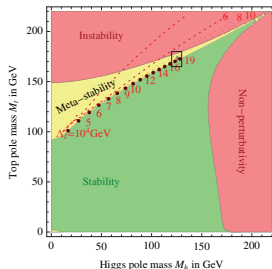
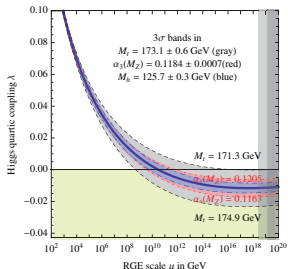
$$\Lambda = M_{\text{Planck}} \rightarrow M_H > (129.1 \pm 1.5) \text{ GeV}$$



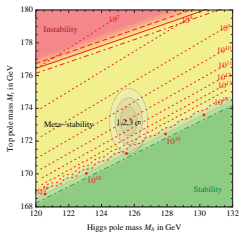
Assumes SM valid all the way up to $\Lambda \leq M_{\text{Planck}}$

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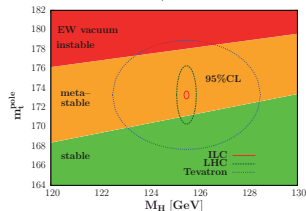
Degrassi et al, 1205.6497, 1307.3536



$$\Lambda = M_{\text{Planck}} \rightarrow M_H > (129.1 \pm 1.5) \text{ GeV} \quad [129.8 \pm 5.6]$$



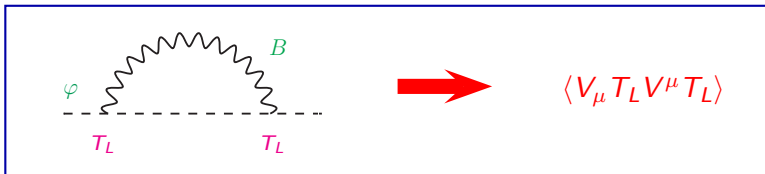
Alekhin et al, 1207.0980



Assumes SM valid all the way up to $\Lambda \leq M_{\text{Planck}}$

Custodial Symmetry Breaking:

$$\hat{B}_\mu \equiv -g' \frac{\sigma_3}{2} B_\mu$$



$$V_\mu \equiv D_\mu U U^\dagger = i \frac{\sqrt{2}}{v} D_\mu \Phi + \dots, \quad T_L \equiv U \frac{\sigma_3}{2} U^\dagger, \quad T_L T_L = \frac{1}{4} I_2$$

$$\begin{aligned} \langle V_\mu T_L V^\mu T_L \rangle &= \langle V_\mu T_L \rangle \langle V^\mu T_L \rangle - \frac{1}{2} \langle V_\mu V^\mu \rangle \langle T_L T_L \rangle \\ &= \langle V_\mu T_L \rangle \langle V^\mu T_L \rangle - \frac{1}{4} \langle V_\mu V^\mu \rangle \end{aligned}$$



$$\mathcal{O}_0 = v^2 \langle V_\mu T_L \rangle \langle V^\mu T_L \rangle$$

$$\varphi^a \varphi^b \rightarrow \varphi^c \varphi^d:$$

- Isospin:**

$$A_0(s, t, u) = 3 A(s, t, u) + A(t, s, u) + A(u, t, s)$$

$$A_1(s, t, u) = A(t, s, u) - A(u, t, s)$$

$$A_2(s, t, u) = A(t, s, u) + A(u, t, s)$$

- Partial Waves:**

$$A_{IJ}(s) = \frac{1}{64\pi} \int_{-1}^{+1} d\cos\theta P_J(\cos\theta) A_I(s, t, u)$$

$$\sigma(s) = \frac{64\pi}{s} \sum_{I,J} (2I+1)(2J+1) |A_{IJ}|^2$$

$$A_{00}(s) = \frac{s}{16\pi v^2} \left\{ 1 + \frac{s}{16\pi^2 v^2} \left[\frac{101}{36} + \frac{64\pi^2}{3} (7 a_4^r + 11 a_5^r) - \frac{25}{18} \log\left(\frac{s}{\mu^2}\right) + i\pi \right] + \dots \right\}$$

$$A_{11}(s) = \frac{s}{96\pi v^2} \left\{ 1 + \frac{s}{16\pi^2 v^2} \left[\frac{1}{9} + 64\pi^2 (a_4^r - 2 a_5^r) + i \frac{\pi}{6} \right] + \dots \right\}$$

$$A_{20}(s) = \frac{-s}{32\pi v^2} \left\{ 1 + \frac{s}{16\pi^2 v^2} \left[-\frac{91}{36} - \frac{256\pi^2}{3} (2 a_4^r + a_5^r) + \frac{10}{9} \log\left(\frac{s}{\mu^2}\right) - i \frac{\pi}{2} \right] + \dots \right\}$$

