

4. High-Energy Dynamics

- **CCWZ Formalism**
- **Heavy Fields**
- **Low-Energy Constants**
- **Asymptotic Behaviour**
- **Signals of Heavy Scales**

Energy Scale

Fields

Effective Theory

M_W

t, b, c
 $s, d, u ; G^a$

$\text{QCD}^{N_f=6}$

$\lesssim m_c$

$s, d, u ; G^a$

$\text{QCD}^{N_f=3}$

Λ_χ

V, A, S, P
 π, K, η

$\text{R}\chi\text{T}$

$\lesssim M_K$

π, K, η

$\chi\text{PT}^{N_f=3}$

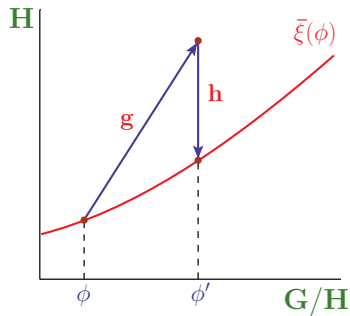
$\lesssim M_\pi$

π

$\chi\text{PT}^{N_f=2}$

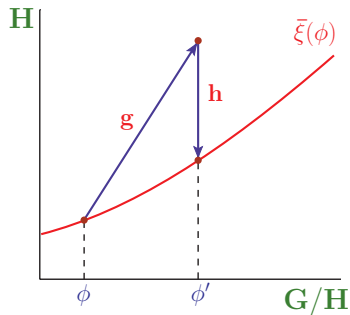
Coset Space Coordinates:

$$G \equiv SU(3)_L \otimes SU(3)_R \xrightarrow{SCSB} H \equiv SU(3)_V$$



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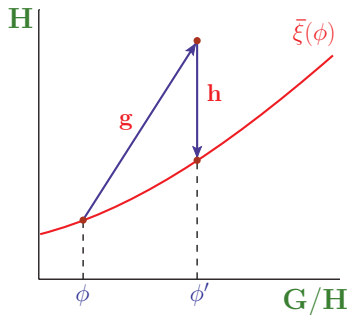
$$\bar{\xi}(\phi) \equiv (\xi_L(\phi), \xi_R(\phi)) \in G$$

$$\xi_L(\phi) \xrightarrow{G} g_L \xi_L(\phi) h^\dagger(\phi, g)$$

$$\xi_R(\phi) \xrightarrow{G} g_R \xi_R(\phi) h^\dagger(\phi, g)$$

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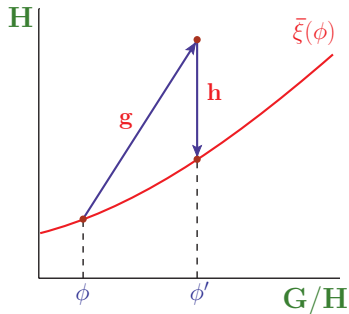
$$\xi_L(\phi) \xrightarrow{G} g_L \xi_L(\phi) h^\dagger(\phi, g)$$

$$\xi_R(\phi) \xrightarrow{G} g_R \xi_R(\phi) h^\dagger(\phi, g)$$

$$U(\phi) \equiv \xi_R(\phi) \xi_L^\dagger(\phi) \xrightarrow{G} g_R U(\phi) g_L^\dagger$$

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Canonical choice:

$$\xi_R(\phi) = \xi_L(\phi)^\dagger \equiv \mathbf{u}(\phi) \xrightarrow{G} g_R \mathbf{u}(\phi) h^\dagger(\phi, g) = h(\phi, g) \mathbf{u}(\phi) g_L^\dagger$$

$$U(\phi) = \mathbf{u}(\phi)^2 = \exp \left\{ i \frac{\sqrt{2}}{f} \Phi \right\}$$

$$\mathbf{u}(\varphi) \xrightarrow{G} g_R \mathbf{u}(\varphi) h^\dagger(\phi, g) = h(\phi, g) \mathbf{u}(\phi) g_L^\dagger$$

