

Introduction to Chiral Perturbation Theory and its Applications

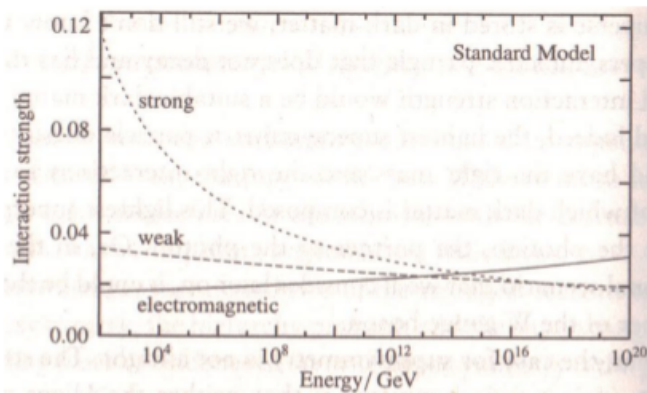
Astrid Hiller Blin

IFIC — Universidad de Valencia
astridblin.4@gmail.com

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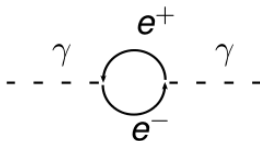
The behaviour of coupling constants



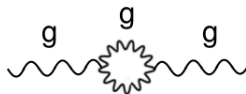
Only the electromagnetic coupling increases at higher energies!

The strength of the forces

- The photon has no charge. Vacuum fluctuations: e^+e^- clouds $\Rightarrow$ larger distance leads to screening of the coupling effect.



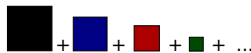
- The gluon has color charge. Vacuum fluctuations: clouds of even more gluons $\Rightarrow$ strengthening of the coupling effect.



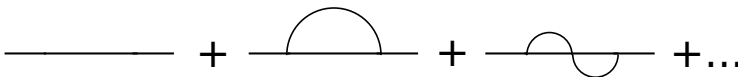
Perturbative regime

- High-energy regime.
- Interaction between quarks and gluons in hadrons — asymptotic freedom.
- α_s is the perturbation parameter.

Taylor:

A diagram representing a Taylor expansion. It consists of four colored squares (black, blue, red, green) followed by an ellipsis, separated by plus signs. The squares decrease in size from left to right.
$$+ + + + \dots$$

$$c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \dots, \quad \alpha_s \ll 1.$$



Non-perturbative regime

- Large distances $\implies$ interaction between hadrons as a whole.
- $\alpha_s \gg 1 \implies$ breakdown of perturbative QCD.



We need a different approach!

- **Effective** field theory: Choice of new expansion parameters — small momenta and masses.

$$m, p \ll \Lambda \approx m_N \approx 1 \text{ GeV}$$

- New degrees of freedom: mesons and baryons instead of quarks and gluons!

Chiral symmetry

- All elementary fermions come in two versions: left- and right-handed.
- For vanishing masses: left and right-handed quarks decouple and show the same transformation properties.

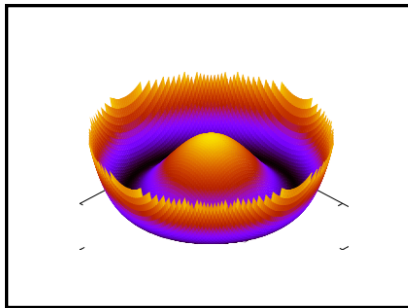
$$q_R \rightarrow e^{i\phi_R} q_R \qquad q_L \rightarrow e^{i\phi_L} q_L$$

Chiral limit!

- In nature, quarks have a mass $\implies$ explicit breaking of the chiral symmetry (small for u and $d \approx 5$ MeV).
- The breaking is introduced in the theory by hand.

