



IFIC, University of Valencia/CSIC, October 28 2014

40 years on a rollercoaster: status and prospects of the Grand unification paradigm

Michal Malinský

IPNP, Charles University in Prague

Outlook

SM B & L violation

BSM perturbative L violation

- Dirac neutrinos and charge de-quantization in SM

BSM perturbative B violation

- proton decay and grand unification

40 years of Grand unification

NLO proton lifetime estimates

- the uniqueness of the minimal non-SUSY SO(10) GUT

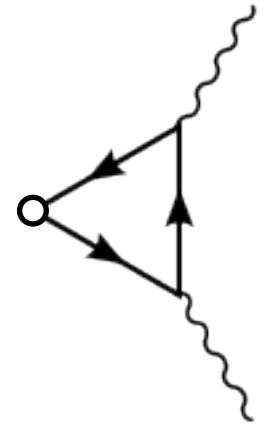
Non-perturbative B & L violation in SM

B & L violation in the Standard model

Expected at some point...

Chiral anomalies:

$$\mathcal{A} \propto \frac{1}{32\pi^2} \text{Tr} (\{T_a, T_b\} T) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$$

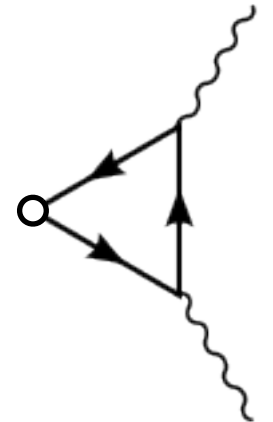


B & L violation in the Standard model

Expected at some point...

Chiral anomalies:

$$\mathcal{A} \propto \frac{1}{32\pi^2} \text{Tr}(\{T_a, T_b\}T) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$$



$$\text{Tr}(\{Y, Y\}L) = \text{Tr}(\{Y, Y\}B) = -\frac{1}{2} \quad \text{Tr}(\{T_L^3, T_L^3\}L) = \text{Tr}(\{T_L^3, T_L^3\}B) = \frac{1}{2}$$

$$\partial^\mu J_\mu^L = \partial^\mu J_\mu^B \neq 0$$

$$\partial^\mu J_\mu^{B-L} = 0$$

$$\partial^\mu J_\mu^{B+L} \neq 0$$

B & L violation in the Standard model

B+L non-conservation: $B + L = i \int d^3x J_{B+L}^0(x)$

$$\Delta(B + L) = N_f (\Delta N_{\text{CS}} - \Delta n_{\text{CS}})$$

Chern-Simons numbers:

$$N_{\text{CS}} = -\frac{g_2^2}{16\pi^2} \int d^3x \, 2\epsilon^{ijk} \text{Tr} \left[\partial_i A_j A_k + i\frac{2}{3}g_2 A_i A_j A_k \right]$$

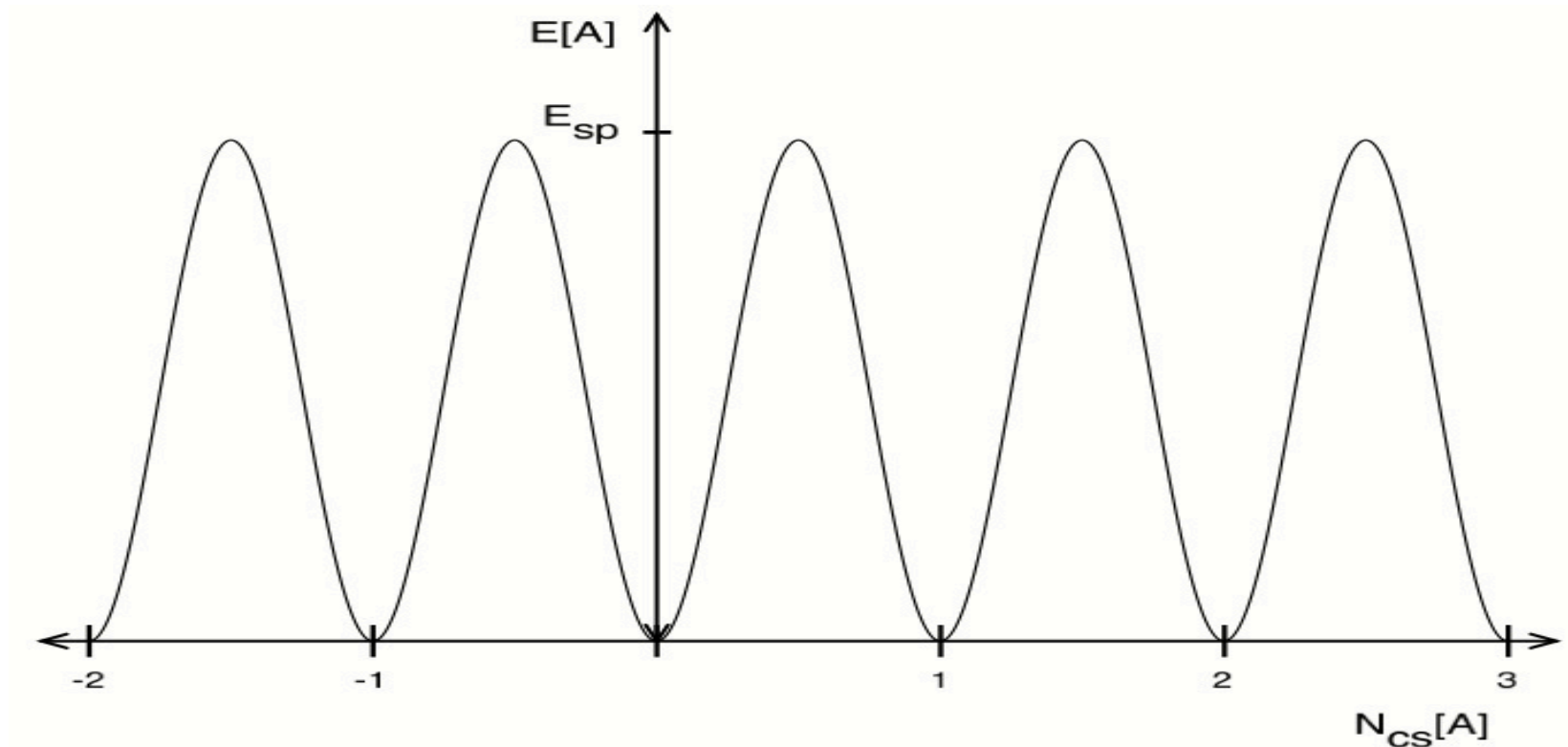
$$n_{\text{CS}} = -\frac{g_1^2}{16\pi^2} \int d^3x \, \epsilon^{ijk} \partial_i B_j B_k,$$

B & L violation in the Standard model

Vacuum structure of non-abelian gauge theories

$$F^{\mu\nu} = 0 \text{ (pure gauge configurations)} \quad A^\mu = U \partial^\mu (U^\dagger) \quad U \in SU(2)$$

$$\pi_3[SU(2)] = \mathbb{Z}$$



Winding number of SU(2) transformations (pure-gauge)

$$n = \frac{1}{24\pi^2} \int d^3x \text{Tr} \left[(\partial_i U) U^{-1} (\partial_j U) U^{-1} (\partial_k U) U^{-1} \right] \epsilon^{ijk}$$

B & L violation in the Standard model

Tunneling between minima with different n 's: **instantons**

$$\Delta(B + L) \propto N_f \times \Delta n$$

“Instanton effects” in the SM

$$9q + 3l \leftrightarrow \emptyset \quad {}^3He \rightarrow e^+ \mu^+ \bar{\nu}_\tau$$

Rates heavily suppressed...

$$\mathcal{A} \sim e^{-2\pi/\alpha_w} \sim 10^{-80}$$

B & L violation in the Standard model

Tunneling between minima with different n 's: **instantons**

$$\Delta(B + L) \propto N_f \times \Delta n$$

“Instanton effects” in the SM

$$9q + 3l \leftrightarrow \emptyset \quad {}^3He \rightarrow e^+ \mu^+ \bar{\nu}_\tau$$

Rates heavily suppressed...

$$\mathcal{A} \sim e^{-2\pi/\alpha_w} \sim 10^{-80}$$

Thermal instantons = sphalerons

Fast thermal transitions

Perturbative L violation

in the SM with massive neutrinos

Standard model with massive neutrinos

charge quantization

and

neutrino masses

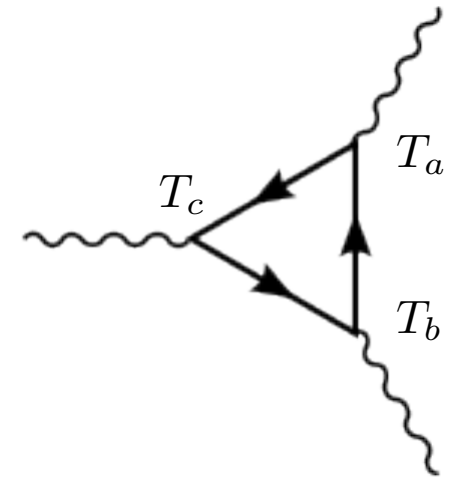
“imply”

perturbative L violation

(Hyper)charge quantization in the Standard model

$SU(3) \times SU(2) \times U(1)$ gauge anomalies

$$\mathcal{A}_c \propto \frac{1}{32\pi^2} \text{Tr}(\{T_a, T_b\}T_c) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$$



(Hyper)charge quantization in the Standard model

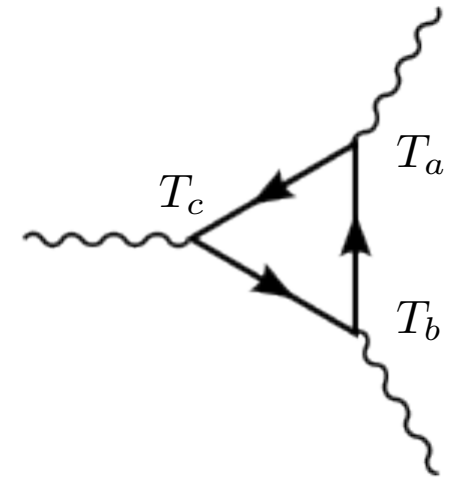
SU(3) x SU(2) x U(1) gauge anomalies $\mathcal{A}_c \propto \frac{1}{32\pi^2} \text{Tr}(\{T_a, T_b\}T_c) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$

SU(2)² U(1):

$$6Y_Q + 2Y_L = 0$$

U(1)³:

$$12Y_Q^3 + 4Y_L^3 - 6Y_U^3 - 6Y_D^3 - 2Y_E^3 = 0$$



(Hyper)charge quantization in the Standard model

SU(3) x SU(2) x U(1) gauge anomalies $\mathcal{A}_c \propto \frac{1}{32\pi^2} \text{Tr}(\{T_a, T_b\}T_c) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$

SU(2)² U(1):

$$6Y_Q + 2Y_L = 0$$

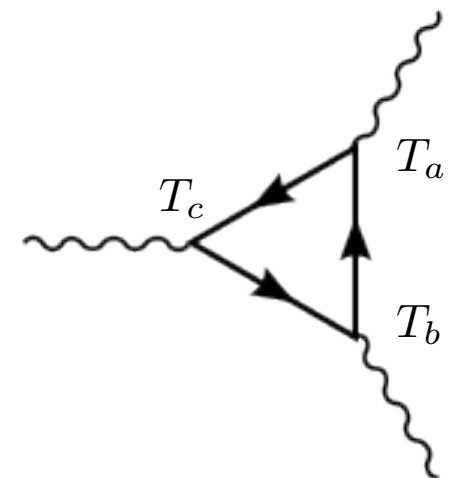
U(1)³: $12Y_Q^3 + 4Y_L^3 - 6Y_U^3 - 6Y_D^3 - 2Y_E^3 = 0$

Yukawas: $Y_{Dij} \overline{Q}_{Li} \langle H \rangle D_{Rj} + Y_{Uij} \overline{Q}_{Li} \langle \tilde{H} \rangle U_{Rj} + Y_{Eij} \overline{L}_{Li} \langle H \rangle E_{Rj}$

$$-Y_Q + Y_D + Y_H = 0$$

$$-Y_L + Y_E + Y_H = 0$$

$$-Y_Q + Y_U - Y_H = 0$$



(Hyper)charge quantization in the Standard model

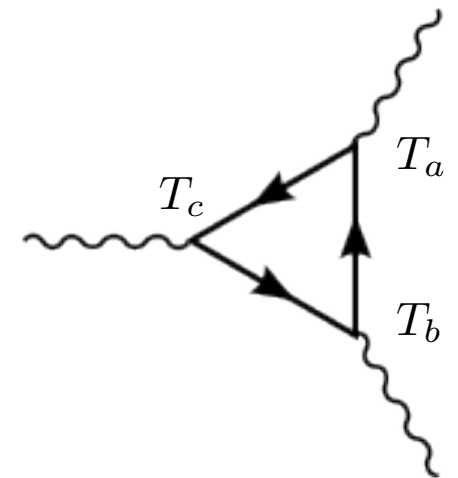
SU(3) x SU(2) x U(1) gauge anomalies $\mathcal{A}_c \propto \frac{1}{32\pi^2} \text{Tr}(\{T_a, T_b\}T_c) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$

SU(2)² U(1):

$$6Y_Q + 2Y_L = 0$$

U(1)³:

$$12Y_Q^3 + 4Y_L^3 - 6Y_U^3 - 6Y_D^3 - 2Y_E^3 = 0$$



Yukawas: $Y_{Dij} \overline{Q}_{Li} \langle H \rangle D_{Rj} + Y_{Uij} \overline{Q}_{Li} \langle \tilde{H} \rangle U_{Rj} + Y_{Eij} \overline{L}_{Li} \langle H \rangle E_{Rj}$

$$-Y_Q + Y_D + Y_H = 0$$

$$-Y_L + Y_E + Y_H = 0$$

$$-Y_Q + Y_U - Y_H = 0$$

Solution:

$$Y_Q = +\frac{1}{6}, \quad Y_U = +\frac{2}{3}, \quad Y_D = -\frac{1}{3},$$

$$Y_L = -\frac{1}{2}, \quad Y_E = -1$$

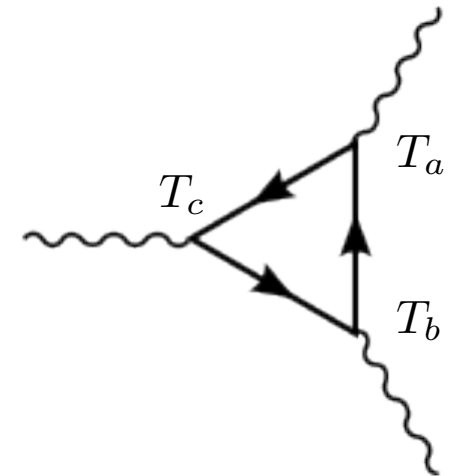
(Hyper)charge quantization in the Standard model

SU(3) x SU(2) x U(1) gauge anomalies $\mathcal{A}_c \propto \frac{1}{32\pi^2} \text{Tr}(\{T_a, T_b\}T_c) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$

Assume Dirac neutrinos: $N_R = (1, 1, Y_N)$

SU(2)² U(1): $6Y_Q + 2Y_L = 0$

U(1)³: $12Y_Q^3 + 4Y_L^3 - 6Y_U^3 - 6Y_D^3 - 2Y_E^3 = 0$



Yukawas: $Y_{Dij} \overline{Q}_{Li} \langle H \rangle D_{Rj} + Y_{Uij} \overline{Q}_{Li} \langle \tilde{H} \rangle U_{Rj} + Y_{Eij} \overline{L}_{Li} \langle H \rangle E_{Rj}$

$$-Y_Q + Y_D + Y_H = 0$$

$$-Y_L + Y_E + Y_H = 0$$

$$-Y_Q + Y_U - Y_H = 0$$

Solution: $Y_Q = +\frac{1}{6}, Y_U = +\frac{2}{3}, Y_D = -\frac{1}{3}, Y_L = -\frac{1}{2}, Y_E = -1$

(Hyper)charge quantization in the Standard model

SU(3) x SU(2) x U(1) gauge anomalies $\mathcal{A}_c \propto \frac{1}{32\pi^2} \text{Tr}(\{T_a, T_b\}T_c) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$

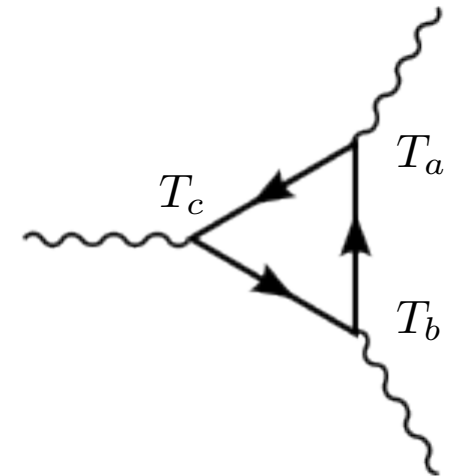
Assume Dirac neutrinos: $N_R = (1, 1, Y_N)$

SU(2)² U(1):

$$6Y_Q + 2Y_L = 0$$

U(1)³:

$$12Y_Q^3 + 4Y_L^3 - 6Y_U^3 - 6Y_D^3 - 2Y_E^3 - 2Y_N^3 = 0$$



Yukawas: $Y_{Dij} \overline{Q}_{Li} \langle H \rangle D_{Rj} + Y_{Uij} \overline{Q}_{Li} \langle \tilde{H} \rangle U_{Rj} + Y_{Eij} \overline{L}_{Li} \langle H \rangle E_{Rj}$

$$-Y_Q + Y_D + Y_H = 0$$

$$-Y_L + Y_E + Y_H = 0$$

$$-Y_Q + Y_U - Y_H = 0$$

Solution:

$$\boxed{\begin{aligned} Y_Q &= +\frac{1}{6} & , & Y_U = +\frac{2}{3} & , & Y_D = -\frac{1}{3} & , \\ Y_L &= -\frac{1}{2} & , & Y_E = -1 \end{aligned}}$$

(Hyper)charge quantization in the Standard model

SU(3) x SU(2) x U(1) gauge anomalies $\mathcal{A}_c \propto \frac{1}{32\pi^2} \text{Tr}(\{T_a, T_b\}T_c) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$

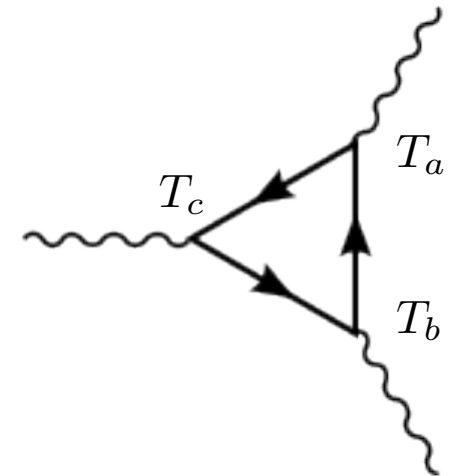
Assume Dirac neutrinos: $N_R = (1, 1, Y_N)$

SU(2)² U(1):

$$6Y_Q + 2Y_L = 0$$

U(1)³:

$$12Y_Q^3 + 4Y_L^3 - 6Y_U^3 - 6Y_D^3 - 2Y_E^3 - 2Y_N^3 = 0$$



Yukawas: $Y_{Dij} \overline{Q}_{Li} \langle H \rangle D_{Rj} + Y_{Uij} \overline{Q}_{Li} \langle \tilde{H} \rangle U_{Rj} + Y_{Eij} \overline{L}_{Li} \langle H \rangle E_{Rj} + Y_{Nij} \overline{L}_{Li} \langle \tilde{H} \rangle N_{Rj}$

$$-Y_Q + Y_D + Y_H = 0$$

$$-Y_L + Y_E + Y_H = 0$$

$$-Y_Q + Y_U - Y_H = 0$$

$$-Y_L + Y_N - Y_H = 0$$

Solution:

$$Y_Q = +\frac{1}{6}, \quad Y_U = +\frac{2}{3}, \quad Y_D = -\frac{1}{3},$$

$$Y_L = -\frac{1}{2}, \quad Y_E = -1$$

(Hyper)charge quantization in the Standard model

SU(3) x SU(2) x U(1) gauge anomalies $\mathcal{A}_c \propto \frac{1}{32\pi^2} \text{Tr}(\{T_a, T_b\}T_c) \tilde{F}_{\mu\nu}^a F^{b\mu\nu}$

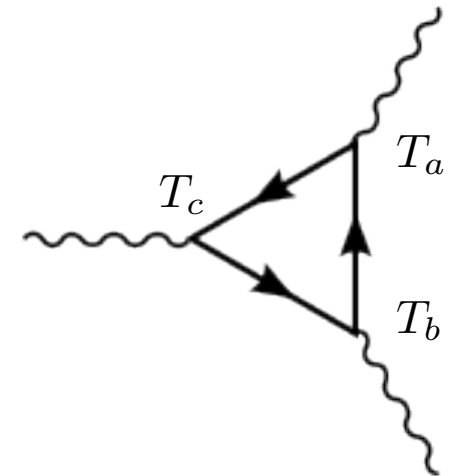
Assume Dirac neutrinos: $N_R = (1, 1, Y_N)$

SU(2)² U(1):

$$6Y_Q + 2Y_L = 0$$

U(1)³:

$$12Y_Q^3 + 4Y_L^3 - 6Y_U^3 - 6Y_D^3 - 2Y_E^3 - 2Y_N^3 = 0$$



Yukawas: $Y_{Dij} \overline{Q}_{Li} \langle H \rangle D_{Rj} + Y_{Uij} \overline{Q}_{Li} \langle \tilde{H} \rangle U_{Rj} + Y_{Eij} \overline{L}_{Li} \langle H \rangle E_{Rj} + Y_{Nij} \overline{L}_{Li} \langle \tilde{H} \rangle N_{Rj}$

$$-Y_Q + Y_D + Y_H = 0$$

$$-Y_L + Y_E + Y_H = 0$$

$$-Y_Q + Y_U - Y_H = 0$$

$$-Y_L + Y_N - Y_H = 0$$

Solution:

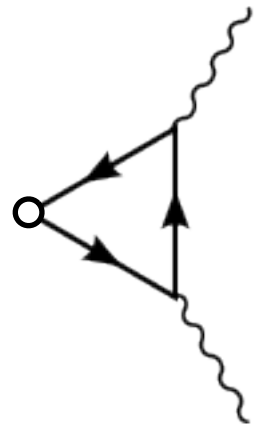
$$Y_Q = +\frac{1}{6} - \frac{1}{3}Y_N, \quad Y_U = +\frac{2}{3} - \frac{1}{3}Y_N, \quad Y_D = -\frac{1}{3} - \frac{1}{3}Y_N,$$

$$Y_L = -\frac{1}{2} + Y_N, \quad Y_E = -1 + Y_N \quad Y_N \in \mathbb{R}$$

A simpler symmetry argument

B and L anomalies & RH neutrino

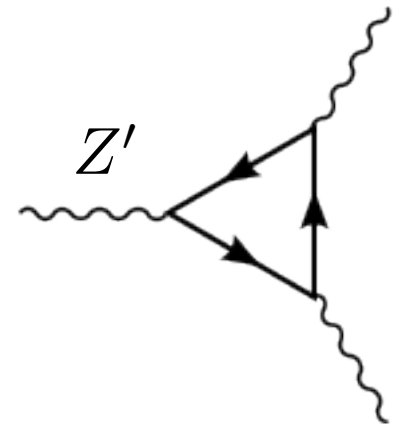
$$\text{Tr}(\{Y, Y\}(B - L)) = 0, \text{Tr}(\{T_L^3, T_L^3\}(B - L)) = 0,$$



A simpler symmetry argument

B and L anomalies & RH neutrino

$$\text{Tr}(\{Y, Y\}(B - L)) = 0, \text{Tr}(\{T_L^3, T_L^3\}(B - L)) = 0,$$



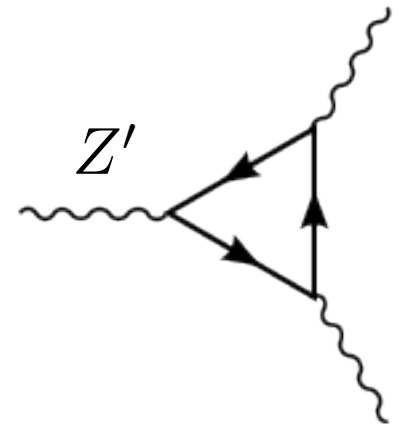
$$\boxed{\text{Tr}(B - L)^3 = 0}$$

B - L can be gauged !

A simpler symmetry argument

B and L anomalies & RH neutrino

$$\text{Tr}(\{Y, Y\}(B - L)) = 0, \text{Tr}(\{T_L^3, T_L^3\}(B - L)) = 0,$$



$$\boxed{\text{Tr}(B - L)^3 = 0}$$

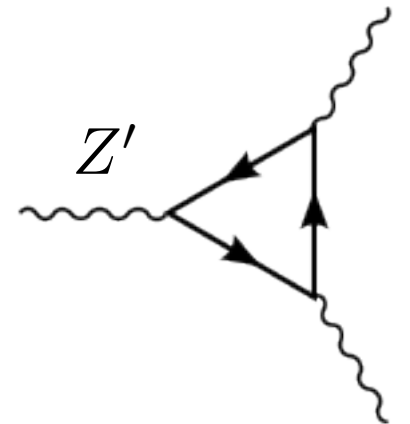
B - L can be gauged !

fuzzy hypercharge $Y \rightarrow Y + \varepsilon(B - L) \quad \varepsilon = -Y_N$

A simpler symmetry argument

B and L anomalies & RH neutrino

$$\text{Tr}(\{Y, Y\}(B - L)) = 0, \text{Tr}(\{T_L^3, T_L^3\}(B - L)) = 0,$$



$$\boxed{\text{Tr}(B - L)^3 = 0}$$

B - L can be gauged !

fuzzy hypercharge $Y \rightarrow Y + \varepsilon(B - L) \quad \varepsilon = -Y_N$

Babu, Mohapatra, Phys.Rev. D41 (1990) 271

Experimentally (neutron neutrality): $|\varepsilon| < 10^{-21}$

Foot, Lew, Volkas 1993

SM with massive neutrinos and quantized charges

Neutrinos are better not Dirac!

$$+ \frac{1}{2} \textcolor{red}{M}_{Rij} \overline{N_{Ri}^c} N_{Rj} + h.c.$$

E. Majorana 1937

Majorana mass = **perturbative L violation**

P. Minkowski, Phys. Lett. B67, 421 (1977)

Lepton number violation in oscillations

B. Pontecorvo, Sov.Phys.JETP 6 (1957) 429

The first approach to neutrino oscillations was “L-violating”!



NB Kaon oscillations in 1957

NB Muon neutrinos only in 1962

M.L. Good, Phys. Rev. 106 (1957) 591

Lederman, Schwarz, Steinberger

Бруно Понтекорво

Lepton number violation in oscillations

B. Pontecorvo, Sov.Phys.JETP 6 (1957) 429

The first approach to neutrino oscillations was “L-violating”!



NB Kaon oscillations in 1957

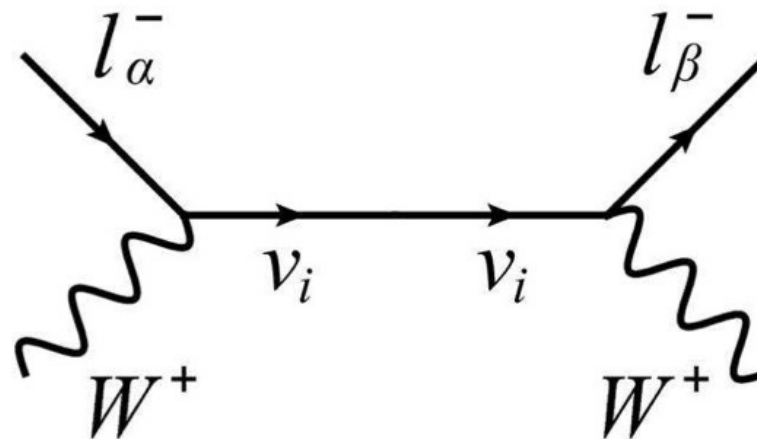
NB Muon neutrinos only in 1962

M.L. Good, Phys. Rev. 106 (1957) 591

Lederman, Schwarz, Steinberger

Diagrammatics:

see e.g. E.Akhmedov, J. Kopp, JHEP 1004 (2010) 008



$$\mathcal{A}(\nu_{\alpha} \rightarrow \nu_{\beta}) \propto \sum_i U_{\alpha i}^{*} U_{\beta i} e^{-i \frac{m_i^2 L}{2E}}$$

Lepton number violation in oscillations

B. Pontecorvo, Sov.Phys.JETP 6 (1957) 429

The first approach to neutrino oscillations was “L-violating”!

NB Kaon oscillations in 1957

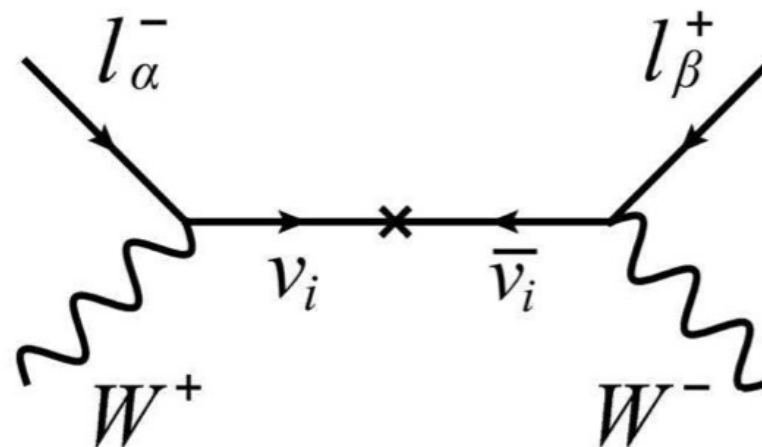
M.L. Good, Phys. Rev. 106 (1957) 591

NB Muon neutrinos only in 1962

Lederman, Schwarz, Steinberger

Diagrammatics:

see e.g. E.Akhmedov, J. Kopp, JHEP 1004 (2010) 008



$$\mathcal{A}(\nu_\alpha \rightarrow \bar{\nu}_\beta) \propto \sum_i U_{\alpha i}^* U_{\beta i}^* \frac{m_i}{E} e^{-i \frac{m_i^2 L}{2E}}$$

Lepton number violation in oscillations

B. Pontecorvo, Sov.Phys.JETP 6 (1957) 429

The first approach to neutrino oscillations was “L-violating”!



NB Kaon oscillations in 1957

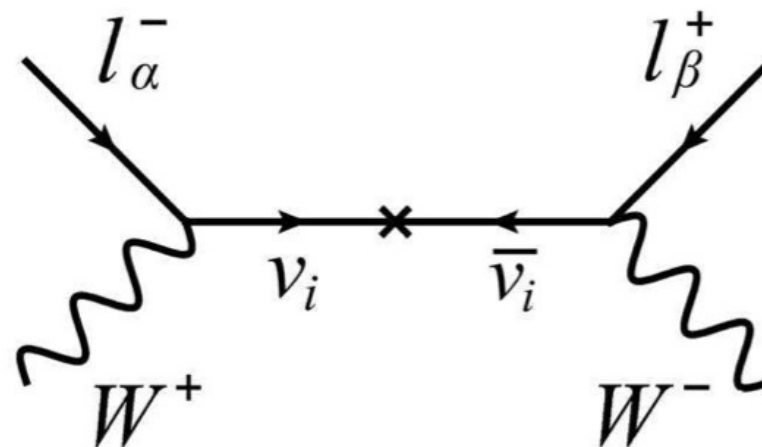
NB Muon neutrinos only in 1962

M.L. Good, Phys. Rev. 106 (1957) 591

Lederman, Schwarz, Steinberger

Diagrammatics:

see e.g. E.Akhmedov, J. Kopp, JHEP 1004 (2010) 008



$$\mathcal{A}(\nu_\alpha \rightarrow \bar{\nu}_\beta) \propto \sum_i U_{\alpha i}^* U_{\beta i}^* \frac{m_i}{E} e^{-i \frac{m_i^2 L}{2E}}$$

Suppressed...

see e.g. Z-z. Xing, arXiv:1301.7654v2

LVN in the SM at d=5 level

Weinberg's d=5 operator $\mathcal{L} \ni \frac{LLHH}{\Lambda}$ S.Weinberg, PRL43, 1566 (1979)

LVN in the SM at d=5 level

Weinberg's d=5 operator $\mathcal{L} \ni \frac{LLHH}{\Lambda}$ S.Weinberg, PRL43, 1566 (1979)

$$\Lambda \sim (10^{12} - 10^{14}) \text{ GeV}$$

LVN in the SM at d=5 level

Weinberg's d=5 operator $\mathcal{L} \ni \frac{LLHH}{\Lambda}$ S.Weinberg, PRL43, 1566 (1979)

$$\Lambda \sim (10^{12} - 10^{14}) \text{ GeV}$$

There is only one d=5 effective operator in the SM!

LVN in the SM at d=5 level

Weinberg's d=5 operator $\mathcal{L} \ni \frac{LLHH}{\Lambda}$ S.Weinberg, PRL43, 1566 (1979)

$$\Lambda \sim (10^{12} - 10^{14}) \text{ GeV}$$

There is only one d=5 effective operator in the SM!

BTW: good to have the “complete Higgs doublet” :-)

(If you prefer ABEHGHKW you rather read “HIGGS”...)

Perturbative B violation

in gauge extensions of the SM

SM as an effective theory at d=6 level

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger \varphi)\Box(\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$
$Q_{\tilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
$Q_{\varphi \tilde{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi \tilde{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \tilde{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
$Q_{\varphi \tilde{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

B. Grzadkowski et al., JHEP 10 (2010) 085, arXiv: 1008.4884

SM as an effective theory at d=6 level

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B -violating			
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s^j q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_p^\alpha)^T C u_r^\beta] [(q_s^{\gamma j})^T C l_t^k]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	Q_{qqqu}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(u_s^\gamma)^T C e_t]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \varepsilon_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma} (\tau^I \varepsilon)_{jk} (\tau^I \varepsilon)_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^\gamma)^T C e_t]$		

B. Grzadkowski et al., JHEP 10 (2010) 085, arXiv: 1008.4884

SM as an effective theory at d=6 level

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B -violating			
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s^j q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_p^\alpha)^T C u_r^\beta] [(q_s^{\gamma j})^T C l_t^k]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(u_s^\gamma)^T C e_t]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \varepsilon_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma} (\tau^I \varepsilon)_{jk} (\tau^I \varepsilon)_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^\gamma)^T C e_t]$		

B. Grzadkowski et al., JHEP 10 (2010) 085, arXiv: 1008.4884

d=6 baryon number violation mediators

Example: $(d_R^T C u_R)(Q_L^T C L_L)$

d=6 baryon number violation mediators

Example: $(d_R^T C u_R)(Q_L^T C L_L)$

Scalar exchange

$$(3, 1, -\frac{1}{3}) \oplus (\bar{3}, 1, +\frac{1}{3})$$



d=6 baryon number violation mediators

Example: $(d_R^T C u_R)(Q_L^T C L_L) \xrightarrow{\text{Fierz}} [(\overline{u_R})^c \gamma_\mu Q][(\overline{d_R})^c \gamma_\mu L]$

Scalar exchange

$$(3, 1, -\frac{1}{3}) \oplus (\bar{3}, 1, +\frac{1}{3})$$



d=6 baryon number violation mediators

Example: $(d_R^T C u_R)(Q_L^T C L_L) \xrightarrow{\text{Fierz}} [(\overline{u_R})^c \gamma_\mu Q][(\overline{d_R})^c \gamma_\mu L]$

Scalar exchange

$$(3, 1, -\frac{1}{3}) \oplus (\bar{3}, 1, +\frac{1}{3})$$
$$\Delta$$

Vector exchange

$$(3, 2, -\frac{5}{6}) \oplus (\bar{3}, 2, +\frac{5}{6})$$
$$X^\mu$$

d=6 baryon number violation mediators

Example: $(d_R^T C u_R)(Q_L^T C L_L) \xrightarrow{\text{Fierz}} [(\overline{u_R})^c \gamma_\mu Q][(\overline{d_R})^c \gamma_\mu L]$

Scalar exchange

$$(3, 1, -\frac{1}{3}) \oplus (\bar{3}, 1, +\frac{1}{3})$$

$$\Delta$$

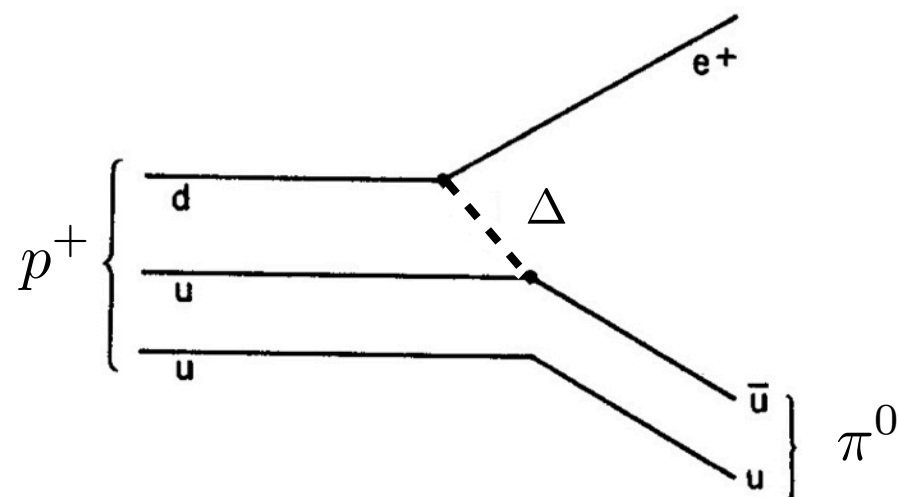
Vector exchange

$$(3, 2, -\frac{5}{6}) \oplus (\bar{3}, 2, +\frac{5}{6})$$

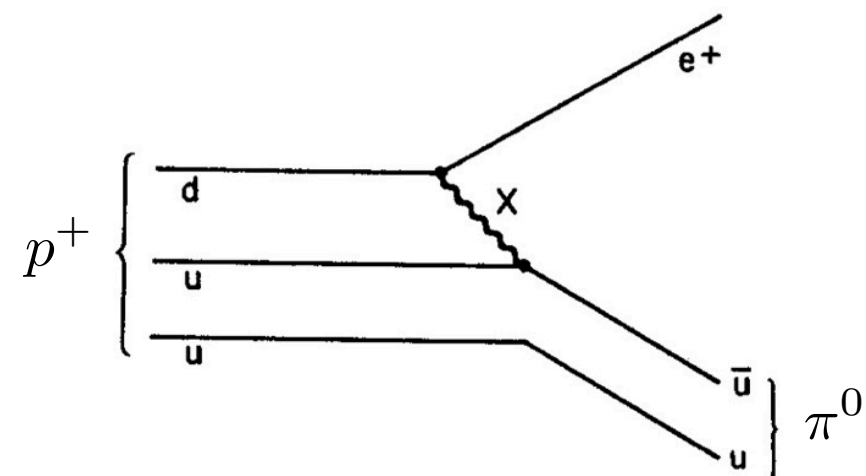
$$X^\mu$$

Proton instability: $\Gamma_p \sim \frac{m_p^5}{M^4} < (10^{34} \text{y})^{-1}$

new Yukawa interactions?



new gauge interactions?



d=6 baryon number violation mediators

Example: $(d_R^T C u_R)(Q_L^T C L_L) \xrightarrow{\text{Fierz}} [(\overline{u_R})^c \gamma_\mu Q][(\overline{d_R})^c \gamma_\mu L]$

Scalar exchange

$$(3, 1, -\frac{1}{3}) \oplus (\bar{3}, 1, +\frac{1}{3})$$

$$\Delta$$

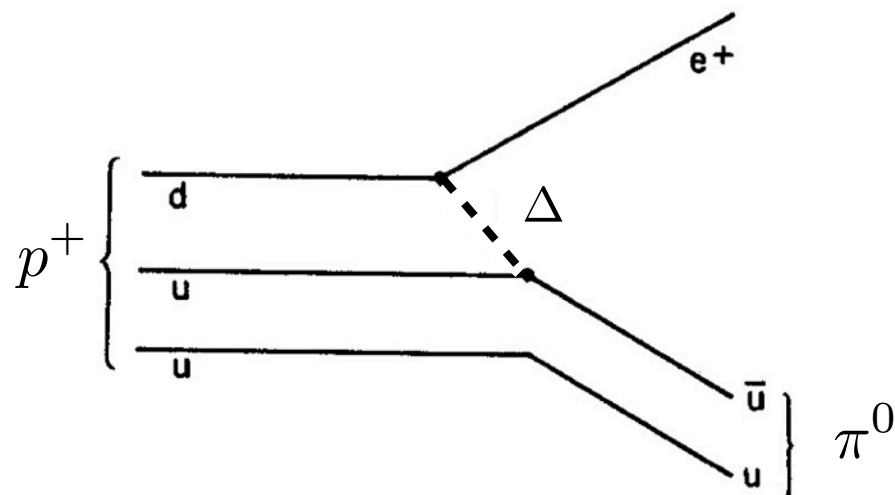
Vector exchange

$$(3, 2, -\frac{5}{6}) \oplus (\bar{3}, 2, +\frac{5}{6})$$

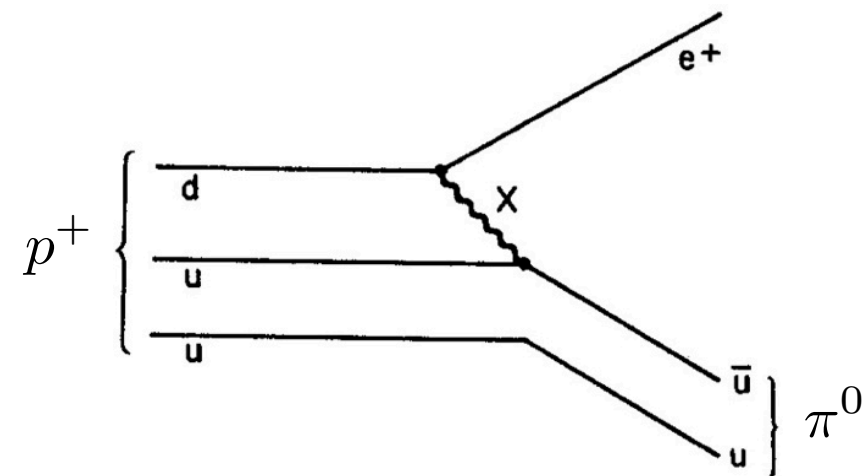
$$X^\mu$$

Proton instability: $\Gamma_p \sim \frac{m_p^5}{M^4} < (10^{34} \text{y})^{-1}$

new Yukawa interactions?



new gauge interactions?