SU(3)_V octets:

$$\mathbf{X} \xrightarrow{G} \mathbf{h}(\phi, g) \mathbf{X} \mathbf{h}(\phi, g)^\dagger$$

$$\mathbf{R} \equiv \frac{1}{2} \lambda^a R^a, \quad \nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R]$$

$$u_\mu \equiv i u^\dagger D_\mu U u^\dagger = u_\mu^\dagger, \quad h^{\mu\nu} = \nabla^\mu u^\nu + \nabla^\nu u^\mu$$

$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u, \quad \chi_\pm \equiv u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$$\Gamma_\mu = \frac{1}{2} \left\{ u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i \ell_\mu) u^\dagger \right\}$$

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$$\chi_\pm \equiv u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$$\Gamma_\mu = \frac{1}{2} \left\{ u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i \ell_\mu) u^\dagger \right\}$$

$$\mathcal{L}_2 = \frac{f^2}{4} \langle u^\mu u_\mu + \chi_+ \rangle$$

Resonance Nonet Multiplets: **V**(1⁻⁻) , **A**(1⁺⁺) , **S**(0⁺⁺) , **P**(0⁻⁻)

$$\mathcal{L}_2^V = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle$$

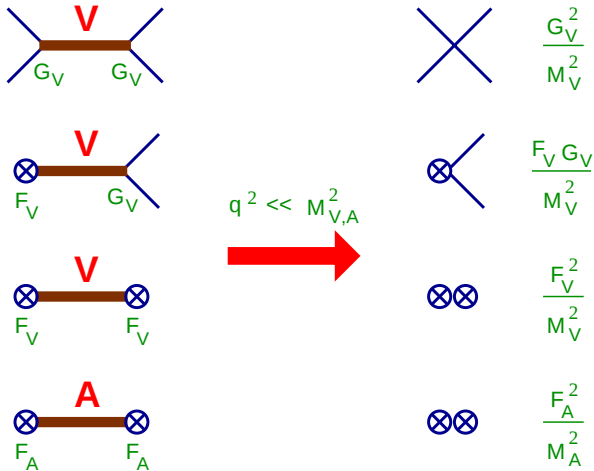
$$\mathcal{L}_2^A = \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle$$

$$\mathcal{L}_2^S = c_d \langle S u^\mu u^\nu \rangle + c_m \langle S \chi_+ \rangle$$

$$\mathcal{L}_2^P = i d_m \langle P \chi_- \rangle$$

$$u_\mu = i u^\dagger D_\mu U u^\dagger = u_\mu^\dagger \quad ; \quad U = u^2$$

$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u \quad ; \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$



$$\left\{ \mathbf{V} [M_V, G_V, F_V] , \mathbf{A} [M_A, F_A] \right\} \longleftrightarrow \left\{ \mathbf{S} [M_S, c_d, c_m] , \mathbf{P} [M_P, d_m] \right\}$$

$O(N_C) :$

$$2 L_1 = L_2 = \sum_i \frac{G_{V_i}^2}{4 M_{V_i}^2} \quad ; \quad L_3 = \sum_i \left\{ -\frac{3 G_{V_i}^2}{4 M_{V_i}^2} + \frac{c_{d_i}^2}{2 M_{S_i}^2} \right\}$$

$$L_5 = \sum_i \frac{c_{d_i} c_{m_i}}{M_{S_i}^2} \quad ; \quad L_8 = \sum_i \left\{ \frac{c_{m_i}^2}{2 M_{S_i}^2} - \frac{d_{m_i}^2}{2 M_{P_i}^2} \right\}$$

$$L_9 = \sum_i \frac{F_{V_i} G_{V_i}}{2 M_{V_i}^2} \quad ; \quad L_{10} = \frac{1}{4} \sum_i \left\{ \frac{F_{A_i}^2}{M_{A_i}^2} - \frac{F_{V_i}^2}{M_{V_i}^2} \right\}$$

