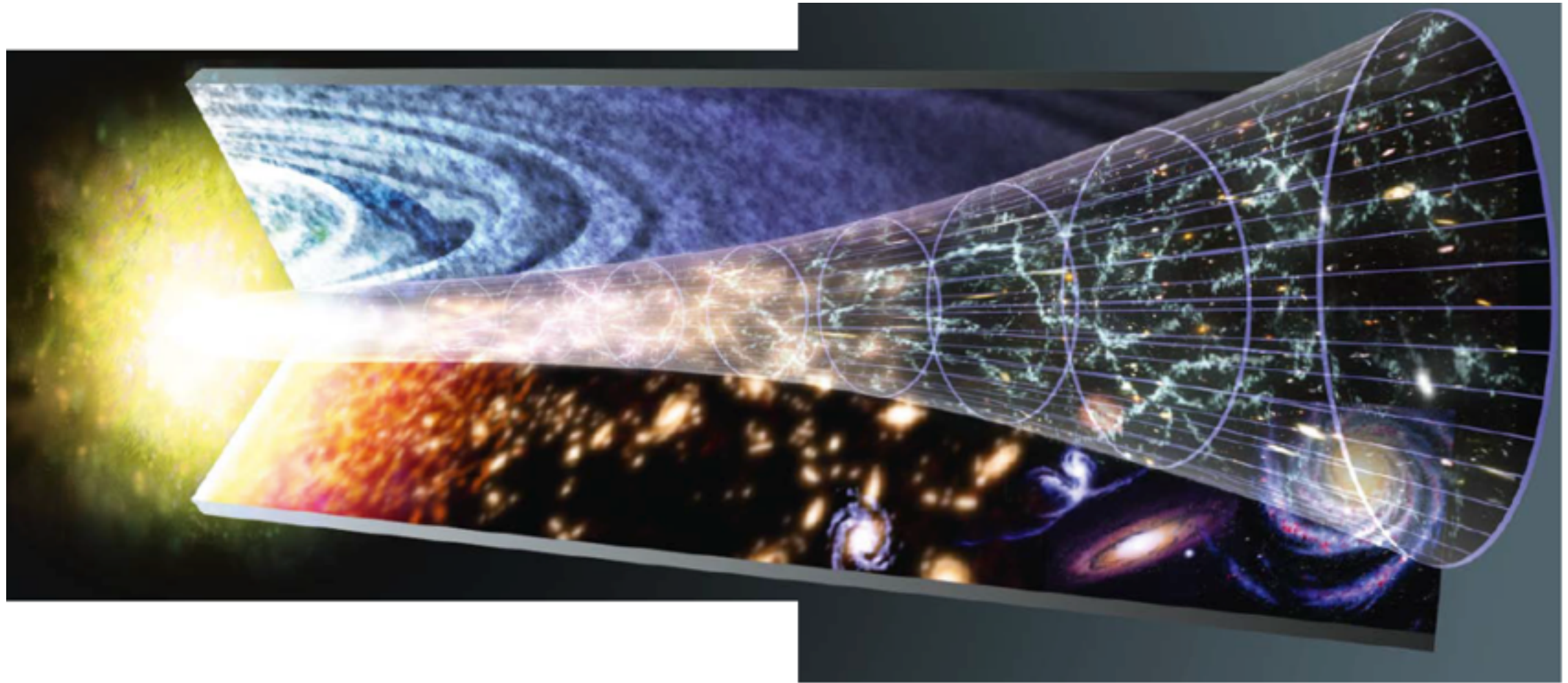




COSMIC **INFLATION** AND THE **REHEATING** OF THE UNIVERSE

Francisco Torrentí - IFT/UAM

Valencia Students Seminars - December 2014

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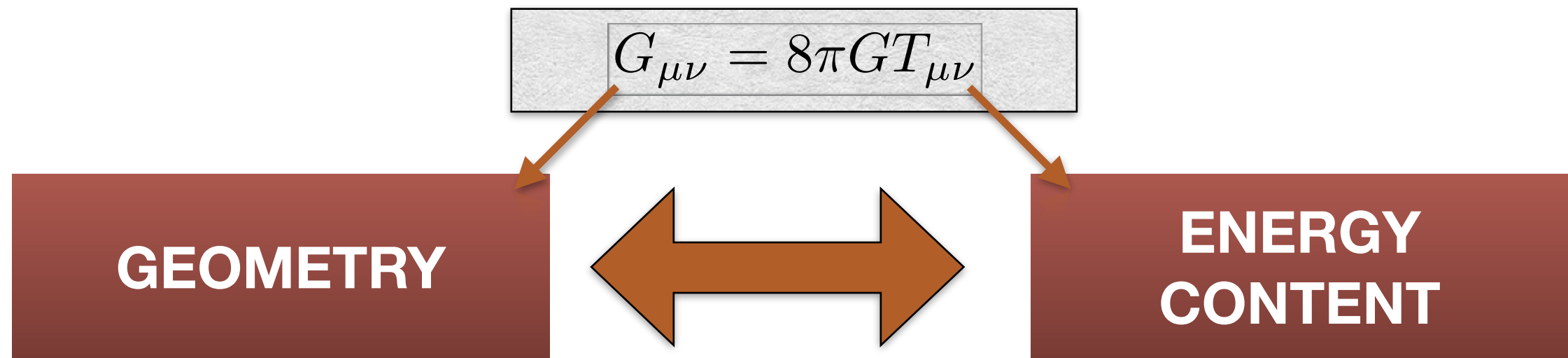
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1. THE FRIEDMANN EQUATIONS

1. The Friedmann equations



The Einstein field equations relate the geometry of a spacetime with its energy content.

Application to the **universe as a whole** $\longrightarrow$ Friedmann equations

Assumption (RHS):

Content of the universe is a **PERFECT FLUID**

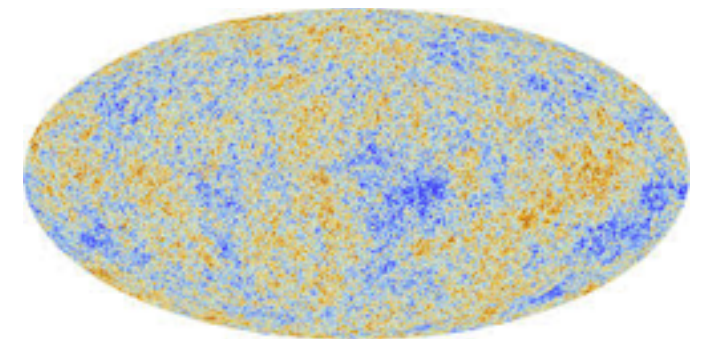
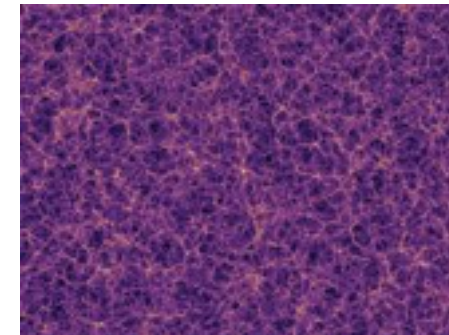
ENERGY CONTENT OF THE UNIVERSE

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & & & \\ 0 & g_{ij}p & & \\ 0 & & & \end{pmatrix}$$

1. The Friedmann equations

Assumption (LHS):

GEOMETRY OF THE UNIVERSE



Homogeneous

(Assumption:
no preferred points
in the universe)

Isotropic

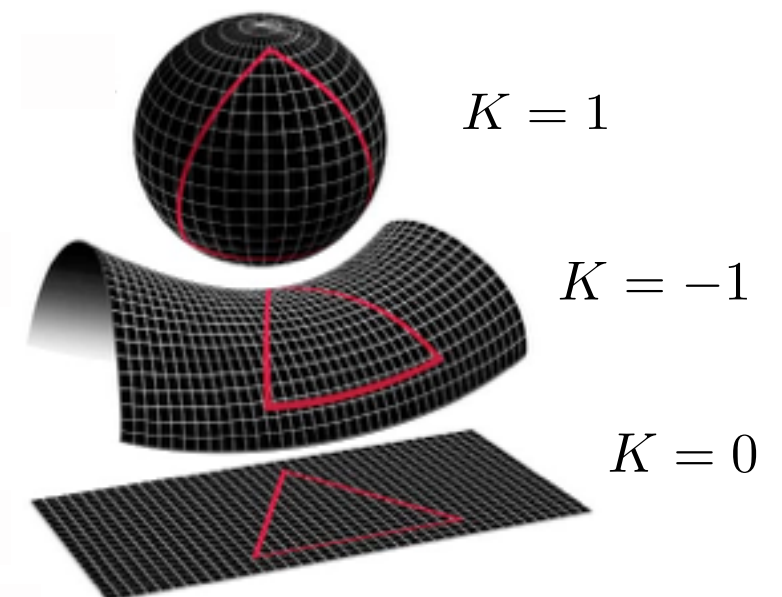
(at one point)
(confirmed by
experiments)