Such a new physics should be above 10^{15} GeV !??

Can SM tell us anything about such a huge-scale dynamics?

Running gauge couplings in the SM

$$\mu \frac{d}{d\mu} g = \beta(g, \dots)$$

calculable in perturbation theory

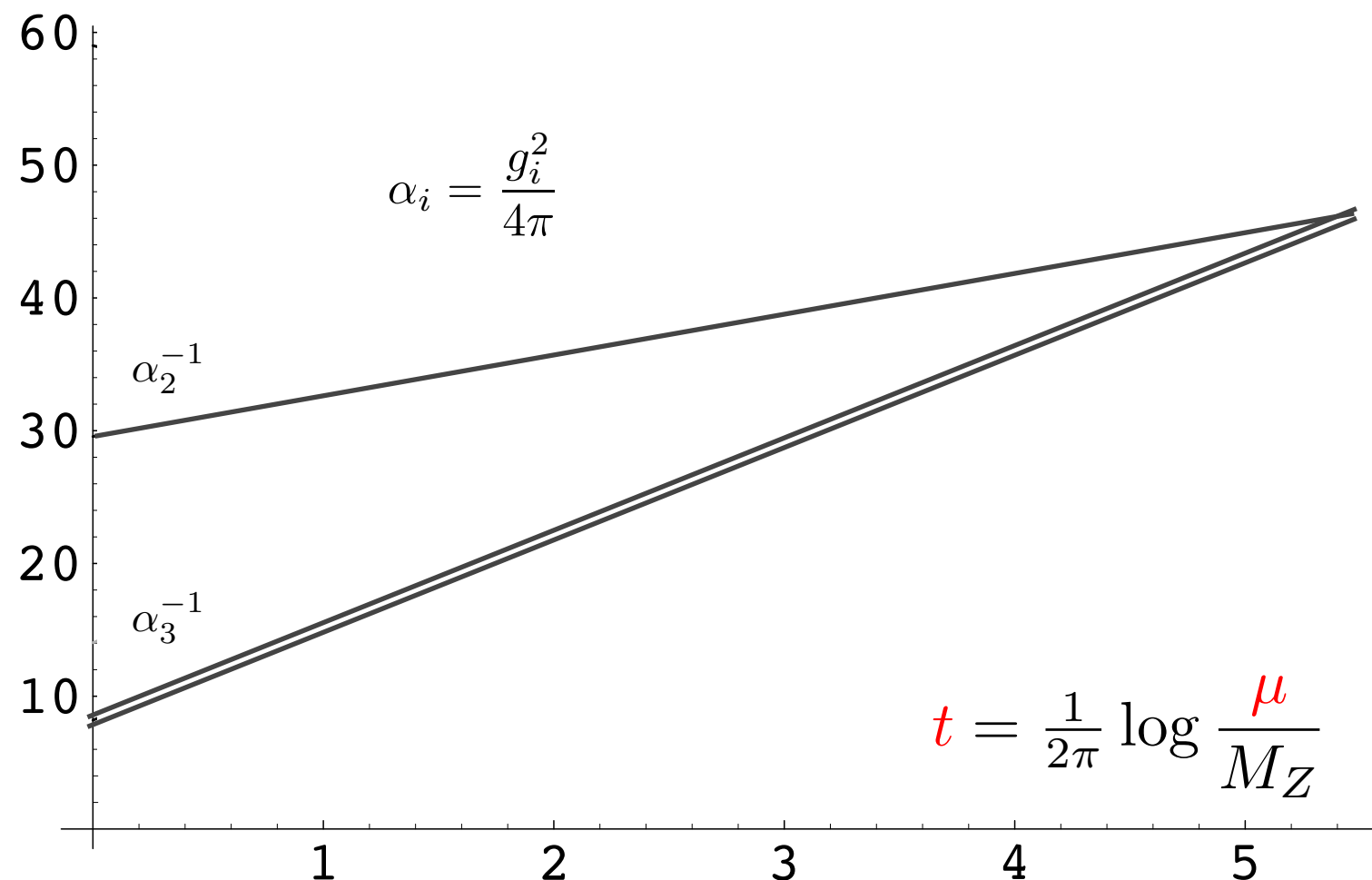
$$\beta = \frac{g^3}{16\pi^2} \left(-\frac{11}{3} C_2(G) + \frac{2}{3} \sum_{f_W} T_2^G(R_{f_W}) + \frac{1}{3} \sum_{s_C} T_2^G(R_{s_C}) \right) + \dots$$

D. Gross, F. Wilczek, D. Politzer 1973

Can SM tell us anything about such a huge-scale dynamics?

Running gauge couplings in the SM

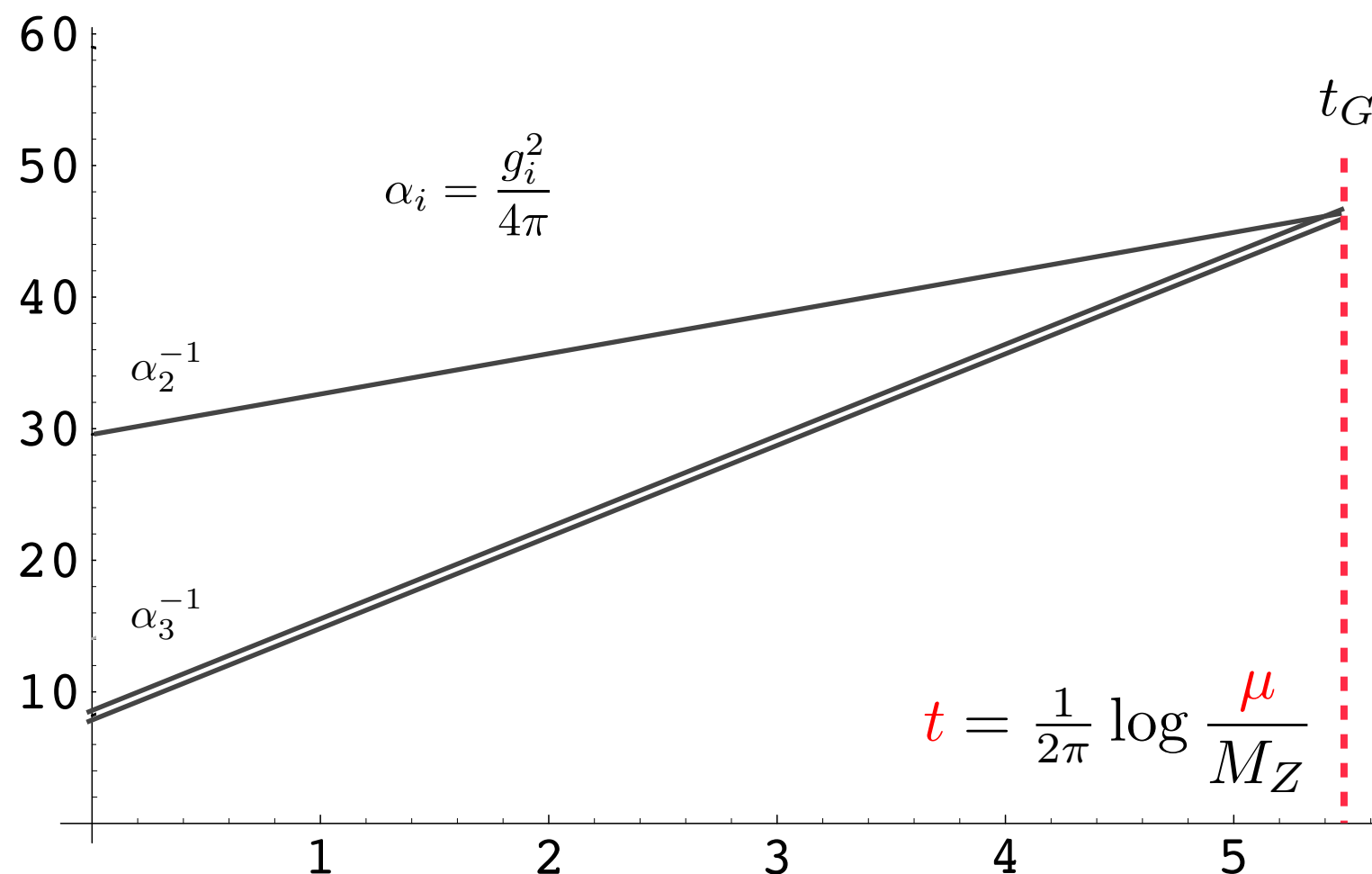
$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{11}{3} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}_{\text{gauge}} + 2 \begin{pmatrix} \frac{10}{3} \\ 2 \\ 2 \end{pmatrix}_{\text{ferm.}} + \frac{1}{3} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}_{\text{scal.}}$$



Can SM tell us anything about such a huge-scale dynamics?

Running gauge couplings in the SM

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{11}{3} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}_{\text{gauge}} + 2 \begin{pmatrix} \frac{10}{3} \\ 2 \\ 2 \end{pmatrix}_{\text{ferm.}} + \frac{1}{3} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}_{\text{scal.}}$$

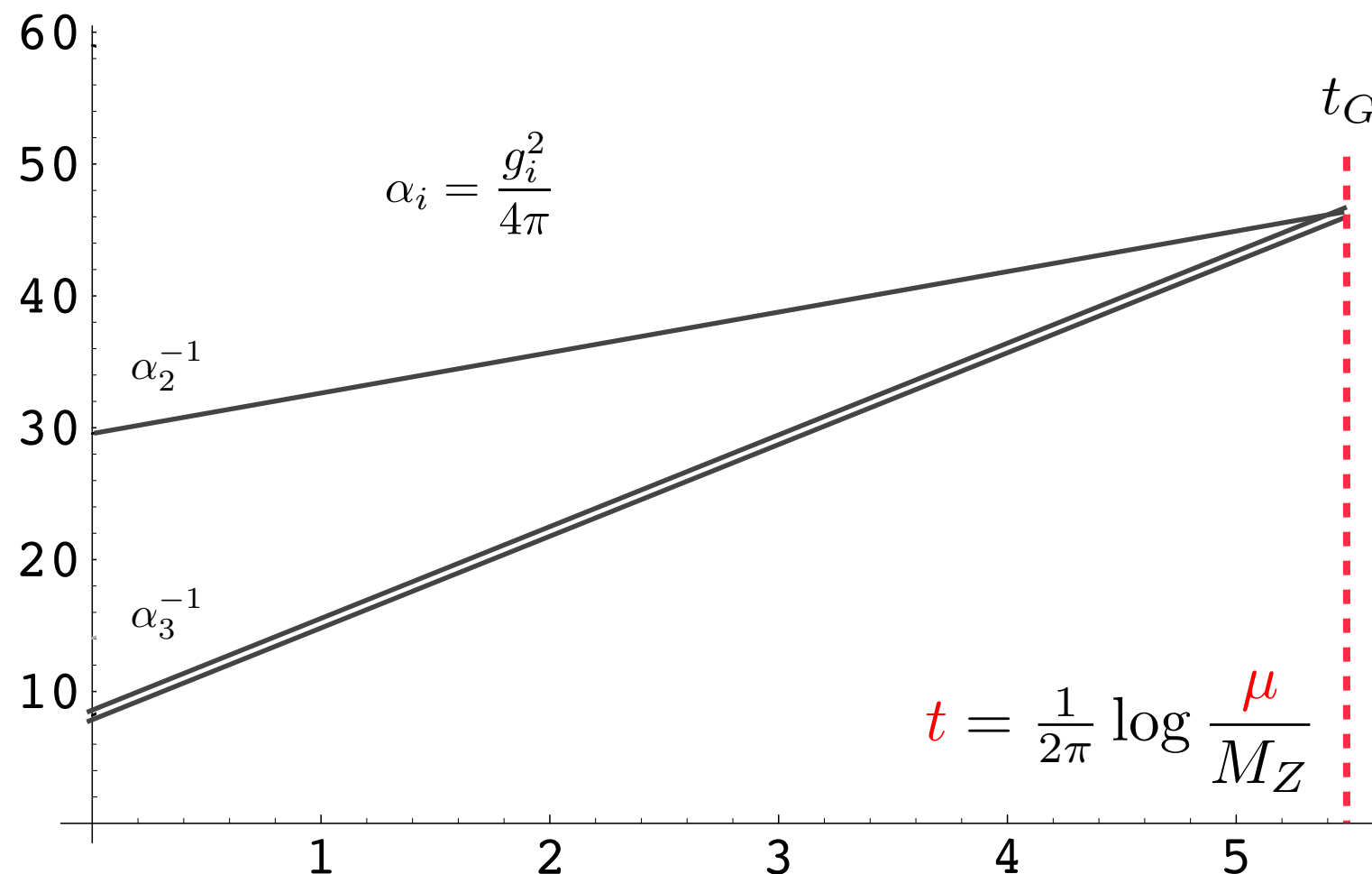


$$M_G \sim 10^{16} \text{ GeV}$$

Can SM tell us anything about such a huge-scale dynamics?

Running gauge couplings in the SM

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{11}{3} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}_{\text{gauge}} + 2 \begin{pmatrix} \frac{10}{3} \\ 2 \\ 2 \end{pmatrix}_{\text{ferm.}} + \frac{1}{3} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}_{\text{scal.}}$$



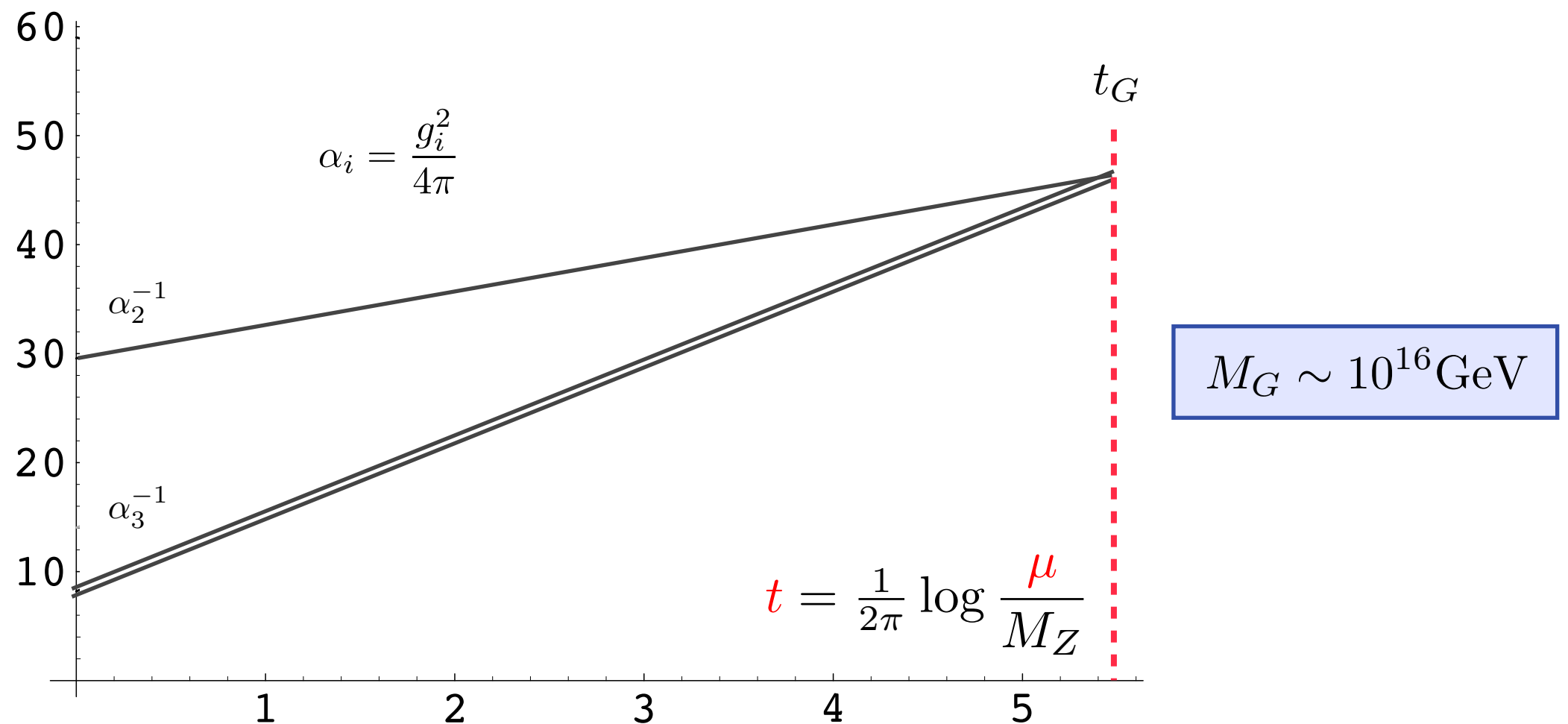
$$M_G \sim 10^{16} \text{ GeV}$$

Can SM tell us anything about such a huge-scale dynamics?

Running gauge couplings in the SM $+ X + \Delta$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{11}{3} \begin{pmatrix} 0 + \frac{25}{3} \\ 2 + 3 \\ 3 + 2 \end{pmatrix}_{\text{gauge}} + 2 \begin{pmatrix} \frac{10}{3} \\ 2 \\ 2 \end{pmatrix}_{\text{ferm.}} + \frac{1}{3} \begin{pmatrix} \frac{1}{2} + \frac{1}{3} \\ \frac{1}{2} \\ 0 + \frac{1}{2} \end{pmatrix}_{\text{scal.}}$$

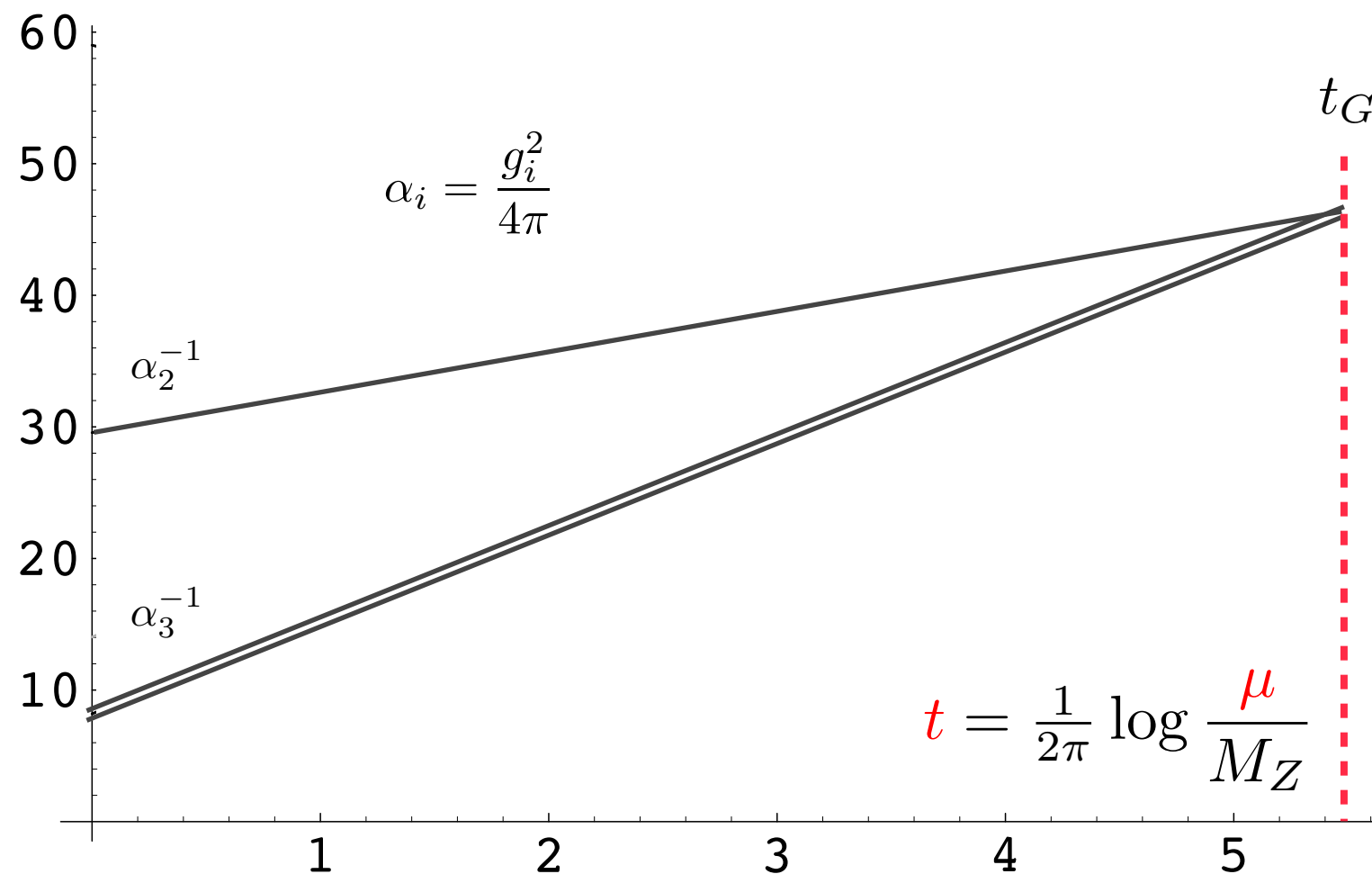
$(3, 2, -\frac{5}{6}) \oplus h.c.$
 $(3, 1, -\frac{1}{3})$



Can SM tell us anything about such a huge-scale dynamics?

Running gauge couplings in the SM $+ X + \Delta$

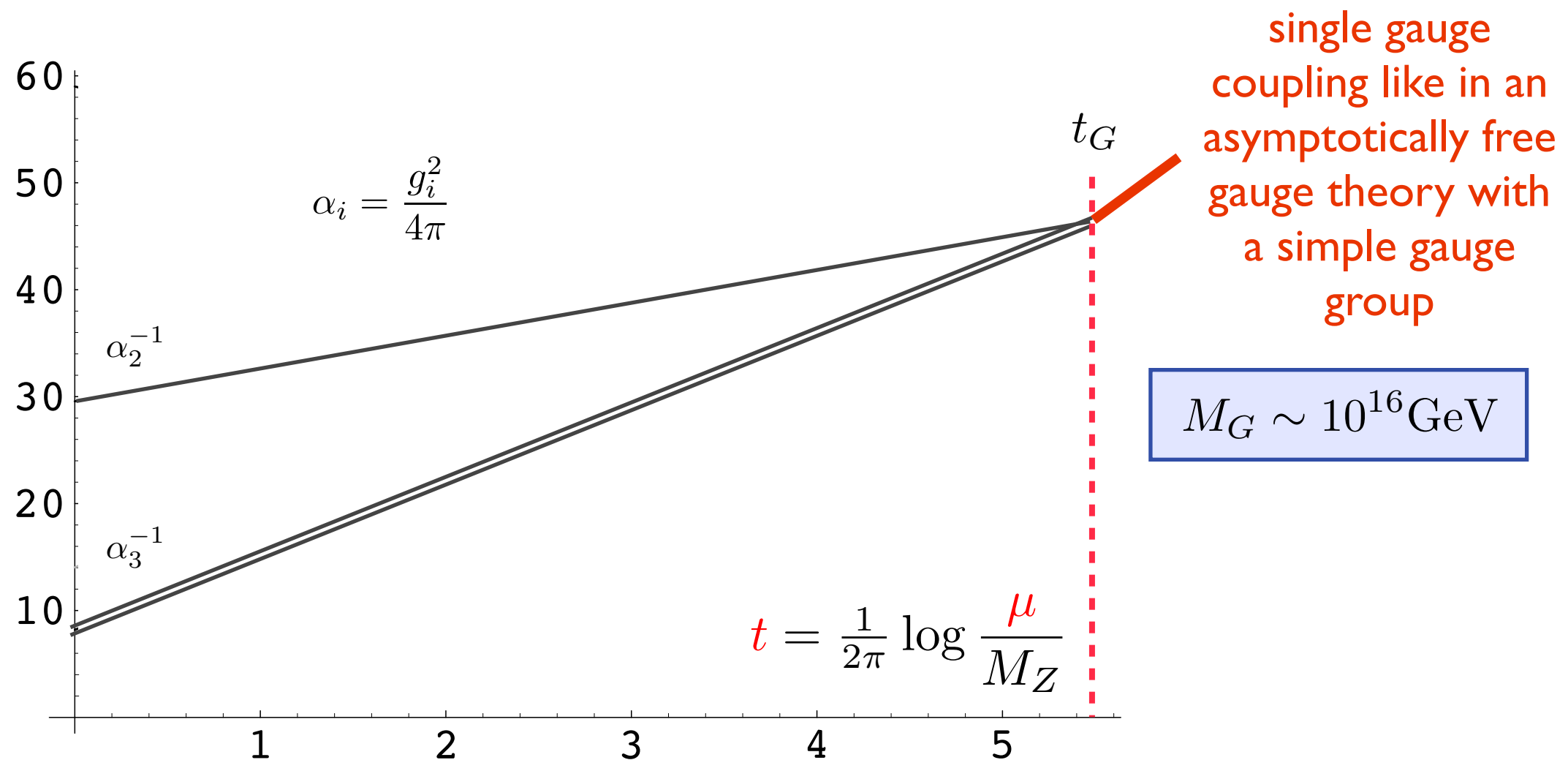
$$\begin{pmatrix} \frac{3}{5}b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{11}{3} \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}_{\text{gauge}} + 2 \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}_{\text{ferm.}} + \frac{1}{3} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}_{\text{scal.}}$$



Can SM tell us anything about such a huge-scale dynamics?

Running gauge couplings in the SM $+ X + \Delta$

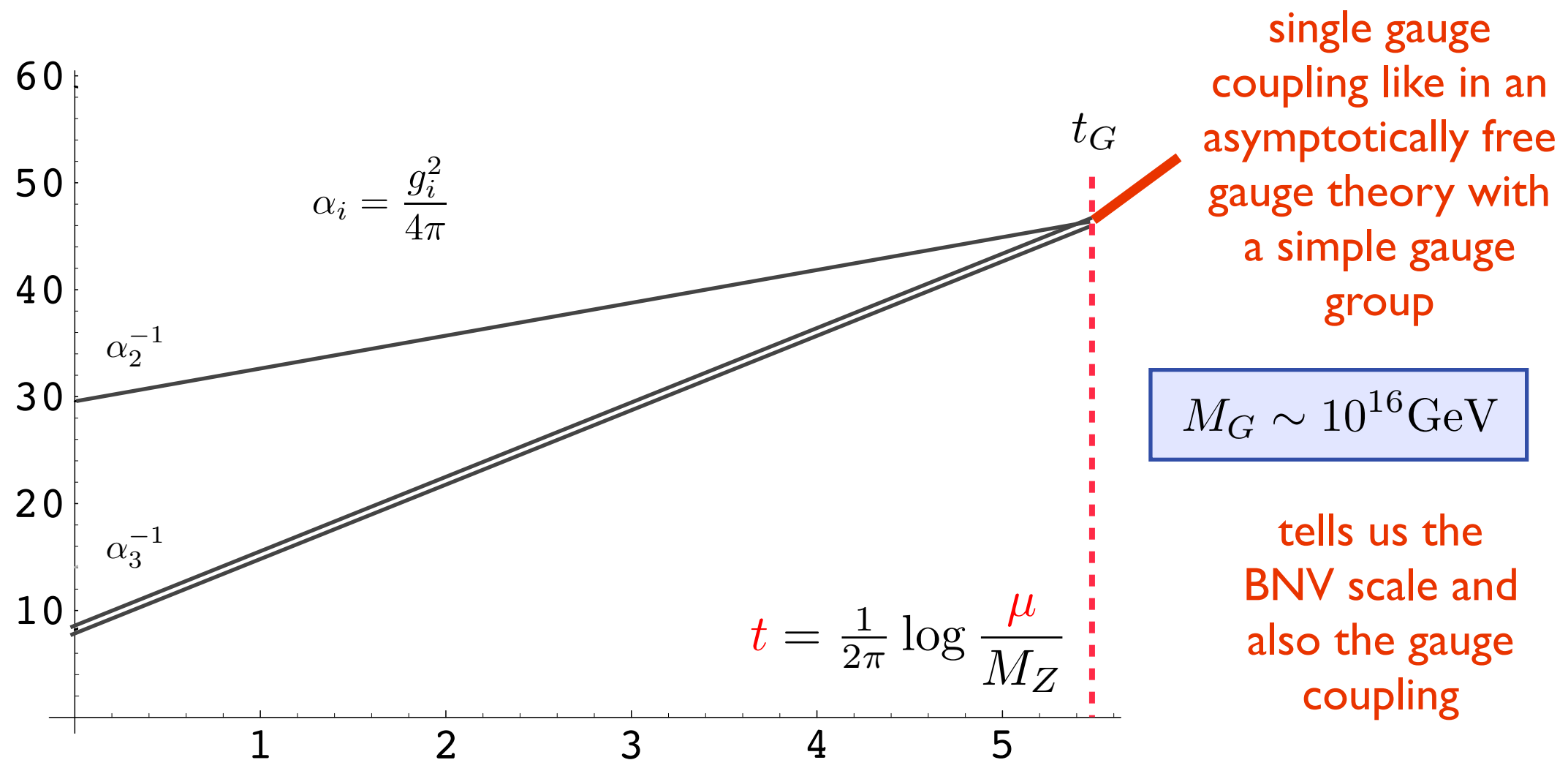
$$\begin{pmatrix} \frac{3}{5}b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{11}{3} \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}_{\text{gauge}} + 2 \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}_{\text{ferm.}} + \frac{1}{3} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}_{\text{scal.}}$$



Can SM tell us anything about such a huge-scale dynamics?

Running gauge couplings in the SM $+X + \Delta$

$$\begin{pmatrix} \frac{3}{5}b_1 \\ b_2 \\ b_3 \end{pmatrix} = -\frac{11}{3} \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}_{\text{gauge}} + 2 \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}_{\text{ferm.}} + \frac{1}{3} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}_{\text{scal.}}$$



Grand unification

Grand Unification

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

GUTs are spontaneously broken BSM gauge theories based on simple compact gauge groups

Grand Unification

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

GUTs are spontaneously broken BSM gauge theories based on simple compact gauge groups

Theories of the $d=6$ BNV operators in the SM...

...and other stuff: magnetic monopoles, charge quantization, LNV etc.