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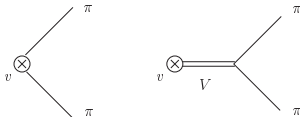
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BUT

$$M_{\eta_1}^2 \sim O\left(\frac{1}{N_C}, \mathcal{M}\right)$$

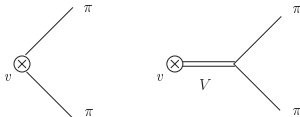
Vector Form Factor



$\langle \pi | \mathbf{v}_\mu | \pi \rangle :$

$$F_V(t) = 1 + \sum_i \frac{F_{V_i} G_{V_i}}{f^2} \frac{t}{M_{V_i}^2 - t}$$

Vector Form Factor

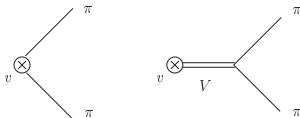


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Short-distance QCD constraint:

$$\lim_{t \rightarrow \infty} F_V(t) = 0$$

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$$\sum_i F_{V_i} G_{V_i} = f^2$$

Weinberg Sum Rules

Chiral Symmetry:

$$\Pi_{LR}^{\mu\nu}(q) \equiv \int d^4x e^{iqx} \langle 0 | T(J_L^\mu(x) J_R^\nu(0)^\dagger) | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{LR}(q^2) = 0$$

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OPE: $\Pi_{LR}^{\mu\nu}(q) \neq 0$ only through **order parameters of EWSB**
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$$\frac{1}{\pi} \int_0^\infty ds [\text{Im} \Pi_{VV}(s) - \text{Im} \Pi_{AA}(s)] = f^2 \quad (1^{\text{st}} \text{ WSR})$$

$$\frac{1}{\pi} \int_0^\infty ds s [\text{Im} \Pi_{VV}(s) - \text{Im} \Pi_{AA}(s)] = 0 \quad (2^{\text{nd}} \text{ WSR})$$

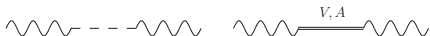
WSRs @ LO



$$\Pi_{LR}(s) = \frac{f^2}{s} + \sum_i \frac{F_{V_i}^2}{M_{V_i}^2 - s} - \sum_i \frac{F_{A_i}^2}{M_{A_i}^2 - s}$$

- 1st WSR: $\sum_i F_{V_i}^2 - \sum_i F_{A_i}^2 = f^2$
- 2nd WSR: $\sum_i F_{V_i}^2 M_{V_i}^2 - \sum_i F_{A_i}^2 M_{A_i}^2 = 0$

WSRs @ LO

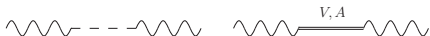


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- 2nd WSR: $\sum_i F_{V_i}^2 M_{V_i}^2 - \sum_i F_{A_i}^2 M_{A_i}^2 = 0$

SRA ➔ $F_V^2 = v^2 \frac{M_A^2}{M_A^2 - M_V^2}, \quad F_A^2 = v^2 \frac{M_V^2}{M_A^2 - M_V^2}, \quad M_A > M_V$

WSRs @ LO



$$\Pi_{LR}(s) = \frac{f^2}{s} + \sum_i \frac{F_{V_i}^2}{M_{V_i}^2 - s} - \sum_i \frac{F_{A_i}^2}{M_{A_i}^2 - s}$$

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SRA $\rightarrow$ $F_V^2 = v^2 \frac{M_A^2}{M_A^2 - M_V^2}$, $F_A^2 = v^2 \frac{M_V^2}{M_A^2 - M_V^2}$, $M_A > M_V$

1st WSR likely valid also in gauge theories with non-trivial UV fixed points

2nd WSR questionable (not valid) in walking (conformal) TC scenarios

Appelquist–Sannino, Orgogozo–Rychkov

Short-Distance Constraints

Vector Form Factor $\langle \pi | v_\mu | \pi \rangle$:

$$F_V(t) = 1 + \sum_i \frac{F_{V_i} G_{V_i}}{f^2} \frac{t}{M_{V_i}^2 - t}$$

$$\lim_{t \rightarrow \infty} F_V(t) = 0$$



$$\sum_i F_{V_i} G_{V_i} = f^2$$

Axial Form Factor $\langle \gamma | a_\mu | \pi \rangle$:

$$G_A(t) = \sum_i \left\{ \frac{2 F_{V_i} G_{V_i} - F_{V_i}^2}{M_{V_i}^2} + \frac{F_{A_i}^2}{M_{A_i}^2 - t} \right\}$$

$$\lim_{t \rightarrow \infty} G_A(t) = 0$$



$$\sum_i (2 F_{V_i} G_{V_i} - F_{V_i}^2) / M_{V_i}^2 = 0$$

Weinberg Sum Rules:

$$\Pi_{LR}(t) = -\frac{f^2}{t} + \sum_i \frac{F_{V_i}^2}{M_{V_i}^2 + t} - \sum_i \frac{F_{A_i}^2}{M_{A_i}^2 + t}$$

$$\lim_{t \rightarrow \infty} t \Pi_{LR}(t) = 0$$

$$\lim_{t \rightarrow \infty} t^2 \Pi_{LR}(t) = 0$$



$$\sum_i (F_{V_i}^2 - F_{A_i}^2) = f^2$$

$$\sum_i (M_{V_i}^2 F_{V_i}^2 - M_{A_i}^2 F_{A_i}^2) = 0$$

Scalar FF:

$$F_{K\pi}^S(s) = 1 + \sum_i \frac{4c_{m_i}}{f^2} \left[c_{d_i} + (c_{m_i} - c_{d_i}) \frac{M_K^2 + M_\pi^2}{M_{S_i}^2} \right] \frac{s}{M_{S_i}^2 - s}$$

$$\lim_{s \rightarrow \infty} F_{K\pi}^S(s) = 0$$



$$4 \sum_i c_{d_i} c_{m_i} = f^2 \quad ; \quad \sum_i \frac{c_{m_i}}{M_{S_i}^2} (c_{m_i} - c_{d_i}) = 0$$

S – P Sum Rules:

$$\Pi_{SS-PP}(t) = 16B_0^2 \left\{ \sum_i \frac{c_{m_i}^2}{M_{S_i}^2 + t} - \sum_i \frac{d_{m_i}^2}{M_{P_i}^2 + t} - \frac{f^2}{8t} \right\}$$

$$\lim_{t \rightarrow \infty} t \Pi_{SS-PP}(t) = 0$$



$$8 \sum_i (c_{m_i}^2 - d_{m_i}^2) = f^2$$

Pseudoscalar Nonet:

$$\mathcal{L}_2 \doteq \frac{f^2}{4} \langle \chi U^\dagger + U \chi^\dagger \rangle \sim -i \frac{f}{\sqrt{24}} \eta_1 \langle \chi_- \rangle$$



$$\tilde{d}_m = -\frac{f}{\sqrt{24}}$$

1-Resonance Approximation:

Ecker, Gasser, Leutwyler, Pich, de Rafael

$$F_V = 2 G_V = \sqrt{2} F_A = \sqrt{2} f \quad ; \quad M_A = \sqrt{2} M_V \quad ; \quad d_m = \frac{1}{2\sqrt{2}} f$$

$$c_m = c_d = \frac{1}{2} f \quad (\text{Jamin, Oller, Pich}) \quad ; \quad M_P \approx \sqrt{2} M_S$$



$$2 L_1 = L_2 = \frac{1}{4} L_9 = -\frac{1}{3} L_{10} = \frac{f^2}{8 M_V^2}$$

$$L_3 = -\frac{3 f^2}{8 M_V^2} + \frac{f^2}{8 M_S^2} \quad ; \quad L_5 = \frac{f^2}{4 M_S^2}$$

$$L_8 = \frac{f^2}{8 M_S^2} - \frac{f^2}{16 M_P^2} \quad ; \quad L_7 = -\frac{f^2}{48 M_{\eta_1}^2}$$