**Homogeneous and
isotropic AT ALL POINTS**

FLRW metric:

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right]$$

a(t): Scale factor



1. The Friedmann equations

The **FRIEDMANN** equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Content of the Universe modeled by: $\omega_i = p_i/\rho_i$ $\rho_i = a^{-3(1+\omega_i)}$

$$\omega_M = 0$$

Matter

$$\omega_R = 1/3$$

Radiation

$$\omega_\Lambda = -1$$

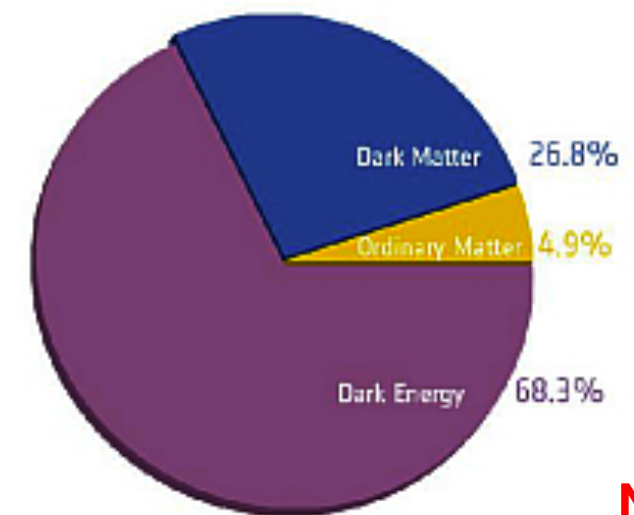
Cosm. Const.

$$\Omega_j = \frac{8\pi G}{3H^2}\rho_j \quad j = M, R, \Lambda$$

Curvature

$$\Omega_c = -\frac{k}{H^2 a^2}$$

$$\sum_i \Omega_i = \Omega_M + \Omega_\Lambda + \Omega_R + \Omega_c = 1$$

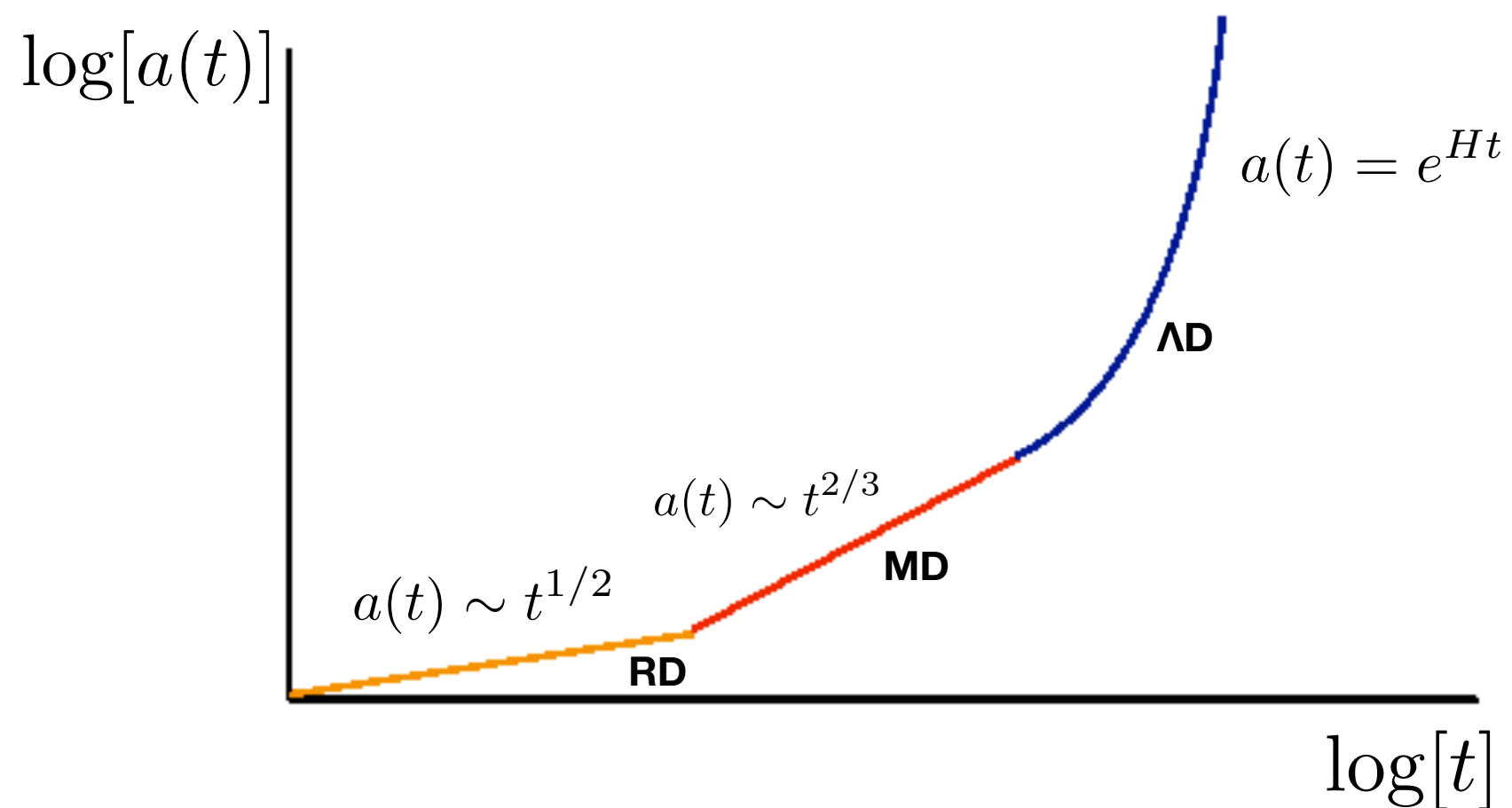


Now

1. The Friedmann equations

The Universe goes through different epochs. In each one, a specific kind of fluid dominated the expansion.

$$\rho_i = a^{-3(1+\omega_i)}$$



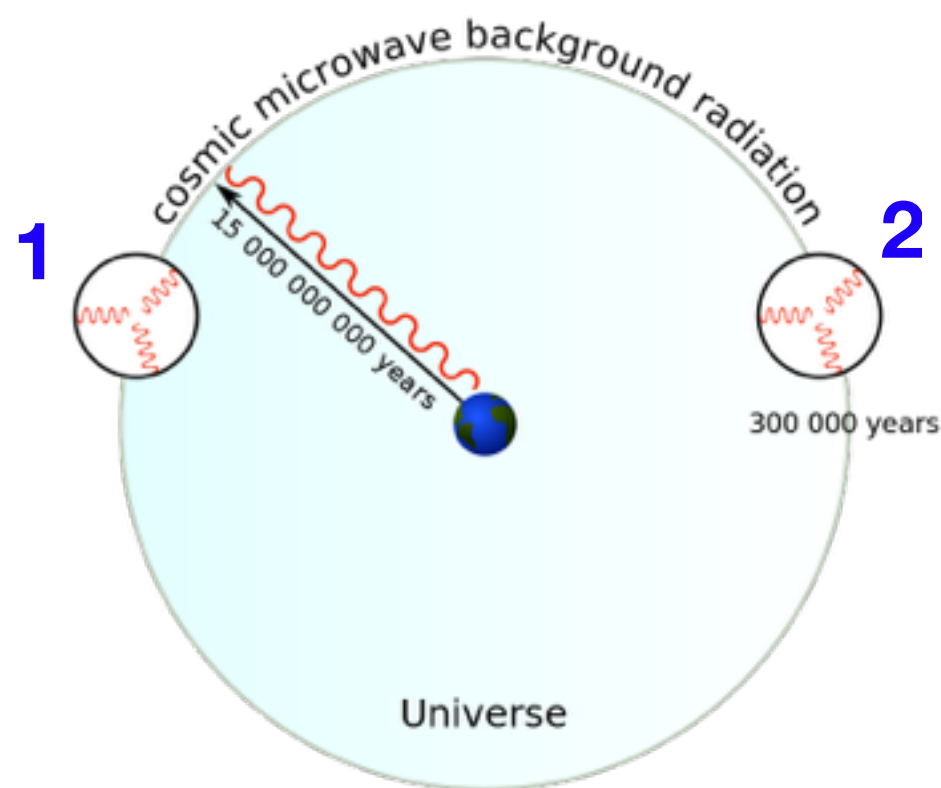
2. INFLATION

2.1. The problems of hBB theory

The three problems of hot Big Bang Theory: $a(t) \sim t^p$ $p = \frac{1}{2}, \frac{2}{3}$

1. The horizon problem

CMB is incredibly homogeneous. Not enough time for light to propagate and get thermal equilibrium.