The minimal SU(5) GUT

VOLUME 32, NUMBER 8

PHYSICAL REVIEW LETTERS

25 FEBRUARY 1974

Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world—that all elementary particle forces (strong, weak, and electromagnetic) are different manifestations of the same fundamental interaction involving a single coupling strength, the fine-structure constant. Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously.

of the GIM mechanism with the notion of colored quarks⁴ keeps the successes of the quark model and gives an important bonus: Lepton and hadron anomalies cancel so that the theory of weak and electromagnetic interactions is renormalizable.⁵

The next step is to include strong interactions. We assume that *strong interactions are mediated by an octet of neutral vector gauge gluons associated with local color SU(3) symmetry*, and that there are no fundamental strongly interacting scalar-meson fields.⁶ This insures that

What Georgi and Glashow showed was the uniqueness of SU(5) @ rank=4

The minimal SU(5) GUT

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

The minimal SU(5) GUT

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

$$(1, 2, -\frac{1}{2}) \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

$$(1, 1, +1) \quad e^c \quad \mu^c$$

$$(3, 2, +\frac{1}{6}) \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix}$$

$$(\bar{3}, 1, -\frac{2}{3}) \quad u^c \quad c^c$$

$$(\bar{3}, 1, +\frac{1}{3}) \quad d^c \quad s^c$$

The minimal SU(5) GUT

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(5)$$

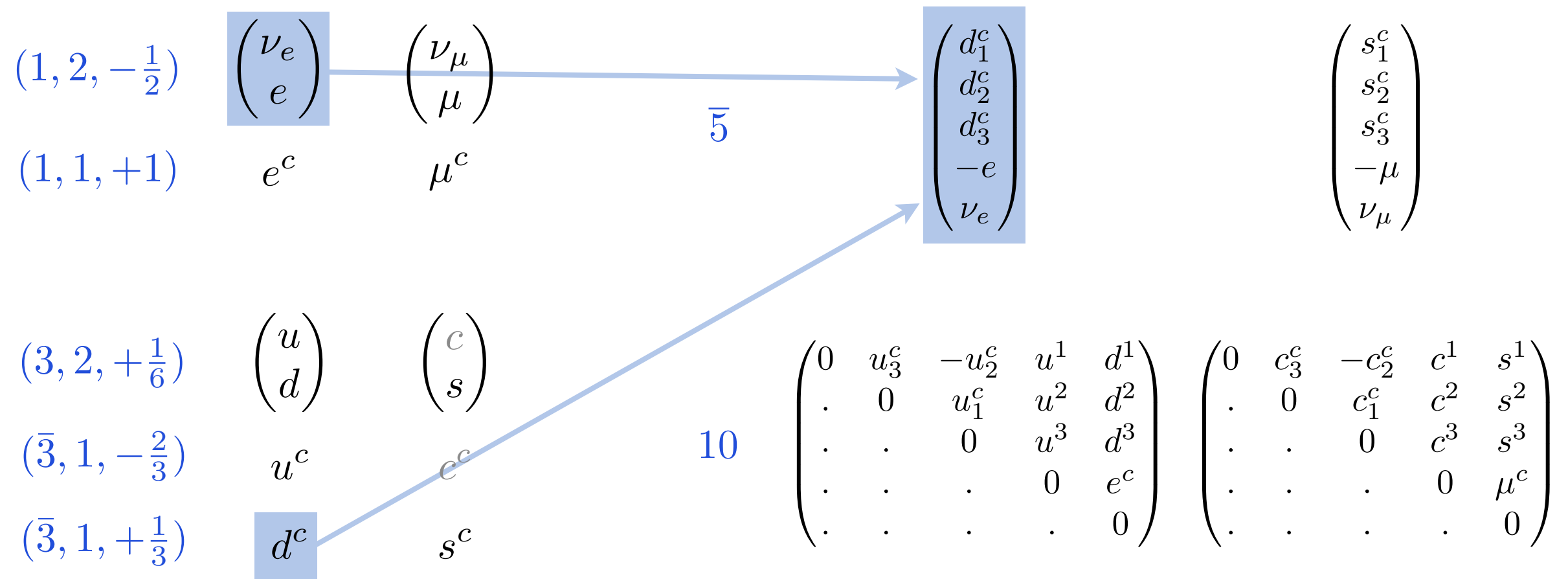
$$\begin{array}{lll} (1, 2, -\frac{1}{2}) & \begin{pmatrix} \nu_e \\ e \end{pmatrix} & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \\ (1, 1, +1) & e^c & \mu^c \end{array} \quad \begin{array}{c} \text{5} \end{array} \quad \begin{array}{c} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ -e \\ \nu_e \end{pmatrix} \\ \begin{pmatrix} s_1^c \\ s_2^c \\ s_3^c \\ -\mu \\ \nu_\mu \end{pmatrix} \end{array}$$

$$\begin{array}{lll} (3, 2, +\frac{1}{6}) & \begin{pmatrix} u \\ d \end{pmatrix} & \begin{pmatrix} c \\ s \end{pmatrix} \\ (\bar{3}, 1, -\frac{2}{3}) & u^c & c^c \\ (\bar{3}, 1, +\frac{1}{3}) & d^c & s^c \end{array} \quad \begin{array}{c} \text{10} \end{array} \quad \begin{array}{c} \begin{pmatrix} 0 & u_3^c & -u_2^c & u^1 & d^1 \\ . & 0 & u_1^c & u^2 & d^2 \\ . & . & 0 & u^3 & d^3 \\ . & . & . & 0 & e^c \\ . & . & . & . & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & c_3^c & -c_2^c & c^1 & s^1 \\ . & 0 & c_1^c & c^2 & s^2 \\ . & . & 0 & c^3 & s^3 \\ . & . & . & 0 & \mu^c \\ . & . & . & . & 0 \end{pmatrix} \end{array}$$

The minimal SU(5) GUT

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(5)$$



The minimal SU(5) GUT

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(5)$$

$(1, 2, -\frac{1}{2})$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\bar{5}$	$\begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ -e \\ \nu_e \end{pmatrix}$	$\begin{pmatrix} s_1^c \\ s_2^c \\ s_3^c \\ -\mu \\ \nu_\mu \end{pmatrix}$
$(1, 1, +1)$	e^c	μ^c			
$(3, 2, +\frac{1}{6})$	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	10	$\begin{pmatrix} 0 & u_3^c & -u_2^c & u^1 & d^1 \\ \cdot & 0 & u_1^c & u^2 & d^2 \\ \cdot & \cdot & 0 & u^3 & d^3 \\ \cdot & \cdot & \cdot & 0 & e^c \\ \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & c_3^c & -c_2^c & c^1 & s^1 \\ \cdot & 0 & c_1^c & c^2 & s^2 \\ \cdot & \cdot & 0 & c^3 & s^3 \\ \cdot & \cdot & \cdot & 0 & \mu^c \\ \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix}$
$(\bar{3}, 1, -\frac{2}{3})$	u^c	c^c			
$(\bar{3}, 1, +\frac{1}{3})$	d^c	s^c			

The minimal SU(5) GUT

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(5)$$

X^μ

The minimal SU(5) GUT

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(5)$$

Gauge sector:

$$24 = (8, 1, 0) \oplus (1, 3, 0) \oplus (1, 1, 0) \oplus (3, 2, -\frac{5}{6}) \oplus (\bar{3}, 2, +\frac{5}{6})$$

$$\left. \begin{matrix} G^\mu & A^\mu \\ & B^\mu \end{matrix} \right\} W^\pm, Z, \gamma \qquad G^\mu \qquad A^\mu \qquad B^\mu \qquad X^\mu$$

The minimal SU(5) GUT

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(5)$$

Gauge sector:

$$24 = (8, 1, 0) \oplus (1, 3, 0) \oplus (1, 1, 0) \oplus (3, 2, -\frac{5}{6}) \oplus (\bar{3}, 2, +\frac{5}{6})$$

$$\left. \begin{matrix} G^\mu & A^\mu \\ & B^\mu \end{matrix} \right\} W^\pm, Z, \gamma \qquad G^\mu \qquad A^\mu \qquad B^\mu \qquad X^\mu$$

Scalar sector:

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q$$

SM Higgs:

$$\bar{5} = \underbrace{(1, \bar{2}, +\frac{1}{2})}_H \oplus \underbrace{(\bar{3}, 1, -\frac{1}{3})}_\Delta$$

The minimal SU(5) GUT

H.Georgi, S.Glashow, Phys.Rev.Lett. 30 (1974)

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(5)$$

Gauge sector:

$$24 = (8, 1, 0) \oplus (1, 3, 0) \oplus (1, 1, 0) \oplus (3, 2, -\frac{5}{6}) \oplus (\bar{3}, 2, +\frac{5}{6})$$

$$\left. \begin{matrix} G^\mu & A^\mu \\ & B^\mu \end{matrix} \right\} W^\pm, Z, \gamma \qquad G^\mu \qquad A^\mu \qquad B^\mu \qquad X^\mu$$

Scalar sector:

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q$$

SM Higgs:

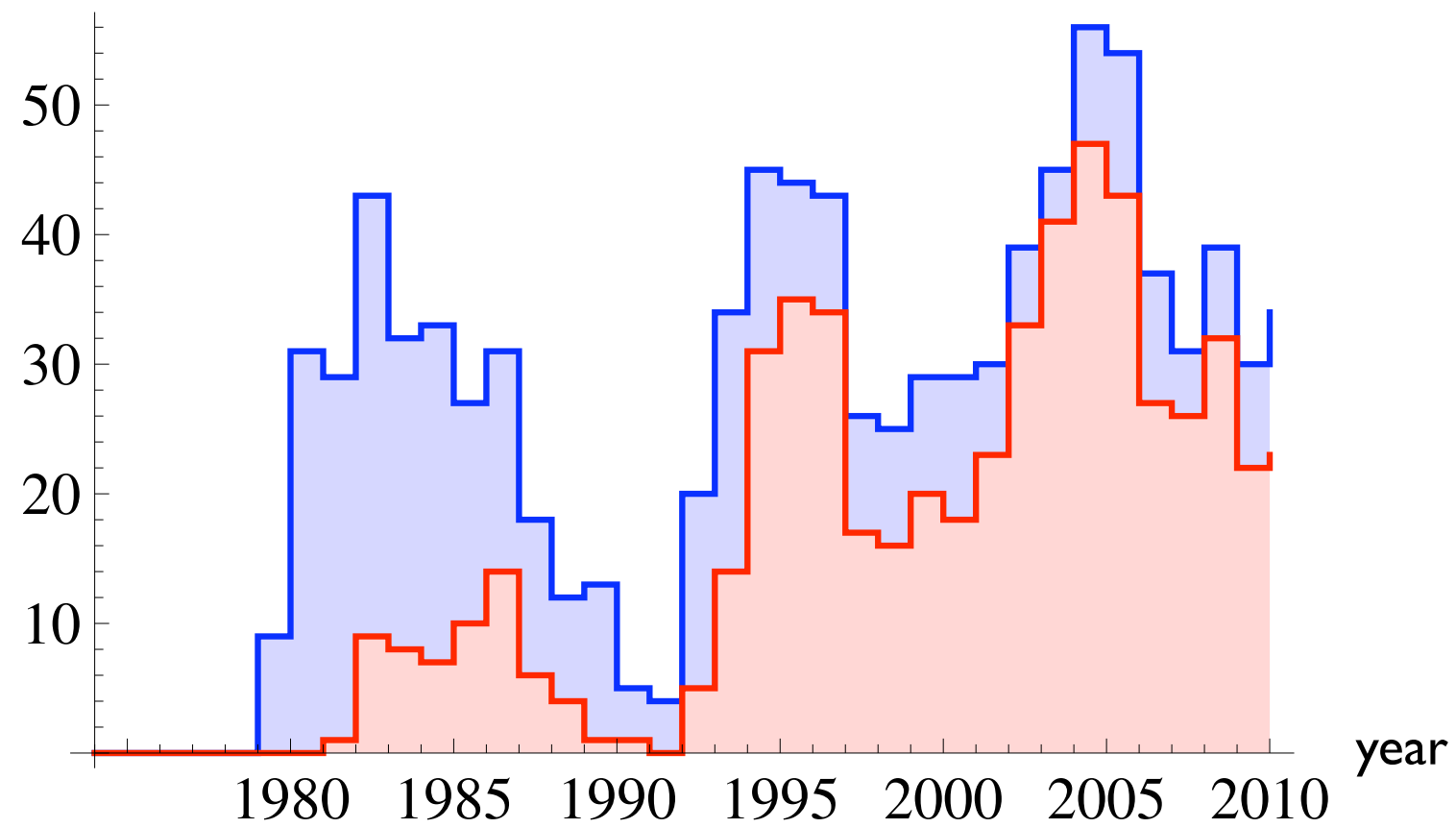
$$\bar{5} = \underbrace{(1, \bar{2}, +\frac{1}{2})}_H \oplus \underbrace{(\bar{3}, 1, -\frac{1}{3})}_\Delta$$

GUT-breaking scalars: $SU(5) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

$$24 = (1, 1, 0) \oplus (8, 1, 0) \oplus (1, 3, 0) \oplus (3, 2, -\frac{5}{6}) \oplus (\bar{3}, 2, +\frac{5}{6})$$

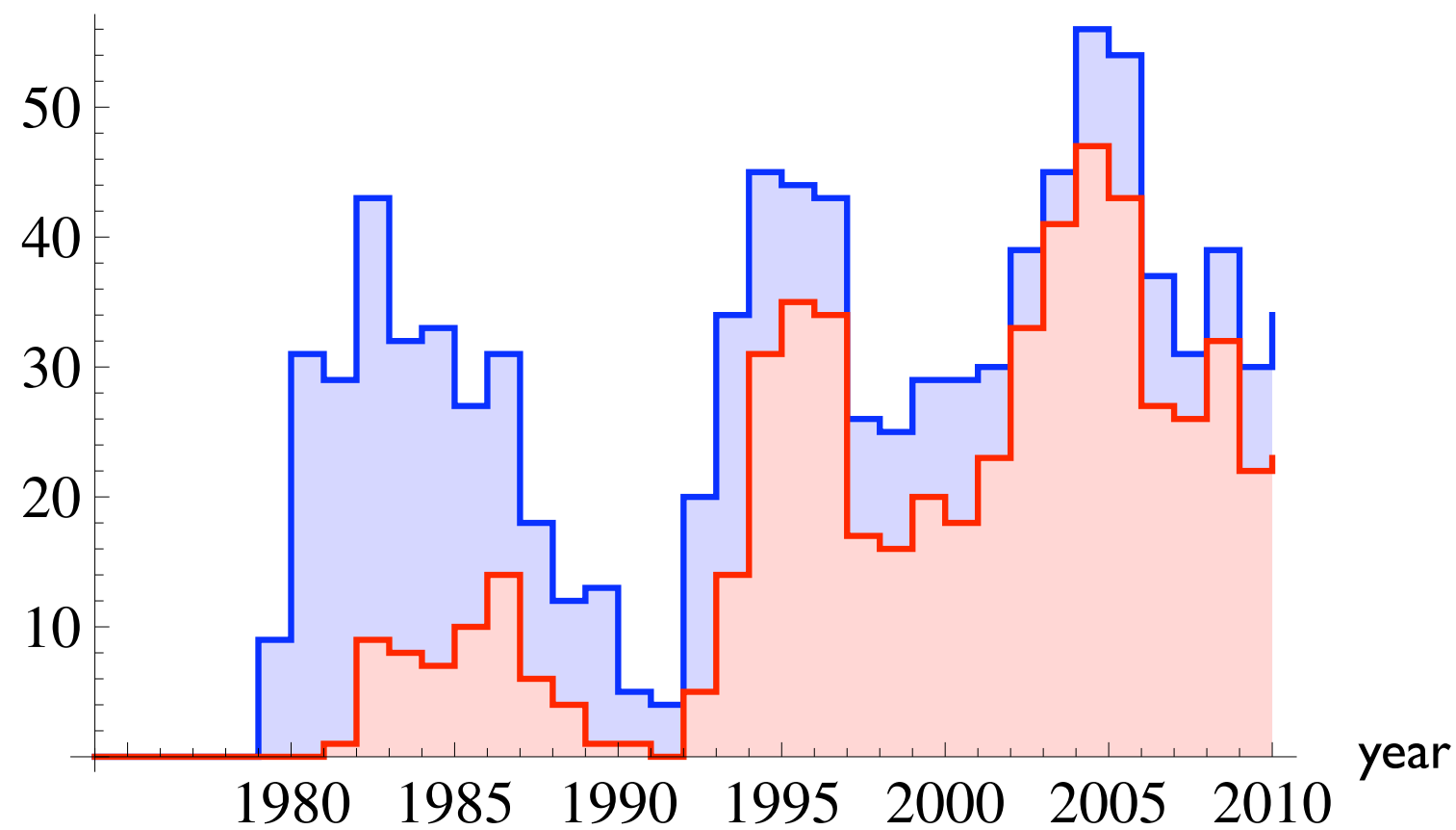
Grand Unifications

of works @ inSPIRE



Grand Unifications

of works @ inSPIRE

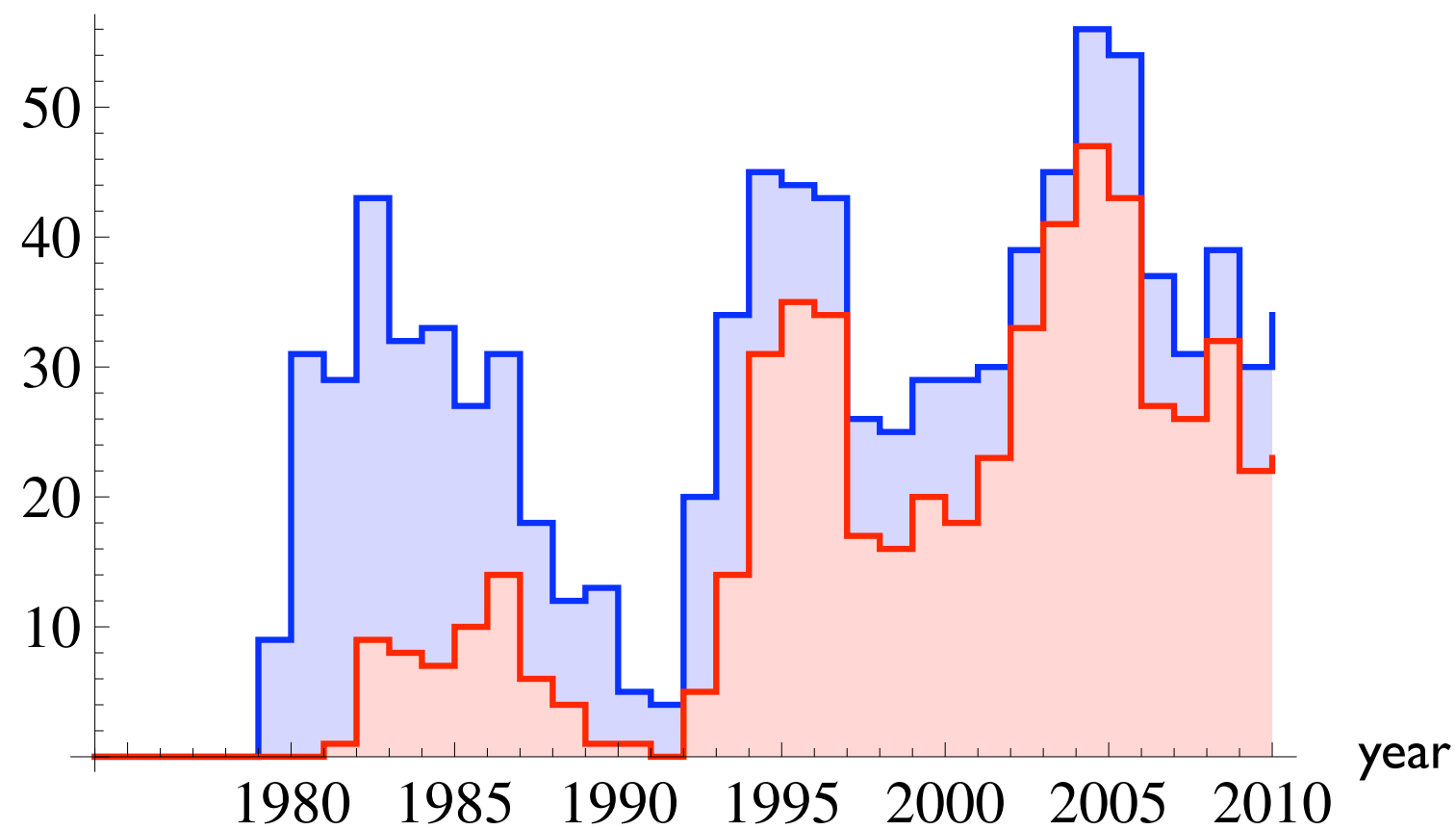


Infancy

Birth: February 25 1974 - Georgi & Glashow PRL32

Grand Unifications

of works @ inSPIRE

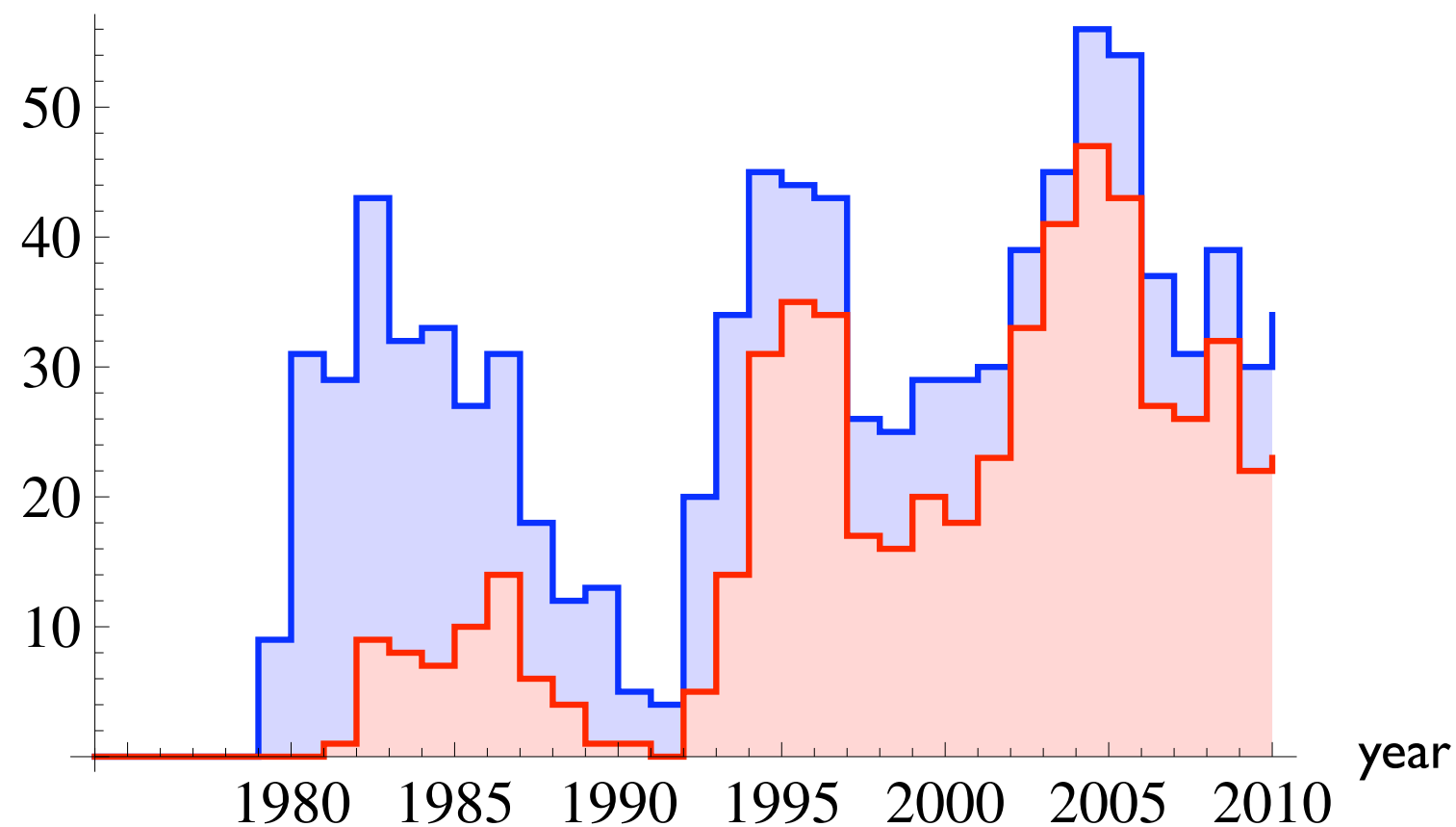


Infancy

Birth: February 25 1974 - Georgi & Glashow PRL32 Experiment: $\tau_p \gtrsim 10^{30}$ yr Reines, Crouch (1974 PRL March 4)

Grand Unifications

of works @ inSPIRE



Infancy

Theory: $\tau_p \sim 10^{31}$ yr Georgi, Quinn, Weinberg 1974

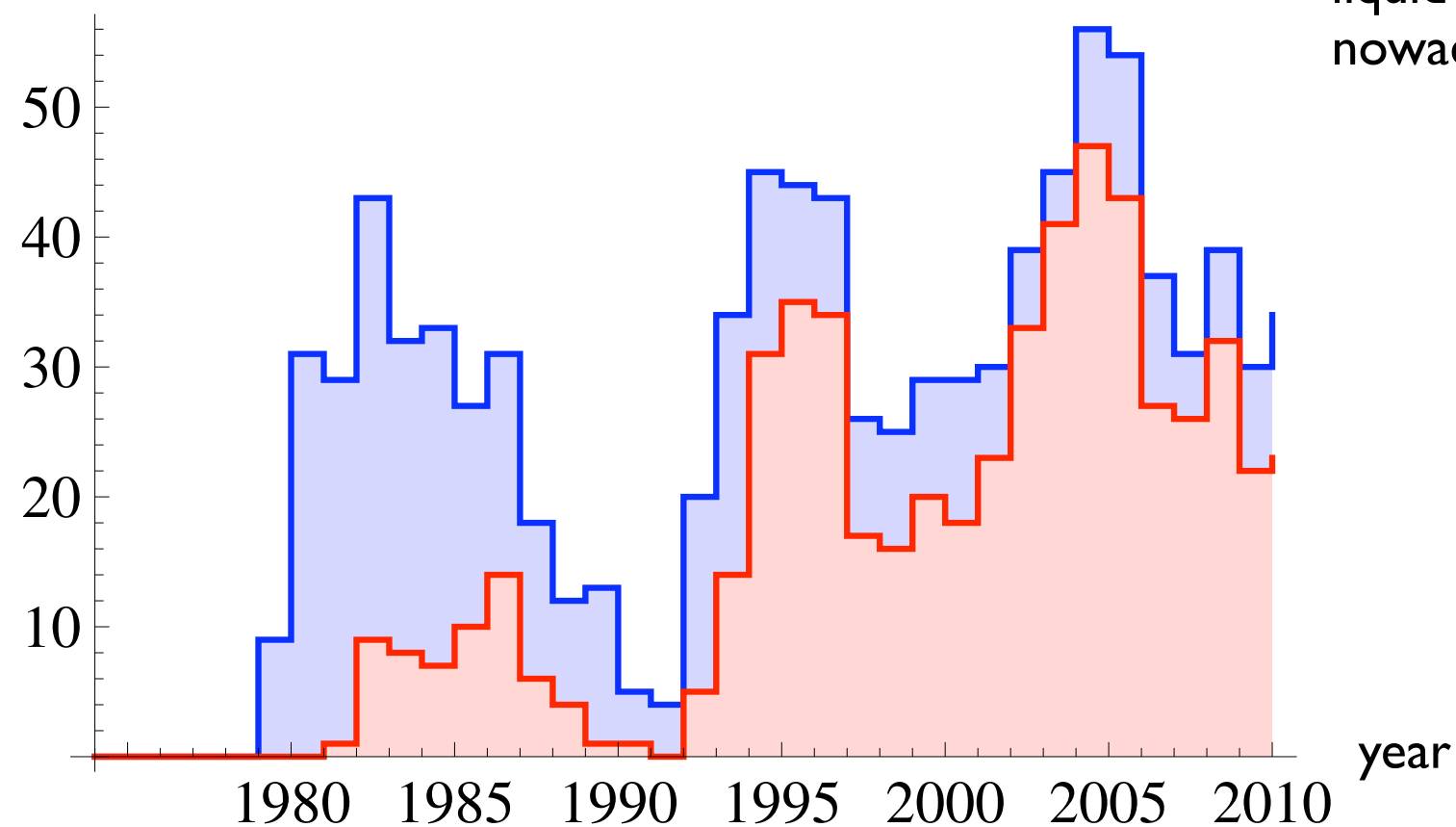
Birth: February 25 1974 - Georgi & Glashow PRL32 Experiment: $\tau_p \gtrsim 10^{30}$ yr Reines, Crouch (1974 PRL March 4)

Grand Unifications

lifetimes enormous $\Rightarrow$ need for large detectors

i.e., water Cherenkov,
liquid scintillators,
nowadays liquid argon...

of works @ inSPIRE



Infancy

Theory: $\tau_p \sim 10^{31}$ yr Georgi, Quinn, Weinberg 1974

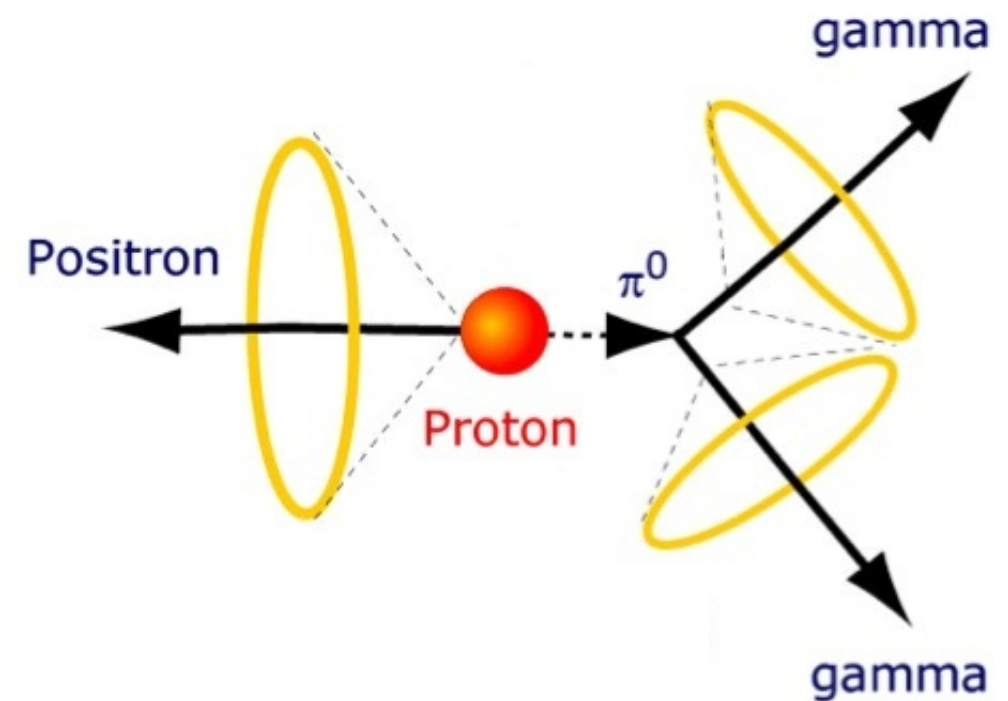
Birth: February 25 1974 - Georgi & Glashow PRL32 Experiment: $\tau_p \gtrsim 10^{30}$ yr Reines, Crouch (1974 PRL March 4)

Proton decay in water

“Golden channel”: $p \rightarrow \pi^0 e^+$
 $\pi^0 \rightarrow 2\gamma$

$$p_\pi = p_e = 459 \text{ MeV}$$

$$p_{\gamma/\pi(\text{rest})} = 68 \text{ MeV}$$

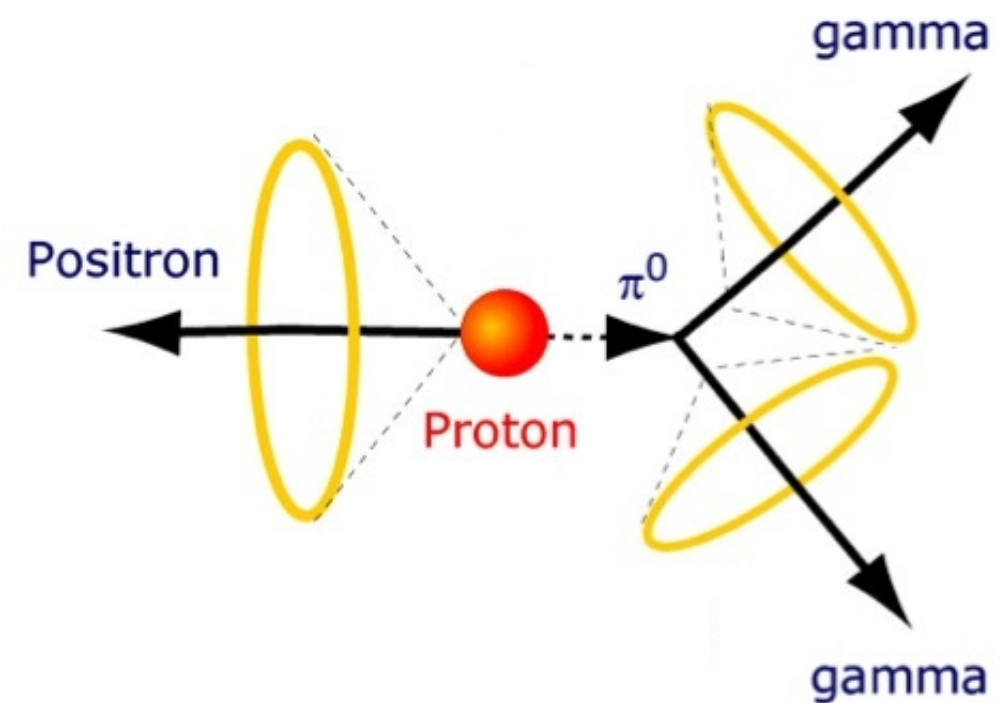
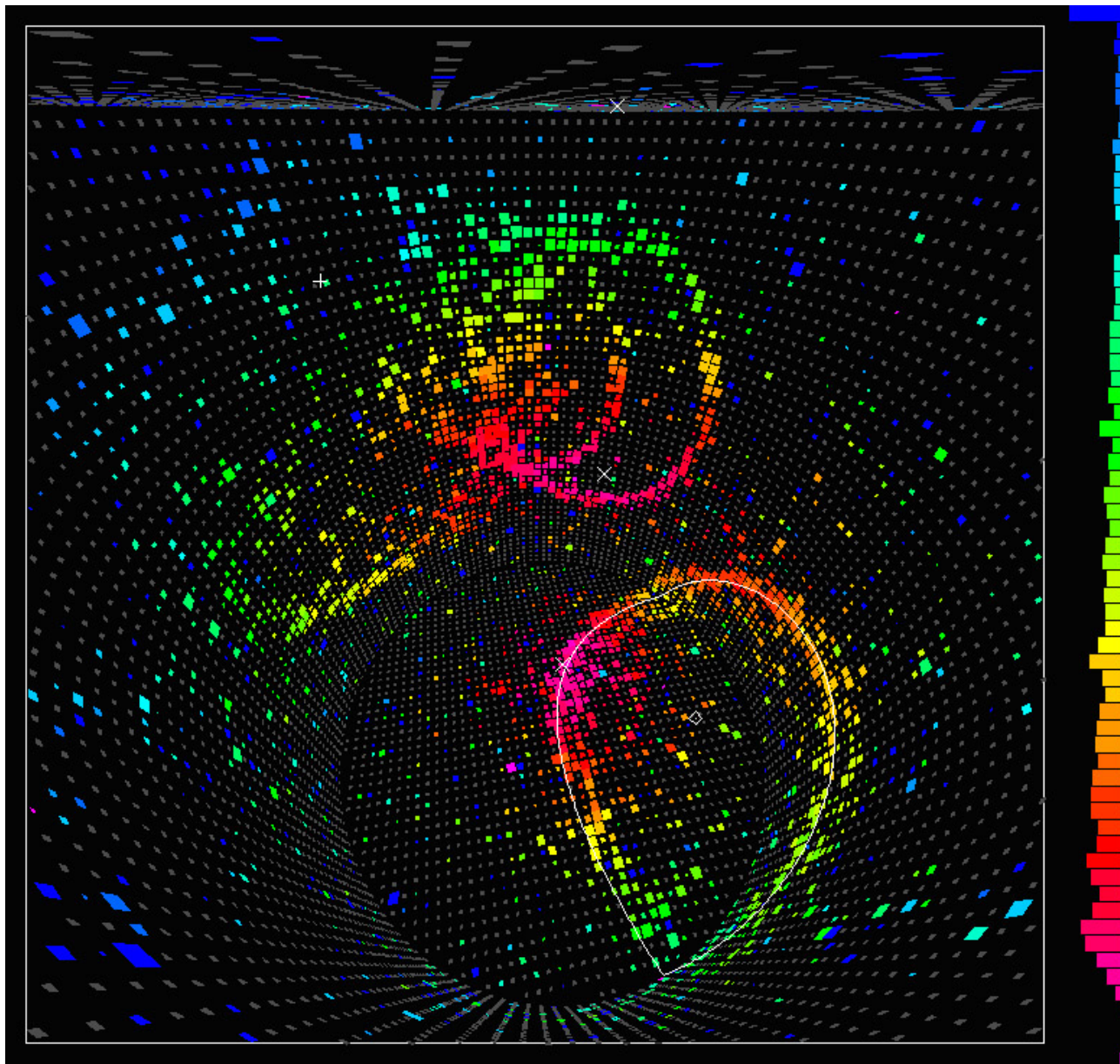


Proton decay in water

“Golden channel”: $p \rightarrow \pi^0 e^+$
 $\pi^0 \rightarrow 2\gamma$

$$p_\pi = p_e = 459 \text{ MeV}$$

$$p_{\gamma/\pi(\text{rest})} = 68 \text{ MeV}$$



Proton decay in water

“Silver channel”: $p \rightarrow K^+ \nu$ $p_K = 340 \text{ MeV}$

Proton decay in water

“Silver channel”: $p \rightarrow K^+ \nu$ $p_K = 340 \text{ MeV}$ Kaons don't shine !

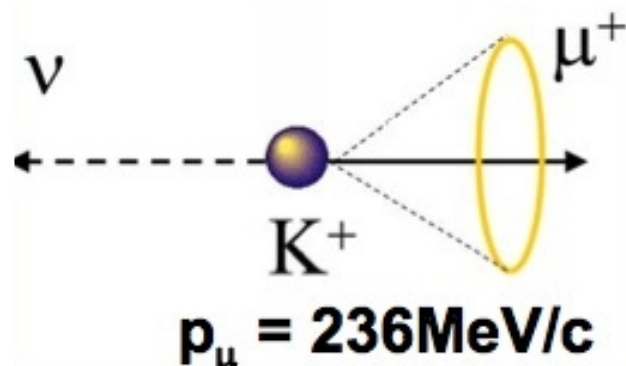
Proton decay in water

“Silver channel”: $p \rightarrow K^+ \nu$

$p_K = 340 \text{ MeV}$

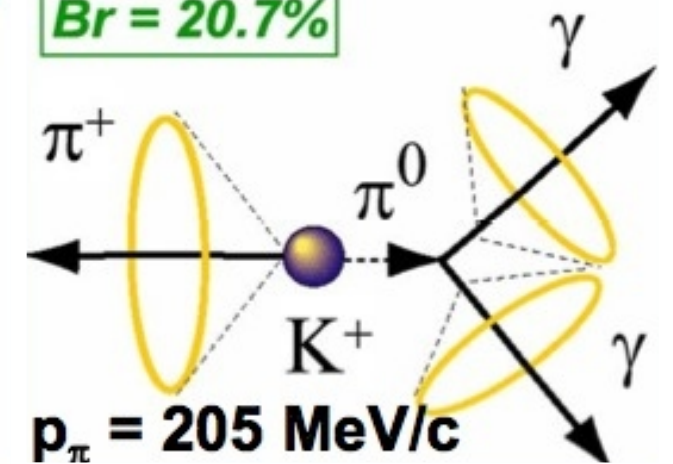
Kaons don't shine !

$K^+ \rightarrow \mu^+ + \bar{\nu}$ $Br = 63.5\%$



- single cone

$K^+ \rightarrow \pi^+ + \pi^0$ $Br = 20.7\%$



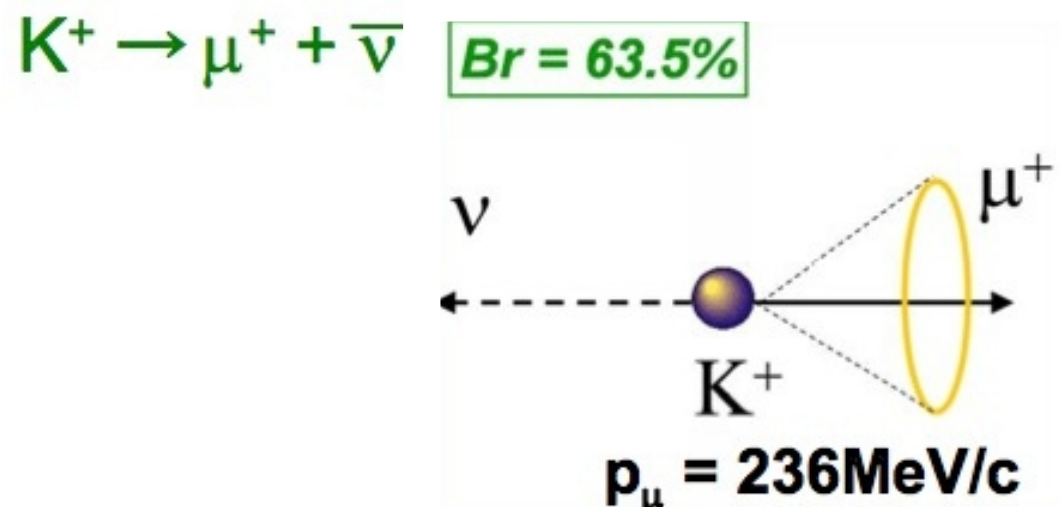
- 2 EM cones
- little opposite-side activity

Proton decay in water

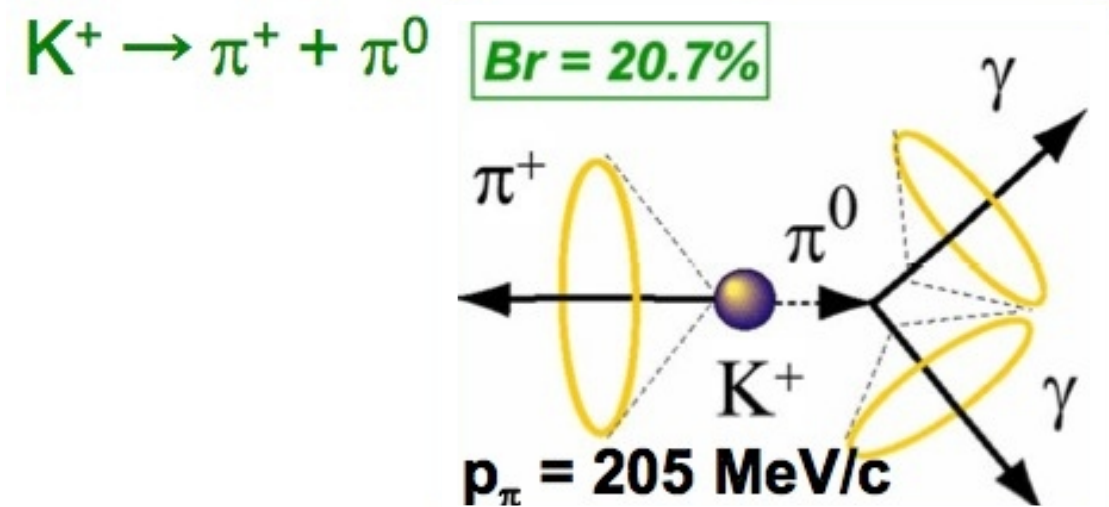
“Silver channel”: $p \rightarrow K^+ \nu$

$p_K = 340 \text{ MeV}$

Kaons don't shine !



