

Effective Field Theory

Antonio Pich

IFIC, Univ. Valencia - CSIC

IFIC, Valencia, 20–23 April 2015

1) General Aspects of EFT

- Dimensional Analysis
- Relevant, Irrelevant, Marginal
- Quantum Loops
- Decoupling. Matching
- Scaling

2) χ PT

- Goldstone Theorem
- Chiral Symmetry
- Effective Goldstone Theory
- Chiral Symmetry Breaking
- Phenomenology

3) EW Effective Theory

- Higgs Mechanism
- Custodial Symmetry
- Equivalence Theorem
- EW Effective Theory
- Linear Realization

4) High-Energy Dynamics

- CCWZ Formalism
- Heavy Fields
- Low-Energy Constants
- Asymptotic Behaviour
- Signals of Heavy Scales

Euler-Heisenberg Lagrangian

Light-by-light scattering in QED at very low energies ($E_\gamma \ll m_e$)

- Gauge, Lorentz, Charge Conjugation & Parity constraints
- Energy expansion (E_γ/m_e)

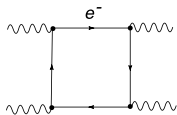
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{a}{m_e^4}(F^{\mu\nu}F_{\mu\nu})^2 + \frac{b}{m_e^4}F^{\mu\nu}F_{\nu\sigma}F^{\sigma\rho}F_{\rho\mu} + \mathcal{O}(F^6/m_e^8)$$

Euler-Heisenberg Lagrangian

Light-by-light scattering in QED at very low energies ($E_\gamma \ll m_e$)

- Gauge, Lorentz, Charge Conjugation & Parity constraints
- Energy expansion (E_γ/m_e)

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{a}{m_e^4}(F^{\mu\nu}F_{\mu\nu})^2 + \frac{b}{m_e^4}F^{\mu\nu}F_{\nu\sigma}F^{\sigma\rho}F_{\rho\mu} + \mathcal{O}(F^6/m_e^8)$$



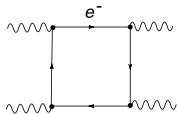
$$a = -\frac{1}{36}\alpha^2, \quad b = \frac{7}{90}\alpha^2$$

Euler-Heisenberg Lagrangian

Light-by-light scattering in QED at very low energies ($E_\gamma \ll m_e$)

- Gauge, Lorentz, Charge Conjugation & Parity constraints
- Energy expansion (E_γ/m_e)

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{a}{m_e^4}(F^{\mu\nu}F_{\mu\nu})^2 + \frac{b}{m_e^4}F^{\mu\nu}F_{\nu\sigma}F^{\sigma\rho}F_{\rho\mu} + \mathcal{O}(F^6/m_e^8)$$



$$\Rightarrow \quad a = -\frac{1}{36}\alpha^2 \quad , \quad b = \frac{7}{90}\alpha^2$$

$$\sigma(\gamma\gamma \rightarrow \gamma\gamma) \propto \frac{\alpha^4 E^6}{m_e^8}$$

Why the sky looks blue?

Why the sky looks blue?

Rayleigh scattering

Low-energy scattering of photons with neutral atoms

$$E_\gamma \ll \Delta E \sim \alpha^2 m_e \ll a_0^{-1} \sim \alpha m_e \ll M_A$$

- Neutral atom + gauge invariance $\Rightarrow F^{\mu\nu} = (\vec{E}, \vec{B})$
- Non-relativistic description

$$\mathcal{L}_{\text{int}} = a_0^3 \psi^\dagger \psi \left(c_1 \vec{E}^2 + c_2 \vec{B}^2 \right) + \dots, \quad c_i \sim \mathcal{O}(1)$$

$$\mathcal{M} \sim c_i a_0^3 E_\gamma^2 \Rightarrow \sigma \propto a_0^6 E_\gamma^4$$

Why the sky looks blue?

Rayleigh scattering

Low-energy scattering of photons with neutral atoms

$$E_\gamma \ll \Delta E \sim \alpha^2 m_e \ll a_0^{-1} \sim \alpha m_e \ll M_A$$

- Neutral atom + gauge invariance $\rightarrow F^{\mu\nu} = (\vec{E}, \vec{B})$
- Non-relativistic description

$$\mathcal{L}_{\text{int}} = a_0^3 \psi^\dagger \psi \left(c_1 \vec{E}^2 + c_2 \vec{B}^2 \right) + \dots, \quad c_i \sim \mathcal{O}(1)$$

$$\mathcal{M} \sim c_i a_0^3 E_\gamma^2 \rightarrow \sigma \propto a_0^6 E_\gamma^4$$

Blue light is scattered more strongly than red one

Dimensions

$$S = \int d^4x \mathcal{L}(x) \quad \rightarrow \quad [\mathcal{L}] = E^4$$

$$\mathcal{L}_{\text{KG}} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi \quad \rightarrow \quad [\phi] = [V^\mu] = [A^\mu] = E$$

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi \quad \rightarrow \quad [\psi] = E^{3/2}$$

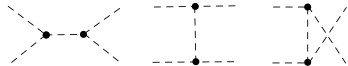
$$[\sigma] = E^{-2} \quad , \quad [\Gamma] = E$$

Scalar Field Theory

- $\mathcal{L}_1 = -\frac{\lambda}{3!} \phi^3 \quad \rightarrow \quad [\lambda] = E$

Scalar Field Theory

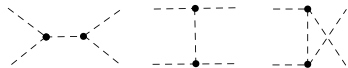
- $\mathcal{L}_1 = -\frac{\lambda}{3!} \phi^3 \quad \Rightarrow \quad [\lambda] = E$



$$\sigma(1+2 \rightarrow 3+4) \sim \frac{\lambda^4}{s^3} \left\{ 1 + \mathcal{O}\left(\frac{\lambda^2}{s}\right) + \dots \right\} \quad (s \gg m^2)$$

Scalar Field Theory

- $\mathcal{L}_1 = -\frac{\lambda}{3!} \phi^3 \quad \rightarrow \quad [\lambda] = E$



$$\sigma(1+2 \rightarrow 3+4) \sim \frac{\lambda^4}{s^3} \left\{ 1 + \mathcal{O}\left(\frac{\lambda^2}{s}\right) + \dots \right\} \quad (s \gg m^2)$$