Power Counting

- Momentum expansion:

$$\Lambda \sim 4\pi v, M_X$$

$$T = \sum_n T_n \left(\frac{p}{\Lambda} \right)^n$$

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- Loop Expansion: A generic L -loop diagram $\mathcal{D}$ scales as

$$\mathcal{D} \sim \frac{(yv)^\nu (gv)^{m+2r+2x+u+z}}{v^{F_L+F_R-2-2\omega}} \frac{p^d}{\Lambda^{2L}} \bar{\psi}_L^{F_L^1} \psi_L^{F_L^2} \bar{\psi}_R^{F_R^1} \psi_R^{F_R^2} \left(\frac{X_{\mu\nu}}{v}\right)^V \left(\frac{\varphi}{v}\right)^B \left(\frac{h}{v}\right)^H$$

$$d \equiv 2L + 2 - \frac{1}{2}(F_L + F_R) - V - \nu - m - 2r - 2x - u - z - 2\omega$$

Buchalla–Catà–Krause

external fields: $F_L = F_L^1 + F_L^2$, $F_R = F_R^1 + F_R^2$, B , H , V (X_μ = gauge boson)

vertices: $m \equiv \sum_l m_l (X_\mu \varphi^l)$, $r \equiv \sum_s r_s (X_\mu^2 \varphi^s)$, $u (X_\mu^3)$, $x (X_\mu^4)$, $\omega \equiv \sum_q \omega_q (h^q)$

$\nu \equiv \sum_k \nu_k (\bar{\psi} \psi \varphi^k) + \sum_{t,b} \tau_{tb} (\bar{\psi} \psi \varphi^t h^b)$, $z \equiv z_L + z_R (\bar{\psi}_\lambda \psi_\lambda X_\mu, \lambda = L, R)$

Too many operators/couplings



Further input needed

$$\mathcal{F}_X(h) = \sum_{n=0} c_X^{(n)} \left(\frac{h}{v} \right)^n = \sum_{n=0} \tilde{c}_X^{(n)} \left(\frac{g_h h}{\Lambda_{\text{NP}}} \right)^n$$

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- **Weak coupling:** $g_h \ll 1$
- **Strong coupling:** $g_h \sim 4\pi = \Lambda_{\text{NP}}/f$  $\mathcal{F}_X(h/f)$
- $v \ll f$  $\xi \equiv \frac{v^2}{f^2}$, $c_X^{(n)} = \tilde{c}_X^{(n)} \xi^{n/2}$

Shifts of SM parameters:

$$\mathcal{L}_{\text{eff}}^{(D=6)} = \sum_i \frac{C_i}{\Lambda^2} \mathcal{O}_i$$

$$V(H) = \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 - \frac{C_\Phi}{\Lambda^2} (\Phi^\dagger \Phi)^3 \quad \rightarrow \quad v_T = \left(1 + \frac{3C_\Phi}{8\lambda} \frac{v^2}{\Lambda^2} \right) v$$

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Canonical Normalization: $H \rightarrow c_H^{\text{kin}} H$, $c_H^{\text{kin}} = \left(C_{\Phi\Box} - \frac{1}{4} C_{\Phi D} \right) \frac{v^2}{\Lambda^2}$

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Flavour: $\mathcal{Y}_f \approx M_f$

$$\mathcal{L} = -\bar{Q}_L^r [Y_d]_{rs} \Phi d_R^s + \dots \quad \rightarrow \quad \begin{aligned} [M_f]_{rs} &= \frac{v_T}{\sqrt{2}} \left([Y_\psi]_{rs} - \frac{v^2}{2\Lambda^2} [C_{f\Phi}]_{rs} \right) \\ [\mathcal{Y}_f]_{rs} &= \frac{1}{v_T} [M_f]_{rs} \left(1 + c_H^{\text{kin}} \right) - \frac{v^2}{\Lambda^2} [C_{f\Phi}]_{rs} \end{aligned}$$

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$$\frac{4 G_F}{\sqrt{2}} = \frac{2}{v_T^2} + \frac{1}{\Lambda^2} \left(2 [C_{\Phi I}^{(3)}]_{ee} + 2 [C_{\Phi I}^{(3)}]_{\mu\mu} - [C_{II}]_{\mu e, e\mu} - [C_{II}]_{e\mu, \mu e} \right)$$

$$\mathcal{L}_{\text{eff}}^{(D=6)} = \sum_i \frac{C_i}{\Lambda^2} \mathcal{O}_i$$

$$\bar{g}' = g' \left(1 + C_{\Phi B} \frac{v_T^2}{\Lambda^2} \right) \quad , \quad \bar{g} = g \left(1 + C_{\Phi W} \frac{v_T^2}{\Lambda^2} \right) \quad , \quad \bar{g}_s = g_s \left(1 + C_{\Phi G} \frac{v_T^2}{\Lambda^2} \right)$$