$$T_{\gamma}(1) \approx T_{\gamma}(2)$$

$$d_{hor}(t) = \int_{t_i}^t \frac{dt'}{a(t')}$$

Comoving horizon distance:
comoving distance travelled by
light since t_i to t_f

RD/MD and $t_i=0$

$$d_{hor}^c(t_{dec}) \sim (a_{dec}H_{dec})^{-1}$$

At decoupling time

$$d_{hor}^c(t_0) \sim (H_0)^{-1}$$

Now

$$\frac{d_{hor}^c(t_{dec})}{d_{hor}^c(t_0)} \sim \frac{a_0 H_0}{a_{dec} H_{dec}} \ll 1$$



2.1. The problems of hBB

2. The flatness problem

Current observations give $\Omega_c \approx 0$. **Unstable point** in Friedmann equations!

$$\Omega_c \left(\frac{\rho_{0,m}}{a(t)} + \frac{\rho_{0,R}}{a^2(t)} \right) = \frac{3k}{8\pi G}$$

$$a(t) \uparrow \longrightarrow \Omega_c(t) \uparrow$$

We need incredible fine-tuning!

$$\Omega_c(t_{pl}) \approx 10^{-60}$$

3. The primordial monopole problem

Not observed cosmic relics predicted by GUT models



2.2. The inflationary idea

An early phase of **exponential expansion** can solve the three problems at once

$$a(t) = e^{Ht}$$

PHYSICAL REVIEW D

VOLUME 23, NUMBER 2

15 JANUARY 1981

Inflationary universe: A possible solution to the horizon and flatness problems

Alan H. Guth*

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 11 August 1980)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally

Alan Guth

**The idea
(but model failed)**

Andrei Linde

**First successful
inflationary model**

A NEW INFLATIONARY UNIVERSE SCENARIO: A POSSIBLE SOLUTION OF THE HORIZON, FLATNESS, HOMOGENEITY, ISOTROPY AND PRIMORDIAL MONOPOLE PROBLEMS

A.D. LINDE

Lebedev Physical Institute, Moscow 117924, USSR

Received 29 October 1981

2.2. The inflationary idea

$$a(t) = e^{Ht}$$

Number of e-folds: $\mathcal{N} \equiv \log \left(\frac{a}{a_i} \right) = Ht$

1. Horizon problem:

Due to the inflationary epoch, all points in the CMB were **causally connected in the past**.

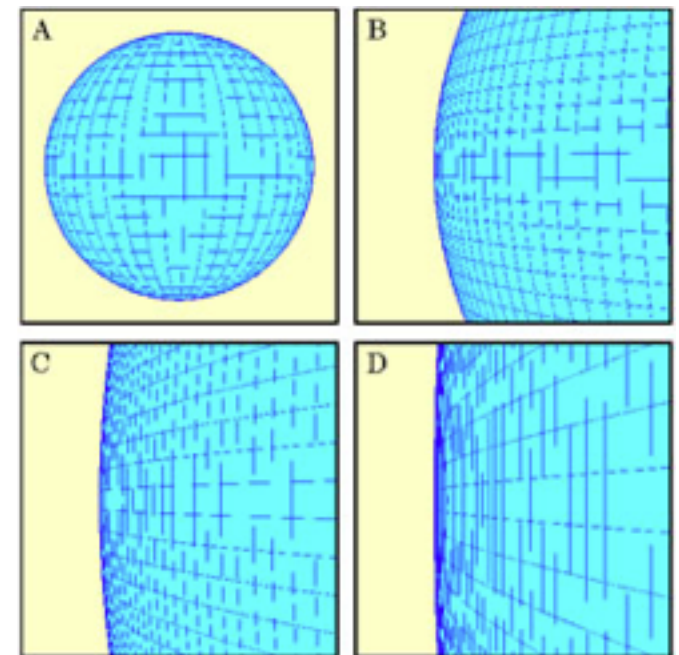
2. Flatness problem:

During inflation, $\Omega_k=0$ is an **attractor point**.

$$\Omega_c \rho_{0,\Lambda} a^2(t) = \frac{3k}{8\pi G} \quad a(t) \uparrow \quad \Omega_c(t) \downarrow$$

3. Primordial monopole problem:

Inflation **washes out any cosmic relics**.



THE THREE PROBLEMS ARE SOLVED WITH

$$\mathcal{N} \approx 60$$

2.3. Slow-roll inflation

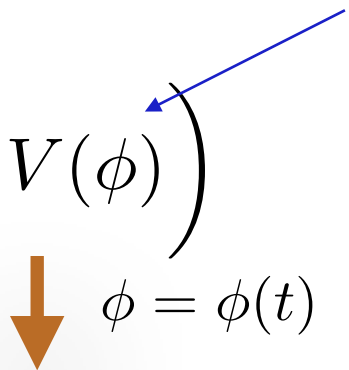
Definition of **INFLATION**:

$$\frac{d^2 a}{dt^2} = 0$$

HOW TO IMPLEMENT IT?

Action of the **inflaton**:

$$S = \int d^4x \sqrt{|g|} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$



Inflationary potential

$$\phi = \phi(t)$$

Field and Friedmann equations:

$$\ddot{\phi} + 3H(t)\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

Energetic content:

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

$$H(t) \equiv \frac{\dot{a}}{a}$$

E.o.m:

$$H^2 = \frac{1}{3m_p^2} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right)$$

2.3. Slow-roll inflation

- First requirement:

$$V(\phi) \gg \frac{1}{2}\dot{\phi}^2 \quad \longrightarrow \quad \boxed{\epsilon = 3 \frac{\frac{1}{2}\dot{\phi}^2}{V(\phi)} \ll 1}$$

Potential energy dominates over kinetic

First SLOW-ROLL parameter

Fr. eqns: $\frac{1}{a} \frac{d^2 a}{dt^2} \approx + \frac{V(\phi)}{3m_p^2}$ $H^2 \approx \frac{V(\phi)}{3m_p^2}$

INFLATION!

$a(t) \approx a_i e^{\int_t H(\phi) dt'}$

Energy: $\omega \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \approx -1 + \frac{2}{3}\epsilon$

(Quasi) de Sitter

2.3. Slow-roll inflation

- **Second requirement: We must ensure that $\epsilon \ll 1$ is sustained for at least 60 e-folds or more.**

The field must not accelerate

$$(\ddot{\phi} \uparrow \uparrow \rightarrow \dot{\phi} \uparrow \uparrow \rightarrow \epsilon \uparrow \uparrow)$$

$$\ddot{\phi} + 3H(t)\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

We need: $|\ddot{\phi}| \ll 3H\dot{\phi}, V'(\phi)$ $\longrightarrow$

$$\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

**Second
SLOW-ROLL
parameter**

SLOW-ROLL CONDITIONS IN TERMS OF POTENTIAL:

$$\epsilon_V \equiv \frac{m_p^2}{2} \left(\frac{V'}{V} \right)^2 \quad \eta_V \equiv m_p^2 \left(\frac{V''}{V} \right)$$

$(\epsilon \approx \epsilon_V) \qquad (\eta \approx \eta_V - \epsilon_V)$

$$\epsilon, \eta \ll 1$$

$$\epsilon_V, \eta_V \ll 1$$

2.3. Slow-roll inflation

Working example:

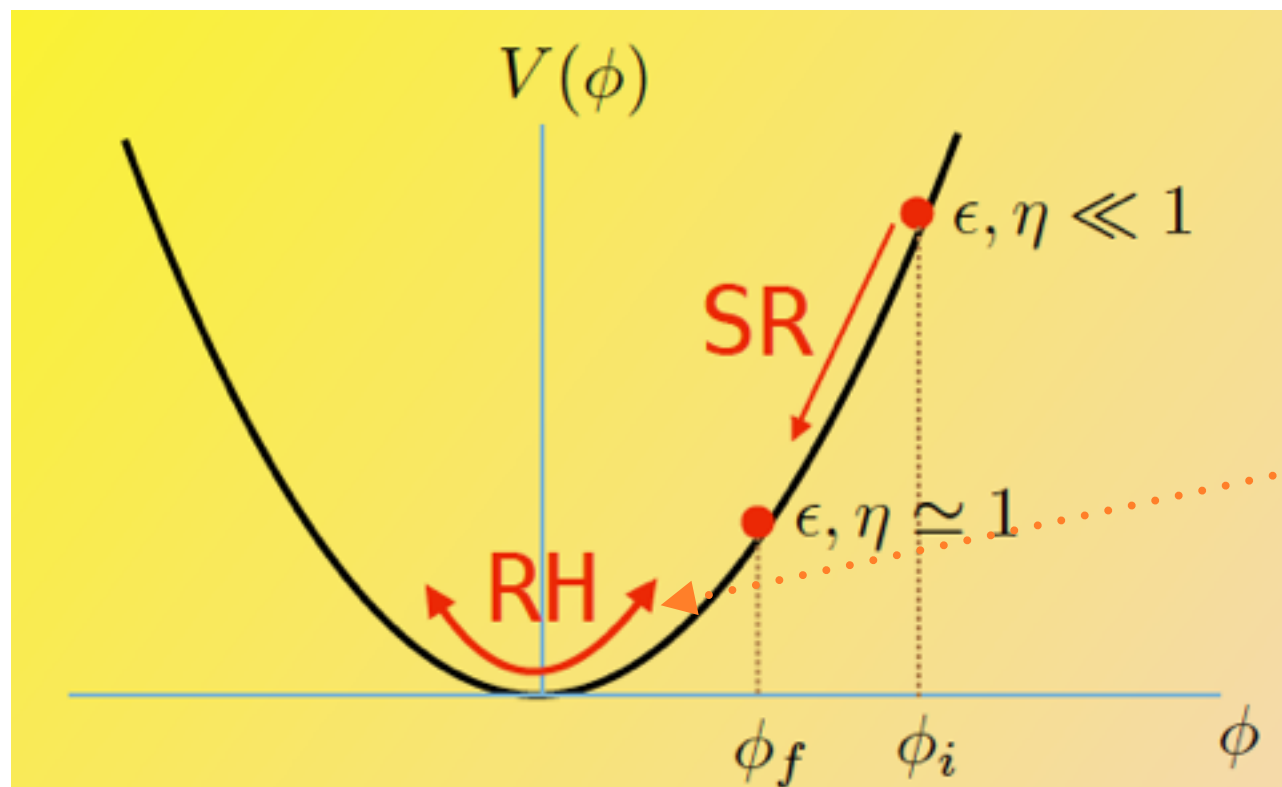
$$V(\phi) = \frac{1}{2}m^2\phi^2$$

$$\epsilon = \frac{2}{m_p^2\phi^2}$$

$$\eta = \frac{2}{m_p^2\phi^2} = \epsilon$$

END of inflation:

$$\phi_{end} = \frac{M_P}{2\sqrt{\pi}} \approx \frac{M_P}{3.5}$$



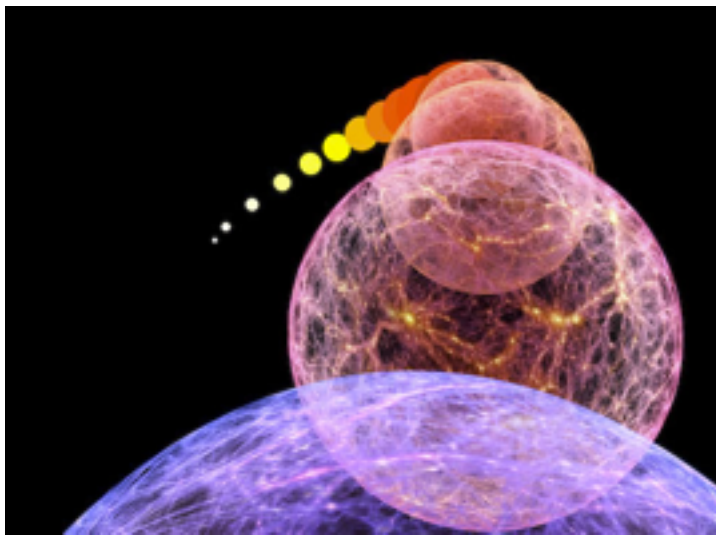
the field starts to oscillate around the minimum of its potential

3. REHEATING

(getting the “bang” from the Big Bang)

3.1. Introduction to reheating

What is the origin of all **matter and radiation** present in our universe today?



During inflation, the universe is empty and cold

$$S \approx 0 \quad M \approx 0 \quad T \approx 0$$

(Inflation dilutes any relic species left from a hypothetical earlier period of the universe)



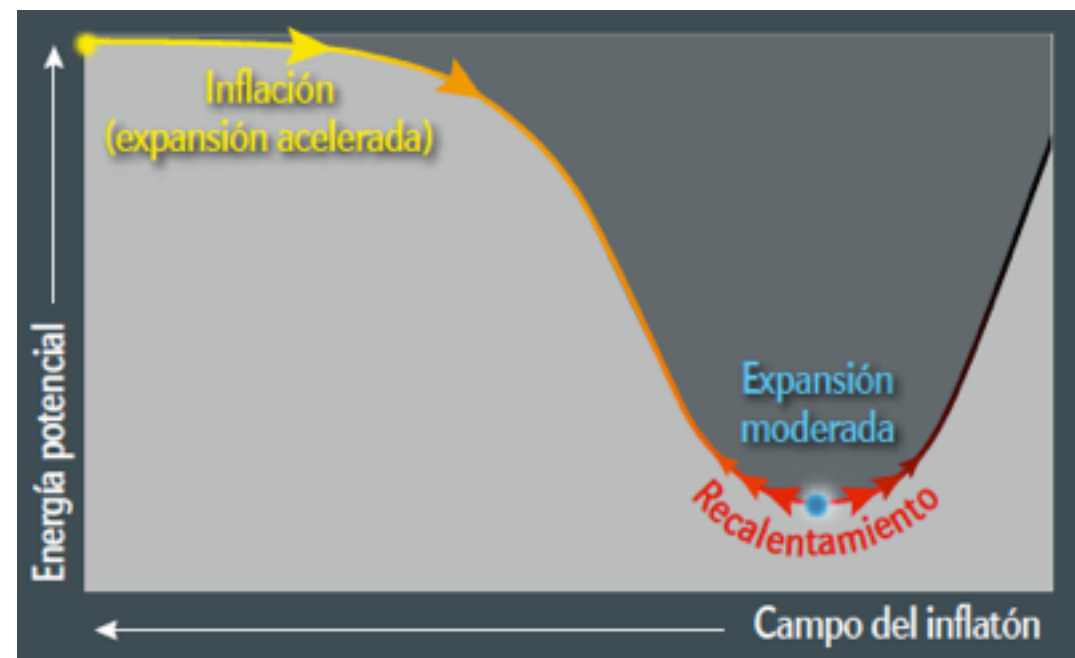
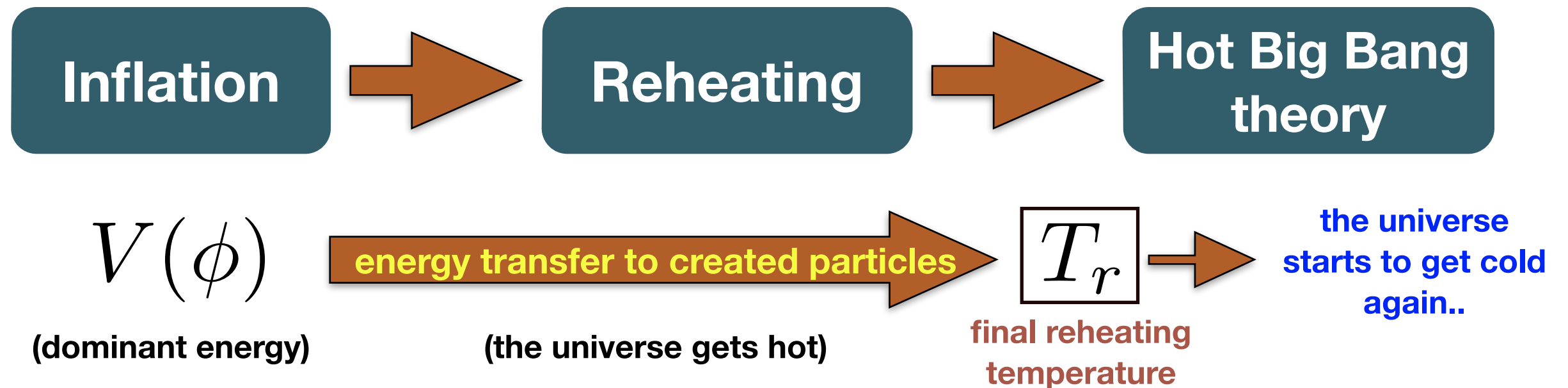
But now...

$$M \approx 10^{23} M_{\odot} \quad S \approx 10^{89}$$

(and $T \gg 0$ in the early universe)

We need to “reheat” the universe after inflation

3.1. Introduction to reheating



3.1. Introduction to reheating

Model for reheating:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \underbrace{V(\phi)}_{\text{Inflaton}} + \text{other fields} + \text{interaction inflaton-fields}$$