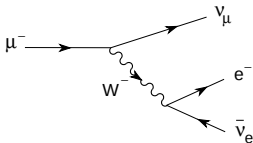
- $\mathcal{L}_1 = -\frac{\lambda}{4!} \phi^4 \quad \rightarrow \quad [\lambda] = E^0$



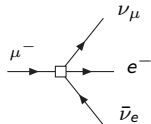
$$\sigma(1+2 \rightarrow 3+4) \sim \frac{\lambda^2}{s} \left\{ 1 + \mathcal{O}(\lambda) + \dots \right\} \quad (s \gg m^2)$$

Fermi Theory

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \xrightarrow{q^2 \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2}$$

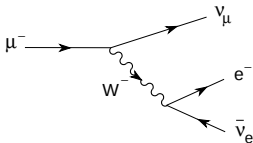


$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \mathcal{J}^\mu + \text{h.c.} \right\}$$

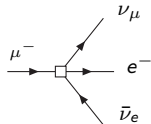
$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \mathcal{J}_\mu^\dagger \mathcal{J}^\mu$$

Fermi Theory

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \xrightarrow{q^2 \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2}$$



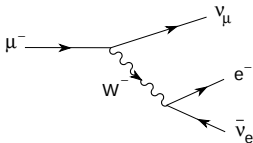
$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \mathcal{J}^\mu + \text{h.c.} \right\}$$

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \mathcal{J}_\mu^\dagger \mathcal{J}^\mu$$

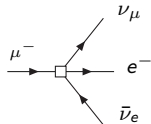
- $\Gamma(l \rightarrow \nu_l l' \bar{\nu}_{l'}) = G_F^2 m_l^5$

Fermi Theory

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \xrightarrow{q^2 \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2}$$



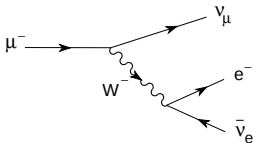
$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \mathcal{J}^\mu + \text{h.c.} \right\}$$

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \mathcal{J}_\mu^\dagger \mathcal{J}^\mu$$

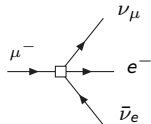
- $\Gamma(l \rightarrow \nu_l l' \bar{\nu}_{l'}) = \frac{G_F^2 m_l^5}{192 \pi^3}$

Fermi Theory

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \xrightarrow{q^2 \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2}$$



$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \mathcal{J}^\mu + \text{h.c.} \right\}$$

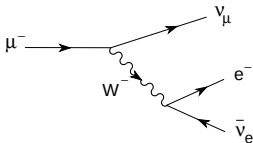
$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \mathcal{J}_\mu^\dagger \mathcal{J}^\mu$$

- $\Gamma(l \rightarrow \nu_l l' \bar{\nu}_{l'}) = \frac{G_F^2 m_l^5}{192\pi^3} f(m_{l'}^2/m_l^2)$

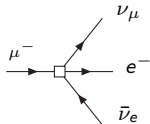
$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

Fermi Theory

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \xrightarrow{q^2 \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2}$$



$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \mathcal{J}^\mu + \text{h.c.} \right\}$$

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \mathcal{J}_\mu^\dagger \mathcal{J}^\mu$$

- $\Gamma(l \rightarrow \nu_l l' \bar{\nu}_{l'}) = \frac{G_F^2 m_l^5}{192\pi^3} f(m_{l'}^2/m_l^2)$

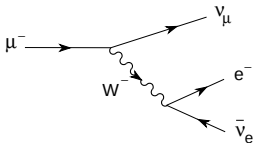
$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$\text{Br}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) = \Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) \tau_\tau = \frac{m_\tau^5}{m_\mu^5} \frac{\tau_\tau}{\tau_\mu} = 17.79\%$$

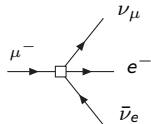
$$\text{Exp: } (17.83 \pm 0.04)\%$$

Fermi Theory

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \xrightarrow{q^2 \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2}$$



$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \mathcal{J}^\mu + \text{h.c.} \right\}$$

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \mathcal{J}_\mu^\dagger \mathcal{J}^\mu$$

- $\Gamma(l \rightarrow \nu_l l' \bar{\nu}_{l'}) = \frac{G_F^2 m_l^5}{192\pi^3} f(m_{l'}^2/m_l^2)$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

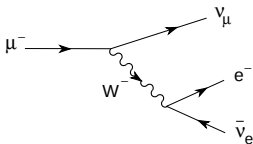
$$\text{Br}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) = \Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) \tau_\tau = \frac{m_\tau^5}{m_\mu^5} \frac{\tau_\tau}{\tau_\mu} = 17.79\%$$

$$\text{Exp: } (17.83 \pm 0.04)\%$$

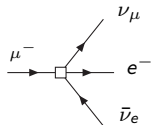
- $\sigma(\nu_\mu e^- \rightarrow \mu^- \nu_e) \sim G_F^2 s$

Fermi Theory

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \xrightarrow{q^2 \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2}$$



$$\mathcal{L}_I = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \mathcal{J}^\mu + \text{h.c.} \right\}$$

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \mathcal{J}_\mu^\dagger \mathcal{J}^\mu$$

- $\Gamma(l \rightarrow \nu_l l' \bar{\nu}_{l'}) = \frac{G_F^2 m_l^5}{192\pi^3} f(m_{l'}^2/m_l^2)$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$\text{Br}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) = \Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) \tau_\tau = \frac{m_\tau^5}{m_\mu^5} \frac{\tau_\tau}{\tau_\mu} = 17.79\%$$

$$\text{Exp: } (17.83 \pm 0.04)\%$$

- $\sigma(\nu_\mu e^- \rightarrow \mu^- \nu_e) \sim G_F^2 s$

Violates unitarity at high energies

Relevant, Irrelevant & Marginal

$$\mathcal{L} = \sum_i c_i O_i \quad , \quad [O_i] = d_i \quad \longrightarrow \quad c_i \sim \frac{1}{\Lambda^{d_i-4}}$$

Low-energy behaviour:

- **Relevant** ($d_i < 4$): $I, \phi^2, \phi^3, \bar{\psi}\psi$

Enhanced by $(\Lambda/E)^{4-d_i}$

- **Marginal** ($d_i = 4$): $m^2\phi^2, m\bar{\psi}\psi, \phi^4, \phi\bar{\psi}\psi, V_\mu\bar{\psi}\gamma^\mu\psi$