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$$\begin{aligned} 2 L_1 &= L_2 = \frac{1}{4} L_9 = -\frac{1}{3} L_{10} = \frac{f^2}{8 M_V^2} \\ L_3 &= -\frac{3 f^2}{8 M_V^2} + \frac{f^2}{8 M_S^2} \quad ; \quad L_5 = \frac{f^2}{4 M_S^2} \\ L_8 &= \frac{f^2}{8 M_S^2} - \frac{f^2}{16 M_P^2} \quad ; \quad L_7 = -\frac{f^2}{48 M_{\eta_1}^2} \end{aligned}$$

Additional SD input:

Pich (*hep-ph/0205030*); Ledwig, Nieves, Pich, Ruiz-Arriola, Ruiz de Elvira

$$\sqrt{2} M_S = \sqrt{2} M_V = M_P = M_A = \sqrt{\frac{48}{N_C}} \pi f$$



$$2 L_1 = L_2 = -\frac{1}{2} L_3 = \frac{1}{2} L_5 = \frac{4}{3} L_8 = \frac{1}{4} L_9 = -\frac{1}{3} L_{10} = \frac{N_C}{192 \pi^2}$$

L_i 's from Resonance Exchange

i	$10^3 \cdot L_i^r(M_\rho)$	V	A	S	η_1	Total		
1	0.7 ± 0.3	0.6	0	0	0	0.6		
2	1.3 ± 0.3	1.2	0	0	0	1.2		
3	-3.5 ± 1.1	-3.6	0	0.6	0	-3.0		
4	-0.3 ± 0.5	0	0	0	0	0.0		
5	1.4 ± 0.5	0	0	1.4 ^{a)}	0	1.4		
6	-0.2 ± 0.3	0	0	0	0	0.0		
7	-0.4 ± 0.2	0	0	0	-0.3	-0.3		
8	0.9 ± 0.3	0	0	0.9 ^{a)}	0	0.9		
9	6.9 ± 0.7	6.9 ^{a)}	0	0	0	6.9		
10	-5.5 ± 0.7	-10.0	4.0	0	0	-6.0		

^a : Input

L_i 's from Resonance Exchange

i	$10^3 \cdot L_i^r(M_\rho)$	V	A	S	η_1	Total	SD1	
1	0.7 ± 0.3	0.6	0	0	0	0.6	0.9	
2	1.3 ± 0.3	1.2	0	0	0	1.2	1.8	
3	-3.5 ± 1.1	-3.6	0	0.6	0	-3.0	-4.3	
4	-0.3 ± 0.5	0	0	0	0	0.0	0.0	
5	1.4 ± 0.5	0	0	1.4 ^{a)}	0	1.4	2.1	
6	-0.2 ± 0.3	0	0	0	0	0.0	0.0	
7	-0.4 ± 0.2	0	0	0	-0.3	-0.3	-0.3	
8	0.9 ± 0.3	0	0	0.9 ^{a)}	0	0.9	0.8	
9	6.9 ± 0.7	6.9 ^{a)}	0	0	0	6.9	7.2	
10	-5.5 ± 0.7	-10.0	4.0	0	0	-6.0	-5.4	

a : Input

SD1: Short-Distance Constraints

L_i 's from Resonance Exchange

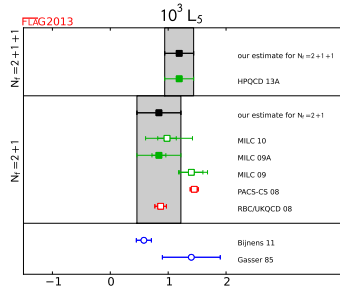
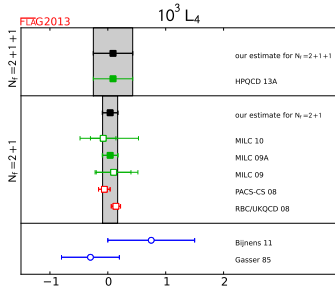
i	$10^3 \cdot L_i^r(M_\rho)$	V	A	S	η_1	Total	SD1	SD2
1	0.7 ± 0.3	0.6	0	0	0	0.6	0.9	0.8
2	1.3 ± 0.3	1.2	0	0	0	1.2	1.8	1.6
3	-3.5 ± 1.1	-3.6	0	0.6	0	-3.0	-4.3	-3.2
4	-0.3 ± 0.5	0	0	0	0	0.0	0.0	0.0
5	1.4 ± 0.5	0	0	$1.4^a)$	0	1.4	2.1	3.2
6	-0.2 ± 0.3	0	0	0	0	0.0	0.0	0.0
7	-0.4 ± 0.2	0	0	0	-0.3	-0.3	-0.3	-0.3
8	0.9 ± 0.3	0	0	$0.9^a)$	0	0.9	0.8	1.2
9	6.9 ± 0.7	$6.9^a)$	0	0	0	6.9	7.2	6.3
10	-5.5 ± 0.7	-10.0	4.0	0	0	-6.0	-5.4	-4.7