Inflaton

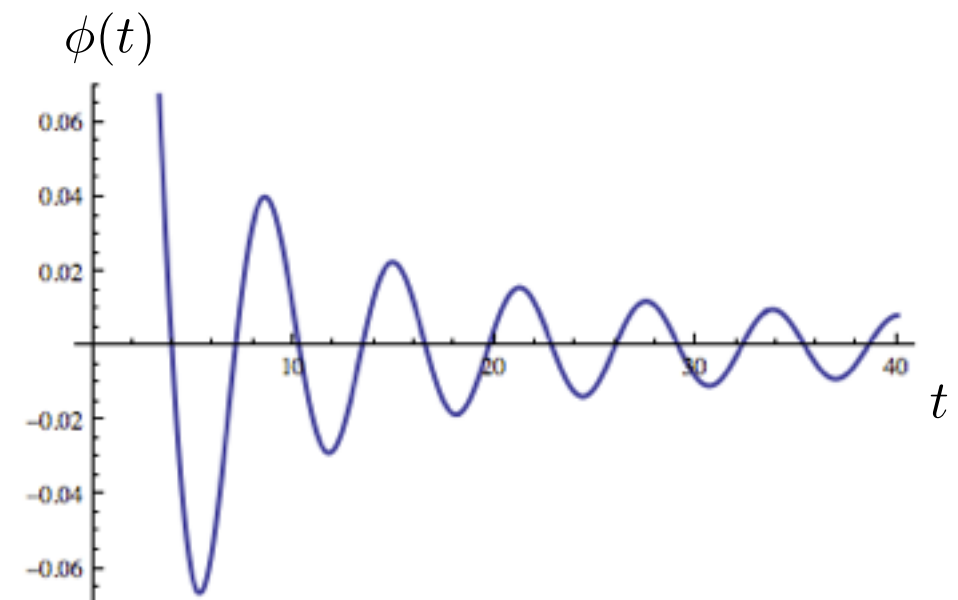
$$V(\phi) = \frac{1}{2}m^2\phi^2 \quad (\text{Parabolic potential})$$

Inflation equation (neglecting interaction):

$$\ddot{\phi} + 3H(t)\dot{\phi} + m^2\phi = 0 \quad (+ \text{ Friedmann eqn.})$$

$$\begin{aligned} \phi(t) &\sim \Phi(t) \sin mt & \Phi(t) &= \frac{\Phi_0}{t} \\ a(t) &\sim t^{2/3} \end{aligned}$$

Matter-dominated

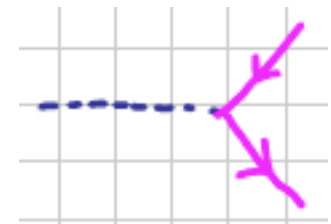


OSCILLATIONS AROUND THE MINIMUM OF THE POTENTIAL

3.2. Perturbative reheating

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)}_{\text{Inflaton}} + \underbrace{\frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m_\chi^2\chi^2}_{\text{Scalar}} + \underbrace{\bar{\psi}(i\gamma^\mu\partial_\mu - m_\psi)\psi}_{\text{Fermion}} + \underbrace{-g\phi\chi^2 - h\phi\bar{\psi}\psi}_{\text{Interaction terms}}$$

Inflaton couples (weakly) to other particles, and then it decays:



$$\left[\begin{aligned} \Gamma(\phi \rightarrow \chi\chi) &= \frac{g^2}{8\pi m} \\ \Gamma(\phi \rightarrow \bar{\psi}\psi) &= \frac{h^2 m}{8\pi} \end{aligned} \right]$$

$$\Gamma_\phi = \Gamma(\phi \rightarrow \chi_i\chi_i) + \Gamma(\phi \rightarrow \bar{\psi}\psi) = \frac{h_{eff}^2 m}{8\pi} \quad h_{eff}^2 = \sum_i \left(h_i^2 + \frac{g_i^2}{m} \right)$$

new friction term

Inflaton effective e.o.m:

$$\ddot{\phi} + 3H(t)\dot{\phi} + \Gamma_\phi\dot{\phi} + m^2\phi = 0$$

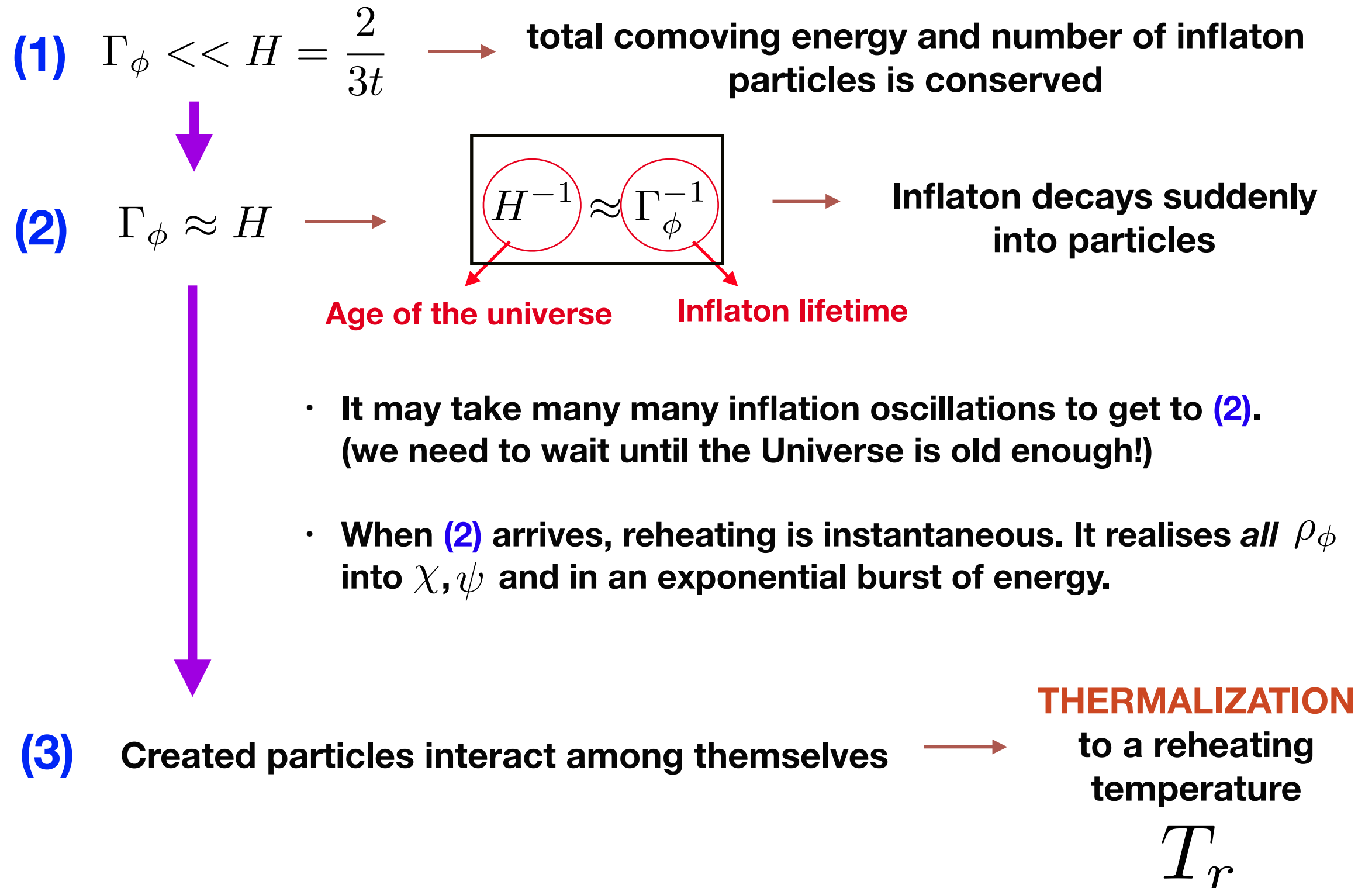
Solution

$$\begin{aligned} \phi(t) &\sim \Phi(t) \sin mt \\ \Phi(t) &= \Phi_0 e^{-\frac{1}{2}(3H+\Gamma)t} \end{aligned}$$

$$\begin{aligned} \frac{d}{dt}(\rho_\phi a^3) &= -\Gamma_\phi \rho_\phi a^3 \\ \frac{d}{dt}(n_\phi a^3) &= -\Gamma_\phi n_\phi a^3 \end{aligned}$$

comoving inflaton energy density and particle number decays into particles

3.2. Perturbative reheating



3.2. Perturbative reheating

Estimation of the reheating temperature:

$$\boxed{t_r \sim \frac{2}{3} \Gamma_\phi^{-1}} \xrightarrow{\hspace{2cm}} \boxed{T_{rh} \approx 0.1 \sqrt{\Gamma_\phi M_p}}$$

Reheating time

For our chaotic model:

$$\Gamma_\phi = \frac{h_{eff}^2 m}{8\pi} \quad \begin{matrix} m \approx 10^{13} GeV \\ h_{eff} \leq 10^{-3} \end{matrix} \xrightarrow{\hspace{2cm}} \boxed{T_{rh} \leq 10^{11} GeV}$$

Reheating temperature

- Problems:**
- 1) Low temperature.
 - 2) In some models, $H > \Gamma_\phi$ always.

BUT: Inflaton is not composed of individual inflaton quanta, it is a coherently oscillating field with large amplitude.

We need new formalism!