- **Irrelevant** ($d_i > 4$): $\bar{\psi}\psi\bar{\psi}\psi, \partial_\mu\phi\bar{\psi}\gamma^\mu\psi, \phi^2\bar{\psi}\psi, \dots$

Suppressed by $(E/\Lambda)^{d_i-4}$

$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \beta_1 \frac{\alpha(Q_0^2)}{2\pi} \log(Q^2/Q_0^2)}$$

QED: $\beta_1 = \frac{2}{3} \sum_f Q_f^2 N_f > 0 \quad \rightarrow \quad \lim_{Q^2 \rightarrow 0} \alpha(Q^2) = 0$

Quantum corrections make **QED irrelevant** at low energies

$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \beta_1 \frac{\alpha(Q_0^2)}{2\pi} \log(Q^2/Q_0^2)}$$

QED: $\beta_1 = \frac{2}{3} \sum_f Q_f^2 N_f > 0 \quad \rightarrow \quad \lim_{Q^2 \rightarrow 0} \alpha(Q^2) = 0$

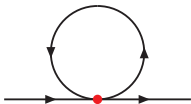
Quantum corrections make **QED irrelevant** at low energies

QCD: $\beta_1 = \frac{2 N_F - 11 N_C}{6} < 0 \quad \rightarrow \quad \lim_{Q^2 \rightarrow 0} \alpha_s(Q^2) = \infty$

Quantum corrections make **QCD relevant** at low energies

Quantum Loops

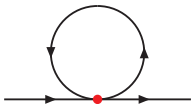
$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - \frac{a}{\Lambda^2} (\bar{\psi}\psi)^2 - \frac{b}{\Lambda^4} (\bar{\psi} \square \psi)(\bar{\psi}\psi) + \dots$$



$$\delta m \sim 2i \frac{a}{\Lambda^2} m \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2}$$

Quantum Loops

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - \frac{a}{\Lambda^2} (\bar{\psi}\psi)^2 - \frac{b}{\Lambda^4} (\bar{\psi} \square \psi)(\bar{\psi}\psi) + \dots$$

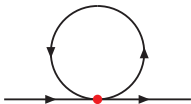


$$\delta m \sim 2i \frac{a}{\Lambda^2} m \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2}$$

- **Cut-off regularization:** $\delta m \sim \frac{m}{\Lambda^2} \Lambda^2 \sim m$ **Not suppressed!**

Quantum Loops

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - \frac{a}{\Lambda^2} (\bar{\psi}\psi)^2 - \frac{b}{\Lambda^4} (\bar{\psi} \square \psi)(\bar{\psi}\psi) + \dots$$



$$\delta m \sim 2i \frac{a}{\Lambda^2} m \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2}$$

- **Cut-off regularization:** $\delta m \sim \frac{m}{\Lambda^2} \Lambda^2 \sim m$ **Not suppressed!**
- **Dimensional regularization:**

$$\Delta_\infty(\mu) = \frac{2\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$$

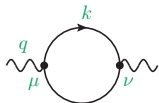
$$\delta m \sim 2a m \frac{m^2}{16\pi^2 \Lambda^2} \left\{ \Delta_\infty(\mu) + \log\left(\frac{m^2}{\mu^2}\right) - 1 + \mathcal{O}(D-4) \right\}$$

Well-defined expansion

(Mass independent)

Vacuum Polarization

($m_f = 0$)



$$i \Pi^{\mu\nu}(q) = i (-q^2 g^{\mu\nu} + q^\mu q^\nu) \Pi(q^2)$$

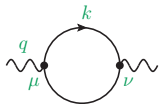
$$\Pi(q^2) = -\frac{\alpha Q_f^2}{3\pi} \left\{ \Delta_\infty(\mu) + \log\left(\frac{-q^2}{\mu^2}\right) - \frac{5}{3} \right\}$$

$$\equiv \Delta\Pi_\epsilon(\mu^2) + \Pi_R(q^2/\mu^2)$$

$$\Delta_\infty(\mu) = \frac{2\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$$

Vacuum Polarization

($m_f = 0$)

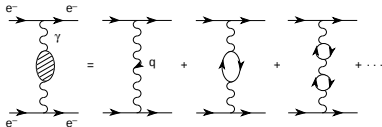


$$i \Pi^{\mu\nu}(q) = i (-q^2 g^{\mu\nu} + q^\mu q^\nu) \Pi(q^2)$$

$$\Pi(q^2) = -\frac{\alpha Q_f^2}{3\pi} \left\{ \Delta_\infty(\mu) + \log\left(\frac{-q^2}{\mu^2}\right) - \frac{5}{3} \right\}$$

$$\equiv \Delta\Pi_\epsilon(\mu^2) + \Pi_R(q^2/\mu^2)$$

$$\Delta_\infty(\mu) = \frac{2\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$$

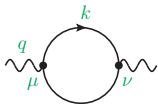


$$\alpha_0 \{1 - \Delta\Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2)\}$$

$$\equiv \alpha_R(\mu^2) \{1 - \Pi_R(q^2/\mu^2)\}$$

Vacuum Polarization

($m_f = 0$)

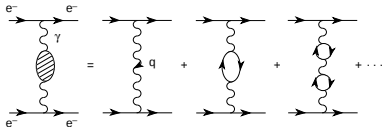


$$i \Pi^{\mu\nu}(q) = i (-q^2 g^{\mu\nu} + q^\mu q^\nu) \Pi(q^2)$$

$$\Pi(q^2) = -\frac{\alpha Q_f^2}{3\pi} \left\{ \Delta_\infty(\mu) + \log\left(\frac{-q^2}{\mu^2}\right) - \frac{5}{3} \right\}$$

$$\equiv \Delta\Pi_\epsilon(\mu^2) + \Pi_R(q^2/\mu^2)$$

$$\Delta_\infty(\mu) = \frac{2\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$$



$$\alpha_0 \{1 - \Delta\Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2)\}$$

$$\equiv \alpha_R(\mu^2) \{1 - \Pi_R(q^2/\mu^2)\}$$

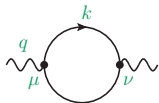
$$\frac{\mu}{\alpha} \frac{d\alpha}{d\mu} \equiv \beta(\alpha) = \beta_1 \frac{\alpha}{\pi} + \dots$$



$$\alpha(Q^2) \approx \frac{\alpha(Q_0^2)}{1 - \beta_1 \frac{\alpha(Q_0^2)}{2\pi} \log(Q^2/Q_0^2)}$$