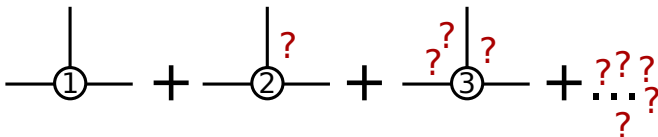
Spontaneous breaking of the symmetry

- For chiral symmetry conservation, the hadron ($\pi^\pm$, π^0 , p , n , ...) masses would vanish.
- Pions have small masses (≈ 150 MeV) and nucleons rather big ones (≈ 1000 MeV).
- Explained by spontaneous breaking of the symmetry!



Chiral perturbation theory

- The expansion is done in terms of $\frac{m_\pi}{\Lambda}$ and $\frac{p}{\Lambda}$.
- In principle there are infinite terms compatible with the symmetry. Coefficients of these terms should be extracted from QCD, but QCD cannot yet be solved!
- The higher the order, the more unknown low-energy constants (LEC) appear $\implies$ fits to data.



Renormalization

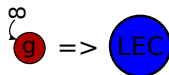
- When calculating amplitudes, there will be divergences of the type

$$\lim_{\text{Dim} \rightarrow 4} \left\{ \frac{1}{\text{Dim} - 4} \right\}.$$

- The LECs are unknown $\implies$ infinities are absorbed into them.
- For instance if we have a coupling constant g , we can redefine

$$g \rightarrow g - \infty,$$

which then cancels the divergence appearing later in the calculation.



The diagram illustrates the renormalization process. On the left, a red circle contains the letter 'g', representing a coupling constant. Above the 'g' is an infinity symbol (∞), indicating a divergence. An arrow points from this red circle to a blue circle on the right. Inside the blue circle are the letters 'LEC', representing a Low Energy Constant. This visualizes the absorption of the infinity into the LEC.

- What we see in the experiment is the g without corrections. The LEC with the same name, additionally has the counterterms to the infinities hidden in it.

Baryonic sector and power-counting breaking

- We want to introduce baryons into the theory $\implies$ the calculation of loops originates terms that break power counting: suddenly terms of lower orders appear!
- This is easy to understand. Where meson loops originated e.g.

$$\mathcal{O}(m_\pi^2/\Lambda^2),$$

analogously a nucleon loop leads to

$$\mathcal{O}(m_N^2/\Lambda^2).$$

- m_π is an expansion parameter (small), but m_N not (big)
 $\implies \mathcal{O}(m_N^2/\Lambda^2) = \mathcal{O}(m_\pi^2/\Lambda^2) - 2.$
- They can also be consistently hidden in the LECs.

Turn a non-predictable theory into a usable one

- Advantage: Divergences and power-counting breaking terms are very easily taken care of.
- Disadvantage: The LECs are in principle unknown.
- Use experimental data to extract the meaning of LECs, e.g. well known in 1st order:

Weak pion decay rate F : $\pi \rightarrow e\nu_e$, $\pi \rightarrow \mu\nu_\mu$;

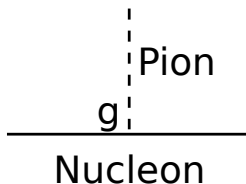
Axial-vector coupling constant g_A .

- After establishing these, OTHER values can be predicted!

Goldberger-Treiman relation

- Relation between strong-interaction quantity and weak processes:

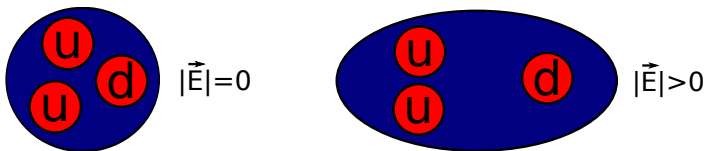
$$g_{\pi N} = \frac{m_N g_A}{F}$$



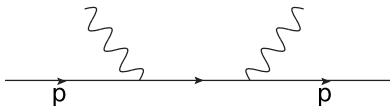
- This is only one example of a couple of interesting low-energy theorems which are pure model-independent predictions.