a : Input

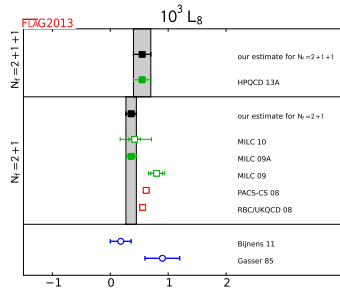
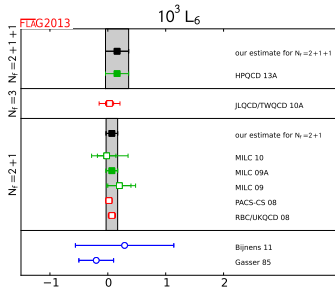
SD1: Short-Distance Constraints

SD2: $L_i \sim \#N_C$

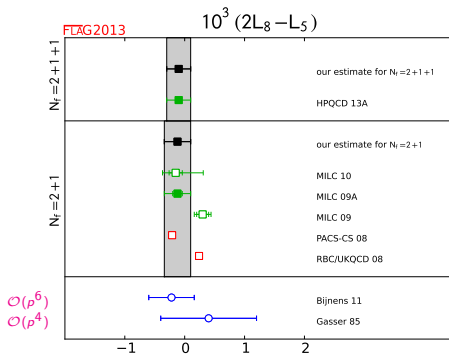
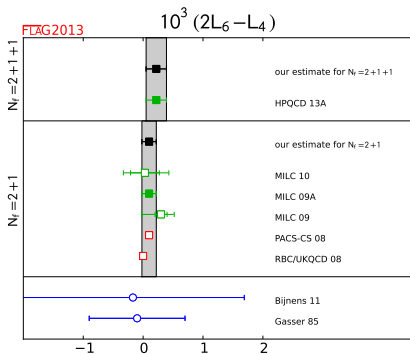
Lattice Determination of SU(3) LECs



$O(p^6)$
 $O(p^4)$

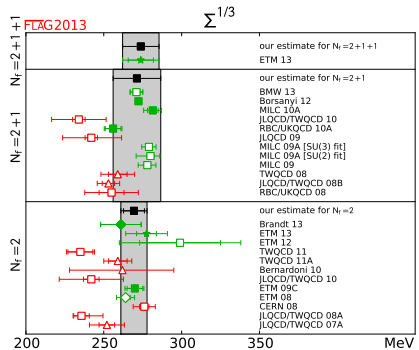
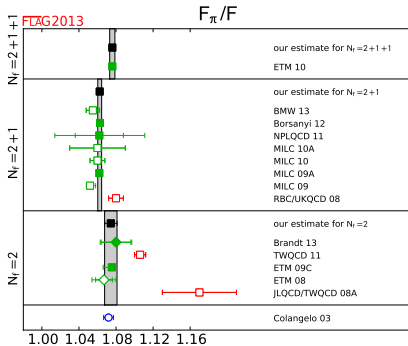