- single cone



- 2 EM cones

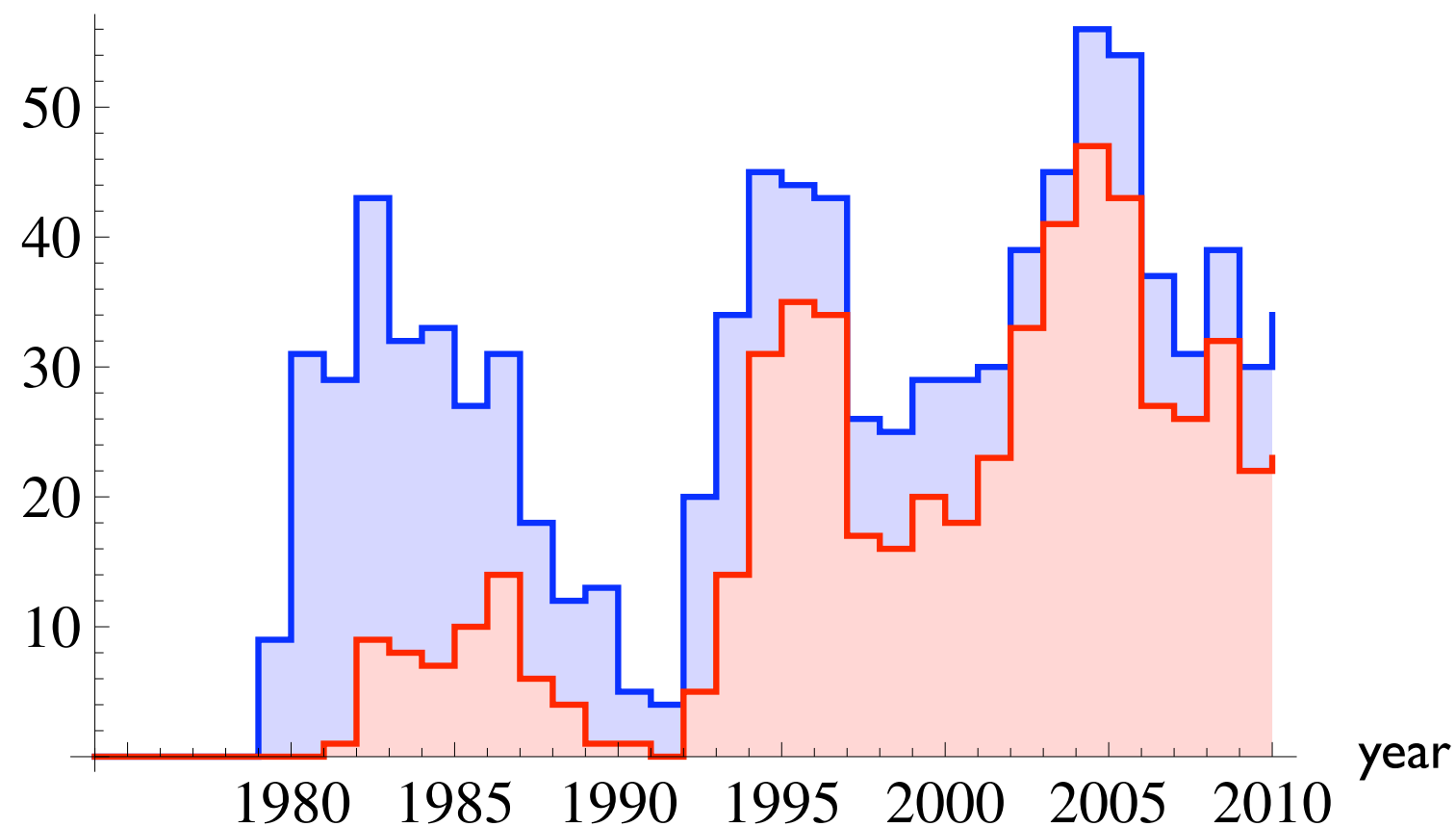
- little opposite-side activity

About one order of magnitude less sensitive than $p \rightarrow \pi^0 e^+$

Grand Unifications

The era of IMB (since 1982) & Kamiokande (since 1983)

of works @ inSPIRE



Golden age

Grand Unifications

FIRST WORKSHOP ON GRAND UNIFICATION

University of New Hampshire, April 1980

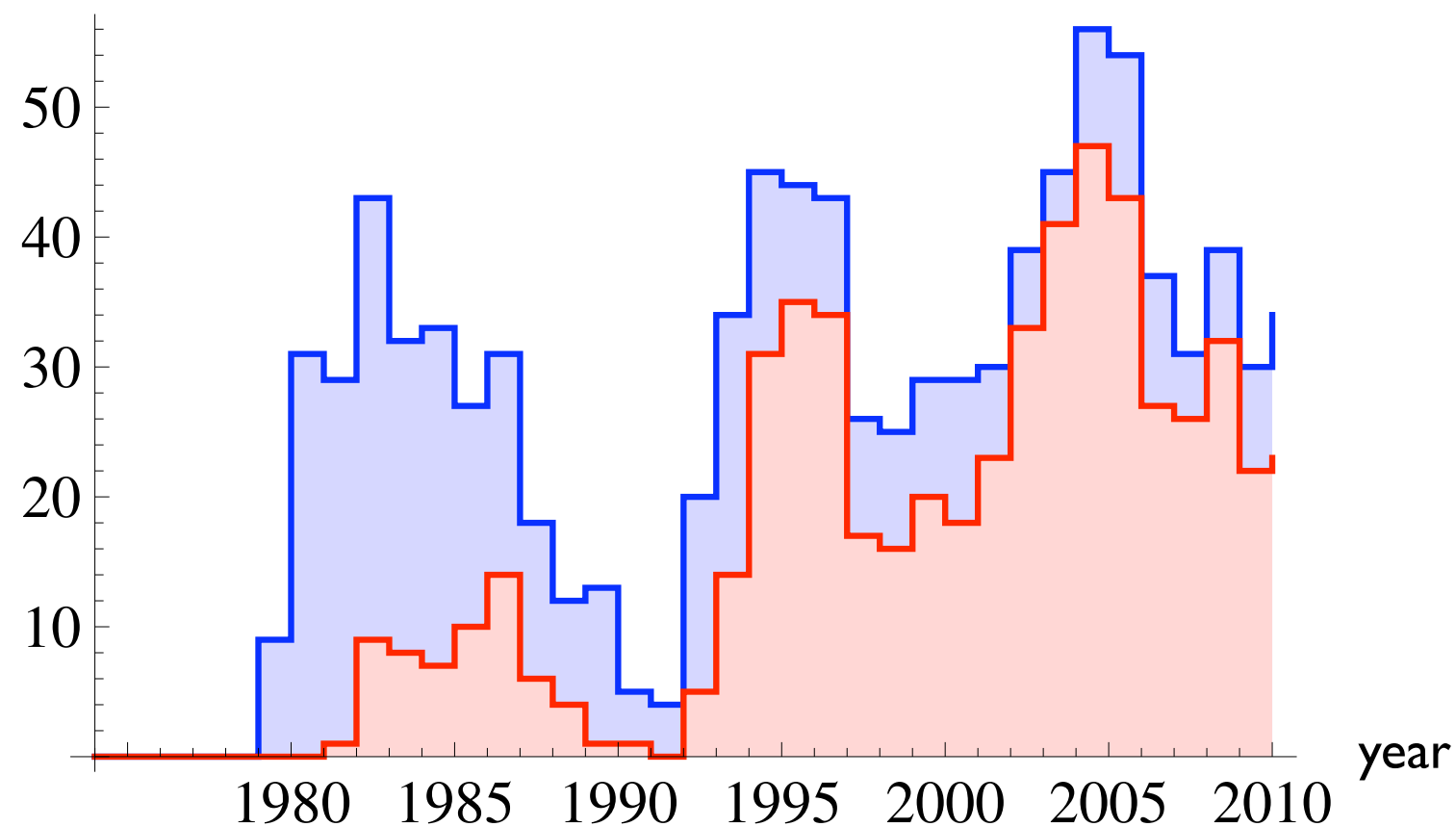
C	L	I	N	E	E	L	L	I	S	F	R	A
M	P	T	O	N	G	E	O	R	G	I	G	L
A	S	H	O	W	G	O	L	D	M	A	N	L
A	N	G	A	C	K	E	R	M	A	R	S	H
A	K	M	O	H	A	P	A	T	R	A	P	A
T	I	R	A	M	O	N	D	R	E	I	N	E
S	R	U	E	G	G	R	U	J	U	L	A	S
A	L	A	M	S	E	C	C	O	S	L	A	N
S	K	Y	S	T	E	I	G	M	A	N	S	T
E	I	N	B	E	R	G	S	U	L	A	K	T
U	R	N	E	R	W	A	L	I	W	E	I	N
B	E	R	G	W	I	L	S	O	N	W	I	N
N	W	I	T	T	E	N	Y	I	L	D	I	Z

Editors: Paul H. Frampton,
Sheldon L. Glashow,
Asim Yildiz.

Grand Unifications

The era of IMB (since 1982) & Kamiokande (since 1983)

of works @ inSPIRE

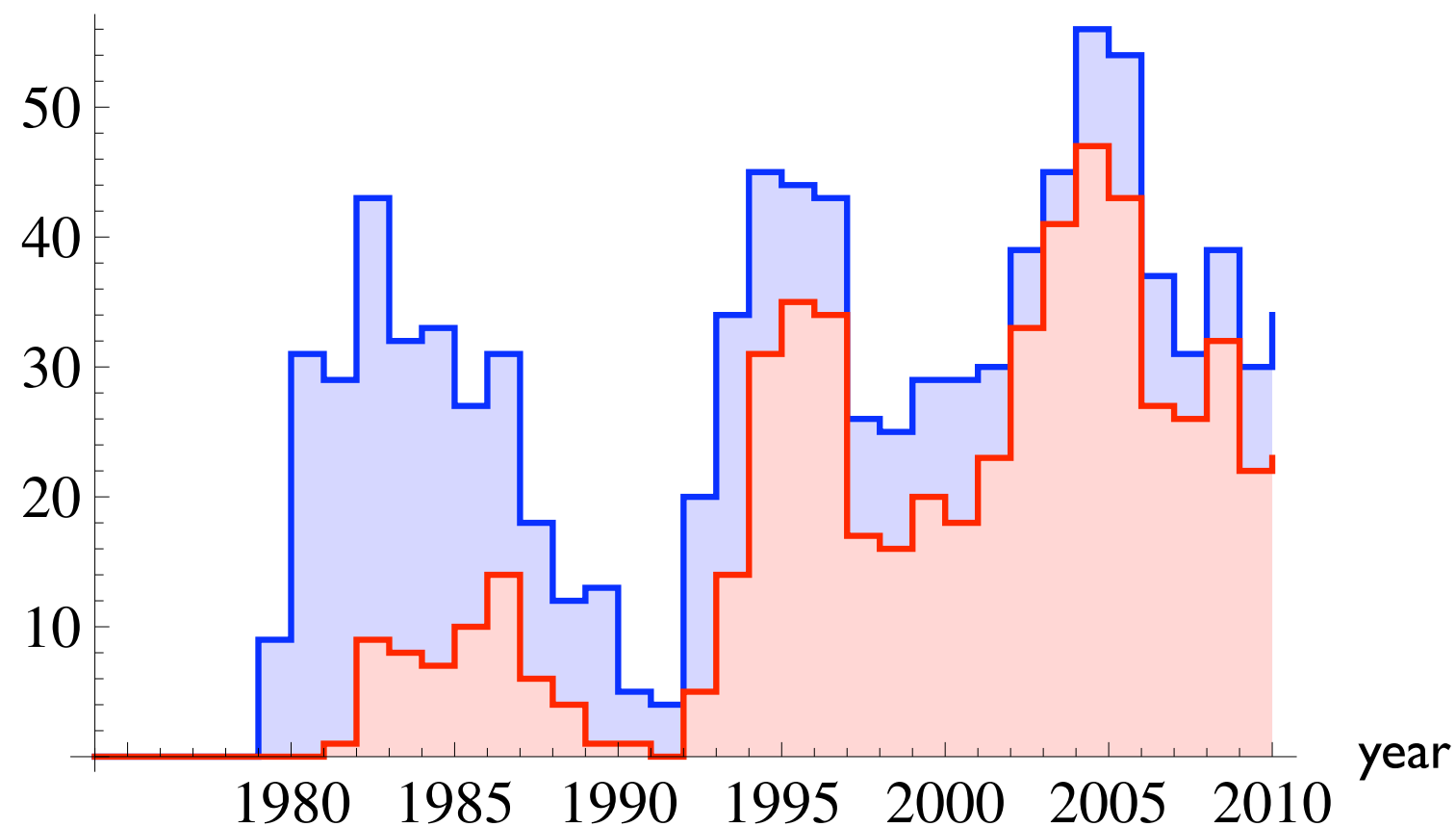


Experiment: $\tau_p \gtrsim 2.6 \times 10^{32}$ yr **Kamiokande (1989)**

Grand Unifications

The era of IMB (since 1982) & Kamiokande (since 1983)

of works @ inSPIRE



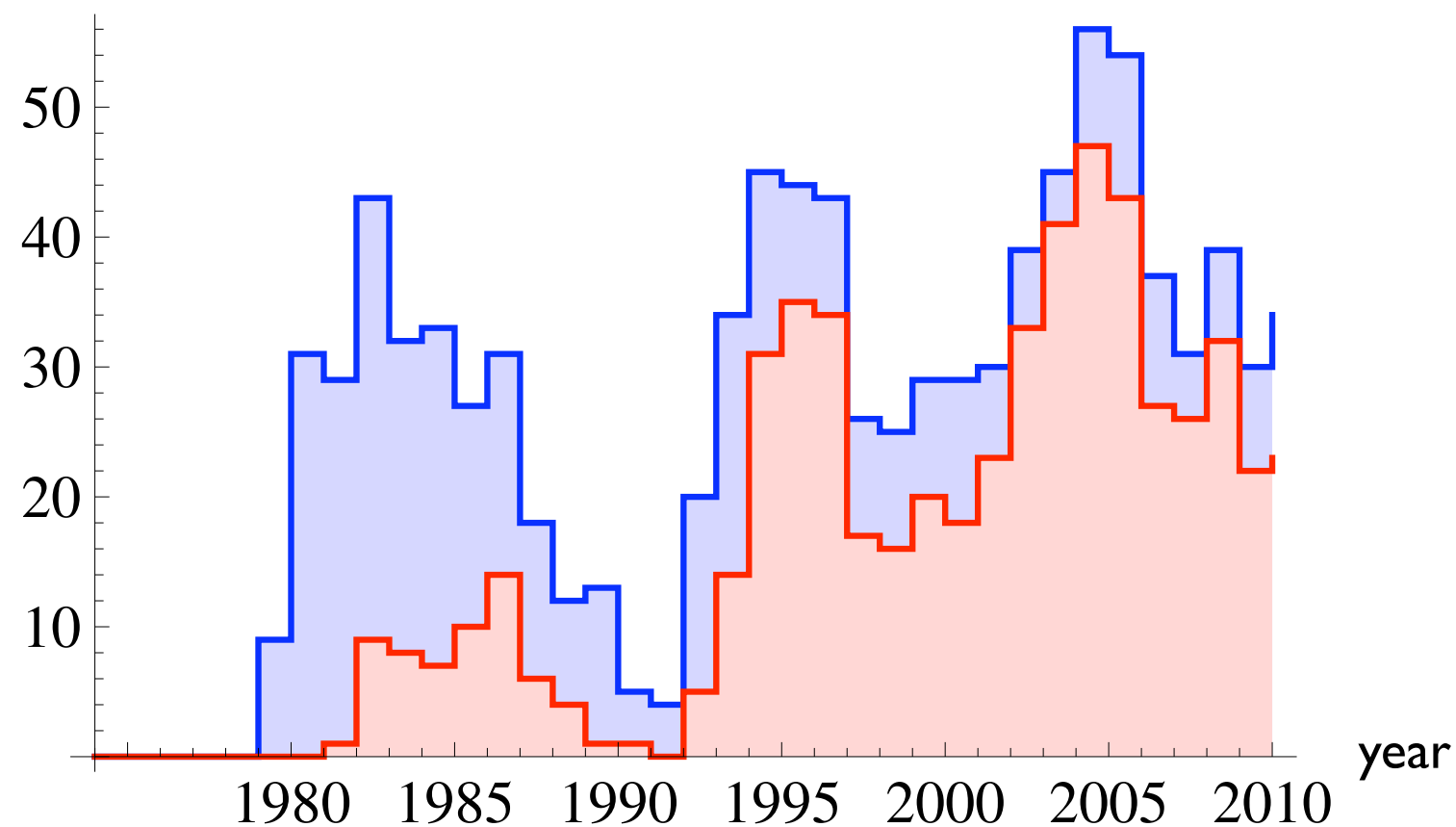
Great
depression

Experiment: $\tau_p \gtrsim 2.6 \times 10^{32} \text{ yr}$ Kamiokande (1989)

Grand Unifications

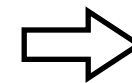
The era of IMB (since 1982) & Kamiokande (since 1983)

of works @ inSPIRE



Great
depression

SPS indicated more than 10 sigma
discrepancy in the weak mixing angle



failure of G-G

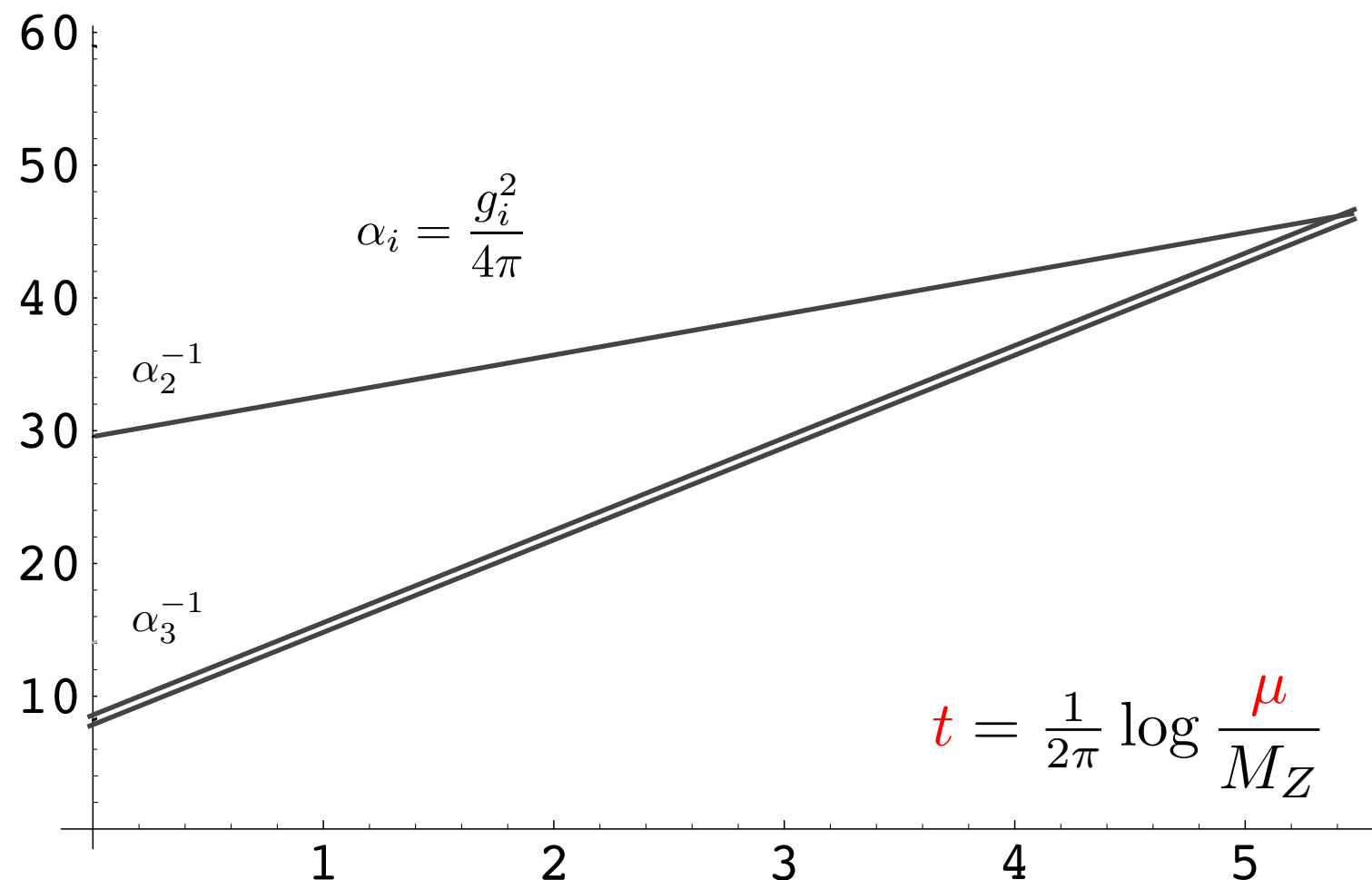
Experiment: $\tau_p \gtrsim 2.6 \times 10^{32} \text{ yr}$ Kamiokande (1989)

Failure of the GG model

Failure of the GG model

Normalization: $\text{Tr}\{T_a, T_b\} = \frac{1}{2}\delta_{ab}$

$$T_{24}^{\bar{5}} = \sqrt{\frac{3}{5}} \begin{pmatrix} +\frac{1}{3} & & & & \\ & +\frac{1}{3} & & & \\ & & +\frac{1}{3} & & \\ & & & -\frac{1}{2} & \\ & & & & -\frac{1}{2} \end{pmatrix}$$

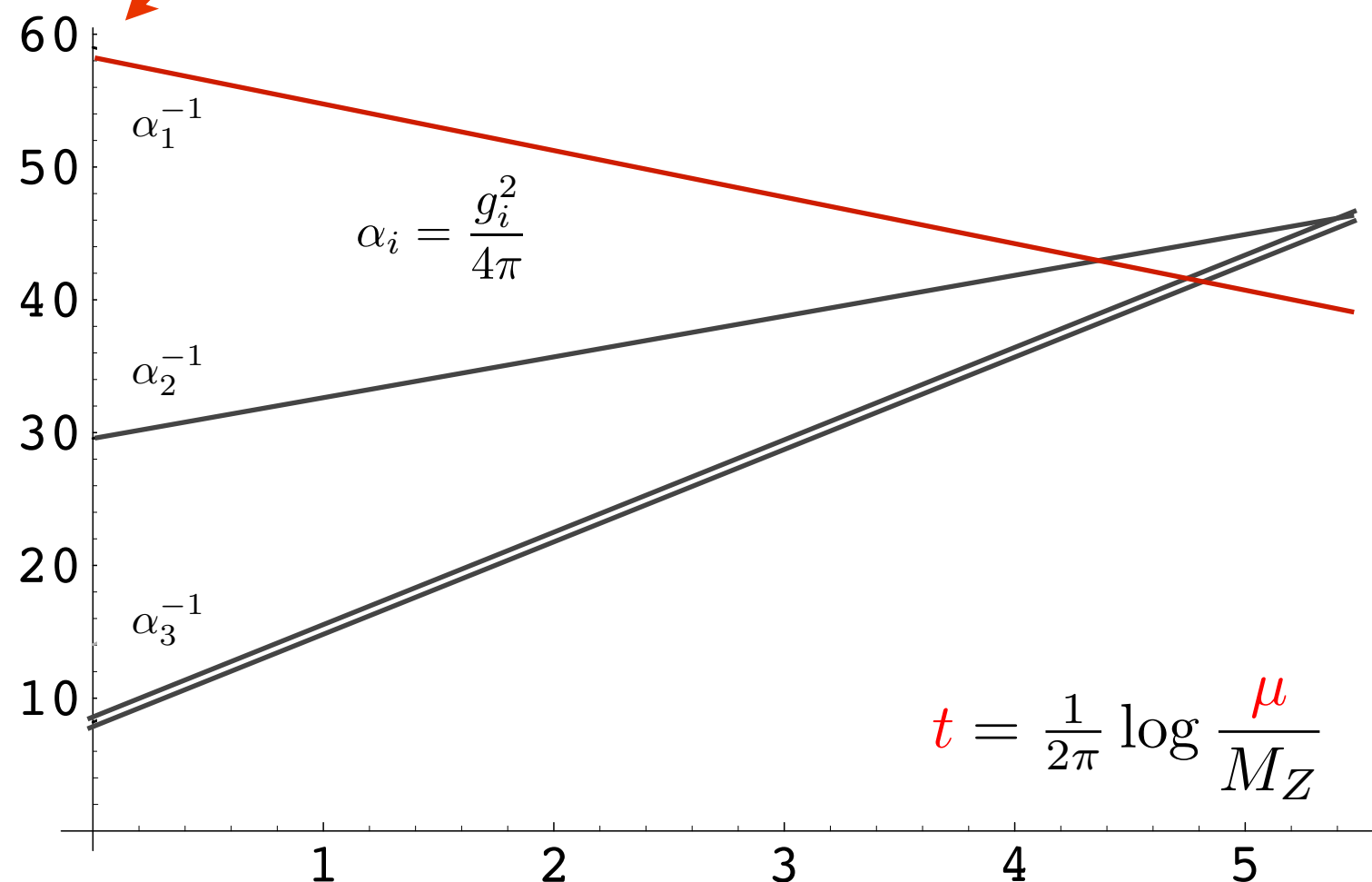


Failure of the GG model

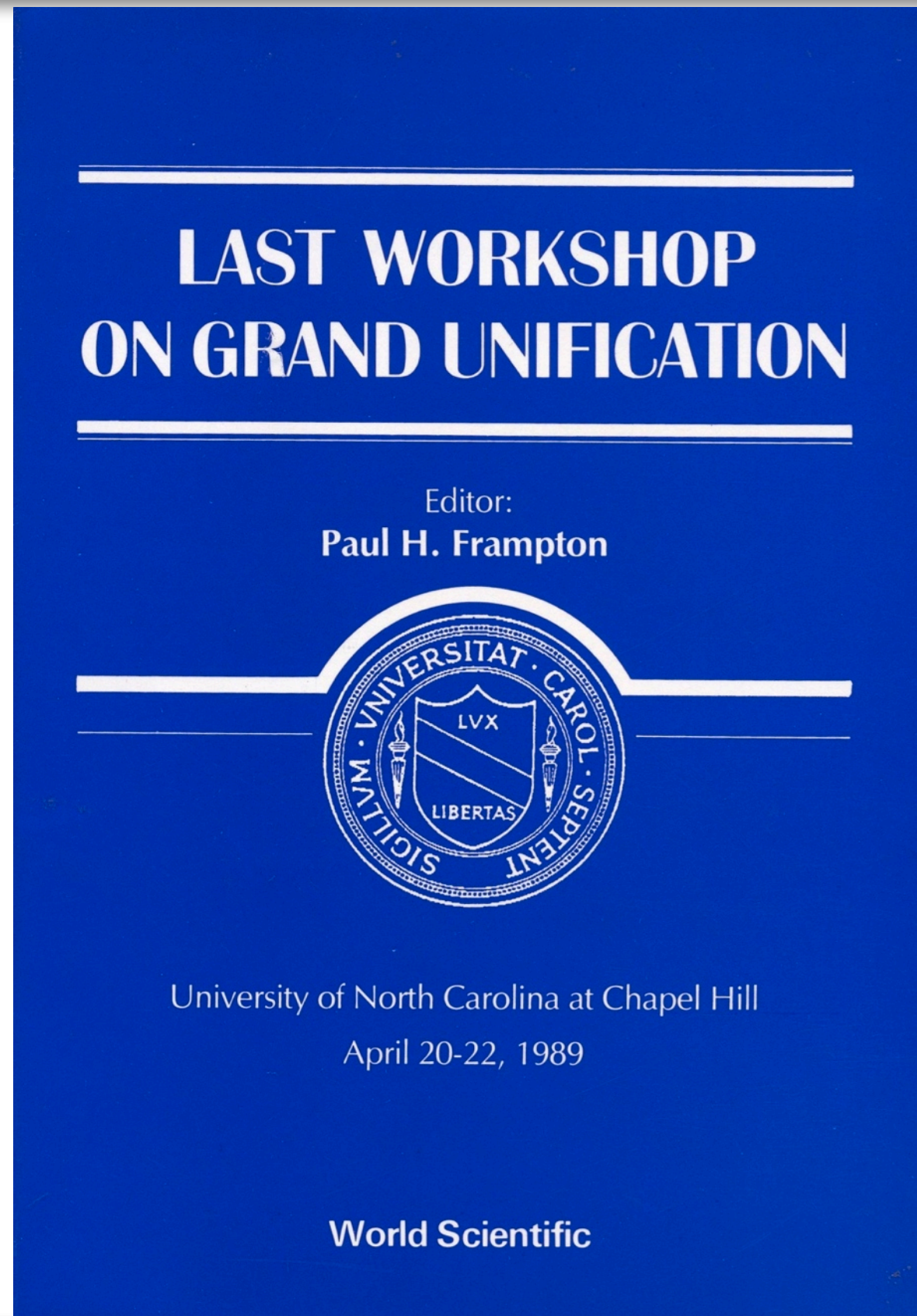
Normalization: $\text{Tr}\{T_a, T_b\} = \frac{1}{2}\delta_{ab}$

$$T_{24}^{\bar{5}} = \sqrt{\frac{3}{5}} \begin{pmatrix} +\frac{1}{3} & & & & \\ & +\frac{1}{3} & & & \\ & & +\frac{1}{3} & & \\ & & & -\frac{1}{2} & \\ & & & & -\frac{1}{2} \end{pmatrix}$$

this sets the initial condition
for the U(1) coupling



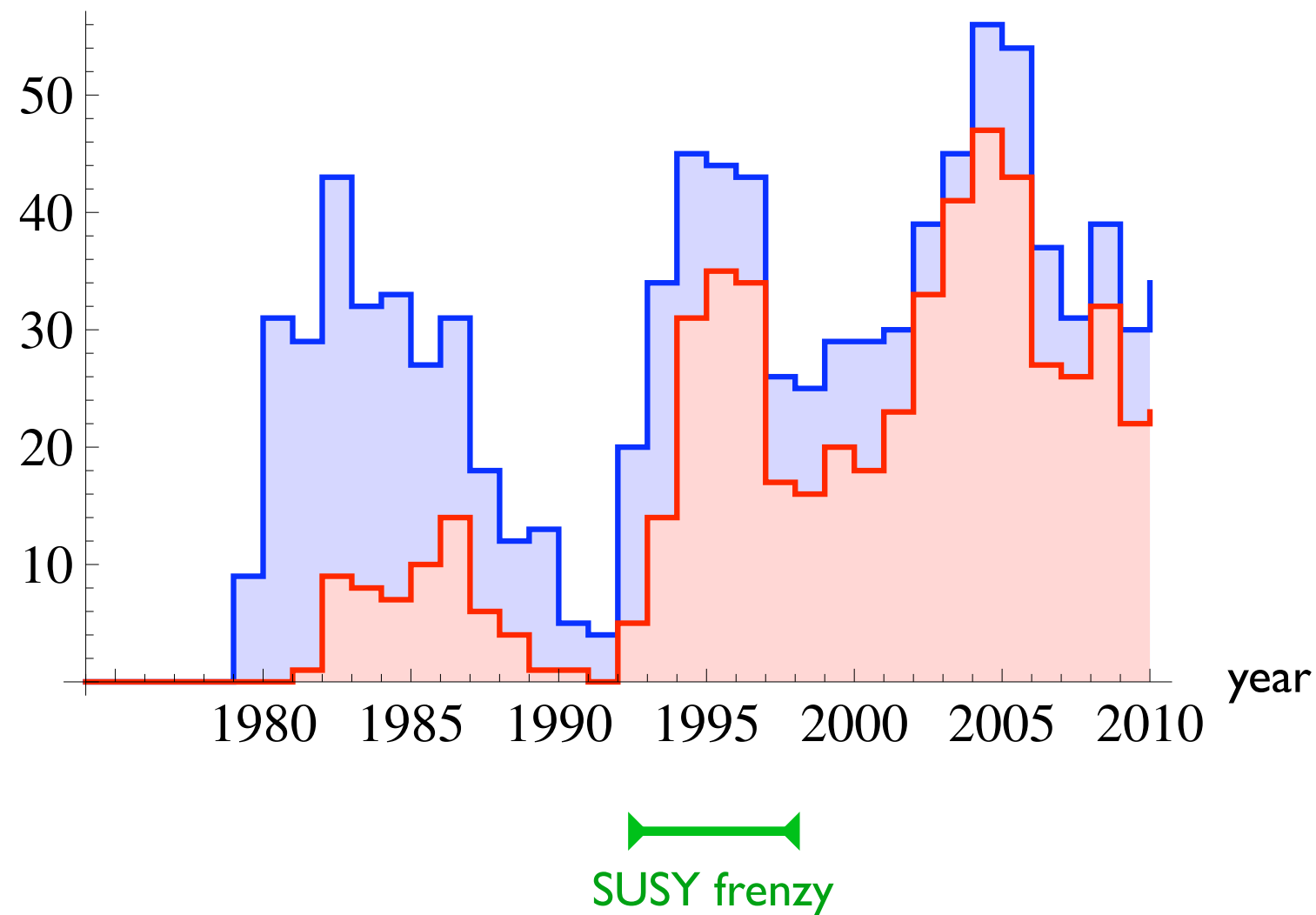
Failure of the GG model



SUSY frenzy

The era of IMB (since 1982) & Kamiokande (since 1983)

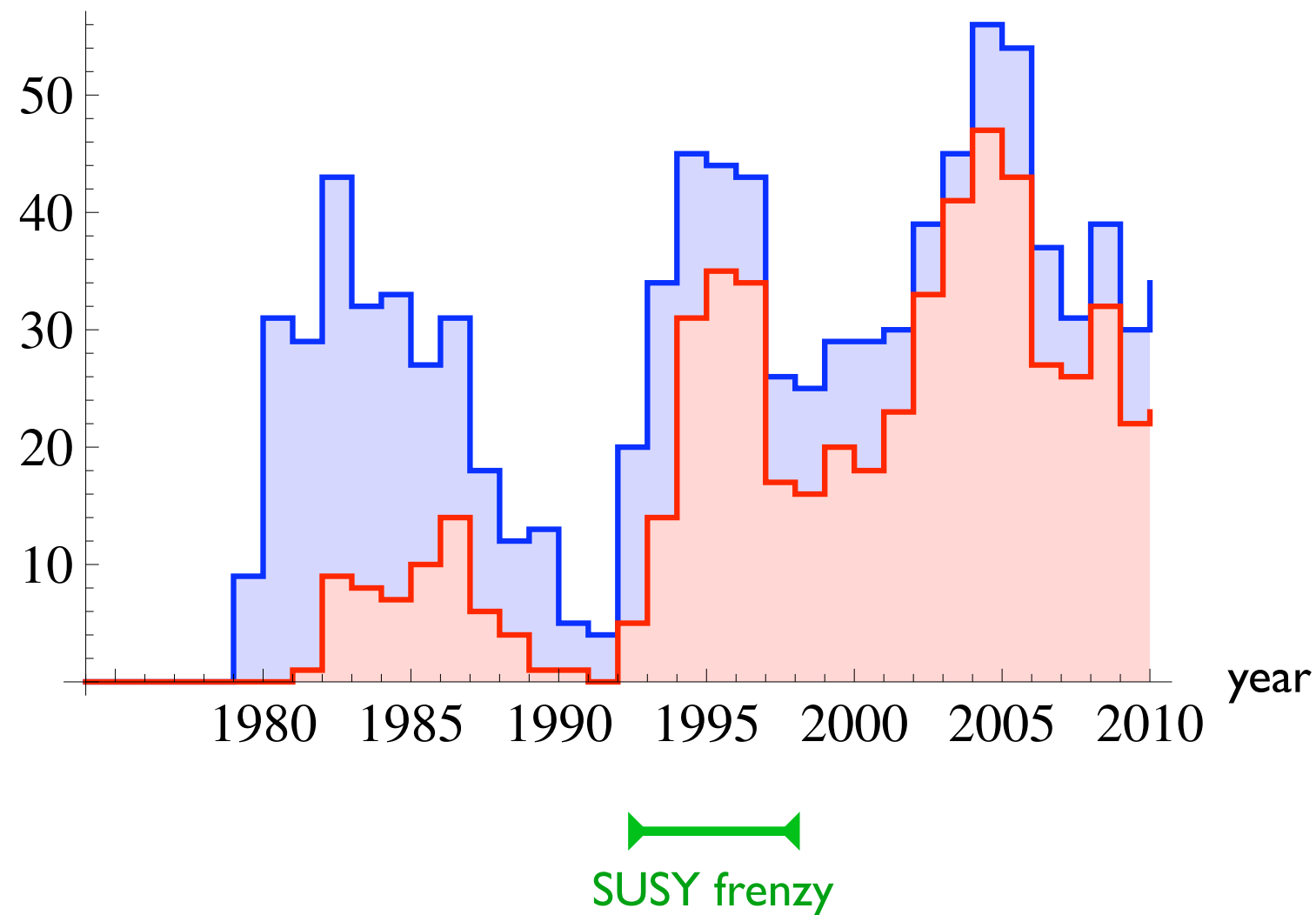
of works @ inSPIRE



SUSY frenzy

The era of IMB (since 1982) & Kamiokande (since 1983)

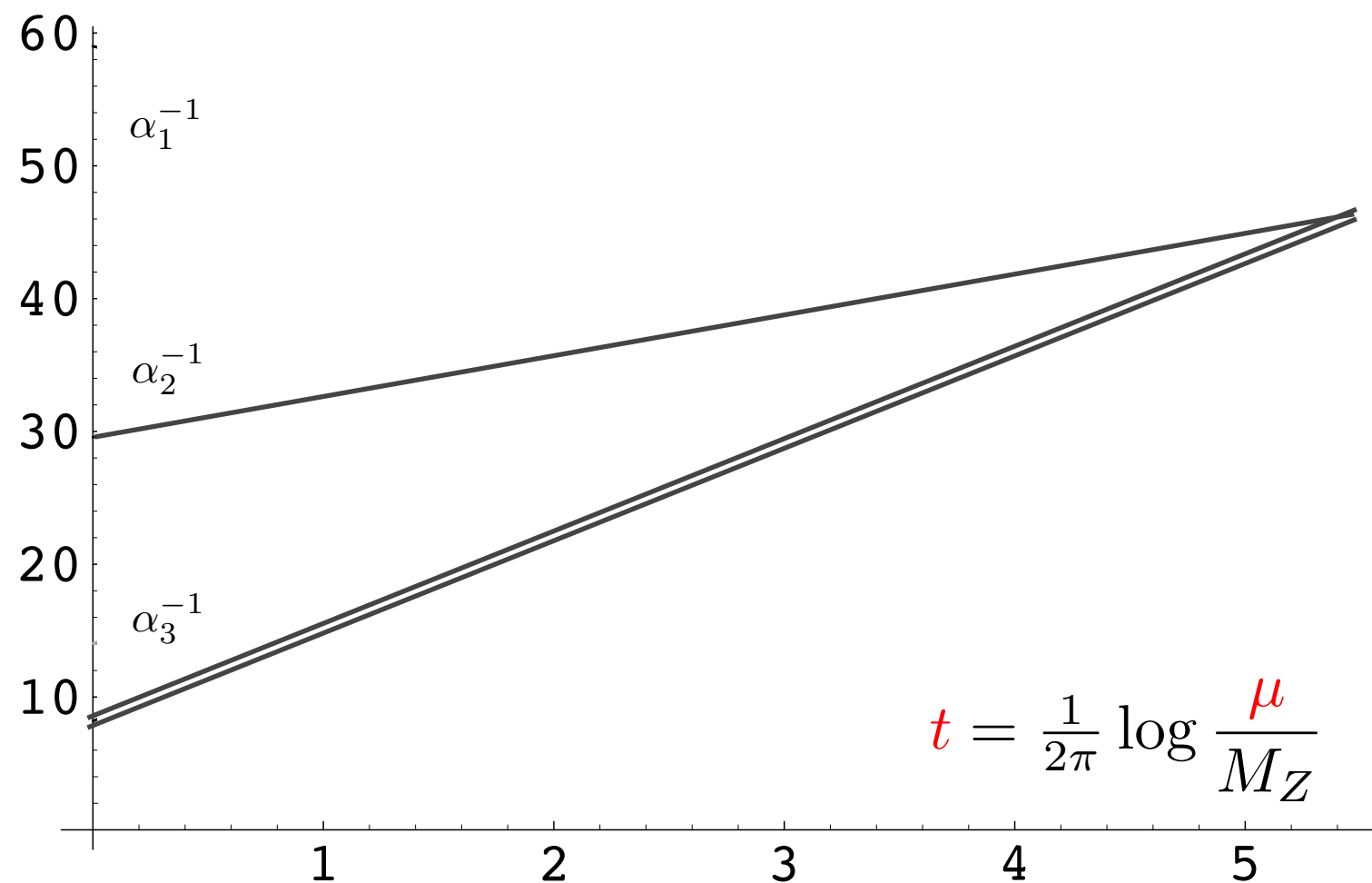
of works @ inSPIRE



LEP precision data support MSSM-like unification $\Rightarrow$ interest in SUSY GUTs

GUTs & TeV-scale supersymmetry...