Vacuum Polarization

($m_f \neq 0$)

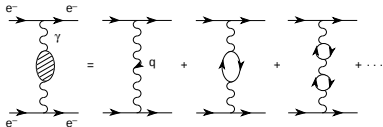


$$i \Pi^{\mu\nu}(q) = i (-q^2 g^{\mu\nu} + q^\mu q^\nu) \Pi(q^2)$$

$$\Pi(q^2) = -\frac{\alpha Q_f^2}{3\pi} \left\{ \Delta_\infty(\mu) + 6 \int_0^1 dx x(1-x) \log \left(\frac{m_f^2 - q^2 x(1-x)}{\mu^2} \right) \right\}$$

$$\equiv \Delta \Pi_\epsilon(\mu^2) + \Pi_R(q^2/\mu^2)$$

$$\Delta_\infty(\mu) = \frac{2\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$$



$$\alpha_0 \{1 - \Delta \Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2)\}$$

$$\equiv \alpha_R(\mu^2) \{1 - \Pi_R(q^2/\mu^2)\}$$

$$\frac{\mu}{\alpha} \frac{d\alpha}{d\mu} \equiv \beta(\alpha) = \beta_1 \frac{\alpha}{\pi} + \dots$$



$$\alpha(Q^2) \approx \frac{\alpha(Q_0^2)}{1 - \beta_1 \frac{\alpha(Q_0^2)}{2\pi} \log(Q^2/Q_0^2)}$$

Mass-Dependent Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \equiv \Pi(-\mu^2)$$

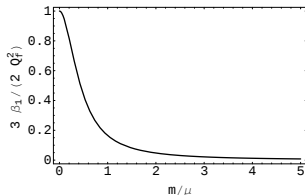
$$\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{m_f^2 + \mu^2 x(1-x)} \right]$$

Mass-Dependent Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \equiv \Pi(-\mu^2)$$

$$\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{m_f^2 + \mu^2 x(1-x)} \right]$$

$$\beta_1 = 4 Q_f^2 \int_0^1 dx \frac{\mu^2 x^2 (1-x)^2}{m_f^2 + \mu^2 x(1-x)}$$

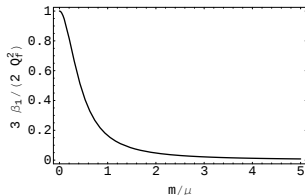


Mass-Dependent Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \equiv \Pi(-\mu^2)$$

$$\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{m_f^2 + \mu^2 x(1-x)} \right]$$

$$\beta_1 = 4 Q_f^2 \int_0^1 dx \frac{\mu^2 x^2 (1-x)^2}{m_f^2 + \mu^2 x(1-x)}$$



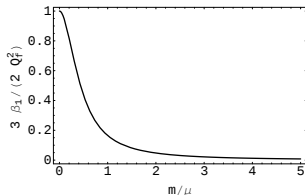
- $m_f^2 \ll \mu^2, q^2$: $\beta_1 = \frac{2}{3} Q_f^2$, $\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log(-q^2/\mu^2)$

Mass-Dependent Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \equiv \Pi(-\mu^2)$$

$$\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{m_f^2 + \mu^2 x(1-x)} \right]$$

$$\beta_1 = 4 Q_f^2 \int_0^1 dx \frac{\mu^2 x^2 (1-x)^2}{m_f^2 + \mu^2 x(1-x)}$$



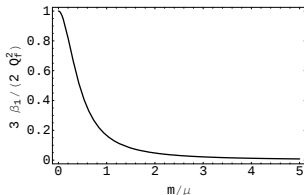
- $m_f^2 \ll \mu^2, q^2$: $\beta_1 = \frac{2}{3} Q_f^2$, $\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log(-q^2/\mu^2)$
- $m_f^2 \gg \mu^2, q^2$: $\beta_1 \sim \frac{2}{15} Q_f^2 \frac{\mu^2}{m_f^2}$, $\Pi_R(q^2/\mu^2) \sim Q_f^2 \frac{\alpha}{15\pi} \frac{q^2 + \mu^2}{m_f^2}$

Mass-Dependent Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \equiv \Pi(-\mu^2)$$

$$\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{m_f^2 + \mu^2 x(1-x)} \right]$$

$$\beta_1 = 4 Q_f^2 \int_0^1 dx \frac{\mu^2 x^2 (1-x)^2}{m_f^2 + \mu^2 x(1-x)}$$



- $m_f^2 \ll \mu^2, q^2$: $\beta_1 = \frac{2}{3} Q_f^2$, $\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log(-q^2/\mu^2)$
- $m_f^2 \gg \mu^2, q^2$: $\beta_1 \sim \frac{2}{15} Q_f^2 \frac{\mu^2}{m_f^2}$, $\Pi_R(q^2/\mu^2) \sim Q_f^2 \frac{\alpha}{15\pi} \frac{q^2 + \mu^2}{m_f^2}$



DECOUPLING

(Appelquist-Carazzone Theorem)

$\overline{\text{MS}}$ Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \equiv -Q_f^2 \frac{\alpha_0}{3\pi} \Delta_\infty(\mu)$$

$$\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx \, x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{\mu^2} \right]$$

$\overline{\text{MS}}$ Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \equiv -Q_f^2 \frac{\alpha_0}{3\pi} \Delta_\infty(\mu)$$

$$\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx \, x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{\mu^2} \right]$$

- $\beta_1 = \frac{2}{3} Q_f^2$

Independent of m_f

Heavy fermions do not decouple

$\overline{\text{MS}}$ Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \equiv -Q_f^2 \frac{\alpha_0}{3\pi} \Delta_\infty(\mu)$$

$$\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx \, x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{\mu^2} \right]$$

- $\beta_1 = \frac{2}{3} Q_f^2$

Independent of m_f

Heavy fermions do not decouple

- $m_f^2 \gg \mu^2, q^2:$ $\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log(m_f^2/\mu^2)$

Perturbation theory breaks down

$\overline{\text{MS}}$ Scheme:

$$\Delta\Pi_\epsilon(\mu^2) \equiv -Q_f^2 \frac{\alpha_0}{3\pi} \Delta_\infty(\mu)$$

$$\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx \, x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{\mu^2} \right]$$