Lattice Determination of SU(3) LECs



$\mathcal{O}(p^6)$
 $\mathcal{O}(p^4)$

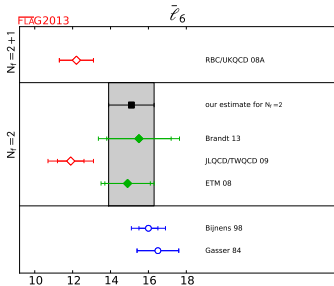
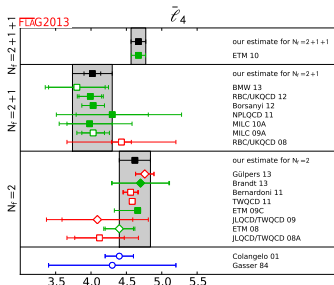
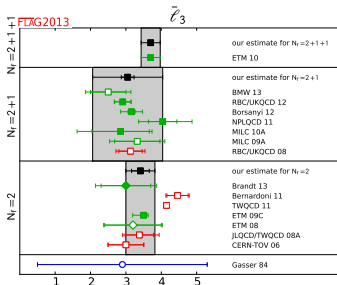
Lattice Determination of SU(2) LECs



$$F = F_\pi|_{m_u=m_d=0},$$

$$\Sigma = -\langle 0|\bar{u}u|0\rangle|_{m_u=m_d=0}$$

Lattice Determination of SU(2) LECs



EW Resonance Effective Theory

- Towers of heavy states are usually present in strongly-coupled models of EWSB: **Technicolour, Walking TC...**
- The low-energy constants (**LECs**) of the Goldstone Lagrangian contain information on the heavier states. **The lightest states not included in the Lagrangian dominate**

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- ① Build $\mathcal{L}_{\text{eff}}(\varphi_i, R_k)$ with the lightest R_k coupled to the φ_i
- ② Require a good **UV behaviour** $\rightarrow$ **Low # of derivatives**
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This program works in QCD: $R\chi T$ (Ecker–Gasser–Leutwyler–Pich–de Rafael)

Good dynamical understanding at large N_C

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\varphi + \sum_R \mathcal{L}_R + \sum_{R,R'} \mathcal{L}_{RR'} + \dots$$

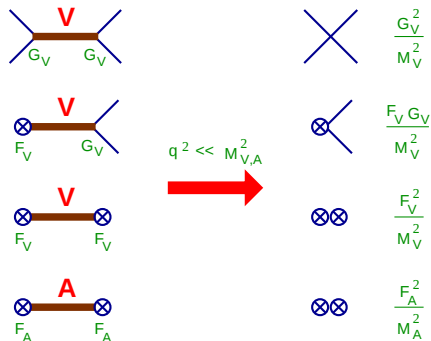
Triples: $\mathbf{V}(1^{--})$, $\mathbf{A}(1^{++})$; **Singlet:** $\mathbf{S}_1(0^{++})$

$$\begin{aligned} \mathcal{L}_{S_1} + \mathcal{L}_A + \mathcal{L}_V &= \frac{v}{2} \kappa_w S_1 \langle u^\mu u_\mu \rangle + \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle \\ &+ \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu u^\nu] \rangle \\ \mathcal{L}_{S_1 A} &= \sqrt{2} \lambda_1^{SA} \partial_\mu S_1 \langle A^{\mu\nu} u_\nu \rangle \end{aligned}$$

Antisymmetric $V_{\mu\nu}$ and $A_{\mu\nu}$ fields (better UV properties):

$$\mathcal{L}_{\text{Kin}} = -\frac{1}{2} \sum_{R=V,A} \langle \nabla^\lambda R_{\lambda\mu} \nabla_\nu R^{\nu\mu} - \frac{1}{2} M_R^2 R_{\mu\nu} R^{\mu\nu} \rangle + \frac{1}{2} \partial^\mu S_1 \partial_\mu S_1 - \frac{1}{2} M_{S_1}^2 S_1^2$$