Running gauge couplings in the SM $+X + \Delta$



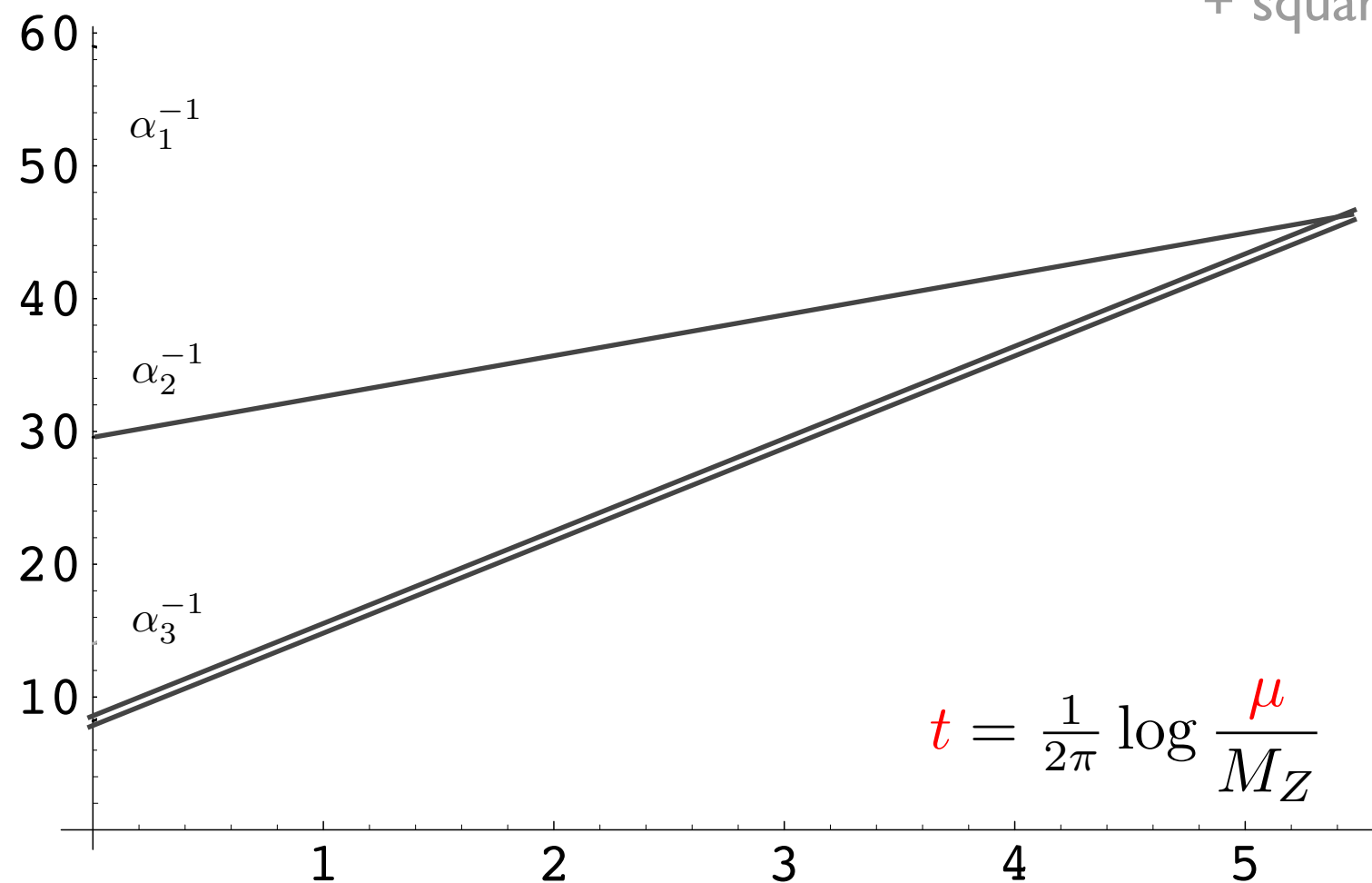
GUTs & TeV-scale supersymmetry...

Running gauge couplings in the SM $+ X + \Delta$ and a TeV-scale supersymmetry

+ gauginos

+ higgsinos

+ squarks and sleptons



$$t = \frac{1}{2\pi} \log \frac{\mu}{M_Z}$$

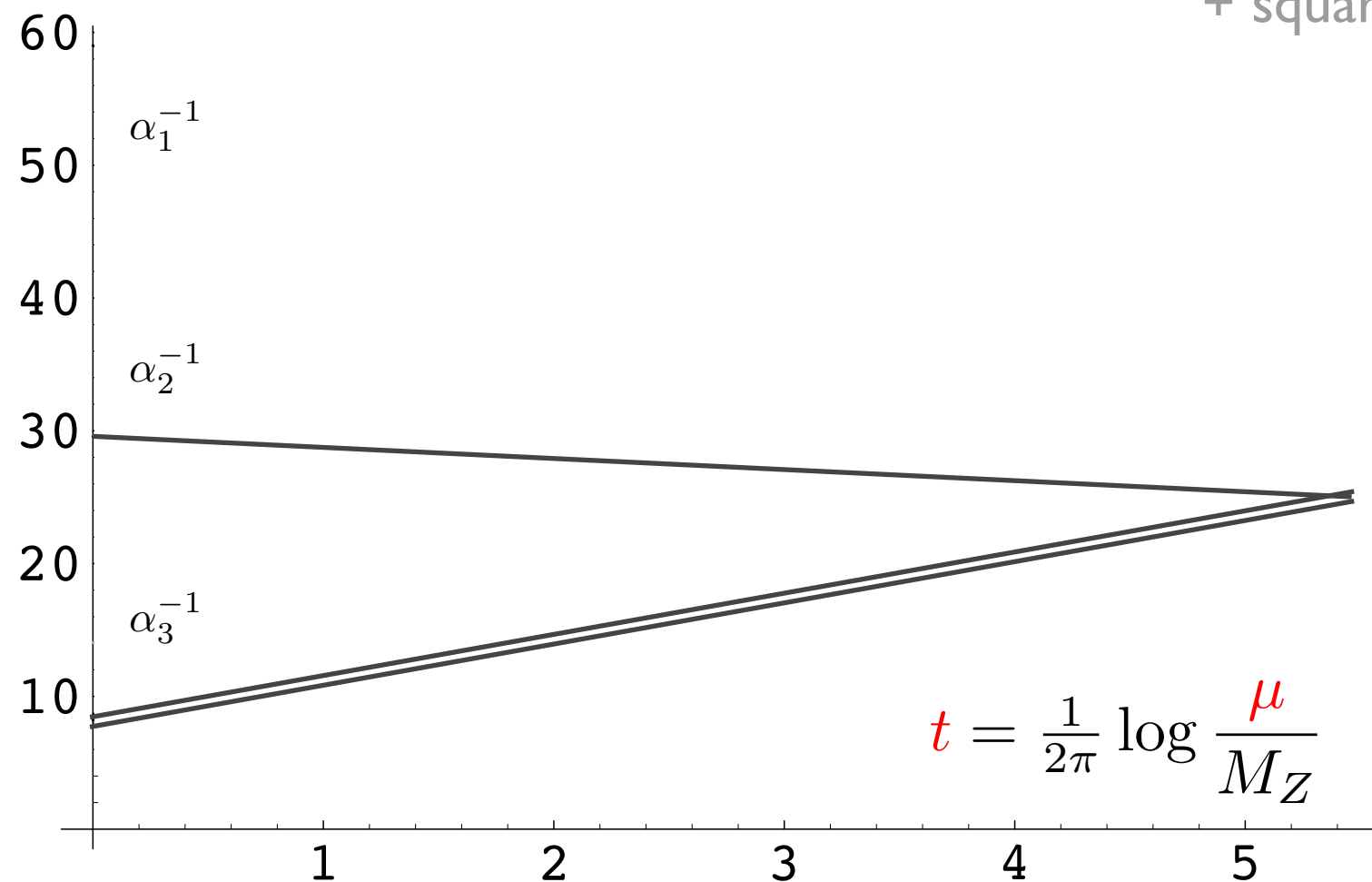
GUTs & TeV-scale supersymmetry...

Running gauge couplings in the SM $+X + \Delta$ and a TeV-scale supersymmetry

+ gauginos

+ higgsinos

+ squarks and sleptons



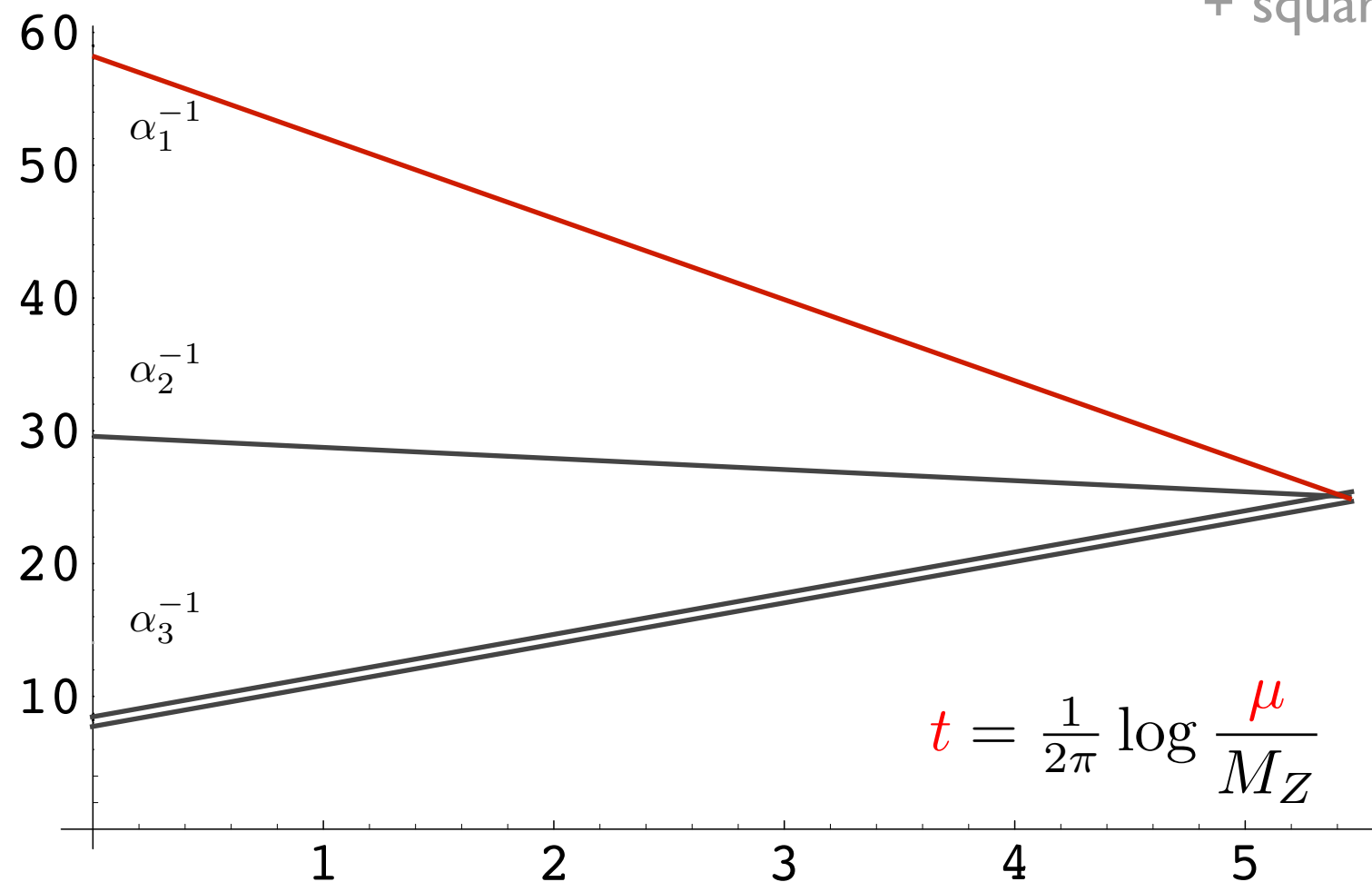
GUTs & TeV-scale supersymmetry...

Running gauge couplings in the SM $+X + \Delta$ and a TeV-scale supersymmetry

+ gauginos

+ higgsinos

+ squarks and sleptons



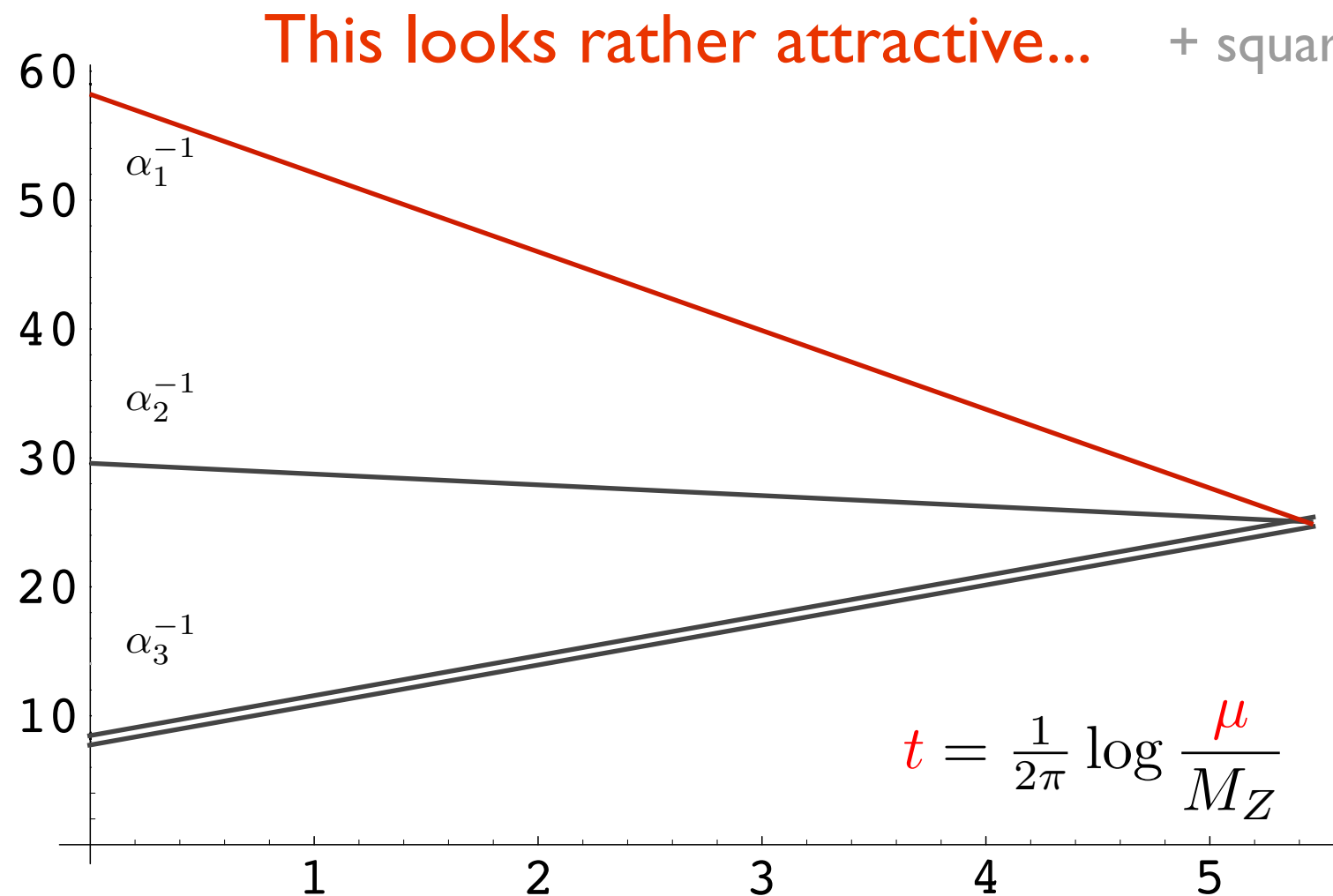
GUTs & TeV-scale supersymmetry...

Running gauge couplings in the SM $+X + \Delta$ and a TeV-scale supersymmetry

+ gauginos

+ higgsinos

+ squarks and sleptons



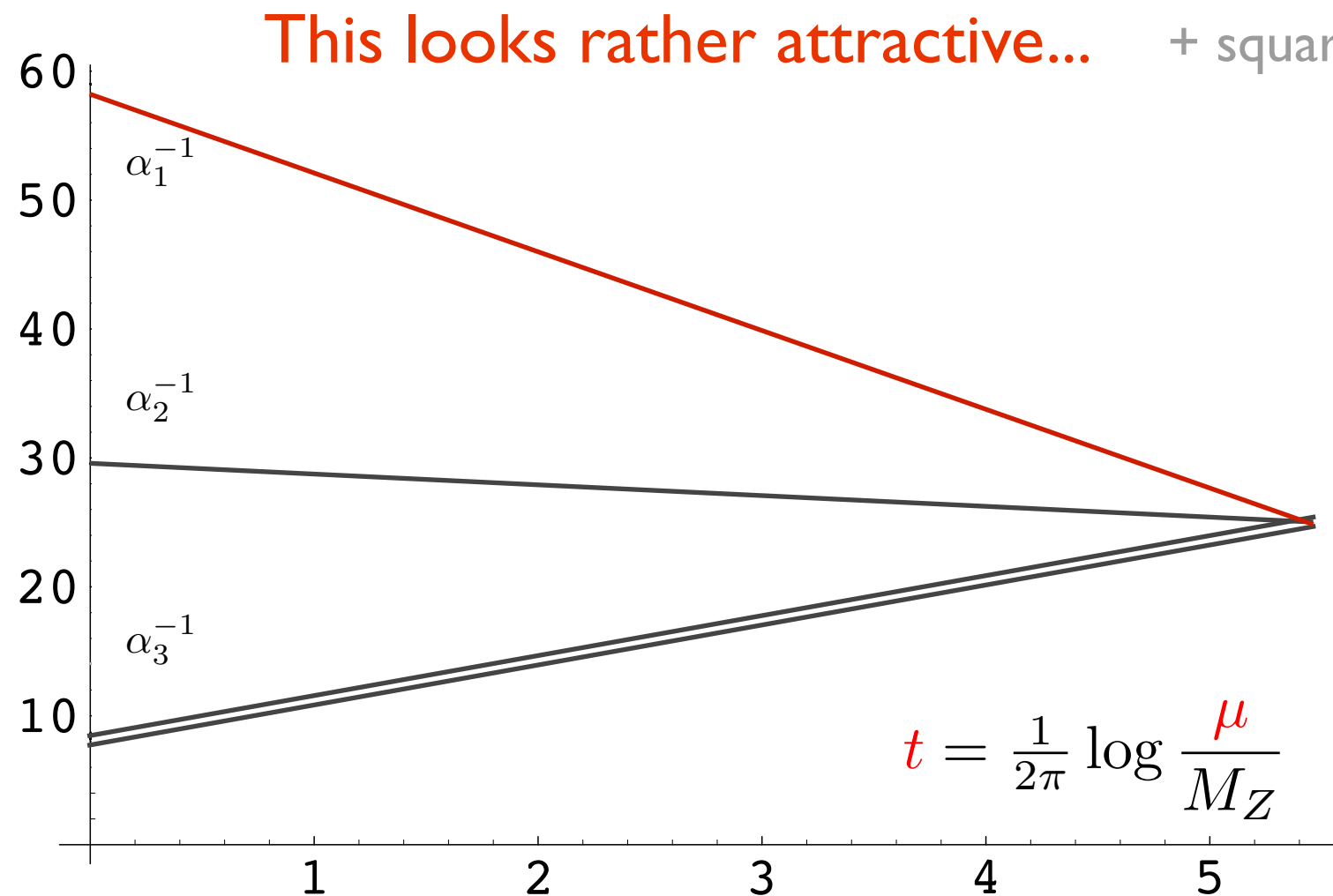
GUTs & TeV-scale supersymmetry...

Running gauge couplings in the SM $+X + \Delta$ and a TeV-scale supersymmetry

+ gauginos

+ higgsinos

+ squarks and sleptons



First reliable calculations...

GUTs & TeV-scale supersymmetry...

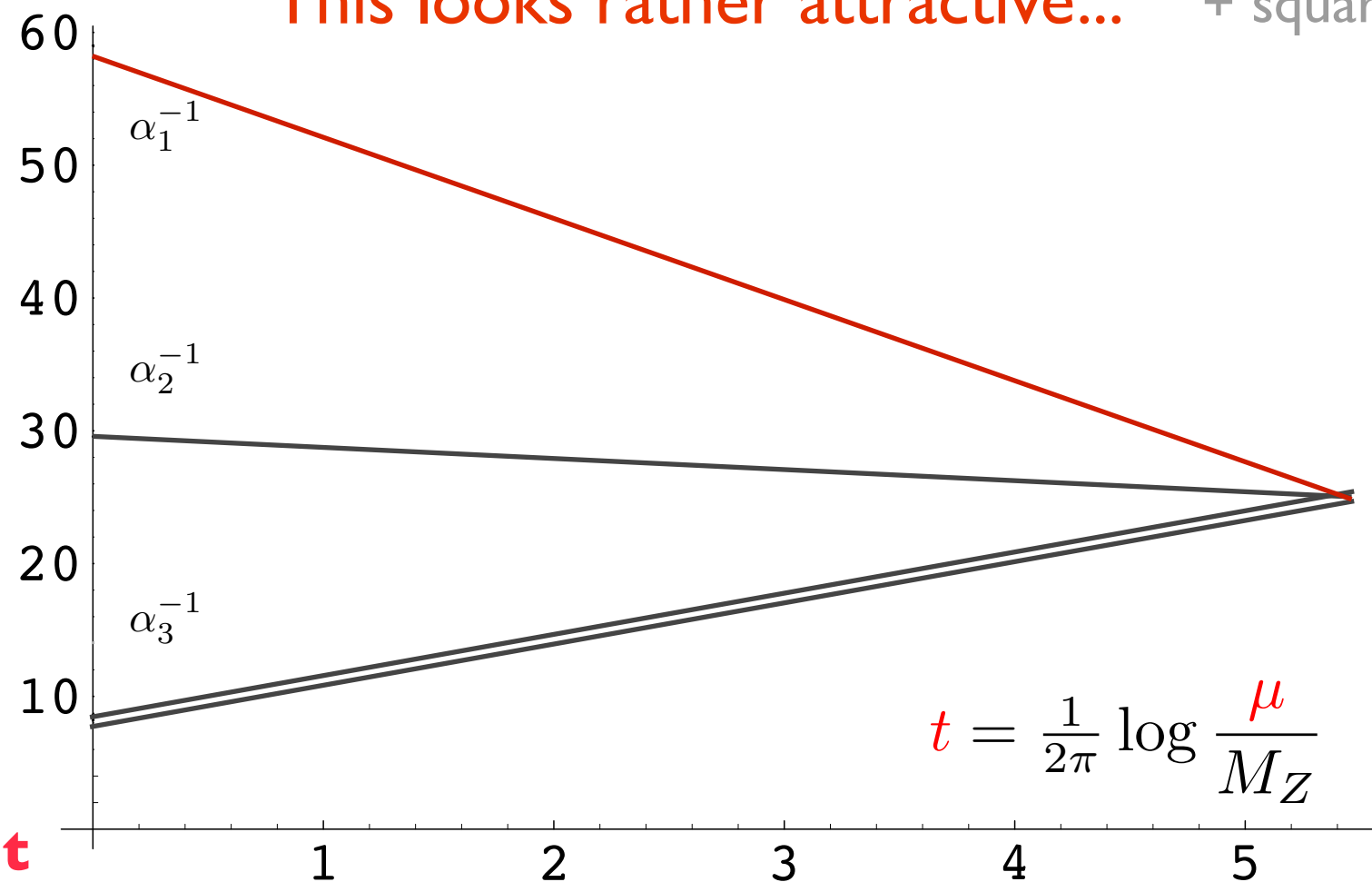
Running gauge couplings in the SM $+ X + \Delta$ and a TeV-scale supersymmetry

+ gauginos

+ higgsinos

+ squarks and sleptons

This looks rather attractive...



to some extent



First reliable calculations...

GUTs & TeV-scale supersymmetry...

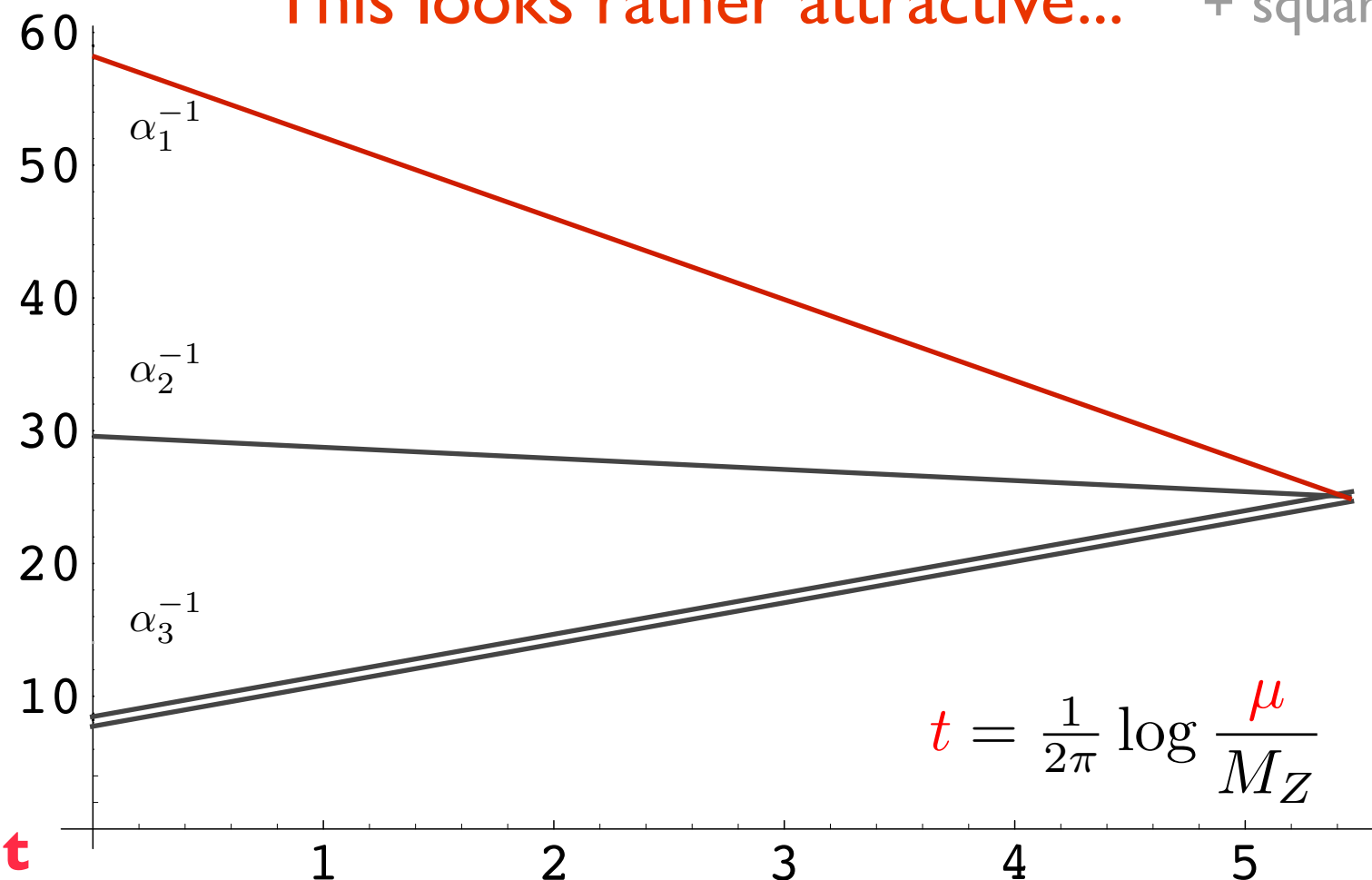
Running gauge couplings in the SM $+ X + \Delta$ and a TeV-scale supersymmetry

+ gauginos

+ higgsinos

+ squarks and sleptons

This looks rather attractive...



to some extent

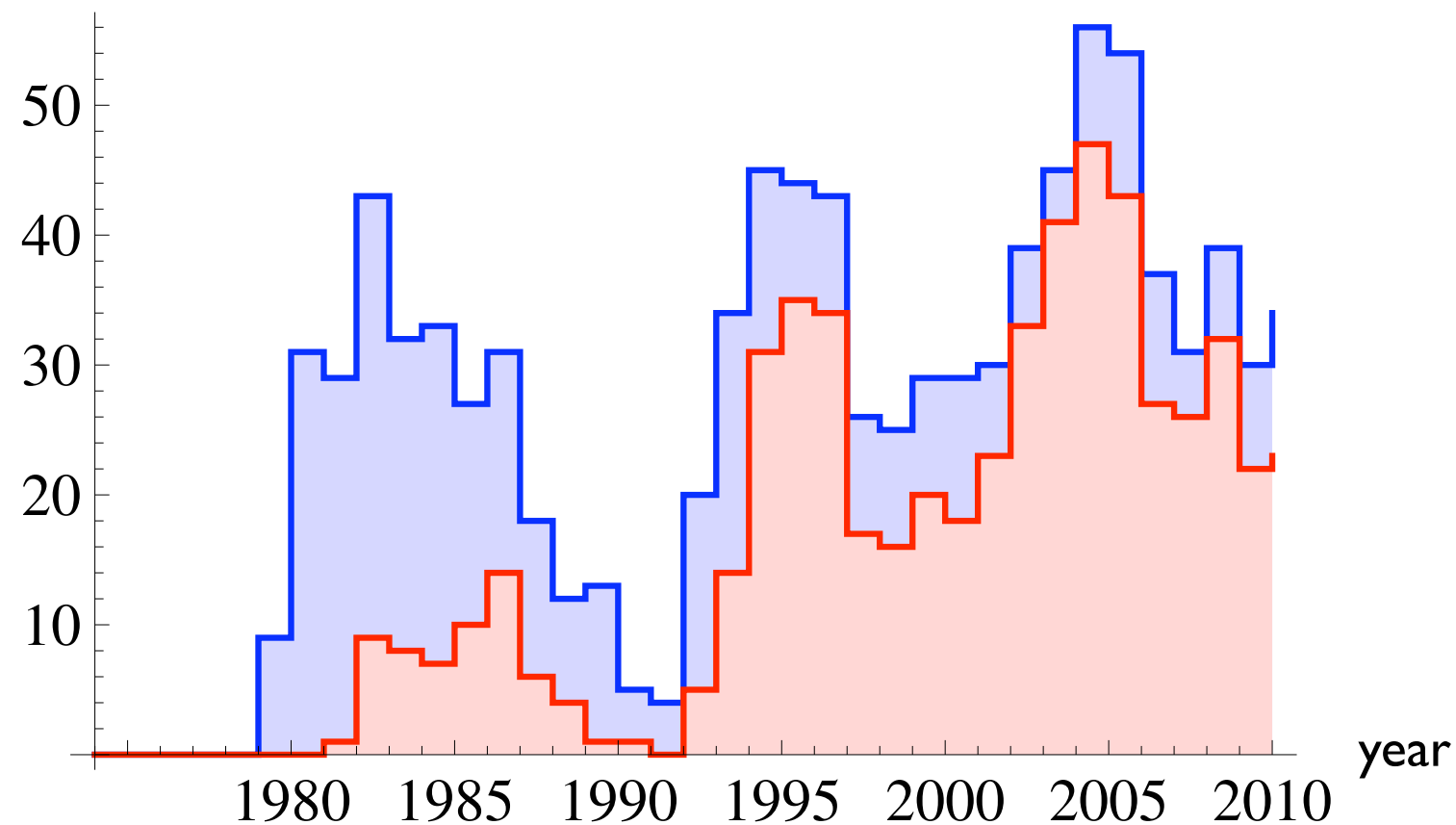


First reliable calculations... SUSY GG SU(5) model in trouble

d=5 proton decay

The era of IMB 3 (beg. of 1990's) & Super-K (since 1996)

of works @ inSPIRE



to some extent



First reliable calculations...