- $\beta_1 = \frac{2}{3} Q_f^2$ **Independent of m_f**

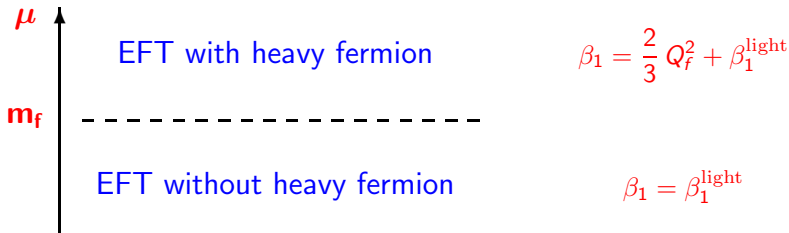
Heavy fermions do not decouple

- $m_f^2 \gg \mu^2, q^2:$ $\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log(m_f^2/\mu^2)$

Perturbation theory breaks down

SOLUTION: Integrate Out Heavy Particles

Matching



- Two different EFTs (with and without the heavy fermion f)
- Same S-matrix elements for light-particle scattering at $\mu = m_f$

Effective Field Theory

$$\mathcal{L}(\phi, \Phi) = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\Phi^2 - \frac{\lambda}{2}\phi^2\Phi$$

Effective Field Theory

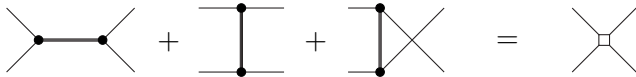
$$\mathcal{L}(\phi, \Phi) = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\Phi^2 - \frac{\lambda}{2}\phi^2\Phi$$

$$\sigma(\phi\phi \rightarrow \phi\phi) \sim \frac{1}{E^2} \times \begin{cases} (\lambda/E)^4, & (m \ll M \ll E) \\ (\lambda/M)^4, & (m, E \ll M) \end{cases}$$

Effective Field Theory

$$\mathcal{L}(\phi, \Phi) = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\Phi^2 - \frac{\lambda}{2}\phi^2\Phi$$

$$\sigma(\phi\phi \rightarrow \phi\phi) \sim \frac{1}{E^2} \times \begin{cases} (\lambda/E)^4, & (m \ll M \ll E) \\ (\lambda/M)^4, & (m, E \ll M) \end{cases}$$



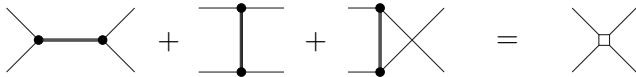
The diagram shows the sum of three Feynman diagrams on the left equaling a contact diagram on the right. The first diagram is a tree-level exchange with two external lines on each side and a horizontal internal line with vertices marked by black dots. The second diagram is a contact interaction with two horizontal external lines and a vertical internal line with vertices marked by black dots. The third diagram is a contact interaction with two horizontal external lines and a vertical internal line with vertices marked by black dots, but with the external lines crossed. The right-hand side is a contact interaction with four external lines meeting at a central square vertex. To the right of the equations is the text $E, m \ll M$.

$$\frac{\lambda^2}{s - M^2} = -\frac{\lambda^2}{M^2} \sum_{n=0} \frac{s^n}{M^{2n}} \quad \rightarrow \quad \mathcal{L}_{\text{eff}}(\phi) = \sum_i c_i O_i(\phi)$$

Effective Field Theory

$$\mathcal{L}(\phi, \Phi) = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\Phi^2 - \frac{\lambda}{2}\phi^2\Phi$$

$$\sigma(\phi\phi \rightarrow \phi\phi) \sim \frac{1}{E^2} \times \begin{cases} (\lambda/E)^4, & (m \ll M \ll E) \\ (\lambda/M)^4, & (m, E \ll M) \end{cases}$$



The diagram shows three terms on the left separated by plus signs, followed by an equals sign and a fourth term. The first term is a tree-level exchange of a massive scalar between two pairs of external lines. The second term is a contact interaction with two external lines and two internal lines. The third term is a contact interaction with two external lines and two internal lines, with a different topology. The fourth term is a contact interaction with four external lines meeting at a central point. To the right of the diagrams is the text $E, m \ll M$.

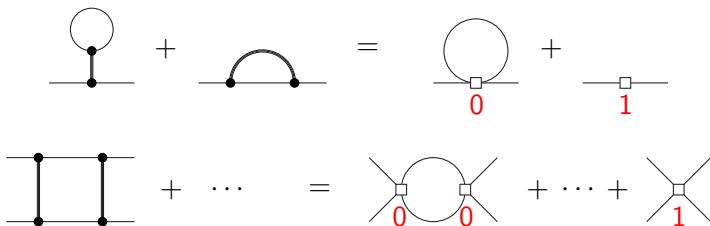
$$\frac{\lambda^2}{s - M^2} = -\frac{\lambda^2}{M^2} \sum_{n=0} \frac{s^n}{M^{2n}} \quad \rightarrow \quad \mathcal{L}_{\text{eff}}(\phi) = \sum_i c_i O_i(\phi)$$

$$[O_i] = d_i \quad ; \quad c_i \sim \frac{\lambda^2}{M^2} \frac{1}{M^{d_i-4}}$$

One Loop:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} a (\partial\phi)^2 - \frac{1}{2} b \phi^2 + c \frac{\lambda^2}{8M^2} \phi^4 + \dots$$

MATCHING



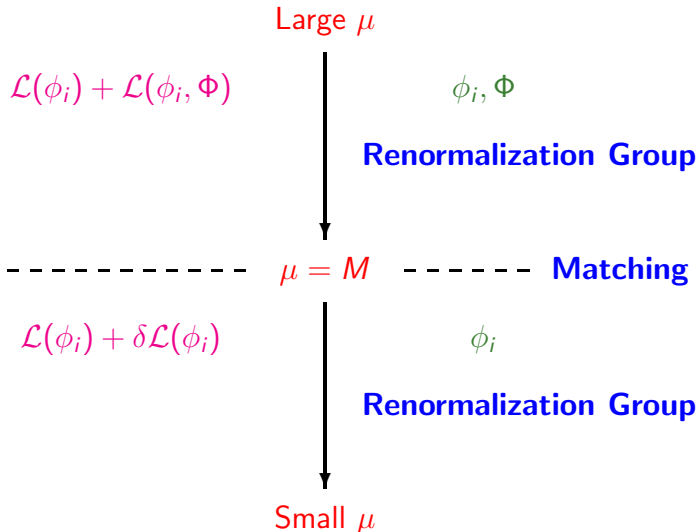
$$a = 1 + a_1 \frac{\lambda^2}{16\pi^2 M^2} + \dots \quad ; \quad b = m^2 + b_1 \frac{\lambda^2}{16\pi^2} + \dots$$