3.3. Preheating

New formalism: **Particle production in the presence of strong background fields.**

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)}_{\text{Inflaton}} + \underbrace{\frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}m_\chi^2 \chi^2}_{\text{Scalar field}} - \underbrace{\frac{1}{2}g^2 \phi^2 \chi^2}_{\text{Interaction term}}$$

**Minkowski
spacetime
(for the moment)**

$\downarrow \phi(t) = \Phi \sin mt$

$$\ddot{\phi} - \nabla^2 \phi + m_\chi^2(t) = 0$$

$$m_\chi^2(t) = m_\chi^2 + g^2 \Phi^2 \sin^2 mt$$

**A free field with
time-dependent mass!**

$\downarrow$ *In momentum space* f_k : (field mode)

$$\frac{d^2 f_k}{dt^2} + \omega_k^2(t) f_k = 0$$

Field-mode equation

$$\omega_k^2(t) = k^2 + m_\chi^2(t)$$

**Time-dependent
frequency**

**Kofman, Linde &
Starobinsky, "Towards
the theory of reheating
after inflation" (1997)**

3.3. Preheating

It can be written as a **Mathieu equation**:

$$\frac{d^2 f_k}{dt^2} + (A_k - 2q \cos(2mt)) f_k = 0$$

$$q = \frac{g^2 \Phi^2}{4m^2} \quad A_k = \frac{k^2 + m_\chi^2}{m^2} + 2q$$

For certain values of (A_k, q) , the solution is **unstable** (exponential growth!):

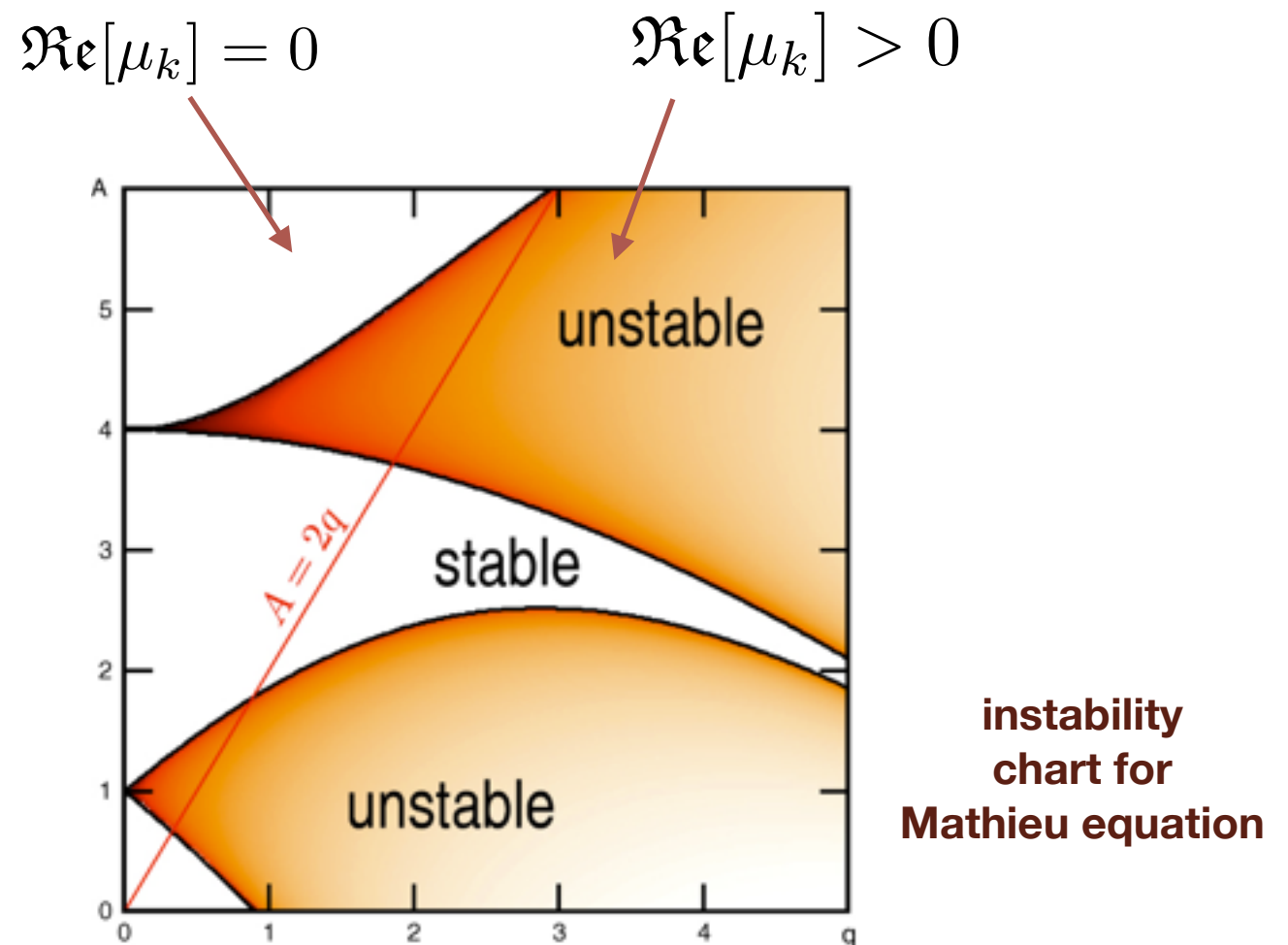
$$f_k \sim e^{\mu_k mt}$$

$$n_k(t) = \frac{1}{2\omega_k} |\dot{f}_k|^2 + \frac{\omega_k}{2} |f_k|^2 - \frac{1}{2}$$

Particle number definition

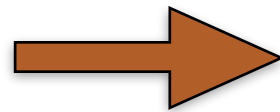
$$n_k(t) \sim e^{2\mu_k mt}$$

**Exponential growth of particle number!!
(Just after inflation ends)**



3.3. Preheating

$$n_k(t) \sim e^{2\mu_k m t}$$

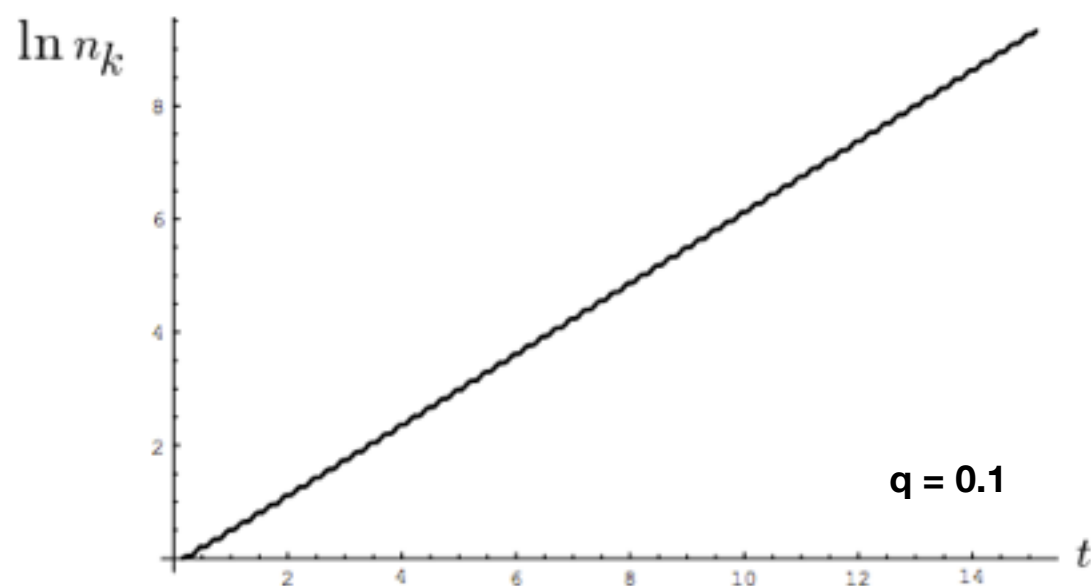


$$\ln n_k(t) = 2\mu_k m t$$

This equations has two regimes:

(for given mode k)