2nd depression: d=5 proton decay

$$\tau_p \gtrsim 8.5 \times 10^{32} \text{ yr}$$

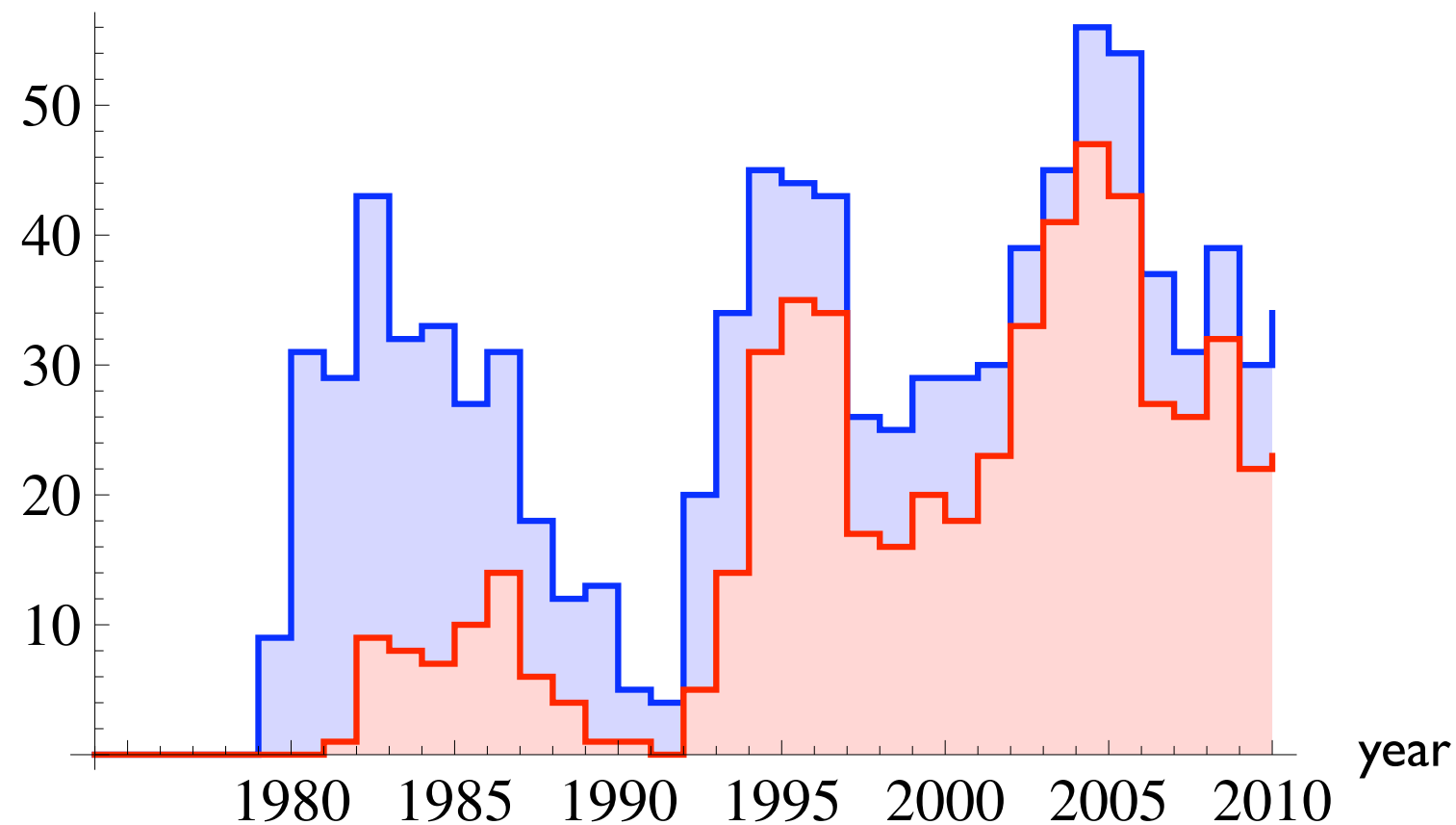
IMB-3 1999

SUSY GG SU(5) model in trouble

Neutrino data

Super-K (since 1996)

of works @ inSPIRE



“SO(10) era”

$$\tau_p \gtrsim 8.2 \times 10^{33} \text{ yr}$$

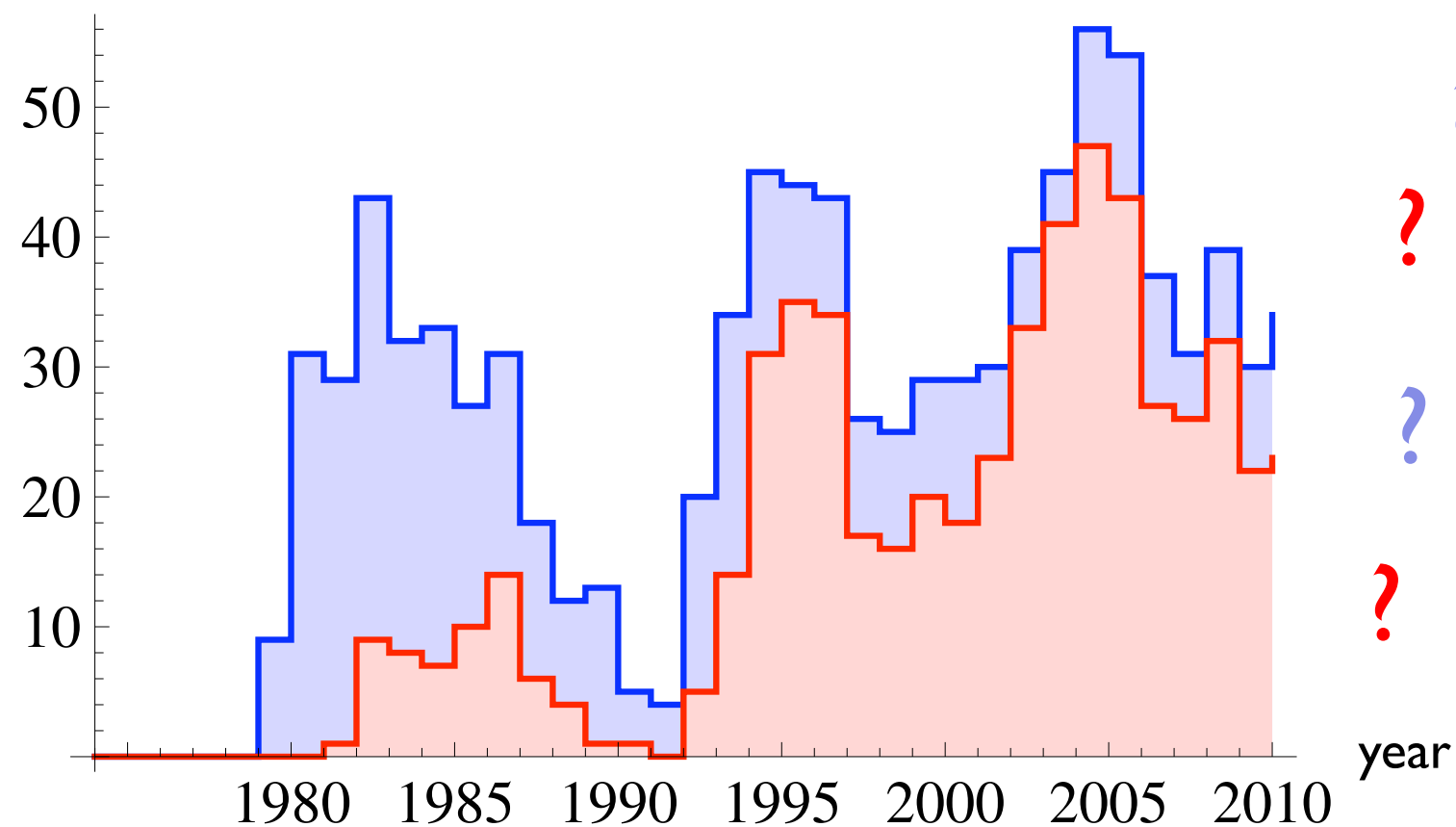
Super-K (2009)

neutrino masses and mixing constraints

Neutrino data

future: Hyper-Kamiokande(?), LBNE (?)

of works @ inSPIRE



“SO(10) era”

$$\tau_p \gtrsim 8.2 \times 10^{33} \text{ yr}$$

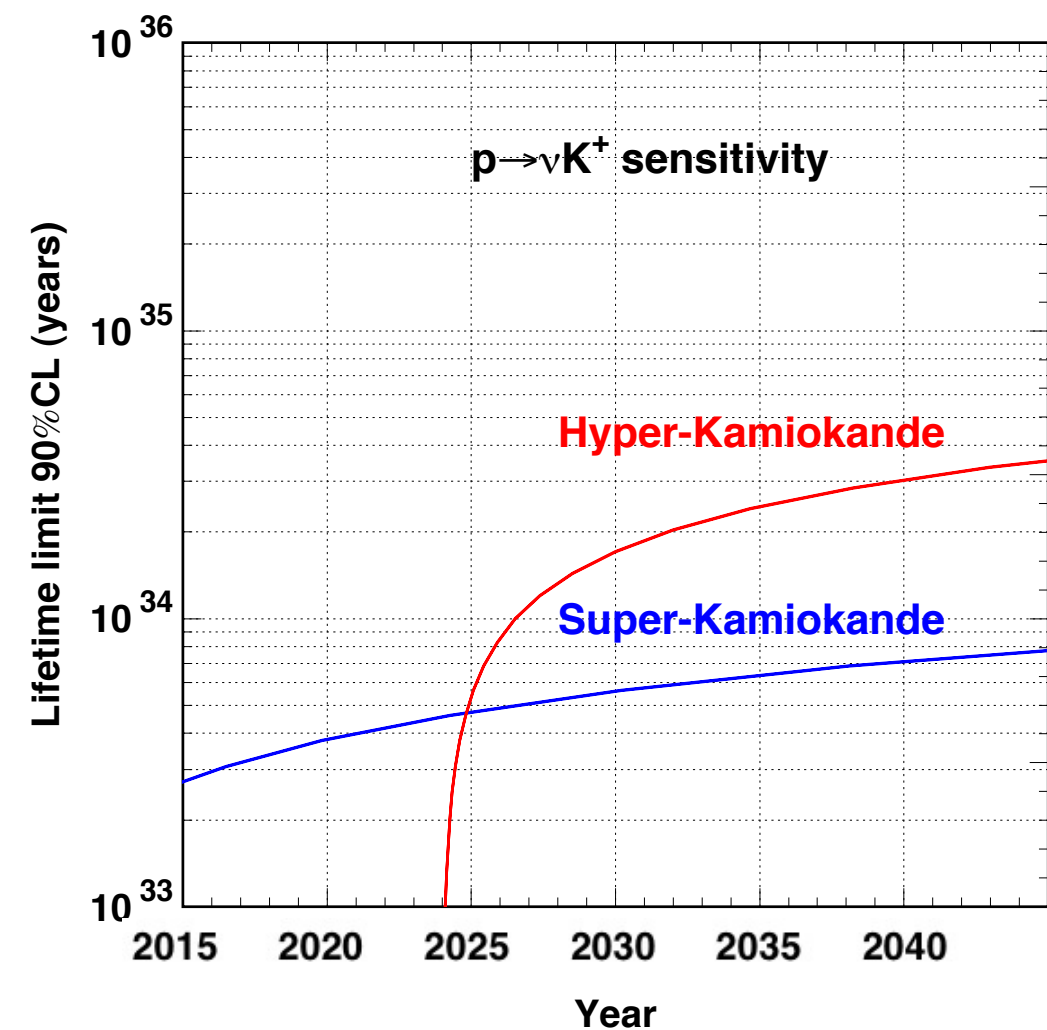
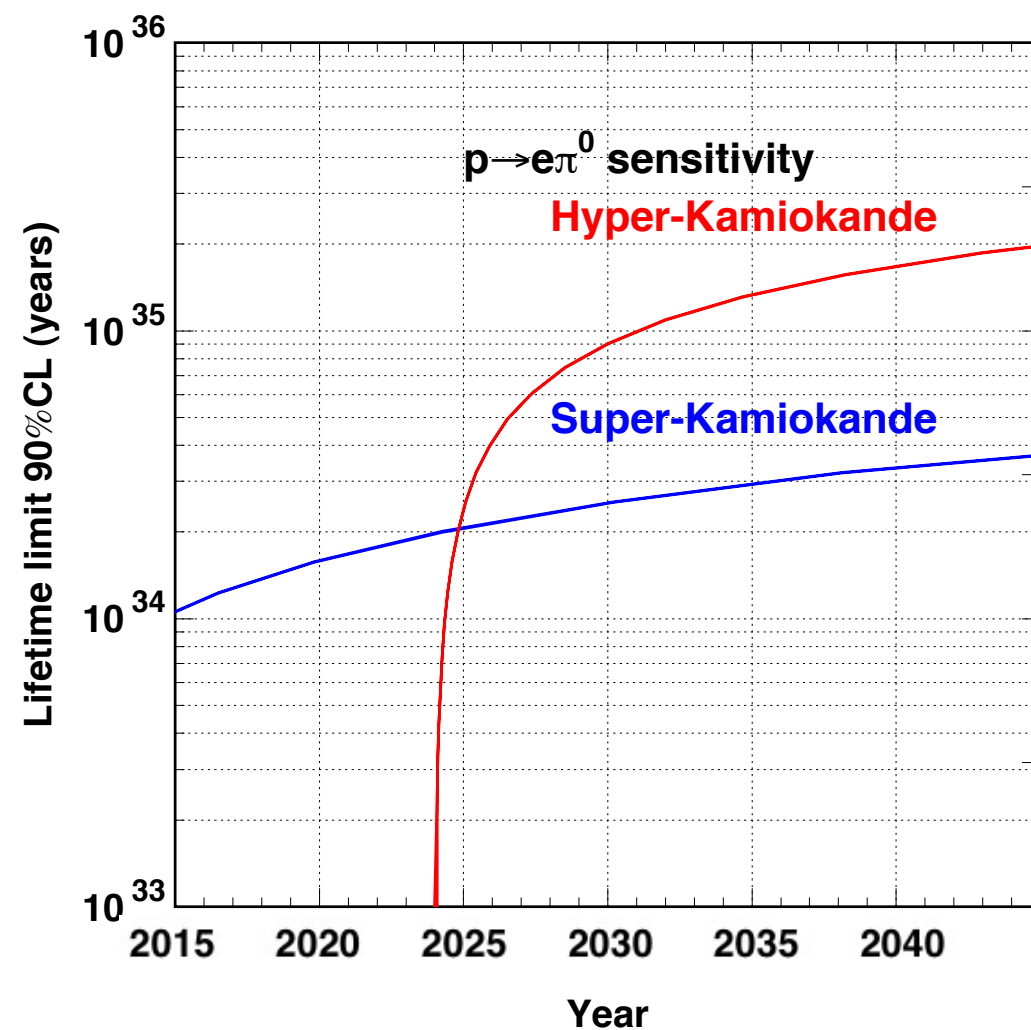
Super-K (2009)

neutrino masses and mixing constraints

Future megaton-scale facilities

Hyper-Kamiokande in 2025 (?)

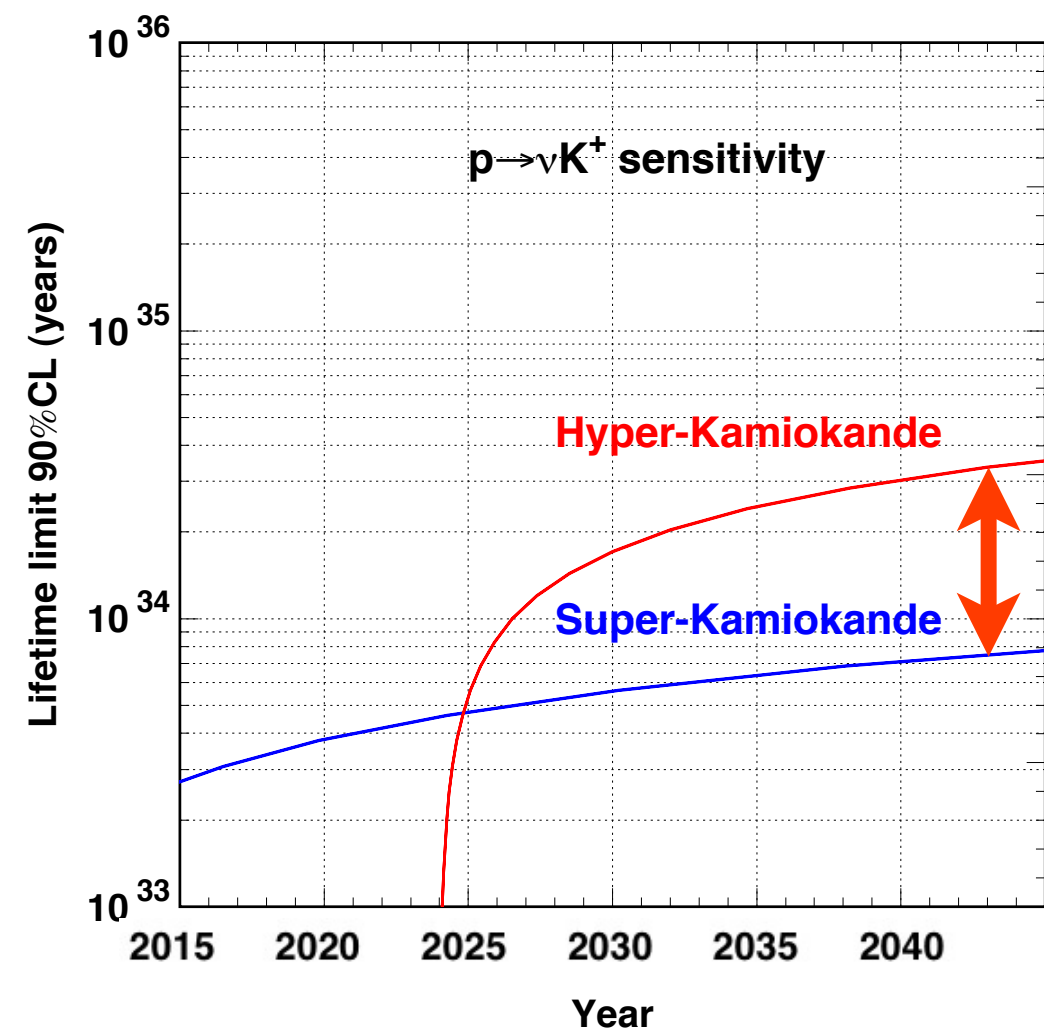
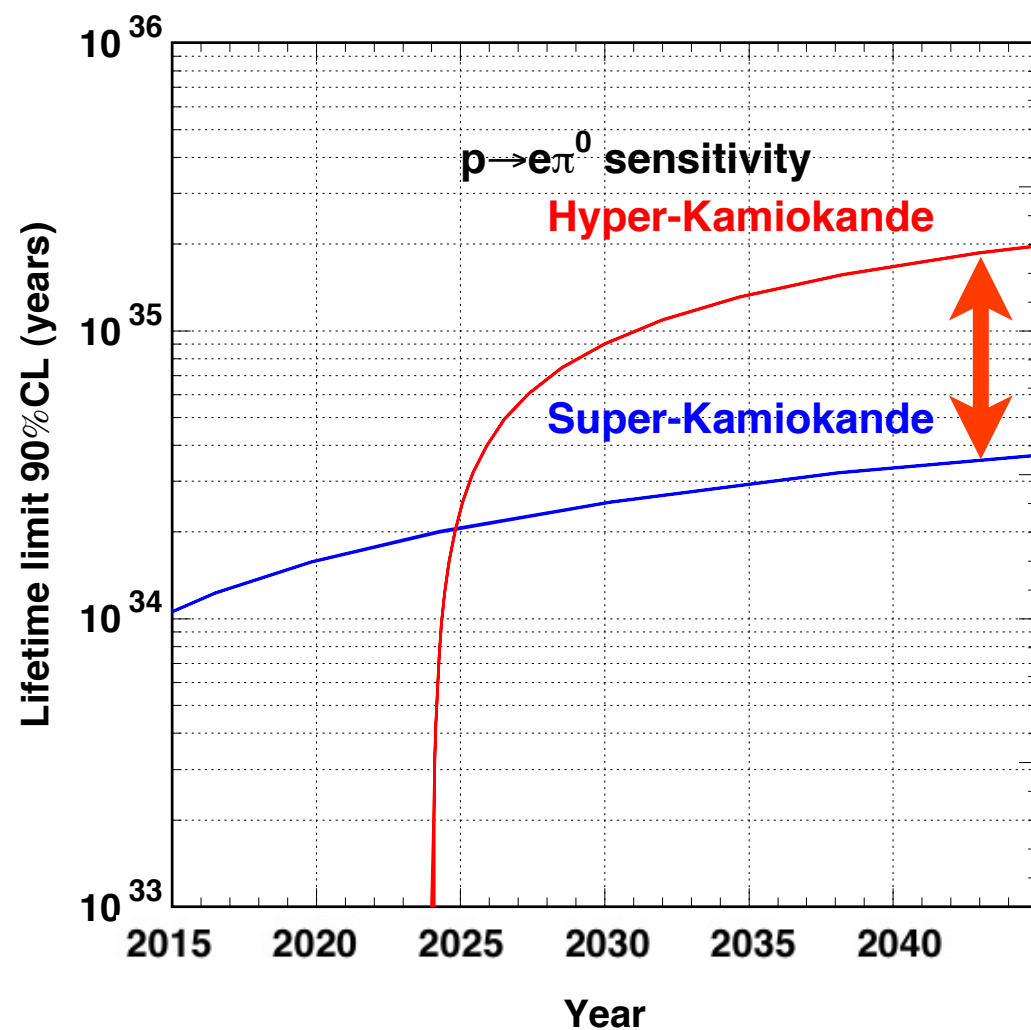
Abe et al., arXiv:1109.3262 [hep-ex]



Future megaton-scale facilities

Hyper-Kamiokande in 2025 (?)

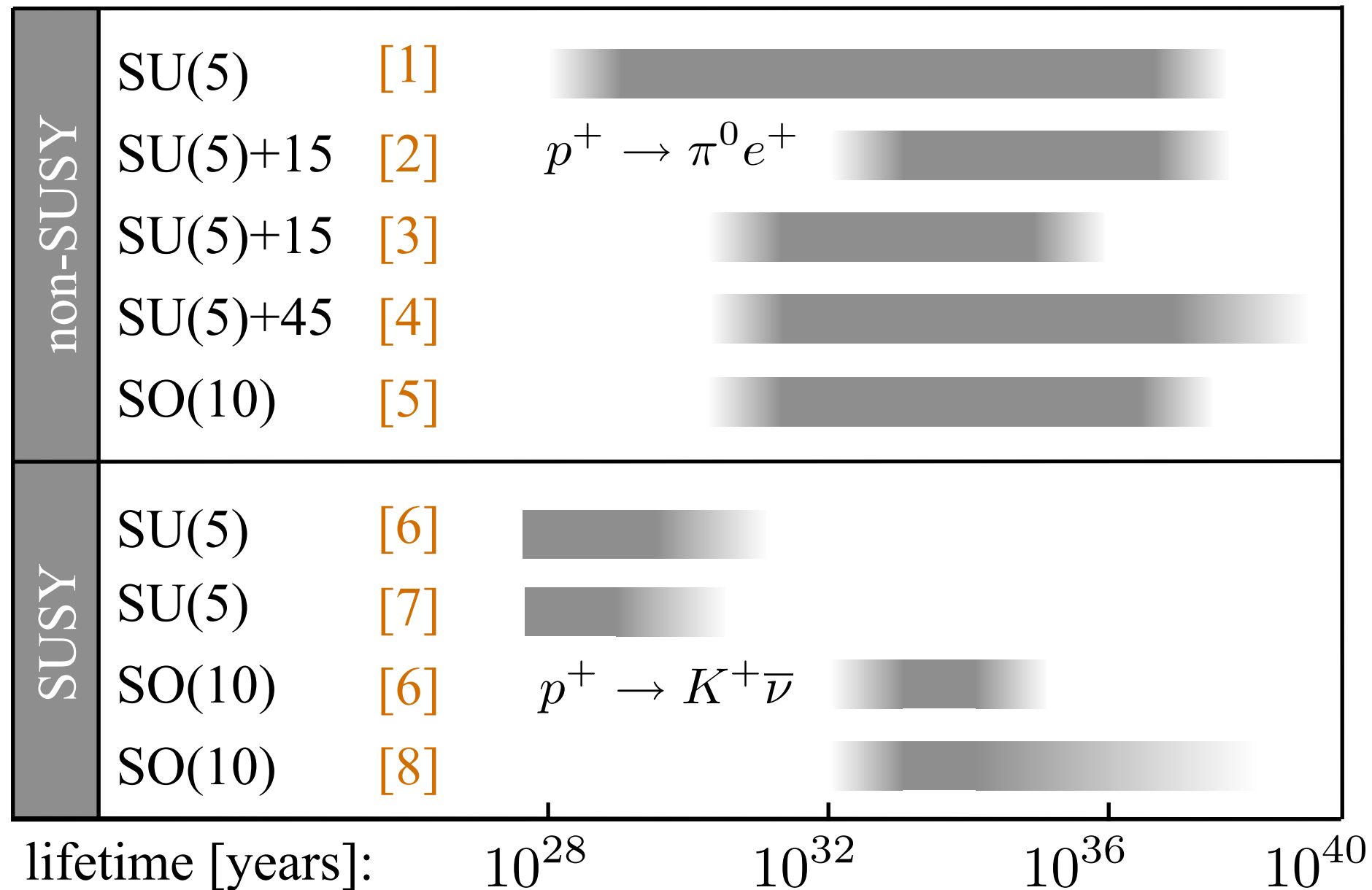
Abe et al., arXiv:1109.3262 [hep-ex]



Accuracy of a **factor of few** in Γ_p needed to make a case !

Proton lifetime estimates in GUTs

Proton lifetime estimates in GUTs



[1] Georgi, Quinn, Weinberg, PRL 33, 451 (1974)

[2] Dorsner, Fileviez Perez, NPB 723, 53 (2005)

[3] Dorsner, Fileviez Perez, Rodrigo, PRD75, 125007 (2007)

[4] Dorsner, Fileviez Perez, PLB 642, 248 (2006)

[5] Lee, Mohapatra, Parida, Rani, PRD 51 (1995)

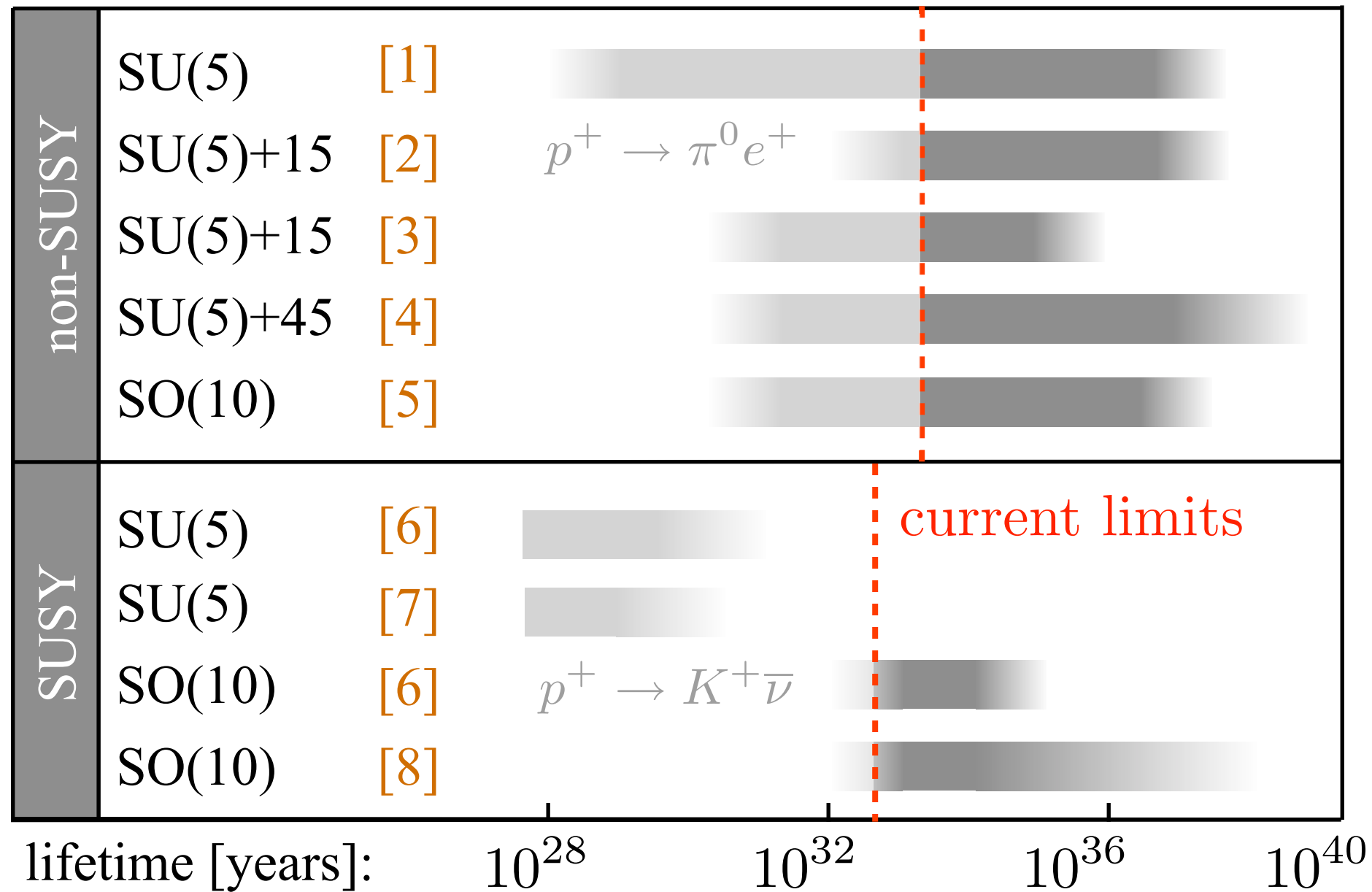
[6] Pati, hep-ph/0507307

[7] Murayama, Pierce, PRD 65, 055009 (2002)

[8] Dutta, Mimura, Mohapatra, PRL 94, 091804 (2005)

... and many more.

Proton lifetime estimates in GUTs



[1] Georgi, Quinn, Weinberg, PRL 33, 451 (1974)

[2] Dorsner, Fileviez Perez, NPB 723, 53 (2005)

[3] Dorsner, Fileviez Perez, Rodrigo, PRD75, 125007 (2007)

[4] Dorsner, Fileviez Perez, PLB 642, 248 (2006)

[5] Lee, Mohapatra, Parida, Rani, PRD 51 (1995)

[6] Pati, hep-ph/0507307

[7] Murayama, Pierce, PRD 65, 055009 (2002)

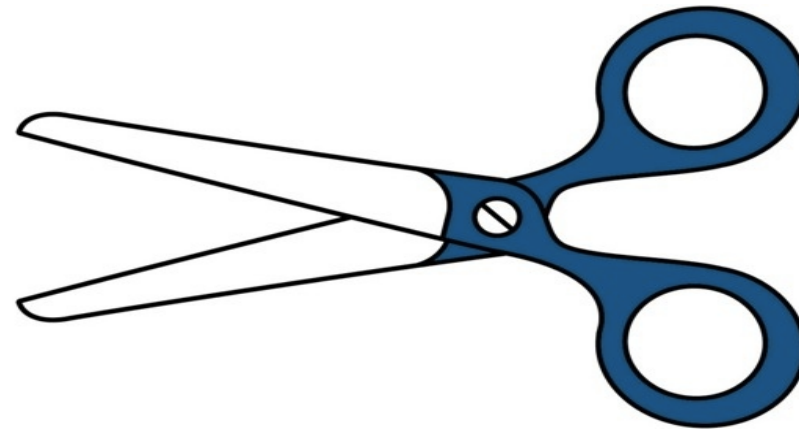
[8] Dutta, Mimura, Mohapatra, PRL 94, 091804 (2005)

... and many more.

Main theoretical uncertainties

GUT scale determination

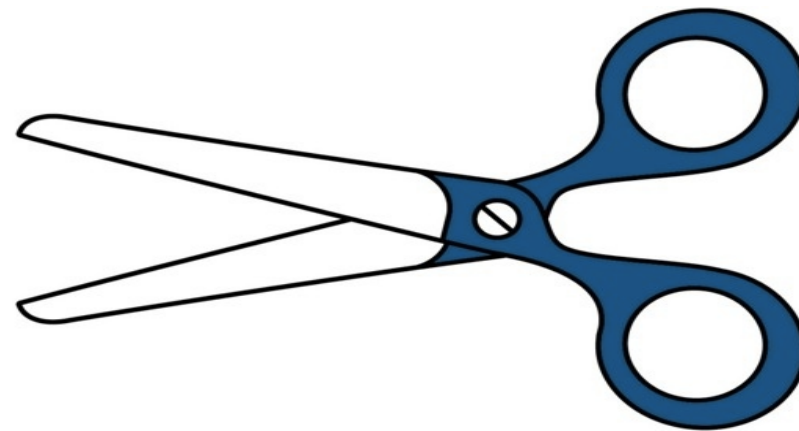
- at least two loops needed to make the case!
- requires a **very good** understanding of the whole spectrum



Main theoretical uncertainties

GUT scale determination

- at least two loops needed to make the case!
- requires a **very good** understanding of the whole spectrum



BTW: no way in SUSY...



My very personal (& perhaps extreme)
view of the hierarchy issue

“The Higgs mass suffers from large radiative corrections!”

The standard (wrong) argument:

$$\Gamma_{hh} \propto \frac{h}{\text{-----}} h \quad p^2 - m_H^2$$

“The Higgs mass suffers from large radiative corrections!”

The standard (wrong) argument:

$$\begin{aligned}
 \Gamma_{hh} \propto & \text{---} h \text{---} h \text{---} & p^2 - m_H^2 \\
 + & \text{---} h \text{---} \text{---} S \text{---} h \text{---} & \frac{1}{16\pi^2} M_S^2 \left(C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right) \\
 + & \text{---} h \text{---} \text{---} S \text{---} \text{---} h \text{---} & \frac{1}{16\pi^2} v^2 \left(C_{UV} - \int_0^1 dx \log \frac{M_S^2 - x(1-x)p^2}{\mu^2} \right) \\
 + & \dots
 \end{aligned}$$

“The Higgs mass suffers from large radiative corrections!”

The standard (wrong) argument:

$$\begin{aligned}
 \Gamma_{hh} \propto & \text{---} h \text{---} & p^2 - m_H^2 \\
 + & \text{---} h \text{---} \text{---} S \text{---} h \text{---} & \frac{1}{16\pi^2} M_S^2 \left(C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right) \\
 + & \text{---} h \text{---} \text{---} S \text{---} \text{---} h & \frac{1}{16\pi^2} v^2 \left(C_{UV} - \int_0^1 dx \log \frac{M_S^2 - x(1-x)p^2}{\mu^2} \right) \\
 + & \dots
 \end{aligned}$$

For large M_S^2 the root shifts enormously...

$$m_H^2 \rightarrow m_H^2 + \frac{1}{16\pi^2} M_S^2 \left(1 - \log \frac{M_S^2}{\mu^2} \right) + \dots$$

so the tree-level mass must be carefully readjusted order by order...

The “hierarchy problem”

“The Higgs mass suffers from large radiative corrections!”

The standard (wrong) argument:

$$\Gamma_{hh} \propto \begin{array}{l} h \\ \text{---} \\ h \end{array} + \begin{array}{l} h \\ \text{---} \text{---} S \text{---} \\ h \end{array} + \begin{array}{l} h \\ \text{---} \text{---} S \text{---} \\ h \end{array}$$

“The Higgs mass suffers from large radiative corrections!”

The standard (wrong) argument:

$$\begin{aligned}
 \Gamma_{hh} \propto & \text{---} h \text{---} & p^2 - m_H^2 \\
 + & \text{---} h \text{---} \text{---} S \text{---} h \text{---} & \frac{1}{16\pi^2} M_S^2 \left(C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right) \\
 + & \text{---} h \text{---} \text{---} S \text{---} h \text{---} & \frac{1}{16\pi^2} v^2 \left(C_{UV} - \int_0^1 dx \log \frac{M_S^2 - x(1-x)p^2}{\mu^2} \right) \\
 \text{SUSY:} & + \text{---} h \text{---} \text{---} \tilde{S} \text{---} h \text{---} & - \frac{1}{16\pi^2} M_S^2 \left(C_{UV} - 1 - \int_0^1 dx \log \frac{M_S^2 - x(1-x)p^2}{\mu^2} \right) + p^2 \text{ terms} \\
 & + \dots &
 \end{aligned}$$

The shift of the root is small even for large M_S^2

“The hierarchy is stabilized by SUSY near M_Z ”

“The Higgs mass suffers from large radiative corrections!”

The devil is in detail:

$$\Gamma_{hh} \propto \frac{h}{p^2 - m_H^2} h + \frac{h}{p^2 - m_H^2} \text{ (loop with } S \text{)} h + \dots$$
$$\frac{1}{16\pi^2} M_S^2 \left(C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right)$$

“The Higgs mass suffers from large radiative corrections!”

The devil is in detail:

$$\Gamma_{hh} \propto \frac{h}{\text{---}\bullet\text{---}} \frac{h}{p^2 - m_H^2} + \frac{h}{\text{---}\bigcirc^S\text{---}} \frac{h}{\frac{1}{16\pi^2} M_S^2 \left(C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right)} + \dots$$

The root is **not the physical mass ! Mind the one-point function !**

Perturbation theory contrived unless $\Gamma_1 = 0$:

“The Higgs mass suffers from large radiative corrections!”

The devil is in detail:

$$\Gamma_{hh} \propto \frac{h \text{---}\bullet\text{---} h}{p^2 - m_H^2} + \frac{h \text{---}\bigcirc^S\text{---} h}{\frac{1}{16\pi^2} M_S^2 \left(C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right)} + \dots$$

The root is **not the physical mass ! Mind the one-point function !**

Perturbation theory contrived unless $\Gamma_1 = 0$:

$$\Gamma_h \propto \frac{h \text{---}\bullet\text{---}\times}{m_H^2 v} + \frac{h \text{---}\times\text{---}\times\text{---}\times}{\lambda v^3}$$

“The Higgs mass suffers from large radiative corrections!”

The devil is in detail:

$$\Gamma_{hh} \propto \frac{h \text{---}\bullet\text{---} h}{p^2 - m_H^2} + \frac{h \text{---}\bigcirc^S\text{---} h}{\frac{1}{16\pi^2} M_S^2 \left(C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right)} + \dots$$

The root is **not the physical mass ! Mind the one-point function !**

Perturbation theory contrived unless $\Gamma_1 = 0$:

$$\Gamma_h \propto \frac{h \text{---}\bullet\text{---}\times}{m_H^2 v} + \frac{h \text{---}\bigtimes\text{---}\times}{\lambda v^3} + \frac{h \text{---}\bigcirc^S\text{---}\times}{\frac{v}{16\pi^2} M_S^2 \left(C_{UV} - 1 + \log \frac{M_S^2}{\mu^2} \right)} + \dots$$

No need to be sorry...

In the **true quantum vacuum** the polynomial M_S^2 -dependence drops out.

The poor Higgs is in the same shape like anybody else in the SM...

“Higgs anti-discrimination act”

Full one-loop EP approach
MM, EPJ C73 (2013) 2415

No need to be sorry...

In the **true quantum vacuum** the polynomial M_S^2 -dependence drops out.

The poor Higgs is in the same shape like anybody else in the SM...

“Higgs anti-discrimination act”

Full one-loop EP approach
MM, EPJ C73 (2013) 2415

Are you scared by the bare charge in QED? No?
So why do you care about hierarchy?

Correlations among observables (= **physics**) **perfectly stable!**

Main theoretical uncertainties

Flavour structure of the BLV currents

Example: $\frac{g^2}{M_{1/6}^2} C_{ijk} \bar{u}^c \gamma^\mu d_i \bar{d}_j^c \gamma_\mu \nu_k \quad C_{ijk} = (V_{d^c}^\dagger V_d)_{ji} (V_u^\dagger V_\nu)_{1k}$

- RH rotations enter here
- simple Yukawa sector desirable!

Main theoretical uncertainties

Planck scale effects

$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle \Phi \rangle F_{\mu\nu}$$

- finite shifts in the gauge matching
- can be as large as $\Delta\alpha_i^{-1} \sim 1$

Larsen, Wilczek, NPB 458, 249 (1996)
G. Veneziano, JHEP 06 (2002) 051
Calmet, Hsu, Reeb, PRD 77, 125015 (2008)
G. Dvali, Fortsch. Phys. 58 (2010) 528-536

Main theoretical uncertainties

Planck scale effects

$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle \Phi \rangle F_{\mu\nu}$$

- finite shifts in the gauge matching
- can be as large as $\Delta\alpha_i^{-1} \sim 1$

Larsen, Wilczek, NPB 458, 249 (1996)
G. Veneziano, JHEP 06 (2002) 051
Calmet, Hsu, Reeb, PRD 77, 125015 (2008)
G. Dvali, Fortsch. Phys. 58 (2010) 528-536

easily an order of magnitude uncertainty in M_G !

IRREDUCIBLE !!!

Main theoretical uncertainties

Planck scale effects

$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle \Phi \rangle F_{\mu\nu}$$

- finite shifts in the gauge matching
- can be as large as $\Delta\alpha_i^{-1} \sim 1$

Larsen, Wilczek, NPB 458, 249 (1996)
G. Veneziano, JHEP 06 (2002) 051
Calmet, Hsu, Reeb, PRD 77, 125015 (2008)
G. Dvali, Fortsch. Phys. 58 (2010) 528-536

Absent if $SO(10)$ is broken by 45!