Resonance Exchange

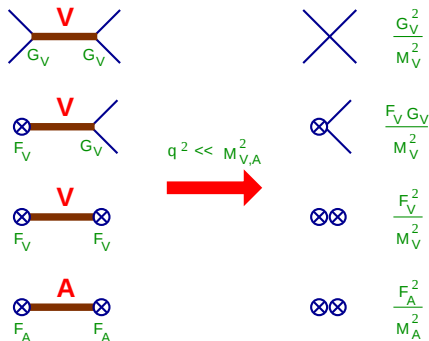


Rosell, Pich, Santos, Sanz-Cillero

$$a_1 = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}, \quad a_2 - a_3 = -\frac{F_V G_V}{2M_V^2}$$

$$a_4 = -a_5 = \frac{G_V^2}{4M_V^2}$$

Resonance Exchange



Rosell, Pich, Santos, Sanz-Cillero

$$a_1 = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2} = -\frac{v^2}{4} \left(\frac{1}{M_V^2} + \frac{1}{M_A^2} \right), \quad a_2 - a_3 = -\frac{F_V G_V}{2M_V^2} = -\frac{v^2}{2M_V^2}$$

$$a_4 = -a_5 = \frac{G_V^2}{4M_V^2} = \frac{v^2}{4} \left(\frac{1}{M_V^2} - \frac{1}{M_A^2} \right)$$

OUTLOOK

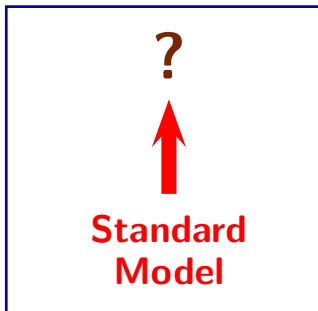
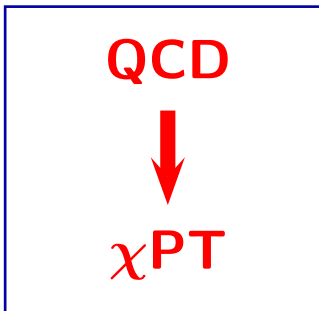
- **Effective Field Theory:** powerful low-energy tool
- **Mass Gap:** $E, m_{\text{light}} \ll \Lambda_{\text{NP}}$
- **Assumption:** relevant symmetries (breakings) & light fields
- **Most general** $\mathcal{L}_{\text{eff}}(\phi_{\text{light}})$ allowed by symmetry
- **Short-distance dynamics** encoded in **LECs**
- **LECs** constrained phenomenologically
- **Goal:** get hints on the underlying fundamental dynamics



New Physics

Learning from QCD experience. **EW problem more difficult**

Fundamental Underlying Theory unknown

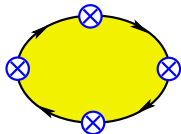


Additional dynamical input (fresh ideas!) needed

Backup Slides

Large- N_C Counting Rules

$$g_s \sim 1/\sqrt{N_C} \quad ; \quad \alpha_s \sim 1/N_C \quad ; \quad \langle T(J_1 \cdots J_n) \rangle \sim N_C$$



- Dominance of planar gluonic exchanges
- Non-planar diagrams suppressed by $1/N_C^2$
- Internal quark loops suppressed by $1/N_C$

Colour Confinement



$$J|0\rangle \sim |1 \text{ Meson}\rangle$$

$$\langle J(k) J(-k) \rangle = \sum_n \frac{f_n^2}{k^2 - M_n^2}$$

- **Infinite** number of mesons ($\sim \ln k^2$)
- $f_n = \langle 0|J|n\rangle \sim \sqrt{N_C}$; $M_n \sim \mathcal{O}(1)$
- Mesons are **free**, **stable** and **non-interacting**

$$\langle J J J \rangle = \Sigma \text{ (triangle diagram) } + \Sigma \text{ (triangle diagram) }$$

$$\langle J J J J \rangle = \Sigma \text{ (box diagram) } + \Sigma \text{ (box diagram) }$$

$$+ \Sigma \text{ (crossing diagram) } + \Sigma \text{ (crossing diagram) }$$

$$\text{Crossing diagram} \sim N_C^{1-\frac{n}{2}}$$

$$\text{Triangle diagram} \sim N_C^{1-\frac{n}{2}}$$

Crossing + Unitarity



Tree Approximation to some Local Effective Meson Lagrangian