$$\tan \bar{\theta}_W = \frac{\bar{g}'}{\bar{g}} + \frac{v_T^2}{2\Lambda^2} \left(1 - \frac{\bar{g}'^2}{\bar{g}^2} \right) C_{\Phi WB}$$

$$M_W^2 = \frac{v_T^2}{4} \bar{g}^2 \quad , \quad M_Z^2 = \frac{v_T^2}{4} (\bar{g}^2 + \bar{g}'^2) + \frac{\bar{v}_T^4}{8\Lambda^2} \{ (\bar{g}^2 + \bar{g}'^2) C_{\Phi D} + 4 \bar{g} \bar{g}' C_{\Phi WB} \}$$

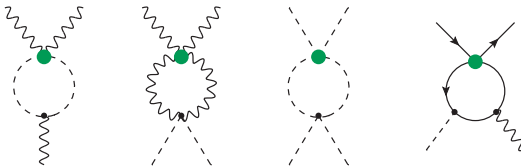
$$\bar{e} = \bar{g} \sin \bar{\theta}_W \left\{ 1 - \cot \bar{\theta}_W \frac{\bar{v}_T^2}{2\Lambda^2} C_{\Phi WB} \right\} \quad , \quad \bar{g}_Z = \frac{\bar{e}}{\sin \bar{\theta}_W \cos \bar{\theta}_W} \left\{ 1 + \frac{\bar{g}^2 + \bar{g}'^2}{2\bar{g}\bar{g}'} \frac{\bar{v}_T^2}{\Lambda^2} C_{\Phi WB} \right\}$$

$$\rho \equiv \frac{\bar{g}^2 M_Z^2}{\bar{g}_Z^2 M_W^2} = 1 + \frac{\bar{v}_T^2}{2\Lambda^2} C_{\Phi D}$$

Renormalization Group Equations

$$\mathcal{L}_{\text{eff}}^{(D=6)} = \sum_i \frac{C_i(\mu)}{\Lambda^2} \mathcal{O}_i(\mu)$$

**Loop
Corrections**



$$\mu \frac{dC_i}{d\mu} = \sum_j \frac{\gamma_{ij}}{16\pi^2} C_j \quad \rightarrow \quad C_i(\mu) = C_i(\Lambda) - \sum_j \frac{\gamma_{ij}}{16\pi^2} C_j(\Lambda) \log \left(\frac{\Lambda}{\mu} \right)$$

Anomalous Dimensions: γ_{ij}

Anomalous Dimensions γ_{ij}

Complete 59×59 matrix computed in the **BW** basis:

Jenkins et al.

$$\gamma \sim O(1, g^2, \lambda, y^2, g^4, g^2\lambda, g^2y^2, \lambda^2, \lambda y^2, y^4, g^6, g^4\lambda, g^6\lambda)$$

	g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
g^3X^3	g^2	0	0	1	0	0	0	0
H^6	$g^6\lambda$	λ, g^2, y^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	$\lambda y^2, y^4$	0	$\lambda y^2, y^4$	0
H^4D^2	g^6	0	g^2, λ, y^2	g^4	y^2	g^2y^2	g^2, y^2	0
$g^2X^2H^2$	g^4	0	1	g^2, λ, y^2	0	y^2	1	0
$y\psi^2H^3$	g^6	0	g^2, λ, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	g^6	0	g^2, y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

(X^2H^2, X^2H^2) submatrix computed in the **HISZ** basis

Chen–Dawson–Zhang

A submatrix computed in the **GGPR** basis

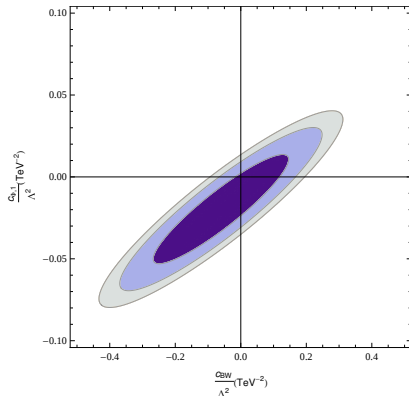
J. Elias-Miro et al.

Oblique Corrections (NLO)

$$\frac{g^2}{16\pi} S \approx -\frac{M_W^2}{\Lambda^2} \left\{ C_{BW}(\mu) + \frac{g^2}{32\pi^2} \left[C_{WW} + \tan^2 \theta_W C_{BB} \right] \left[1 - 2 \log \left(\frac{M_H^2}{\mu^2} \right) \right] \right\}$$

$$\frac{g^2 \sin^2 \theta_W}{4\pi} T \approx -\frac{v^2}{2\Lambda^2} C_{\Phi,1} \quad (\text{HISZ basis})$$

Mebane et al., Chen–Dawson–Zhang



**Tree-level
Bounds**