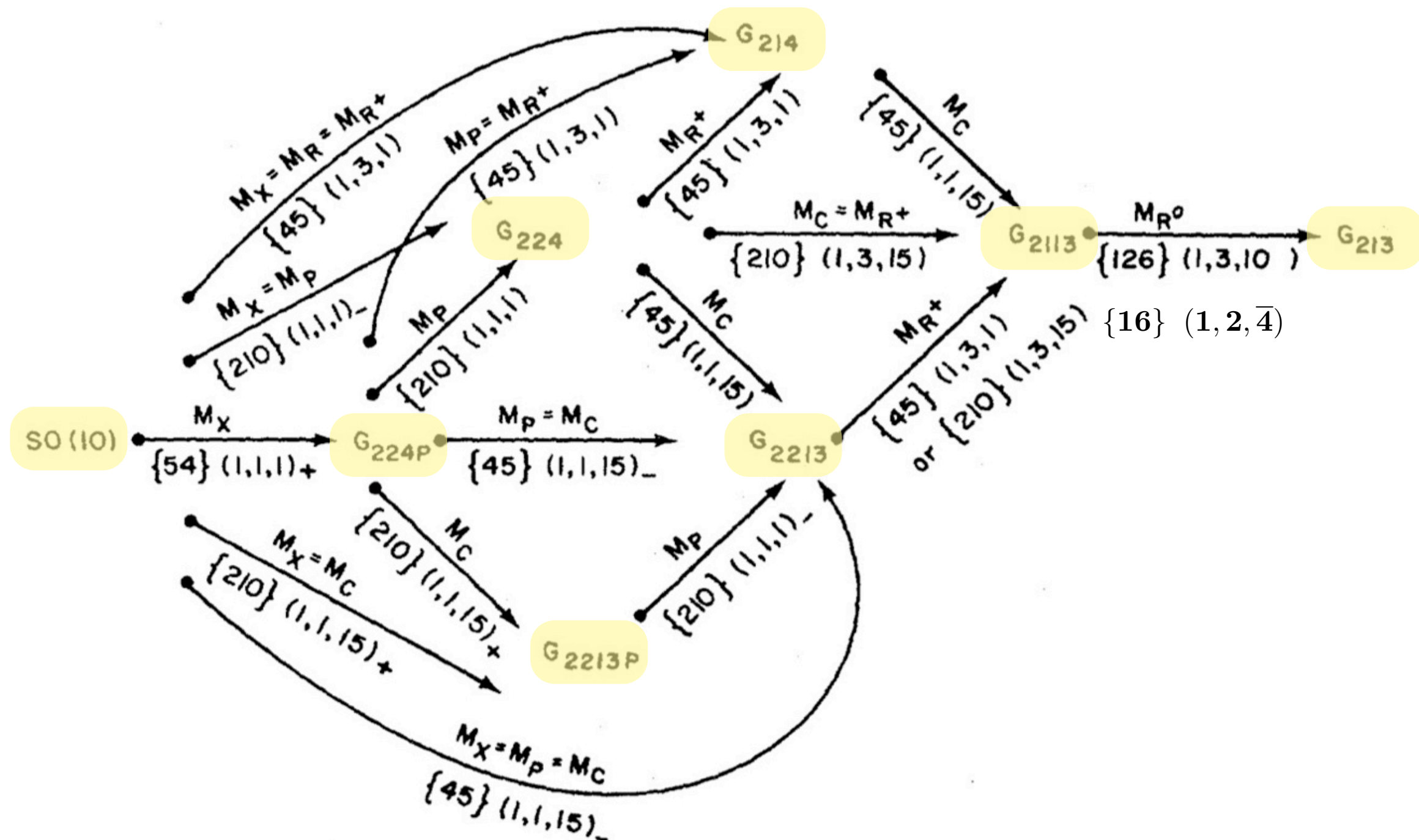
$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle 45 \rangle F_{\mu\nu} = 0$$

The minimal non-SUSY SO(10) GUT

The minimal SO(10) unification

Chang, Mohapatra, Gipson, Marshak, Parida 1985

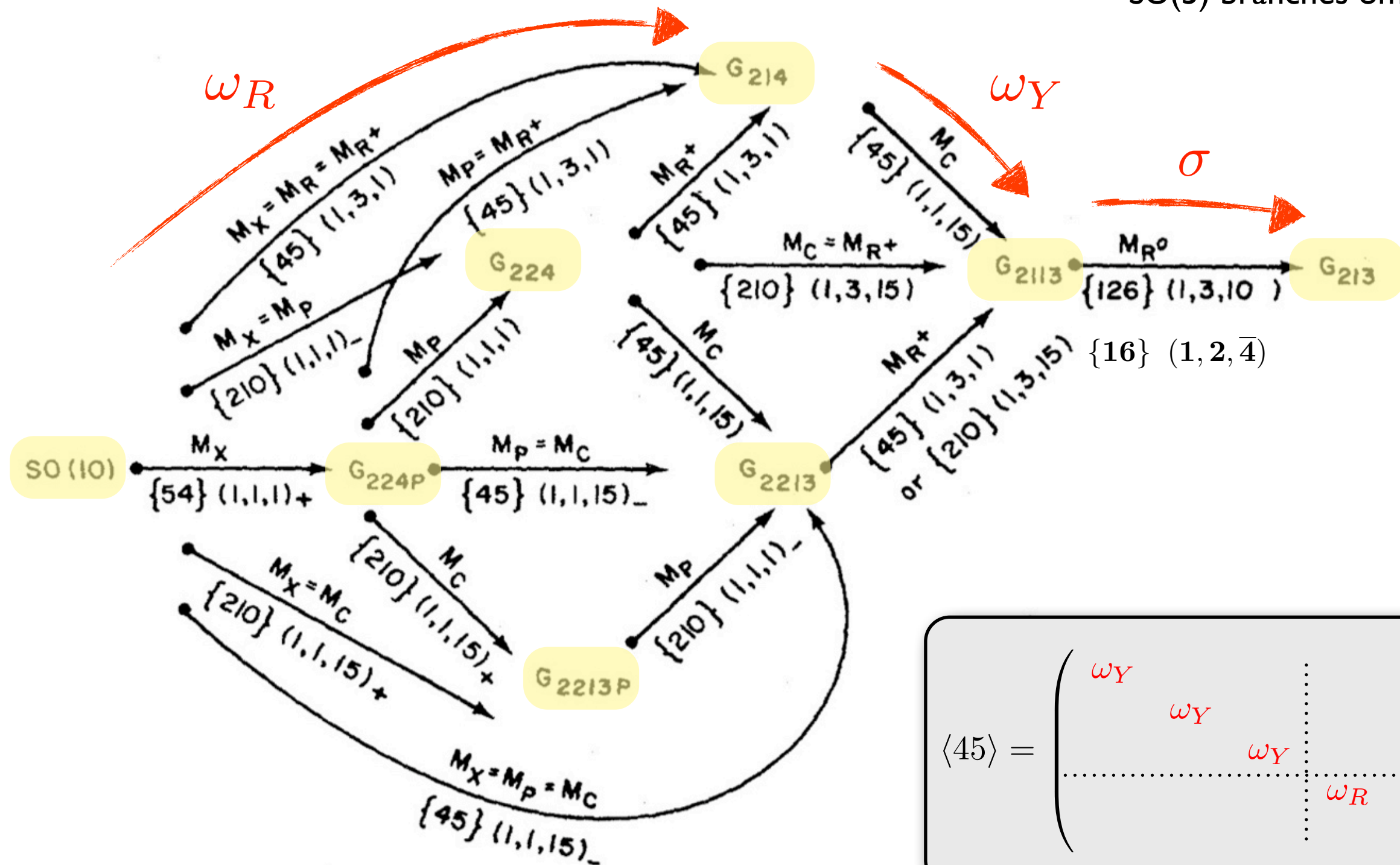
SU(5) branches omitted



The minimal SO(10) unification

Chang, Mohapatra, Gipson, Marshak, Parida 1985

SU(5) branches omitted

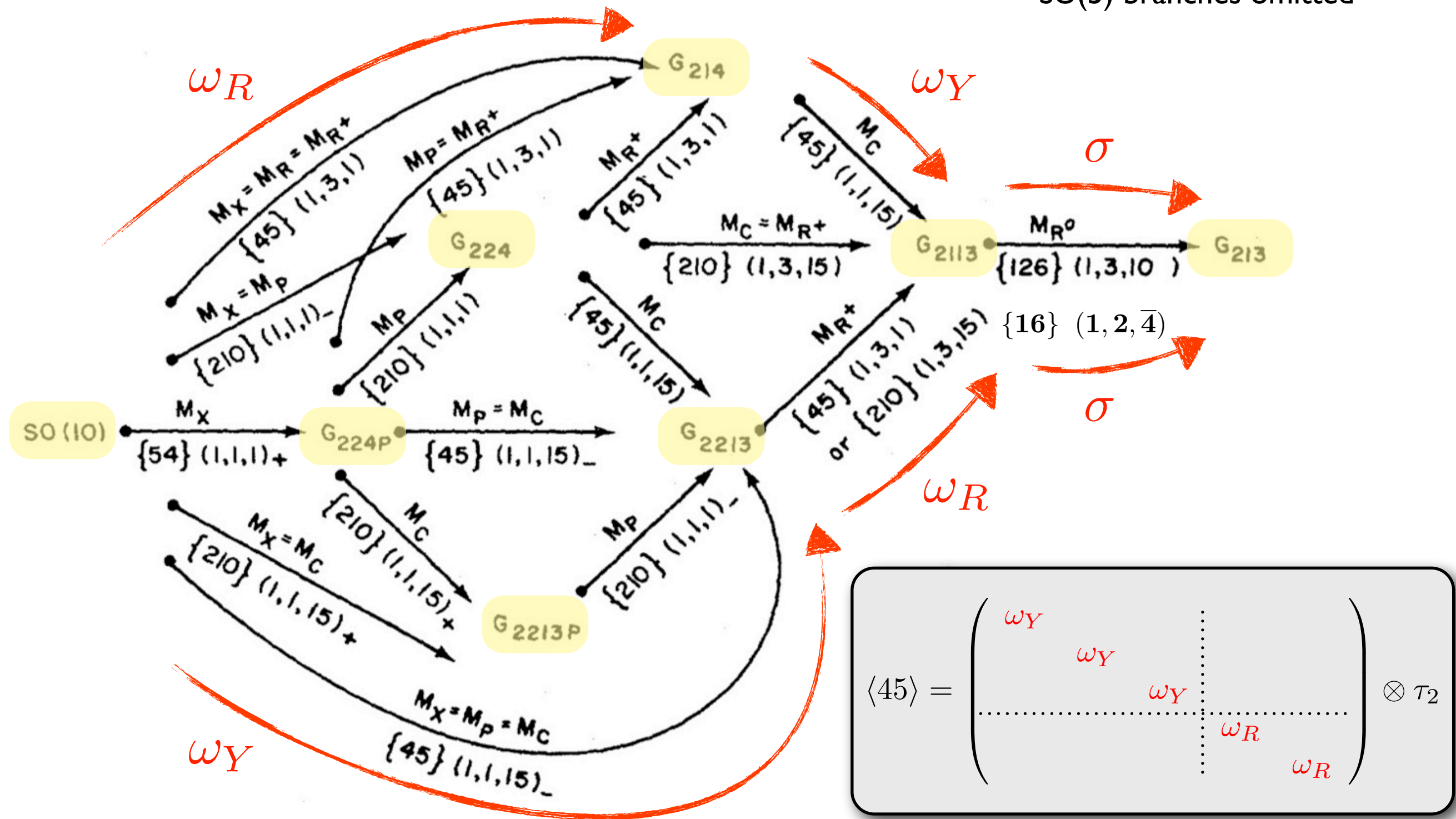


$$\langle 45 \rangle = \begin{pmatrix} \omega_Y & & & \\ & \omega_Y & & \\ & & \omega_Y & \\ \hline & & & \omega_R \\ & & & & \omega_R \end{pmatrix} \otimes \tau_2$$

The minimal SO(10) unification

Chang, Mohapatra, Gipson, Marshak, Parida 1985

SU(5) branches omitted



The minimal SO(10) unification

SO(10) broken by 45, rank reduced by 126

Scalar potential: $V = V_{45} + V_{126} + V_{\text{mix}}$

$$V_{45} = -\frac{\mu^2}{2}(\phi\phi)_0 + \frac{a_0}{4}(\phi\phi)_0(\phi\phi)_0 + \frac{a_2}{4}(\phi\phi)_2(\phi\phi)_2,$$

$$\begin{aligned} V_{126} = & -\frac{\nu^2}{5!}(\Sigma\Sigma^*)_0 \\ & + \frac{\lambda_0}{(5!)^2}(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 + \frac{\lambda_2}{(4!)^2}(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 \\ & + \frac{\lambda_4}{(3!)^2(2!)^2}(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 + \frac{\lambda'_4}{(3!)^2}(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} \\ & + \frac{\eta_2}{(4!)^2}(\Sigma\Sigma)_2(\Sigma\Sigma)_2 + \frac{\eta_2^*}{(4!)^2}(\Sigma^*\Sigma^*)_2(\Sigma^*\Sigma^*)_2, \end{aligned}$$

$$\begin{aligned} V_{\text{mix}} = & \frac{i\tau}{4!}(\phi)_2(\Sigma\Sigma^*)_2 + \frac{\alpha}{2 \cdot 5!}(\phi\phi)_0(\Sigma\Sigma^*)_0 \\ & + \frac{\beta_4}{4 \cdot 3!}(\phi\phi)_4(\Sigma\Sigma^*)_4 + \frac{\beta'_4}{3!}(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} \\ & + \frac{\gamma_2}{4!}(\phi\phi)_2(\Sigma\Sigma)_2 + \frac{\gamma_2^*}{4!}(\phi\phi)_2(\Sigma^*\Sigma^*)_2. \end{aligned}$$

$$(\phi\phi)_0(\phi\phi)_0 \equiv \phi_{ij}\phi_{ij}\phi_{kl}\phi_{kl}$$

$$(\phi\phi)_2(\phi\phi)_2 \equiv \phi_{ij}\phi_{ik}\phi_{lj}\phi_{lk}$$

$$(\phi\phi)_0 \equiv \phi_{ij}\phi_{ij}, \quad (\Sigma\Sigma^*)_0 \equiv \Sigma_{ijklm}\Sigma_{ijklm}^*$$

$$(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 \equiv \Sigma_{ijklm}\Sigma_{ijklm}^*\Sigma_{nopqr}\Sigma_{nopqr}^*$$

$$(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 \equiv \Sigma_{ijklm}\Sigma_{ijkln}^*\Sigma_{opqrm}\Sigma_{opqrm}^*$$

$$(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 \equiv \Sigma_{ijklm}\Sigma_{ijkno}^*\Sigma_{pqrlm}\Sigma_{pqrlm}^*$$

$$(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} \equiv \Sigma_{ijklm}\Sigma_{ijkno}^*\Sigma_{pqrln}\Sigma_{pqrln}^*$$

$$(\Sigma\Sigma)_2(\Sigma\Sigma)_2 \equiv \Sigma_{ijklm}\Sigma_{ijkln}\Sigma_{opqrm}\Sigma_{opqrm}$$

$$(\phi)_2(\Sigma\Sigma^*)_2 \equiv \phi_{ij}\Sigma_{klmni}\Sigma_{klmni}^*$$

$$(\phi\phi)_0(\Sigma\Sigma^*)_0 \equiv \phi_{ij}\phi_{ij}\Sigma_{klmno}\Sigma_{klmno}^*$$

$$(\phi\phi)_4(\Sigma\Sigma^*)_4 \equiv \phi_{ij}\phi_{kl}\Sigma_{mnoij}\Sigma_{mnoij}^*$$

$$(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} \equiv \phi_{ij}\phi_{kl}\Sigma_{mnoik}\Sigma_{mnoik}^*$$

$$(\phi\phi)_2(\Sigma\Sigma)_2 \equiv \phi_{ij}\phi_{ik}\Sigma_{lmnoj}\Sigma_{lmnoj}$$

$$(\phi\phi)_2(\Sigma^*\Sigma^*)_2 \equiv \phi_{ij}\phi_{ik}\Sigma_{lmnoj}^*\Sigma_{lmnoj}^*$$

The minimal SO(10) unification *nightmare*

SO(10) broken by 45, rank reduced by 126

Scalar potential: $V = V_{45} + V_{126} + V_{\text{mix}}$

$$V_{45} = -\frac{\mu^2}{2}(\phi\phi)_0 + \frac{a_0}{4}(\phi\phi)_0(\phi\phi)_0 + \frac{a_2}{4}(\phi\phi)_2(\phi\phi)_2,$$

$$V_{126} = -\frac{\nu^2}{5!}(\Sigma\Sigma^*)_0 + \frac{\lambda_0}{(5!)^2}(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 + \frac{\lambda_2}{(4!)^2}(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 + \frac{\lambda_4}{(3!)^2(2!)^2}(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 + \frac{\lambda'_4}{(3!)^2}(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} + \frac{\eta_2}{(4!)^2}(\Sigma\Sigma)_2(\Sigma\Sigma)_2 + \frac{\eta_2^*}{(4!)^2}(\Sigma^*\Sigma^*)_2(\Sigma^*\Sigma^*)_2,$$

$$V_{\text{mix}} = \frac{i\tau}{4!}(\phi)_2(\Sigma\Sigma^*)_2 + \frac{\alpha}{2 \cdot 5!}(\phi\phi)_0(\Sigma\Sigma^*)_0 + \frac{\beta_4}{4 \cdot 3!}(\phi\phi)_4(\Sigma\Sigma^*)_4 + \frac{\beta'_4}{3!}(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} + \frac{\gamma_2}{4!}(\phi\phi)_2(\Sigma\Sigma)_2 + \frac{\gamma_2^*}{4!}(\phi\phi)_2(\Sigma^*\Sigma^*)_2.$$

$$(\phi\phi)_0(\phi\phi)_0 \equiv \phi_{ij}\phi_{ij}\phi_{kl}\phi_{kl}$$

$$(\phi\phi)_2(\phi\phi)_2 \equiv \phi_{ij}\phi_{ik}\phi_{lj}\phi_{lk}$$

$$(\phi\phi)_0 \equiv \phi_{ij}\phi_{ij}, \quad (\Sigma\Sigma^*)_0 \equiv \Sigma_{ijklm}\Sigma_{ijklm}^*$$

$$(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 \equiv \Sigma_{ijklm}\Sigma_{ijklm}^*\Sigma_{nopqr}\Sigma_{nopqr}^*$$

$$(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 \equiv \Sigma_{ijklm}\Sigma_{ijkln}^*\Sigma_{opqrm}\Sigma_{opqrm}^*$$

$$(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 \equiv \Sigma_{ijklm}\Sigma_{ijkno}^*\Sigma_{pqrlm}\Sigma_{pqrlm}^*$$

$$(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} \equiv \Sigma_{ijklm}\Sigma_{ijkno}^*\Sigma_{pqrln}\Sigma_{pqrln}^*$$

$$(\Sigma\Sigma)_2(\Sigma\Sigma)_2 \equiv \Sigma_{ijklm}\Sigma_{ijkln}\Sigma_{opqrm}\Sigma_{opqrm}$$

$$(\phi)_2(\Sigma\Sigma^*)_2 \equiv \phi_{ij}\Sigma_{klmni}\Sigma_{klmni}^*$$

$$(\phi\phi)_0(\Sigma\Sigma^*)_0 \equiv \phi_{ij}\phi_{ij}\Sigma_{klmno}\Sigma_{klmno}^*$$

$$(\phi\phi)_4(\Sigma\Sigma^*)_4 \equiv \phi_{ij}\phi_{kl}\Sigma_{mnoij}\Sigma_{mnoij}^*$$

$$(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} \equiv \phi_{ij}\phi_{kl}\Sigma_{mnoik}\Sigma_{mnoik}^*$$

$$(\phi\phi)_2(\Sigma\Sigma)_2 \equiv \phi_{ij}\phi_{ik}\Sigma_{lmnoj}\Sigma_{lmnoj}$$

$$(\phi\phi)_2(\Sigma^*\Sigma^*)_2 \equiv \phi_{ij}\phi_{ik}\Sigma_{lmnoj}^*\Sigma_{lmnoj}^*$$

The minimal $SO(10)$ unification ~~nightmare~~

“Ruled out” in 1980’s

$$m_{(8,1,0)}^2 = 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y)$$

$$m_{(1,3,0)}^2 = 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R)$$

Yasue 1981, Anastaze, Derendinger, Buccella 1983, Babu, Ma 1985

The minimal $SO(10)$ unification ~~nightmare~~

“Ruled out” in 1980’s

$$\begin{aligned} m_{(8,1,0)}^2 &= 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y) \\ m_{(1,3,0)}^2 &= 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R) \end{aligned}$$

Yasue 1981, Anastaze, Derendinger, Buccella 1983, Babu, Ma 1985

Aaarrggh... tachyonic spectrum unless $\frac{1}{2} < |\omega_Y/\omega_R| < 2$

The minimal $SO(10)$ unification ~~nightmare~~

“Ruled out” in 1980’s

$$\begin{aligned}m_{(8,1,0)}^2 &= 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y) \\ m_{(1,3,0)}^2 &= 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R)\end{aligned}$$

Yasuè 1981, Anastaze, Derendinger, Buccella 1983, Babu, Ma 1985

Aaarrggh... tachyonic spectrum unless $\frac{1}{2} < |\omega_Y/\omega_R| < 2$

$$\langle 45 \rangle = \begin{pmatrix} \omega_Y & & & & \\ & \omega_Y & & & \\ & & \omega_Y & & \\ & & & \omega_R & \\ & & & & \omega_R \end{pmatrix} \otimes \tau_2$$

$SU(5)$ -like vacua only, not far from the “SM running”!

The minimal $SO(10)$ unification *nightmare*

“Ruled out” in 1980’s

$$m_{(8,1,0)}^2 = 2a_2(\omega_R - \omega_Y)(\omega_R + 2\omega_Y)$$

$$m_{(1,3,0)}^2 = 2a_2(\omega_Y - \omega_R)(\omega_Y + 2\omega_R)$$

Yasue 1981, Anastaze, Derendinger, Buccella 1983, Babu, Ma 1985

Aaarrggh... tachyonic spectrum unless $\frac{1}{2} < |\omega_Y/\omega_R| < 2$

$$\langle 45 \rangle = \begin{pmatrix} \omega_Y & & & & \\ & \omega_Y & & & \\ & & \omega_Y & & \\ & & & \omega_R & \\ & & & & \omega_R \end{pmatrix} \otimes \tau_2$$

SU(5)-like vacua only, not far from the “SM running”!



The minimal $SO(10)$ unification ~~nightmare~~

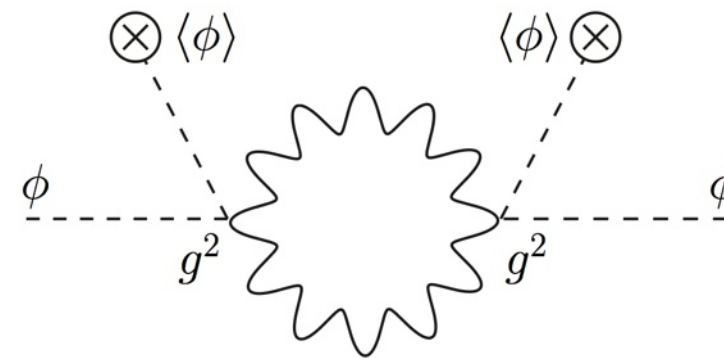
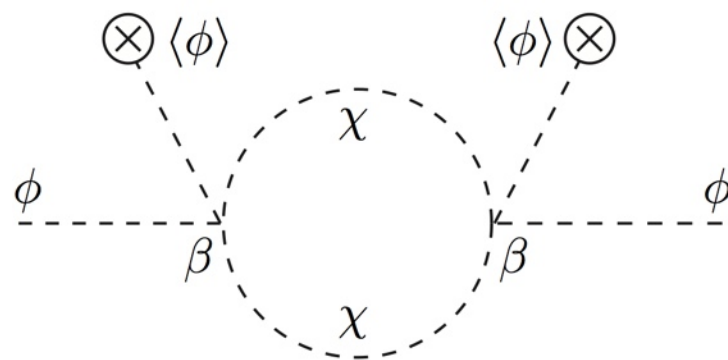
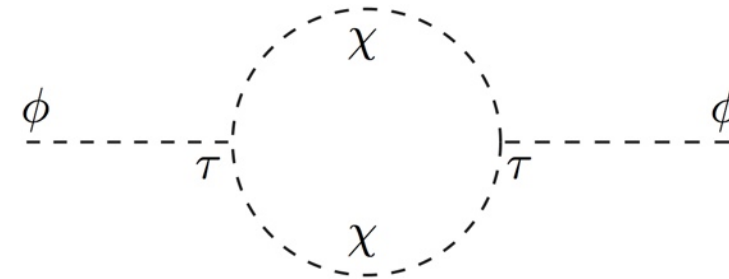
**“Do not trust arguments based
on the lowest order of perturbation theory.”**

S.Weinberg , “Why RG is a good thing”
in “Asymptotic Realm of Physics”, MIT press 1983

The minimal SO(10) unification ~~nightmare~~

Quantum salvation in 2010

One-loop effective potential:



$$\Delta m_{(1,3,0)}^2 = \frac{1}{4\pi^2} \left[\tau^2 + \beta^2 (2\omega_R^2 - \omega_R \omega_Y + 2\omega_Y^2) + g^4 (16\omega_R^2 + \omega_Y \omega_R + 19\omega_Y^2) \right] + \text{logs},$$

$$\Delta m_{(8,1,0)}^2 = \frac{1}{4\pi^2} \left[\tau^2 + \beta^2 (\omega_R^2 - \omega_R \omega_Y + 3\omega_Y^2) + g^4 (13\omega_R^2 + \omega_Y \omega_R + 22\omega_Y^2) \right] + \text{logs},$$

Bertolini, Di Luzio, MM, PRD 81, 035015 (2010)

The minimal consistent SO(10) unification

*"Consistency is the last refuge
of people without imagination"*
Oscar Wilde

The minimal consistent SO(10) unification

Chang, Mohapatra, Gipson, Marshak, Parida (1985)

Deshpande, Keith, Pal (1993)

Bertolini, Di Luzio, MM (2009)

*"Consistency is the last refuge
of people without imagination"*
Oscar Wilde

Simple estimates: $M_{\text{seesaw}} \sim 10^{10} \text{ GeV} \Rightarrow$ too heavy LH neutrinos!?
multiple Yukawa finetuning?

The minimal consistent SO(10) unification

Chang, Mohapatra, Gipson, Marshak, Parida (1985)

Deshpande, Keith, Pal (1993)

Bertolini, Di Luzio, MM (2009)

*“Consistency is the last refuge
of people without imagination”
Oscar Wilde*

Simple estimates: $M_{\text{seesaw}} \sim 10^{10} \text{ GeV} \Rightarrow$ too heavy LH neutrinos!
multiple Yukawa finetuning?

NO! Enough to make the unavoidable finetuning elsewhere.

The minimal consistent SO(10) unification

Chang, Mohapatra, Gipson, Marshak, Parida (1985)

Deshpande, Keith, Pal (1993)

Bertolini, Di Luzio, MM (2009)

*“Consistency is the last refuge
of people without imagination”
Oscar Wilde*

Simple estimates: $M_{\text{seesaw}} \sim 10^{10} \text{ GeV} \Rightarrow$ too heavy LH neutrinos!?
multiple Yukawa finetuning?

NO! Enough to make the unavoidable finetuning elsewhere.

Two other potentially realistic minimally fine-tuned consistent scenarios

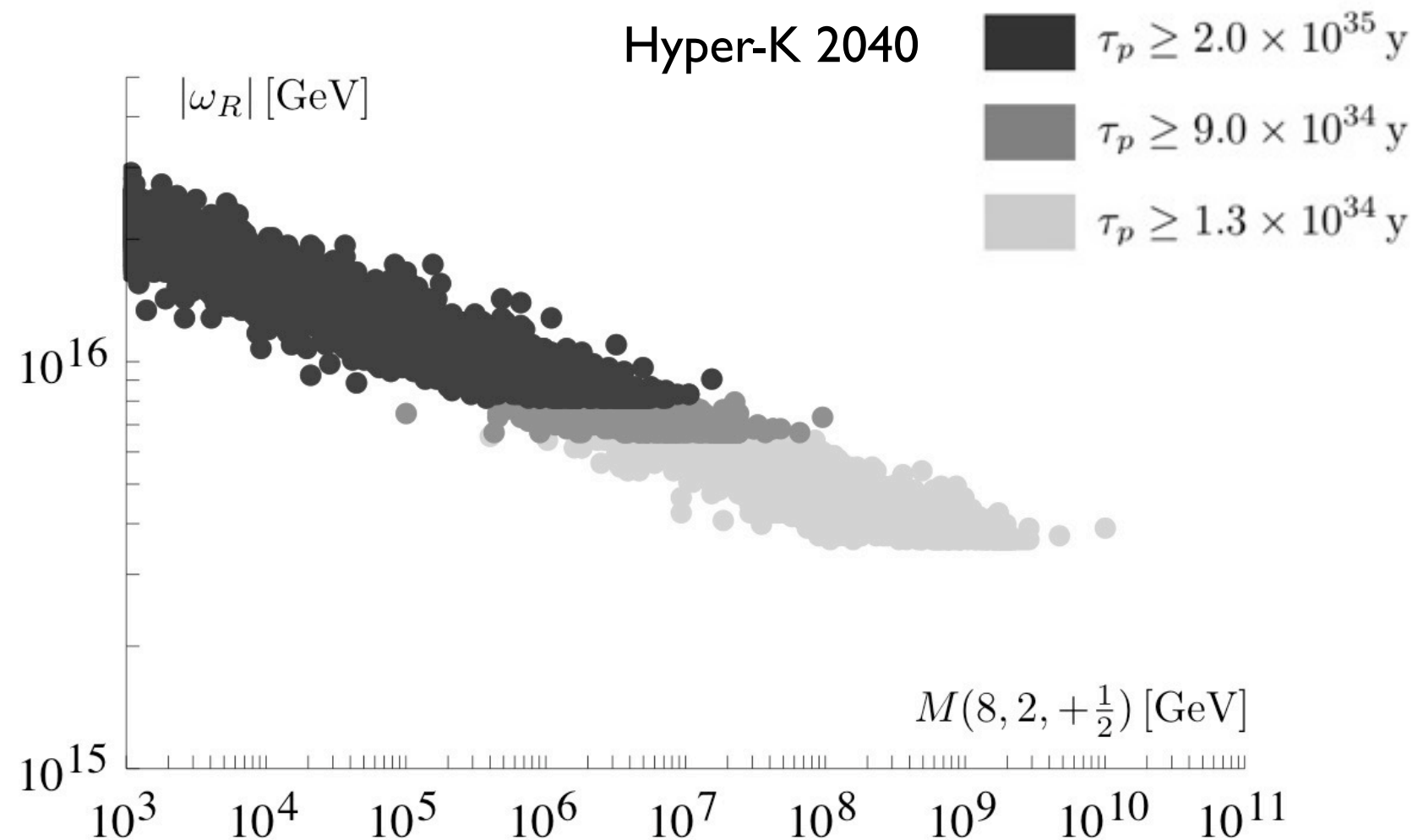
Bertolini, Di Luzio, MM, PRD85 095014 2012

Towards a consistent & potentially realistic SO(10) scenario

Case I: light $(8, 2, +\frac{1}{2})$ @ one loop Bertolini, Di Luzio, MM, PRD 85, 095014 (2012)

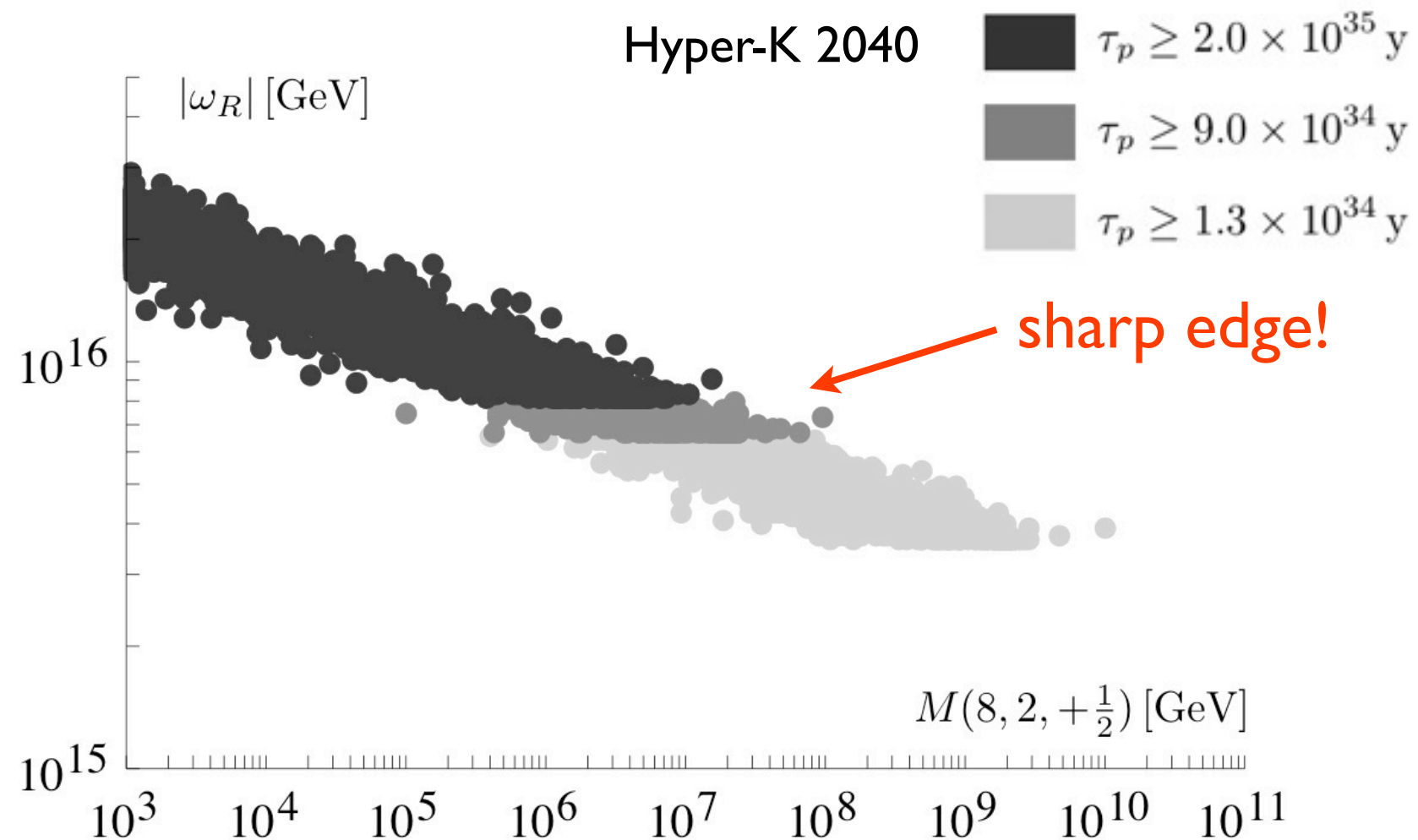
Towards a consistent & potentially realistic SO(10) scenario

Case I: light $(8, 2, +\frac{1}{2})$ @ one loop Bertolini, Di Luzio, MM, PRD 85, 095014 (2012)



Towards a consistent & potentially realistic SO(10) scenario

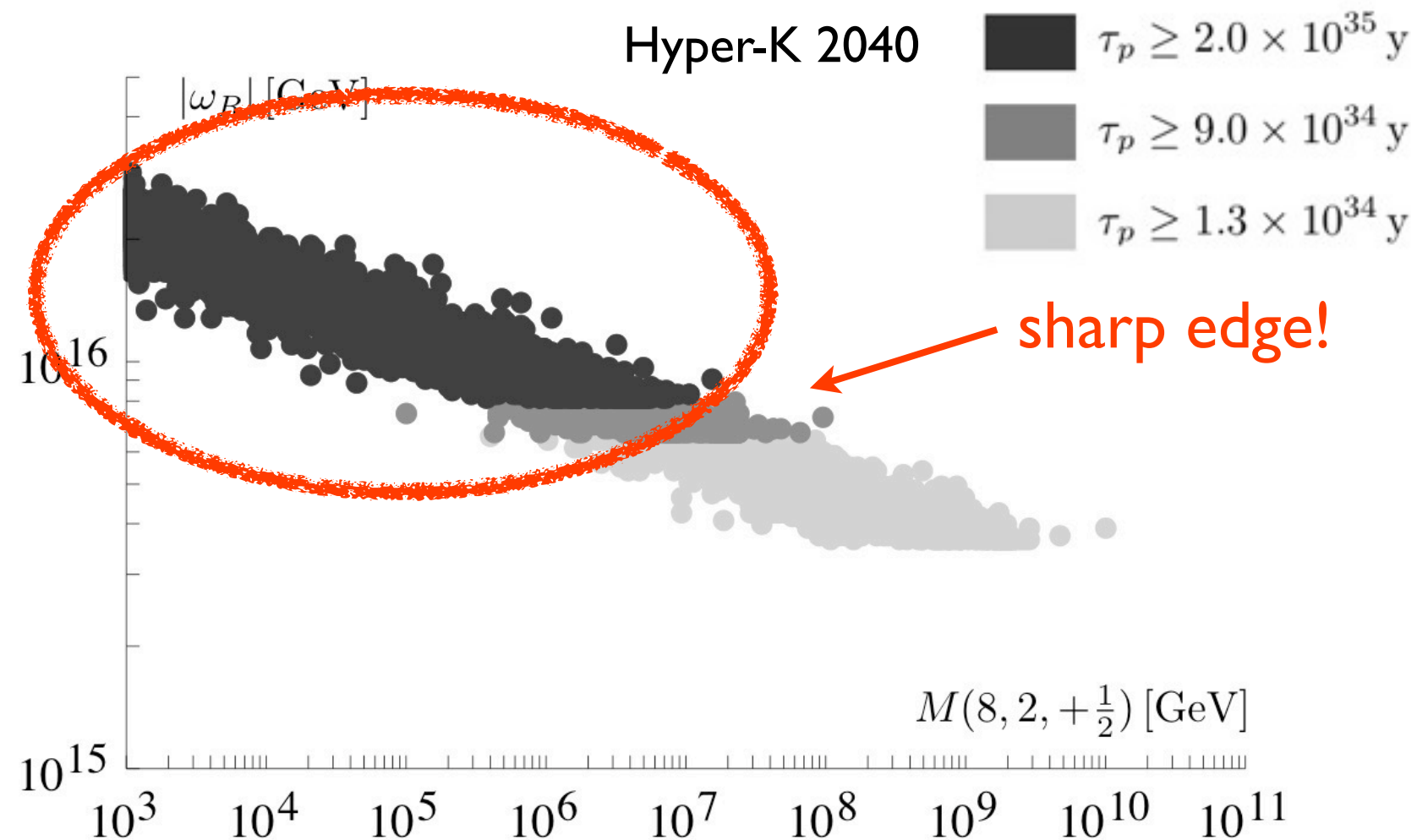
Case I: light $(8, 2, +\frac{1}{2})$ @ one loop Bertolini, Di Luzio, MM, PRD 85, 095014 (2012)



The octet should be light!!!

Towards a consistent & potentially realistic SO(10) scenario

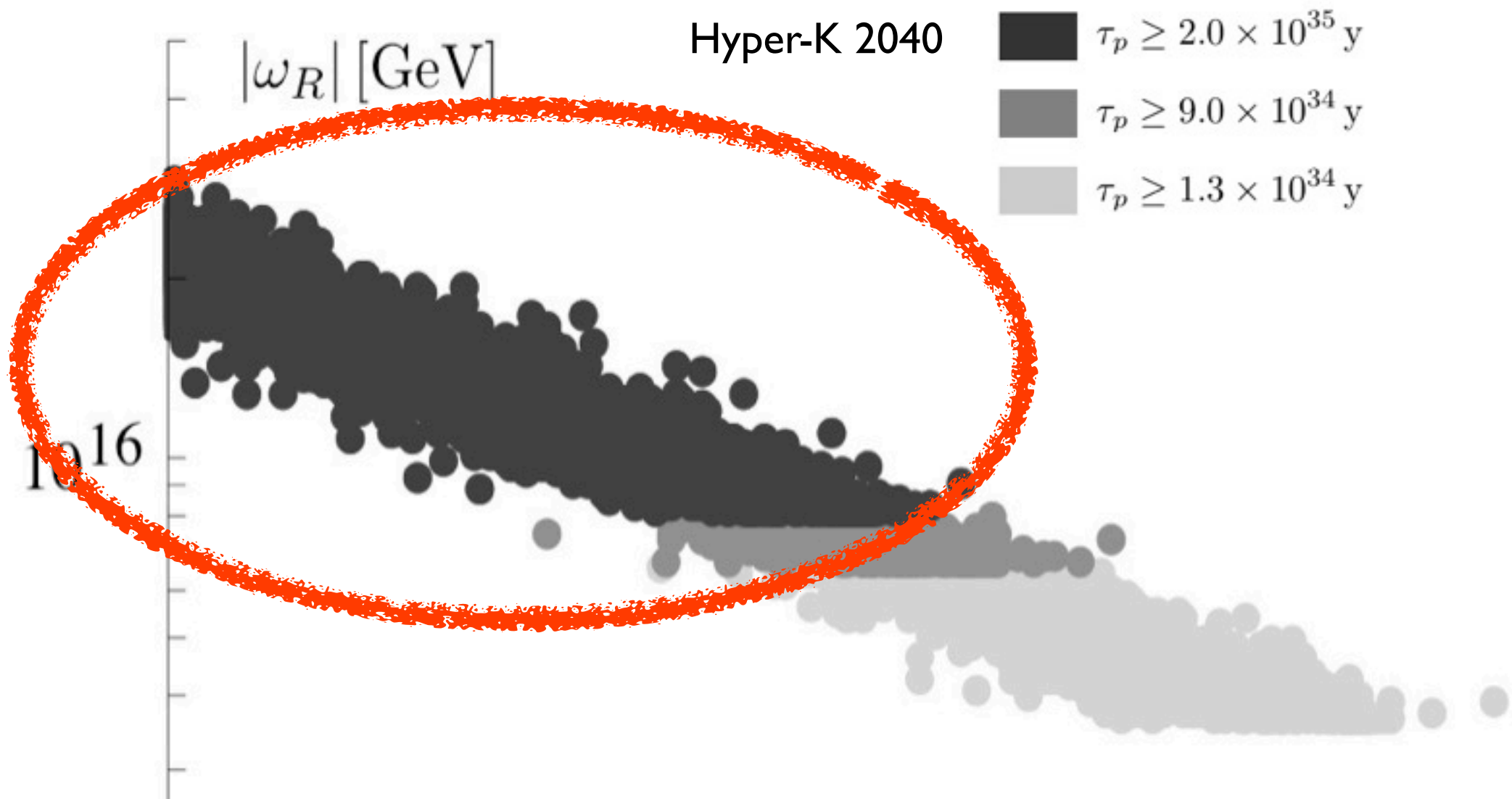
Case I: light $(8, 2, +\frac{1}{2})$ @ one loop Bertolini, Di Luzio, MM, PRD 85, 095014 (2012)



The octet should be light!!!