$$c = 1 + c_1 \frac{\lambda^2}{16\pi^2 M^2} + \dots \quad ; \quad \dots$$

Principles of Effective Field Theory

- **Low-energy dynamics** independent of details at high energies
- Appropriate physics description at the analyzed scale
(degrees of freedom)
- **Energy gaps:** $0 \leftarrow m \ll E \ll M \rightarrow \infty$
- Non-local heavy-particle exchanges replaced by a **tower of local interactions** among the light particles
- **Accuracy:** $(E/M)^{(d_i-4)} \gtrsim \epsilon \iff d_i \lesssim 4 + \frac{\log(1/\epsilon)}{\log(M/E)}$
- **Same infrared** (but different ultraviolet) **behaviour** than the underlying fundamental theory
- The only remnants of the high-energy dynamics are in the **low-energy couplings** and in the **symmetries** of the EFT

Evolution from High to Low Scales



QCD Matching

$$(\mu > M) \quad \mathcal{L}_{\text{QCD}}^{(N_F)} \quad \longleftrightarrow \quad \mathcal{L}_{\text{QCD}}^{(N_F-1)} + \sum_{d_i > 4} \frac{c_i}{M^{d_i-4}} O_i \quad (\mu < M)$$

QCD Matching

$$(\mu > M) \quad \mathcal{L}_{\text{QCD}}^{(N_F)} \quad \longleftrightarrow \quad \mathcal{L}_{\text{QCD}}^{(N_F-1)} + \sum_{d_i > 4} \frac{C_i}{M^{d_i-4}} O_i \quad (\mu < M)$$

$$\alpha_s^{(N_F)}(\mu^2) = \alpha_s^{(N_F-1)}(\mu^2) \left\{ 1 + \sum_{k=1}^{\infty} C_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi} \right]^k \right\}$$

$L \equiv \ln(\mu^2/M^2)$

$$m_q^{(N_F)}(\mu^2) = m_q^{(N_F-1)}(\mu^2) \left\{ 1 + \sum_{k=1}^{\infty} H_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi} \right]^k \right\}$$

QCD Matching

$$(\mu > M) \quad \mathcal{L}_{\text{QCD}}^{(N_F)} \quad \longleftrightarrow \quad \mathcal{L}_{\text{QCD}}^{(N_F-1)} + \sum_{d_i > 4} \frac{C_i}{M^{d_i-4}} O_i \quad (\mu < M)$$

$$\alpha_s^{(N_F)}(\mu^2) = \alpha_s^{(N_F-1)}(\mu^2) \left\{ 1 + \sum_{k=1}^{\infty} C_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi} \right]^k \right\}$$

$L \equiv \ln(\mu^2/M^2)$

$$m_q^{(N_F)}(\mu^2) = m_q^{(N_F-1)}(\mu^2) \left\{ 1 + \sum_{k=1}^{\infty} H_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi} \right]^k \right\}$$

- **Matching conditions known to 4 loops:** $C_{1,2,3,4}$, $H_{1,2,3,4}$
(Schroder-Steinhauser, Chetyrkin et al, Larin et al, Liu-Steinhauser)
- $\alpha_s(\mu^2)$ **is not continuous at threshold**

Wilson Coefficients:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} \mathcal{O}_i$$

Wilson Coefficients:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$$

$$\langle O_i \rangle_B = Z_i(\epsilon, \mu) \langle O_i(\mu) \rangle_R \quad ; \quad \mu \frac{d}{d\mu} \langle O_i \rangle_B = 0$$

Wilson Coefficients:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$$

$$\langle O_i \rangle_B = Z_i(\epsilon, \mu) \langle O_i(\mu) \rangle_R \quad ; \quad \mu \frac{d}{d\mu} \langle O_i \rangle_B = 0$$

$$\left(\mu \frac{d}{d\mu} + \gamma_{O_i} \right) \langle O_i \rangle_R = 0 \quad ; \quad \gamma_{O_i} \equiv \frac{\mu}{Z_i} \frac{dZ_i}{d\mu} = \gamma_{O_i}^{(1)} \frac{\alpha}{\pi} + \gamma_{O_i}^{(2)} \left(\frac{\alpha}{\pi} \right)^2 + \dots$$

Wilson Coefficients:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$$

$$\langle O_i \rangle_B = Z_i(\epsilon, \mu) \langle O_i(\mu) \rangle_R \quad ; \quad \mu \frac{d}{d\mu} \langle O_i \rangle_B = 0$$

$$\left(\mu \frac{d}{d\mu} + \gamma_{O_i} \right) \langle O_i \rangle_R = 0 \quad ; \quad \gamma_{O_i} \equiv \frac{\mu}{Z_i} \frac{dZ_i}{d\mu} = \gamma_{O_i}^{(1)} \frac{\alpha}{\pi} + \gamma_{O_i}^{(2)} \left(\frac{\alpha}{\pi} \right)^2 + \dots$$

$$\mu \frac{d}{d\mu} c_i(\mu) \langle O_i \rangle_R = 0$$

Wilson Coefficients:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$$

$$\langle O_i \rangle_B = Z_i(\epsilon, \mu) \langle O_i(\mu) \rangle_R \quad ; \quad \mu \frac{d}{d\mu} \langle O_i \rangle_B = 0$$

$$\left(\mu \frac{d}{d\mu} + \gamma_{O_i} \right) \langle O_i \rangle_R = 0 \quad ; \quad \gamma_{O_i} \equiv \frac{\mu}{Z_i} \frac{dZ_i}{d\mu} = \gamma_{O_i}^{(1)} \frac{\alpha}{\pi} + \gamma_{O_i}^{(2)} \left(\frac{\alpha}{\pi} \right)^2 + \dots$$

$$\mu \frac{d}{d\mu} c_i(\mu) \langle O_i \rangle_R = 0 \quad \rightarrow \quad \left(\mu \frac{d}{d\mu} - \gamma_{O_i} \right) c_i(\mu) = 0$$