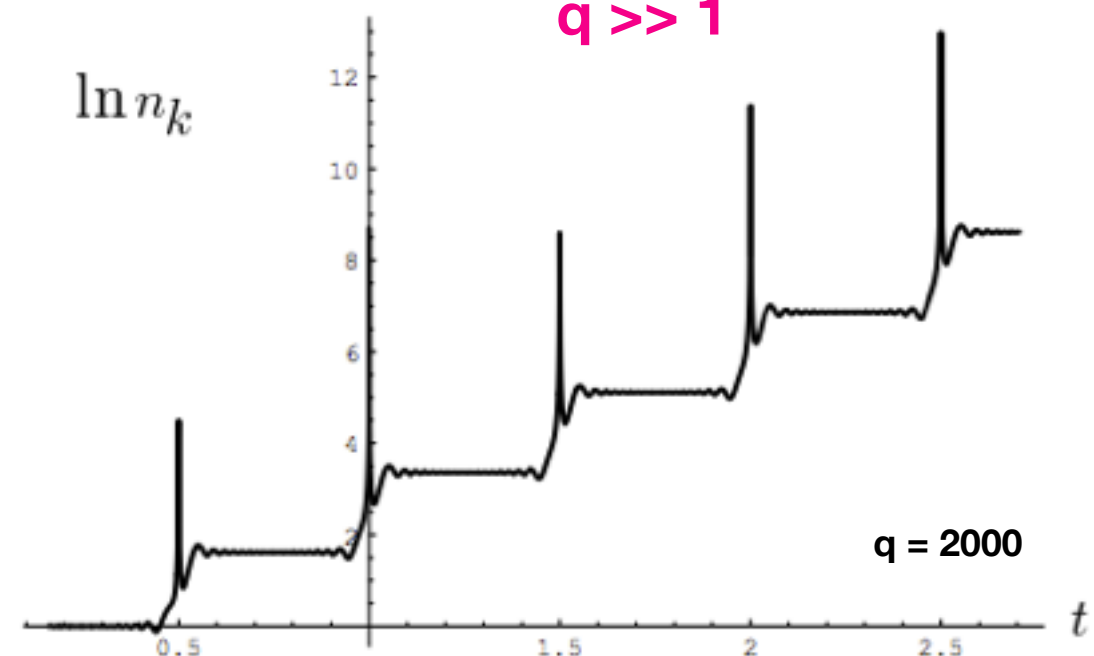
$q < 1$



Narrow resonance

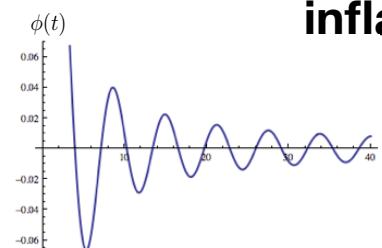
Continuous growth of particle number

$q \gg 1$



Broad resonance

Explosive particle creation when inflaton crosses the minimum of its potential.



3.3. Preheating

$$n_k^{j+1} = f(n_k^j)$$



PROPERTIES:

- 1. Step-like.** Particles are produced only when inflation crosses zero.
(when *effective frequency* changes rapidly)

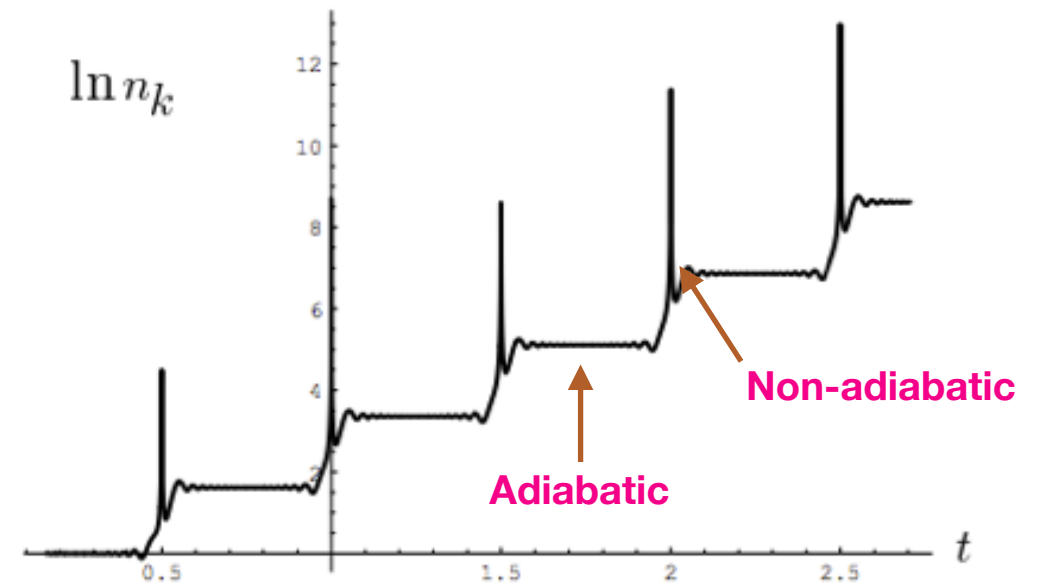
$$\left| \frac{\dot{\omega}_k}{\omega_k^2} \right| \gg 1 \quad \text{Non-adiabatic regime}$$

- 2. Non-perturbative.** $n_k^1 \propto e^{-\frac{k^2}{g}}$

- 3. Infrared effect.** $n_k^{j+1} \simeq n_k^j \simeq 0$
($k \rightarrow \infty$)

Kofman, Linde & Starobinsky:
analytical model

$$q \gg 1$$



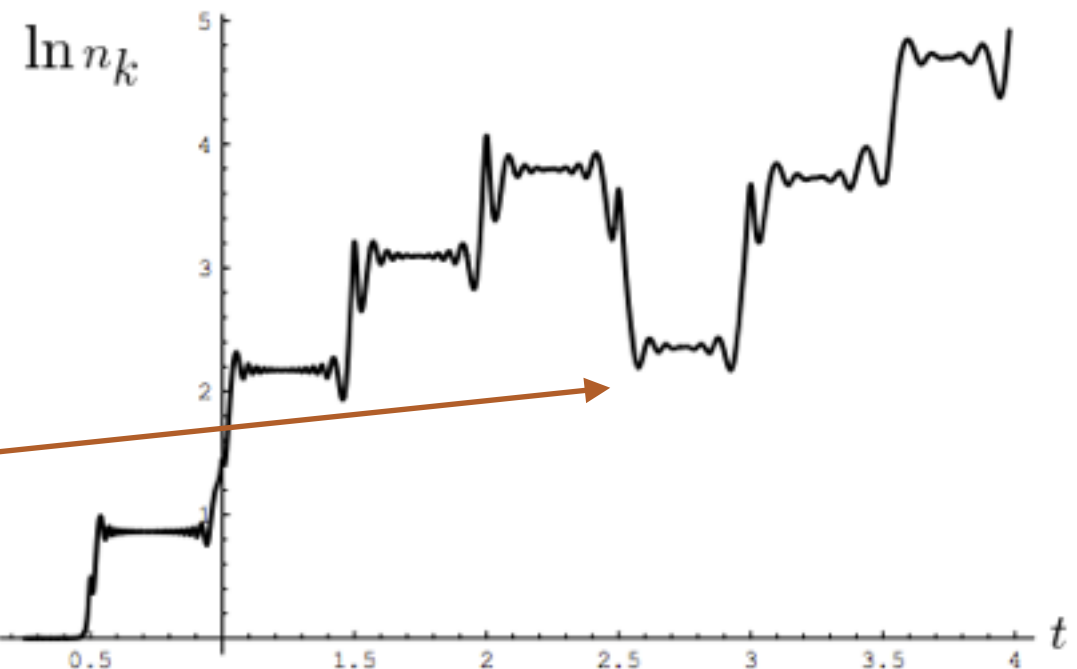
- 4.** We can create particles with mass greater than the inflaton mass
(forbidden in perturbative reheating).
- 5. Band structure.** Only some momenta are excited (also valid for other potentials).

3.3. Preheating

Including expansion...

The natural momentum redshifts: $\frac{k}{a(t)}$

6. **Stochastic preheating.** The steps go down ~25% times)

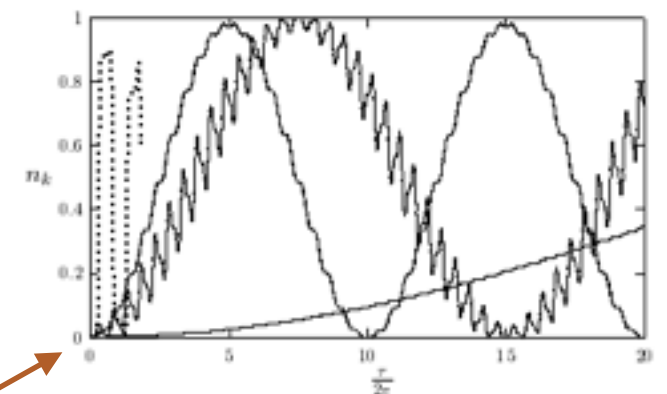


7. **Redshifts.** For a given momentum, the parameter q redshifts, entering eventually to the narrow resonance regime ($q < 1$)

$$q = \frac{g^2 \Phi^2(t)}{4m^2} \propto \frac{1}{t^2}$$

8. **Behaviour**

- **Bosons:** $n_k(t) \sim e^{2\mu_k m t}$
- **Fermions:** $n_k(t) < 1$ (due to exclusion principle)



Greene, Kofman (2000)