Polarizabilities

- Polarizabilities — response of non-elementary particles to external electromagnetic fields.

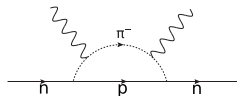
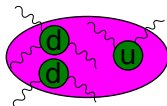
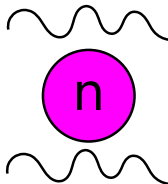


- They are extracted via Compton scattering: energies of the photon of the order of the pion mass.



Advantages of studying polarizabilities

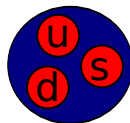
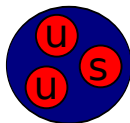
- The photon does not couple to the neutron directly, at 1st order, but to its inner structure.



- Protons and neutrons experimentally accessible: good for testing the theory and fitting unknown constants.
- At the order we are considering, the needed pieces of the theoretical amplitude do NOT have power counting or infinity problems $\Rightarrow$ no renormalization, real predictions!

We want to use this information to make cooler stuff

- So we tested the theory. Now we want to actually predict results before the experimentalists can think of it!
- Hyperons have too short lifetimes to allow actual experimental tests being done on them $\implies$ theoretical predictions only.



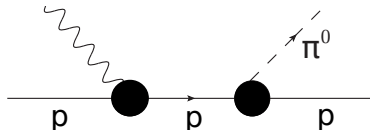
Extending ChPT to include the strange quarks

- ChPT was initially developed for the lightest quarks, u and d (current-quark masses ≈ 5 MeV).
- Later it was extended to the next-lightest quark, s (mass ≈ 100 MeV).
- It is still far below the chosen scale Λ , but of course leads to higher imprecision $\implies$ greater care has to be taken.

$$\left| \begin{array}{c} S=0 \\ p \\ n \end{array} \right| \longrightarrow \left| \begin{array}{ccc} S=0 & S=1 & S=2 \\ p & \Sigma^+ & \\ n & \Sigma^0, \Lambda & \Xi^0 \\ & \Sigma^- & \Xi^- \end{array} \right|$$

Photopion production

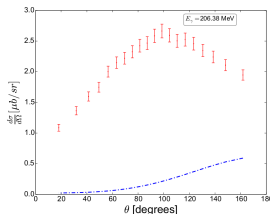
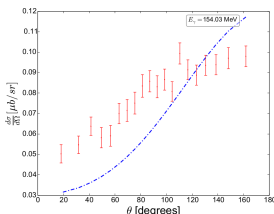
- Also ChPT energy regime.



- Here the idea is to reproduce the cross sections obtained empirically.

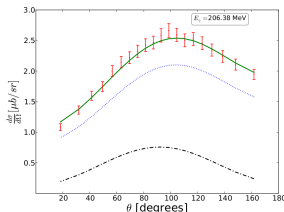
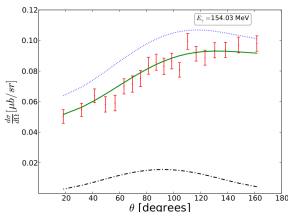
What the hell are we actually doing when we're working?

- We have experimental data and our theoretical curve at lowest order. And we compare them.



- And we see they're awful. So we go to higher orders.

Introducing corrections



- At higher orders the fits are already much better (blue curves).
- Increase accordance even further: e.g. consider Δ resonances.
- Experimentally very probable states appearing when working on protons. Theoretically more complicated because of spin 3/2 (Rarita-Schwinger) instead of spin 1/2 (Dirac).
- The theories get more complicated, but always better until... The perfect fit is obtained! (green curves).

Summary and outlook

- At the low-energy limit of high-energy particle physics QCD is non-perturbative.
- ChPT is an effective theory describing hadrons, mesons and their interactions at energies below $\Lambda = 1 \text{ GeV}$.
- An expansion in masses and small momenta is performed.
- Unknown constants appear, whose numerical value is determined by data fits.
- Using these constants, predictions can be made about not yet experimentally explored/explorable nuclear processes.