Wilson Coefficients:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$$

$$\langle O_i \rangle_B = Z_i(\epsilon, \mu) \langle O_i(\mu) \rangle_R \quad ; \quad \mu \frac{d}{d\mu} \langle O_i \rangle_B = 0$$

$$\left(\mu \frac{d}{d\mu} + \gamma_{O_i} \right) \langle O_i \rangle_R = 0 \quad ; \quad \gamma_{O_i} \equiv \frac{\mu}{Z_i} \frac{dZ_i}{d\mu} = \gamma_{O_i}^{(1)} \frac{\alpha}{\pi} + \gamma_{O_i}^{(2)} \left(\frac{\alpha}{\pi} \right)^2 + \dots$$

$$\mu \frac{d}{d\mu} c_i(\mu) \langle O_i \rangle_R = 0 \quad \rightarrow \quad \left(\mu \frac{d}{d\mu} - \gamma_{O_i} \right) c_i(\mu) = 0$$


$$\begin{aligned} c_i(\mu) &= c_i(\mu_0) \exp \left\{ \int_{\alpha_0}^{\alpha} \frac{d\alpha}{\alpha} \frac{\gamma_{O_i}(\alpha)}{\beta(\alpha)} \right\} \\ &= c_i(\mu_0) \left[\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)} \right]^{\gamma_{O_i}^{(1)}/\beta_1} \left\{ 1 + \dots \right\} \end{aligned}$$

Operator Mixing:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$$

$$\langle O_i \rangle_B = \sum_j \mathbf{Z}_{ij}(\epsilon, \mu) \langle O_j(\mu) \rangle_R \quad ; \quad \gamma_O \equiv \mathbf{Z}^{-1} \mu \frac{d}{d\mu} \mathbf{Z}$$

Operator Mixing:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$$

$$\langle O_i \rangle_B = \sum_j \mathbf{Z}_{ij}(\epsilon, \mu) \langle O_j(\mu) \rangle_R \quad ; \quad \gamma_O \equiv \mathbf{Z}^{-1} \mu \frac{d}{d\mu} \mathbf{Z}$$

$$\left(\mu \frac{d}{d\mu} + \gamma_O \right) \langle \vec{O} \rangle_R = 0 \quad ; \quad \left(\mu \frac{d}{d\mu} - \gamma_O^T \right) \langle \vec{c} \rangle_R = 0$$

Operator Mixing:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$$

$$\langle O_i \rangle_B = \sum_j \mathbf{Z}_{ij}(\epsilon, \mu) \langle O_j(\mu) \rangle_R \quad ; \quad \gamma_O \equiv \mathbf{Z}^{-1} \mu \frac{d}{d\mu} \mathbf{Z}$$

$$\left(\mu \frac{d}{d\mu} + \gamma_O \right) \langle \vec{O} \rangle_R = 0 \quad ; \quad \left(\mu \frac{d}{d\mu} - \gamma_O^T \right) \langle \vec{c} \rangle_R = 0$$

Diagonalization: $\left(\mathbf{U}^{-1} \gamma_O^T \mathbf{U} \right)_{ij} = \tilde{\gamma}_{O_i} \delta_{ij} \quad ; \quad \tilde{c}_i = \mathbf{U}_{ij}^{-1} c_j$

Operator Mixing:

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$$

$$\langle O_i \rangle_B = \sum_j \mathbf{Z}_{ij}(\epsilon, \mu) \langle O_j(\mu) \rangle_R \quad ; \quad \gamma_O \equiv \mathbf{Z}^{-1} \mu \frac{d}{d\mu} \mathbf{Z}$$

$$\left(\mu \frac{d}{d\mu} + \gamma_O \right) \langle \vec{O} \rangle_R = 0 \quad ; \quad \left(\mu \frac{d}{d\mu} - \gamma_O^T \right) \langle \vec{c} \rangle_R = 0$$

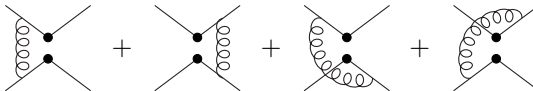
Diagonalization: $\left(\mathbf{U}^{-1} \gamma_O^T \mathbf{U} \right)_{ij} = \tilde{\gamma}_{O_i} \delta_{ij} \quad ; \quad \tilde{c}_i = \mathbf{U}_{ij}^{-1} c_j$



$$c_i(\mu) = \sum_{j,k} \mathbf{U}_{ij} \exp \left\{ \int_{\alpha_0}^{\alpha} \frac{d\alpha}{\alpha} \frac{\tilde{\gamma}_{O_j}(\alpha)}{\beta(\alpha)} \right\} \mathbf{U}_{jk}^{-1} c_k(\mu_0)$$

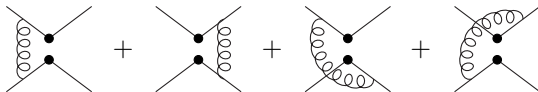
Wilson Coeff. in the Fermi EFT

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{12} V_{43}^* O_{\{1,2;3,4\}} \quad ; \quad O_{\{1,2;3,4\}} \equiv [\bar{q}_1 \gamma^\mu (1 - \gamma_5) q_2] [\bar{q}_3 \gamma_\mu (1 - \gamma_5) q_4]$$



Wilson Coeff. in the Fermi EFT

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{12} V_{43}^* O_{\{1,2;3,4\}} \quad ; \quad O_{\{1,2;3,4\}} \equiv [\bar{q}_1 \gamma^\mu (1 - \gamma_5) q_2] [\bar{q}_3 \gamma_\mu (1 - \gamma_5) q_4]$$



Colour:

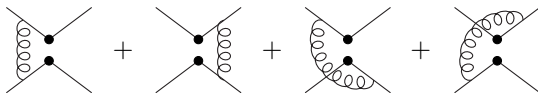
$$\sum_a \lambda_{ij}^a \lambda_{kl}^a = -\frac{2}{N_C} \delta_{ij} \delta_{kl} + 2 \delta_{il} \delta_{kj}$$

Fierz:

$$[\gamma^\mu (1 - \gamma_5)]_{\alpha\beta} [\gamma_\mu (1 - \gamma_5)]_{\gamma\delta} = -[\gamma^\mu (1 - \gamma_5)]_{\alpha\delta} [\gamma_\mu (1 - \gamma_5)]_{\gamma\beta}$$

Wilson Coeff. in the Fermi EFT

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{12} V_{43}^* O_{\{1,2;3,4\}} \quad ; \quad O_{\{1,2;3,4\}} \equiv [\bar{q}_1 \gamma^\mu (1 - \gamma_5) q_2] [\bar{q}_3 \gamma_\mu (1 - \gamma_5) q_4]$$