3.3. Preheating

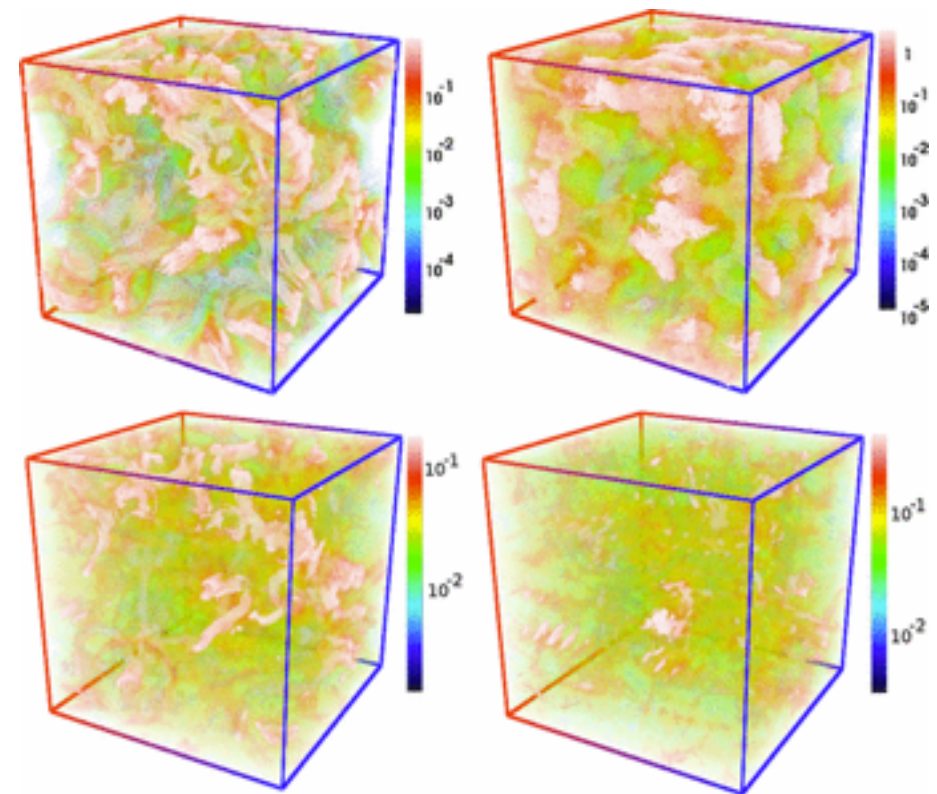
Analytical approach to reheating has [limitations](#). It does not take into account:

- **Backscattering of the created particles** to the inflaton field and expansion rate of the universe.
- **Spatial distributions** of fields (**structure formation**).

To go beyond, we need **LATTICE TECHNIQUES**.

- Solve the differential field equations numerically in a **finite box**.
- Quantum expectation values are spatial averages.

(Example: **LATTICEEASY** program)



4. REHEATING PHENOMENOLOGY

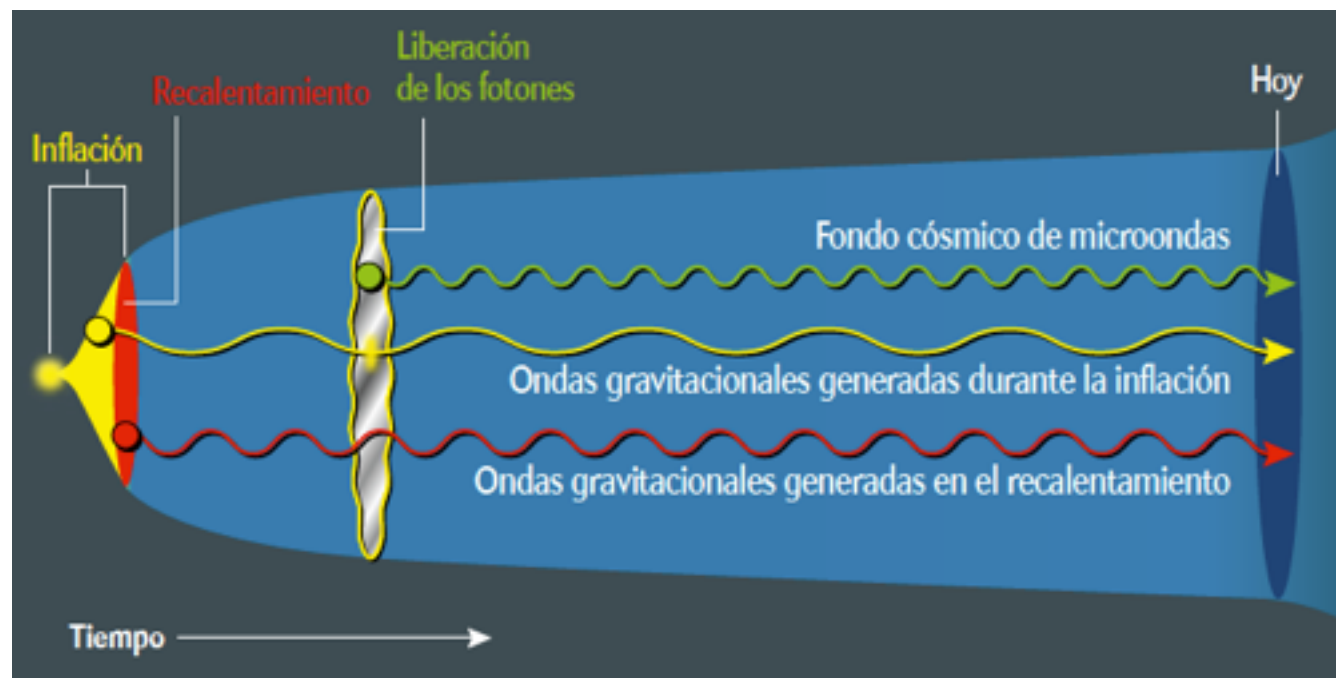
4.1. Primordial gravitational waves

Preheating in the early universe is a **very violent phenomenon**

(huge masses colliding at nearly the speed of light)



A source of primordial GRAVITATIONAL WAVES



Can these GW be detected by any experiment in the future?

**Inflationary
GW:
Scale-invariant**

**(p)Reheating
GW:
Non
scale-invariant**



High-energy (p)reheating

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4.2. Reheating the universe from the SM

In order to do reheating phenomenology, we need to assume:

1. An inflationary model.
2. A set of couplings.

ASSUMPTIONS:

1. The Standard Model is approximately correct up to inflationary energies
2. EW vacuum is stable up to inflationary energies
3. The Higgs does not couple to the inflation (which is a beyond-the-SM particle)

4.2. Reheating the universe from the SM

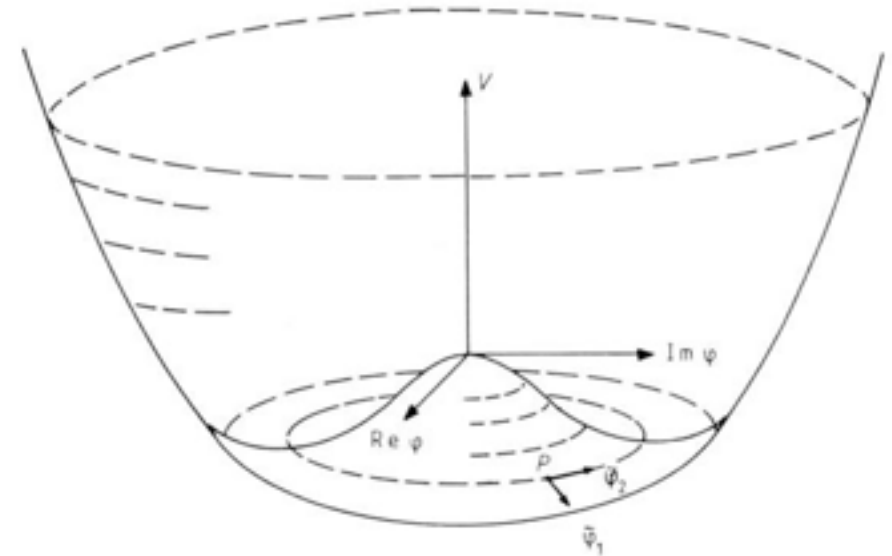
The Higgs starts to oscillate around the minimum of its potential a short time after inflation.



It can reheat the universe

$$V(h) \sim \frac{\lambda(h)}{4} h^4$$

running coupling constant



Well-known
SM couplings to the
other particles

Fermions
(quarks, leptons)

Already studied

Enqvist, Figueroa,
Meriniemi (2012)

Gauge bosons
(W, Z, γ)



**(we're working
on it!)**

Summary

- **Inflation provides a natural solution to the horizon and flatness problems of classical hot Big Bang theory (and also explains large-scale structure of the Universe).**
- **Reheating is a key process of the primordial universe. It requires non-ordinary techniques of QFT outside the equilibrium.**
- **GW emitted during reheating constitute a possible window to the primordial universe.**
- **Reheating takes place even in the Standard Model.**

Thank you for your attention!