Colour:

$$\sum_a \lambda_{ij}^a \lambda_{kl}^a = -\frac{2}{N_C} \delta_{ij} \delta_{kl} + 2 \delta_{il} \delta_{kj}$$

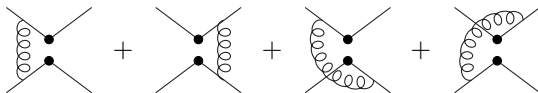
Fierz:

$$[\gamma^\mu (1 - \gamma_5)]_{\alpha\beta} [\gamma_\mu (1 - \gamma_5)]_{\gamma\delta} = -[\gamma^\mu (1 - \gamma_5)]_{\alpha\delta} [\gamma_\mu (1 - \gamma_5)]_{\gamma\beta}$$

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{2\sqrt{2}} V_{12} V_{43}^* \{c_+(\mu) Q_+ + c_-(\mu) Q_-\} \quad ; \quad Q_\pm \equiv O_{\{1,2;3,4\}} \pm O_{\{1,4;3,2\}}$$

Wilson Coeff. in the Fermi EFT

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{12} V_{43}^* O_{\{1,2;3,4\}} \quad ; \quad O_{\{1,2;3,4\}} \equiv [\bar{q}_1 \gamma^\mu (1 - \gamma_5) q_2] [\bar{q}_3 \gamma_\mu (1 - \gamma_5) q_4]$$



Colour:

$$\sum_a \lambda_{ij}^a \lambda_{kl}^a = -\frac{2}{N_C} \delta_{ij} \delta_{kl} + 2 \delta_{il} \delta_{kj}$$

Fierz:

$$[\gamma^\mu (1 - \gamma_5)]_{\alpha\beta} [\gamma_\mu (1 - \gamma_5)]_{\gamma\delta} = -[\gamma^\mu (1 - \gamma_5)]_{\alpha\delta} [\gamma_\mu (1 - \gamma_5)]_{\gamma\beta}$$

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{2\sqrt{2}} V_{12} V_{43}^* \{c_+(\mu) Q_+ + c_-(\mu) Q_-\} \quad ; \quad Q_\pm \equiv O_{\{1,2;3,4\}} \pm O_{\{1,4;3,2\}}$$

$$\gamma_\pm = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \frac{\alpha_s}{\pi}$$



$$c_\pm(\mu) \approx \left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)} \right)^{a_\pm}, \quad a_\pm = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \frac{6}{33 - 2N_f}$$

$$\mathbf{K} \rightarrow (2\pi)_I$$

$$\text{Bose: } I = 0, 2 \quad (\Delta I = \frac{1}{2}, \frac{3}{2})$$

$$A[K^0 \rightarrow \pi^+ \pi^-] \equiv A_0 e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2 e^{i\chi_2}$$

$$A[K^0 \rightarrow \pi^0 \pi^0] \equiv A_0 e^{i\chi_0} - \sqrt{2} A_2 e^{i\chi_2}$$

$$A[K^+ \rightarrow \pi^+ \pi^0] \equiv \frac{3}{2} A_2 e^{i\chi_2}$$

$$\omega \equiv \frac{\text{Re}(A_2)}{\text{Re}(A_0)} \approx \frac{1}{22}$$

$$\Delta I = 1/2 \text{ Rule}$$

$$K \rightarrow (2\pi)_I$$

$$\text{Bose: } I = 0, 2 \quad (\Delta I = \frac{1}{2}, \frac{3}{2})$$

$$A[K^0 \rightarrow \pi^+ \pi^-] \equiv A_0 e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2 e^{i\chi_2}$$

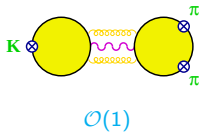
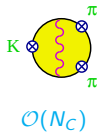
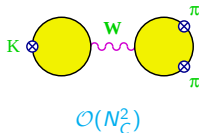
$$A[K^0 \rightarrow \pi^0 \pi^0] \equiv A_0 e^{i\chi_0} - \sqrt{2} A_2 e^{i\chi_2}$$

$$A[K^+ \rightarrow \pi^+ \pi^0] \equiv \frac{3}{2} A_2 e^{i\chi_2}$$

$$\omega \equiv \frac{\text{Re}(A_2)}{\text{Re}(A_0)} \approx \frac{1}{22}$$

$$\Delta I = 1/2 \text{ Rule}$$

**Weak Currents
Factorize
at Large N_C**



$$A[K^0 \rightarrow \pi^0 \pi^0] = 0 \quad \rightarrow \quad A_0 = \sqrt{2} A_2$$

$$K \rightarrow (2\pi)_I$$

$$\text{Bose: } I = 0, 2 \quad (\Delta I = \frac{1}{2}, \frac{3}{2})$$

$$A[K^0 \rightarrow \pi^+ \pi^-] \equiv A_0 e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2 e^{i\chi_2}$$

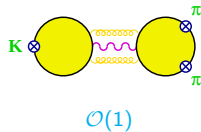
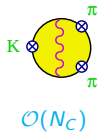
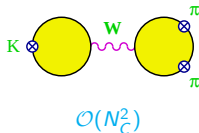
$$A[K^0 \rightarrow \pi^0 \pi^0] \equiv A_0 e^{i\chi_0} - \sqrt{2} A_2 e^{i\chi_2}$$

$$A[K^+ \rightarrow \pi^+ \pi^0] \equiv \frac{3}{2} A_2 e^{i\chi_2}$$

$$\omega \equiv \frac{\text{Re}(A_2)}{\text{Re}(A_0)} \approx \frac{1}{22}$$

$\Delta I = 1/2$ Rule

**Weak Currents
Factorize
at Large N_C**



$$A[K^0 \rightarrow \pi^0 \pi^0] = 0 \quad \rightarrow \quad A_0 = \sqrt{2} A_2$$

Short-distance enhancement: $Q_{\pm} \sim [\bar{u}_L \gamma^{\mu} s_L] [\bar{d}_L \gamma_{\mu} u_L] \pm [\bar{u}_L \gamma^{\mu} u_L] [\bar{d}_L \gamma_{\mu} s_L]$

$$c_{\pm}(\mu) \approx \left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)} \right)^{a_{\pm}} \approx \begin{cases} 0.75 \\ 1.76 \end{cases} \quad \text{at } \mu = 1 \text{ GeV}$$