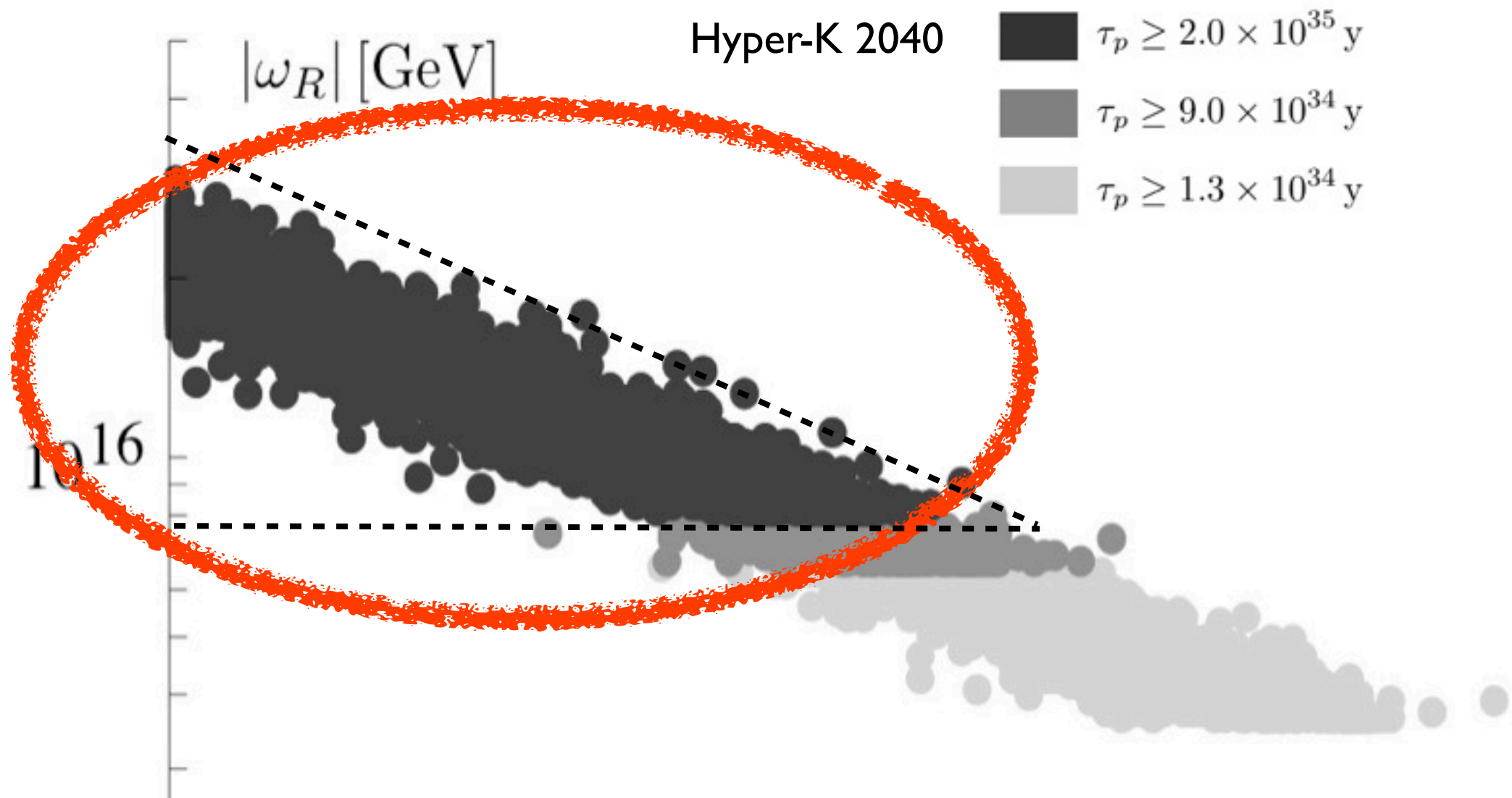
Towards a consistent & potentially realistic SO(10) scenario

Case I: light $(8, 2, +\frac{1}{2})$ @ one loop Bertolini, Di Luzio, MM, PRD 85, 095014 (2012)



Towards a consistent & potentially realistic SO(10) scenario

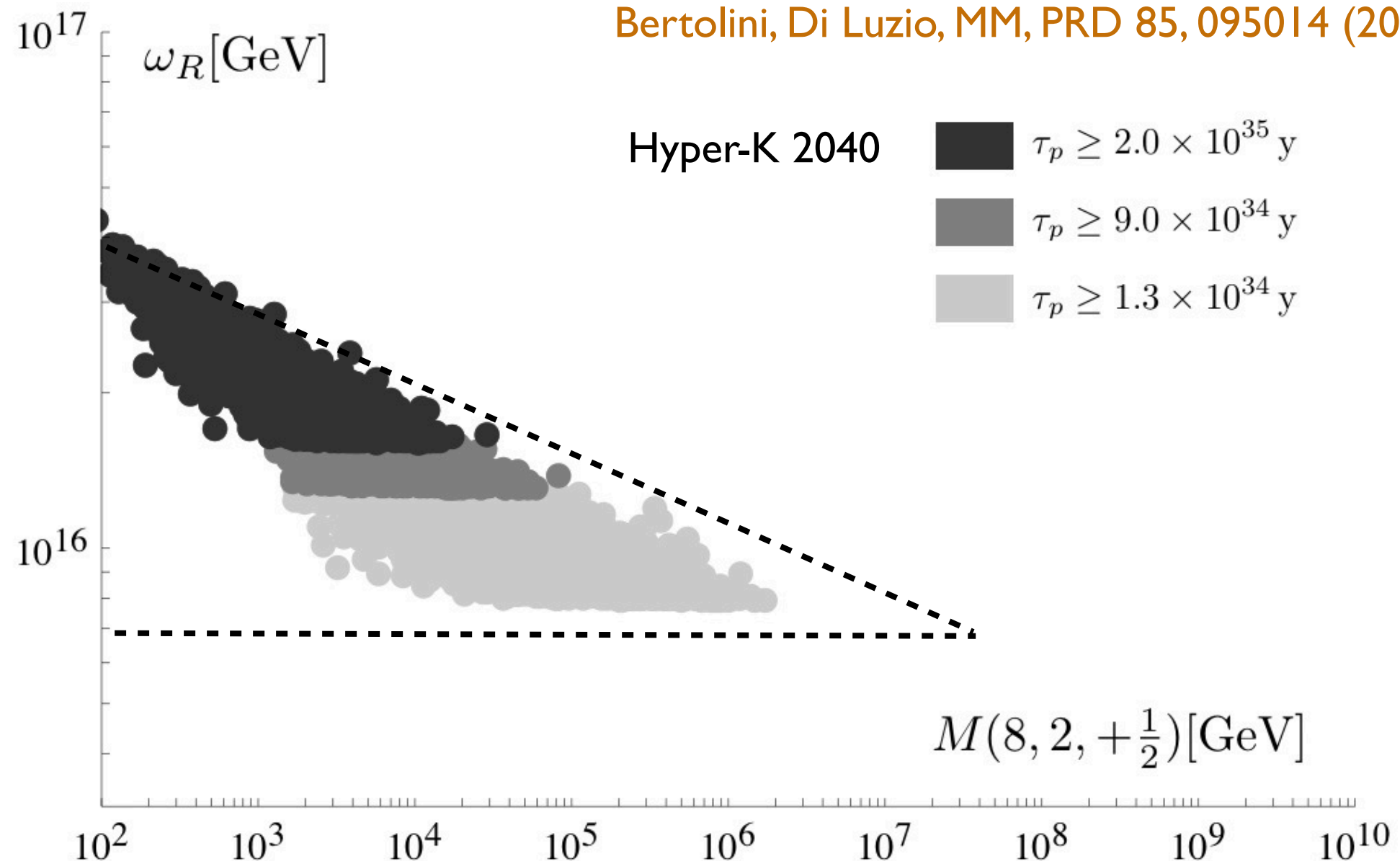
Case I: light $(8, 2, +\frac{1}{2})$ @ one loop Bertolini, Di Luzio, MM, PRD 85, 095014 (2012)



Towards a consistent & potentially realistic SO(10) scenario

Case I: light $(8, 2, +\frac{1}{2})$ @ two loops + improved proton decay

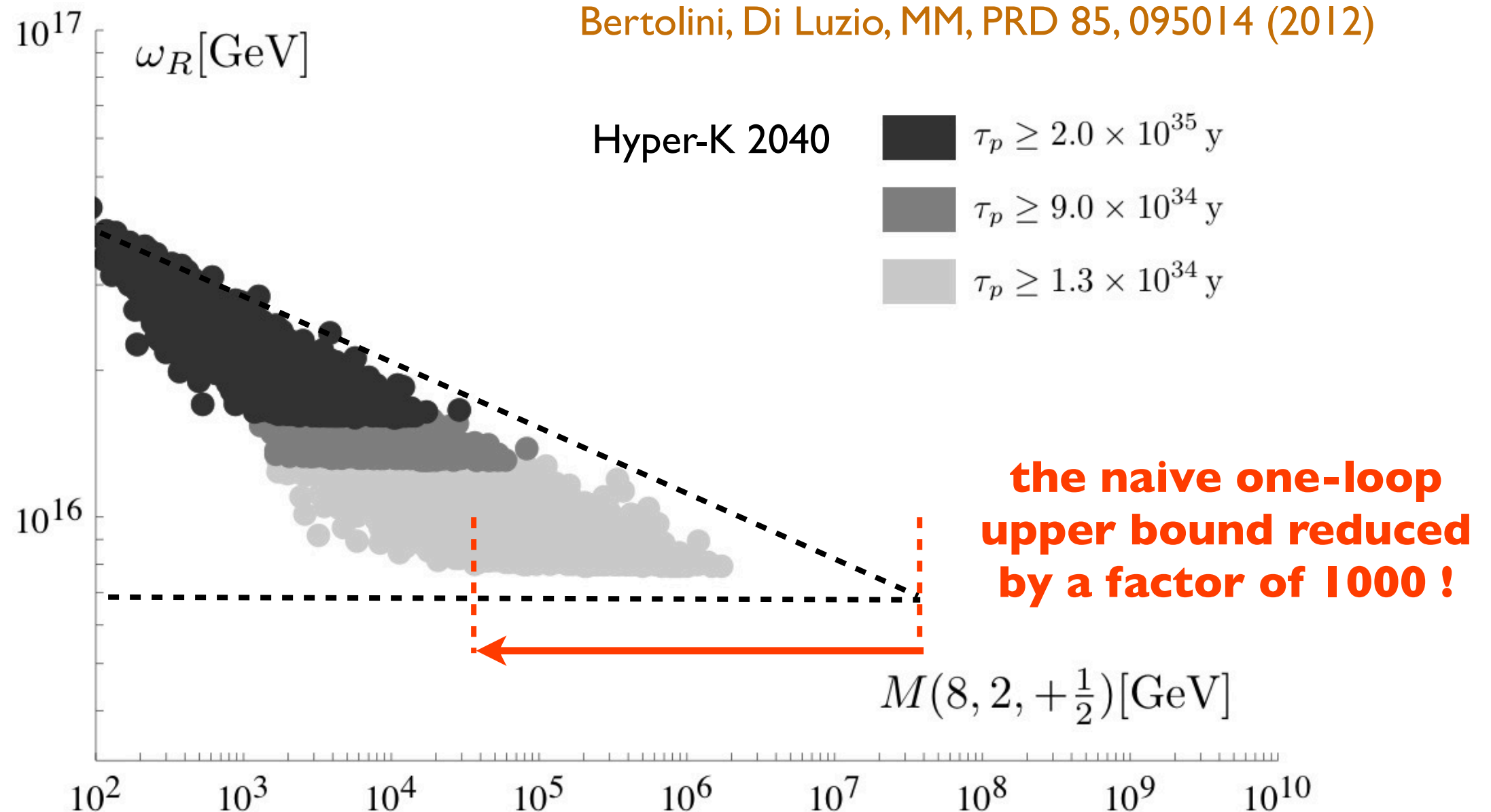
Bertolini, Di Luzio, MM, PRD 85, 095014 (2012)



Towards a consistent & potentially realistic SO(10) scenario

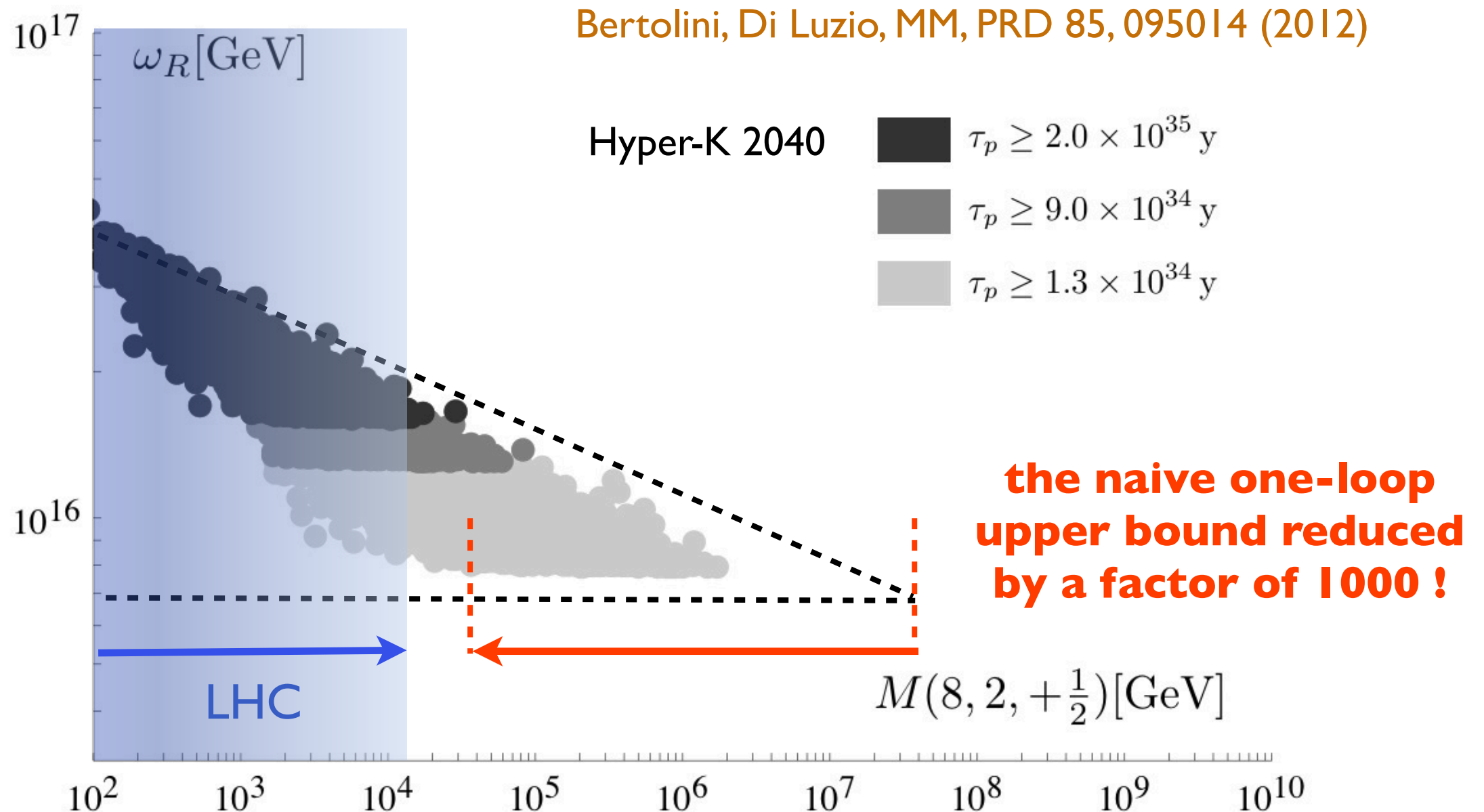
Case I: light $(8, 2, +\frac{1}{2})$ @ two loops + improved proton decay

Bertolini, Di Luzio, MM, PRD 85, 095014 (2012)



Towards a consistent & potentially realistic SO(10) scenario

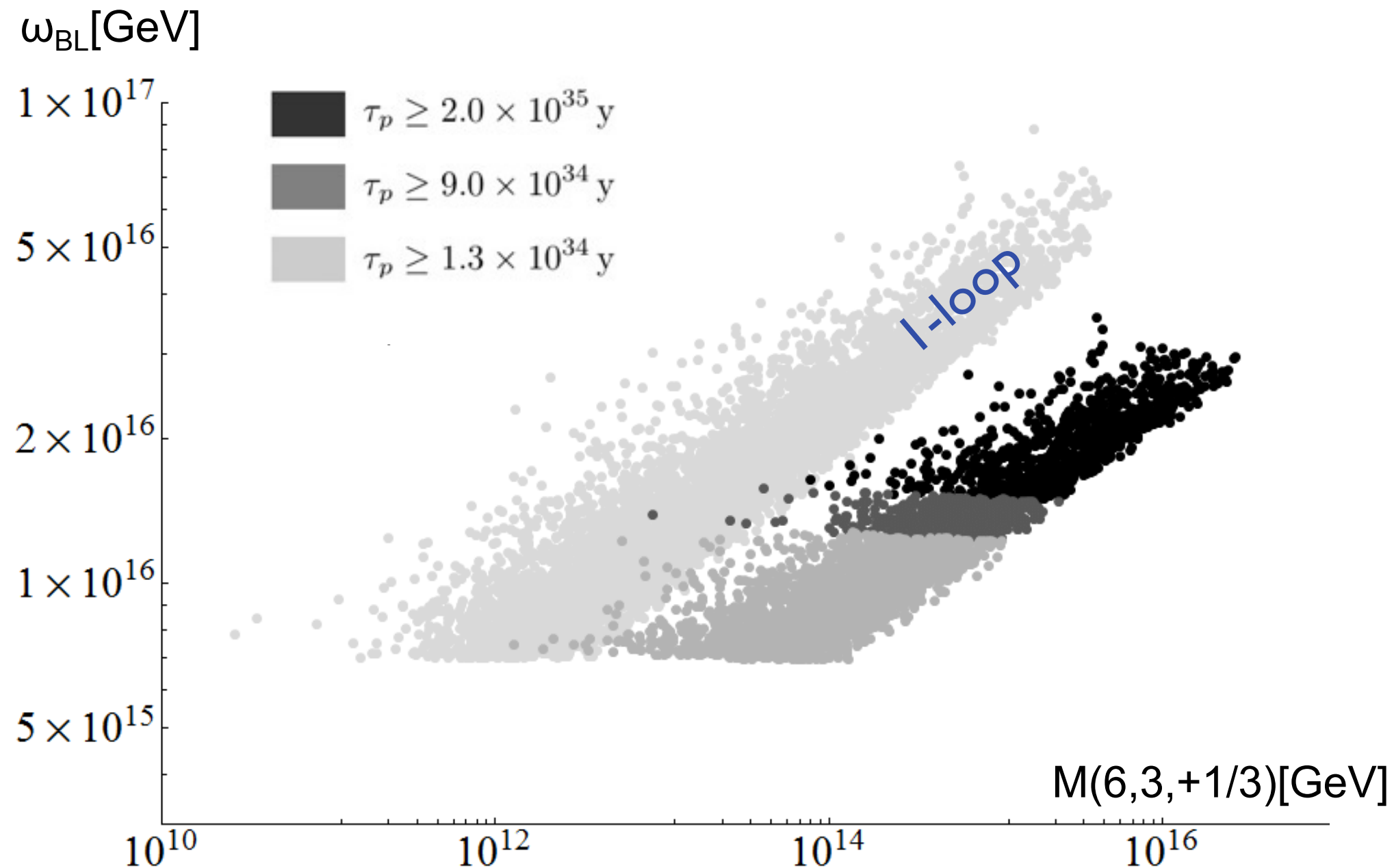
Case I: light $(8, 2, +\frac{1}{2})$ @ two loops + improved proton decay



Towards a consistent & potentially realistic SO(10) scenario

Case II: light $(6, 3, +\frac{1}{3})$ @ two loops

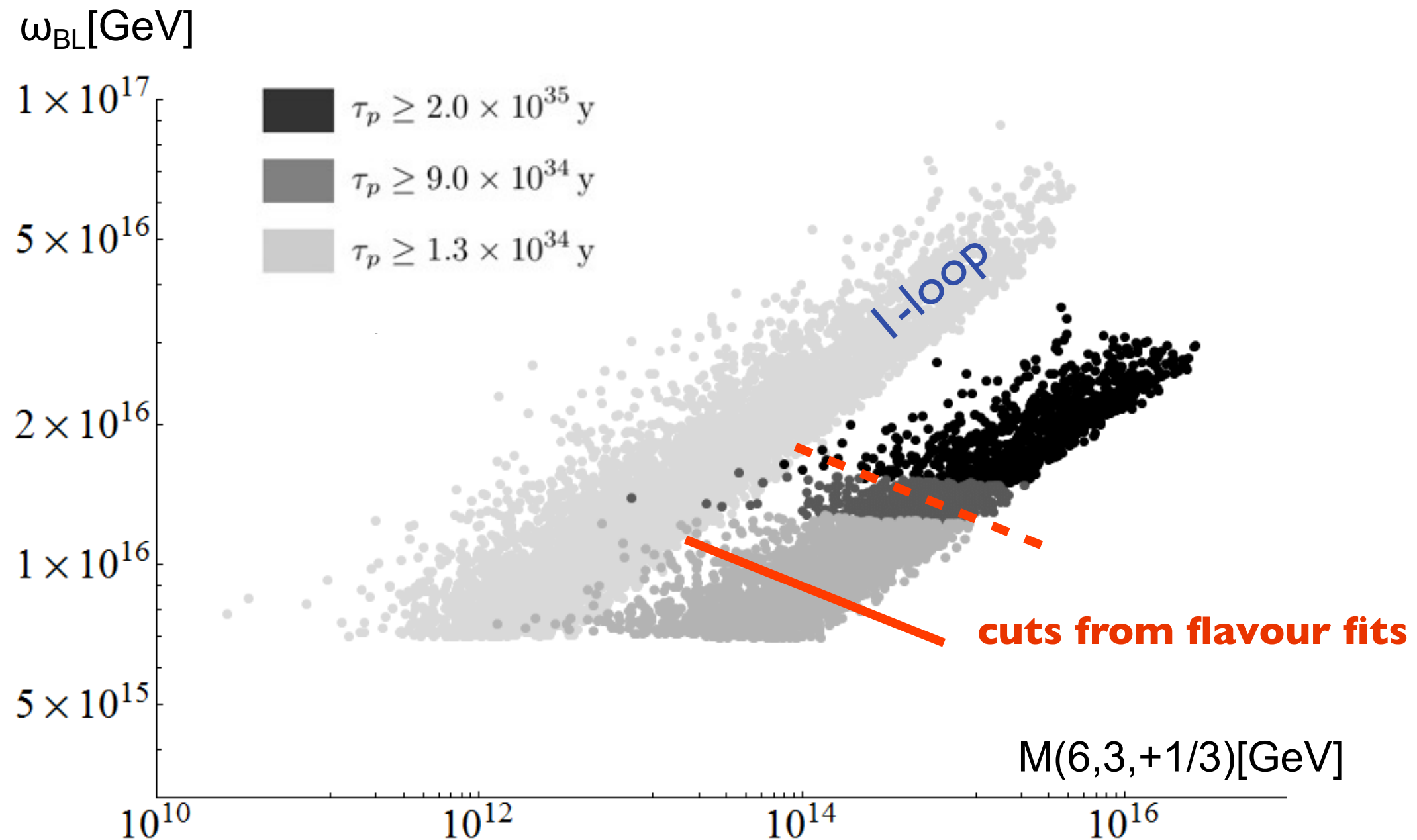
H. Kolečová, MM, arXiv:1409.4961



Towards a consistent & potentially realistic SO(10) scenario

Case II: light $(6, 3, +\frac{1}{3})$ @ two loops

H. Kolečová, MM, arXiv:1409.4961



Conclusions / outlook

It's almost impossible to calculate proton lifetime accurately

The minimal non-SUSY $SO(10)$ broken by 45 is very special

At the vast majority of the allowed parameter space it says that

either

one should see a scalar color octet @ LHC

or

one should see proton decay @ Hyper-K

Thank you for your kind attention!

Experimental affairs

First large water-Cherenkov detectors

KamiokaNDE

Kamioka-cho, Gifu, Japan

3,000 tons of pure water, about 1,000 PMs

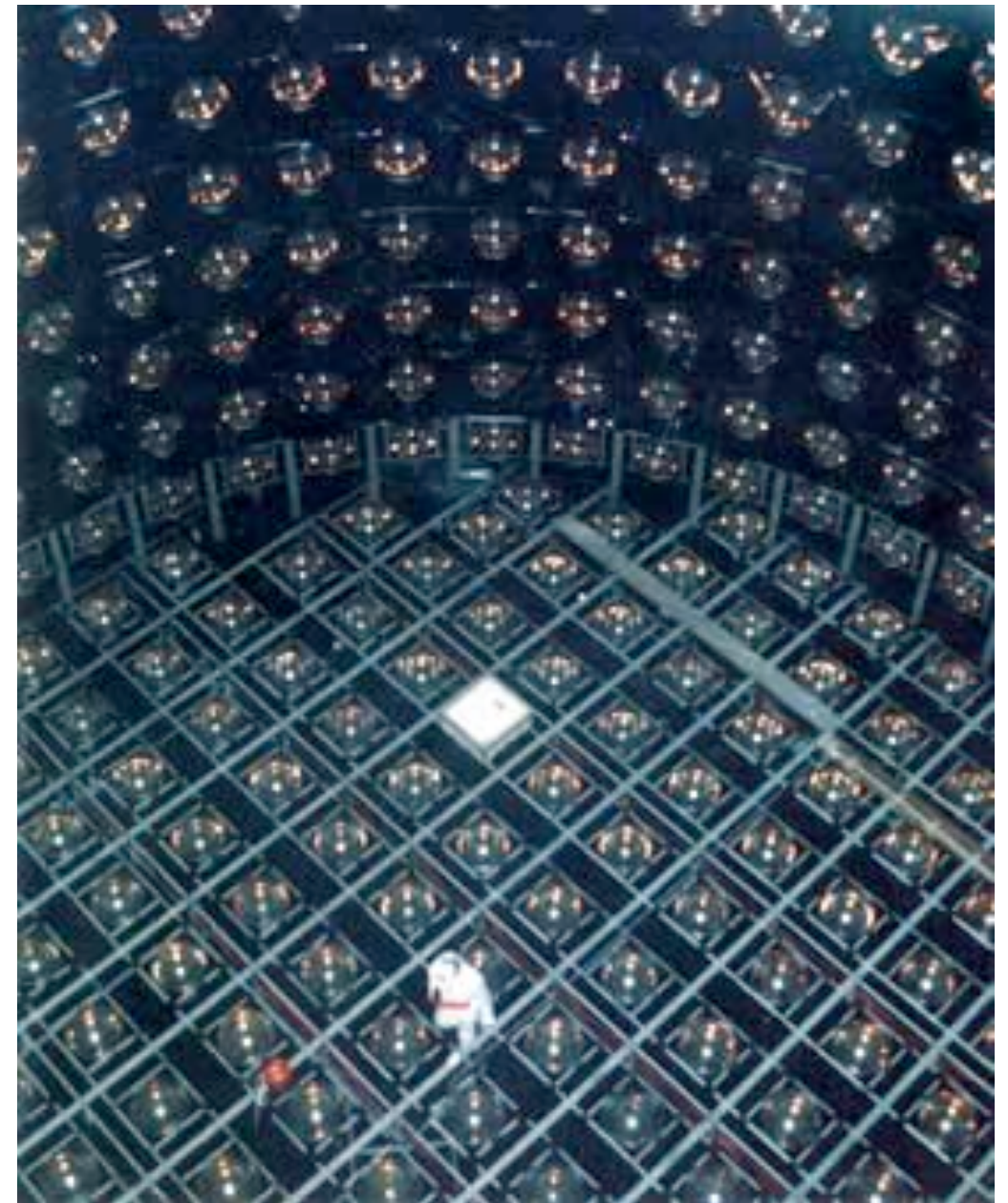
1983-1985 - first phase (proton decay focused)

1987-1990 - solar neutrino deficit measurements

Feb. 23 1987 07:35 - 12 out of 10^{58} neutrinos
from SN 1987A (170,000 ly)

1989 $\tau_p \gtrsim 2.6 \times 10^{32} \text{ yr}$

1990 Solar neutrino deficit confirmation



2002 Nobel prize for Masatoshi Koshiba

First large water-Cherenkov detectors

Kamioka

3,000 tons

1983-1990

1987-1990

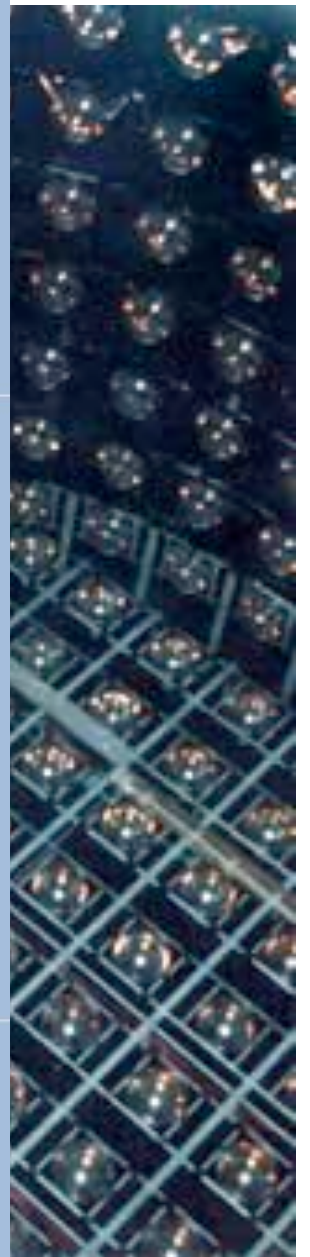
Feb. 23

from SN

1989 τ_1

1990 Solar neutrino deficit confirmation

2002 Nobel prize for Masatoshi Koshiba



First large water-Cherenkov detectors

KamiokaNDE

Kamioka-cho, Gifu, Japan

3,000 tons of pure water, about 1,000 PMs

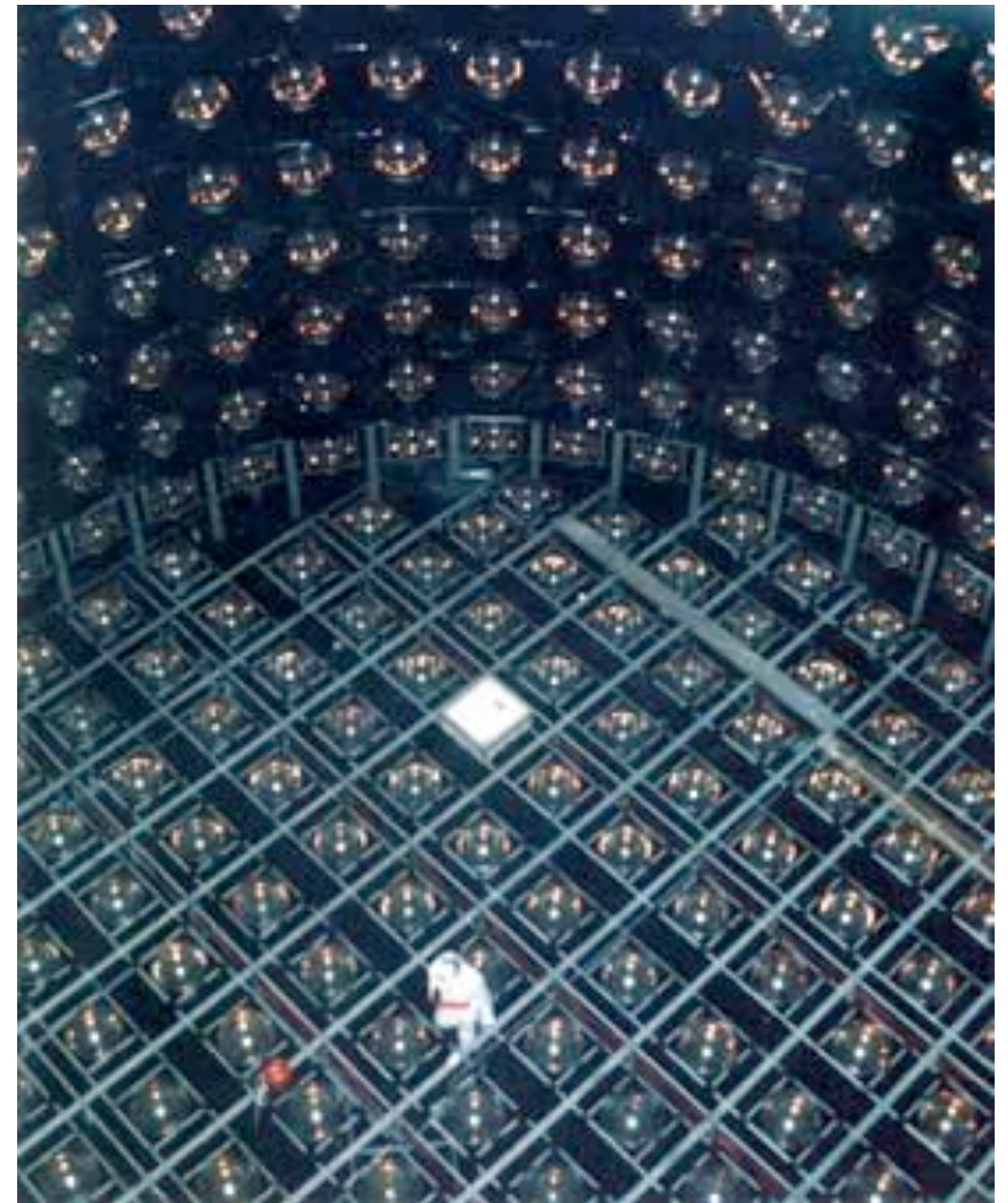
1983-1985 - first phase (proton decay focused)

1987-1990 - solar neutrino deficit measurements

Feb. 23 1987 07:35 - 12 out of 10^{58} neutrinos
from SN 1987A (170,000 ly)

1989 $\tau_p \gtrsim 2.6 \times 10^{32} \text{ yr}$

1990 Solar neutrino deficit confirmation



2002 Nobel prize for Masatoshi Koshiba

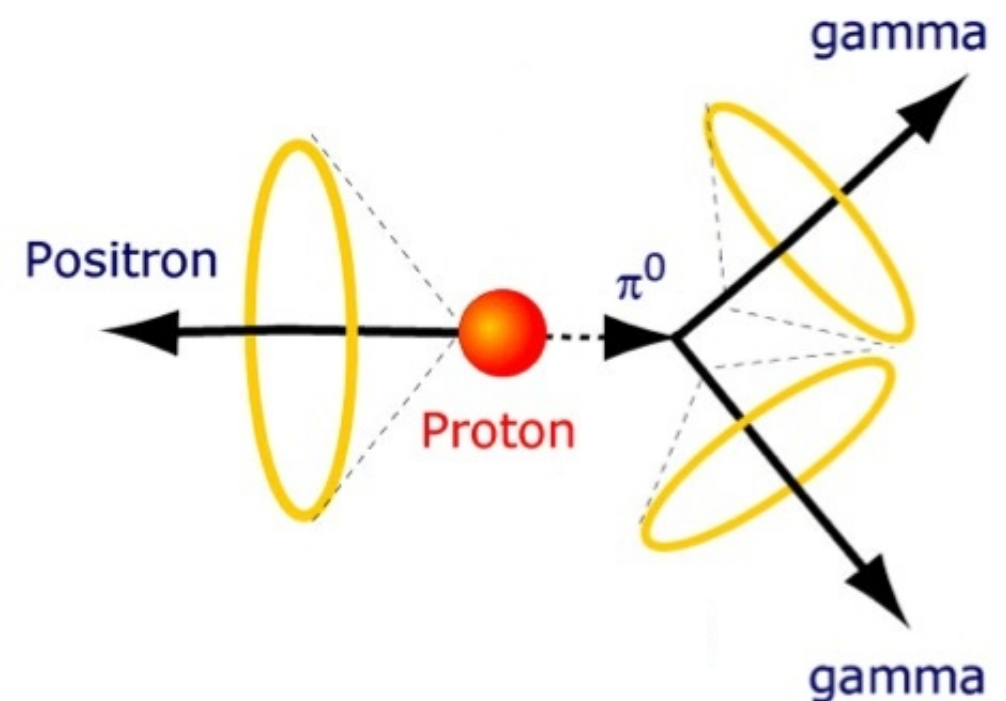
Proton decay in water

“Golden channel”: $p \rightarrow \pi^0 e^+$
 $\pi^0 \rightarrow 2\gamma$

$$p_\pi = p_e = 459 \text{ MeV}$$

$$p_{\gamma/\pi R} = 68 \text{ MeV}$$

Main background: $\nu N \rightarrow N e^+ + \# \pi$ inelastic CC scattering of atmospheric neutrinos



Proton decay in water

“Golden channel”: $p \rightarrow \pi^0 e^+$
 $\pi^0 \rightarrow 2\gamma$

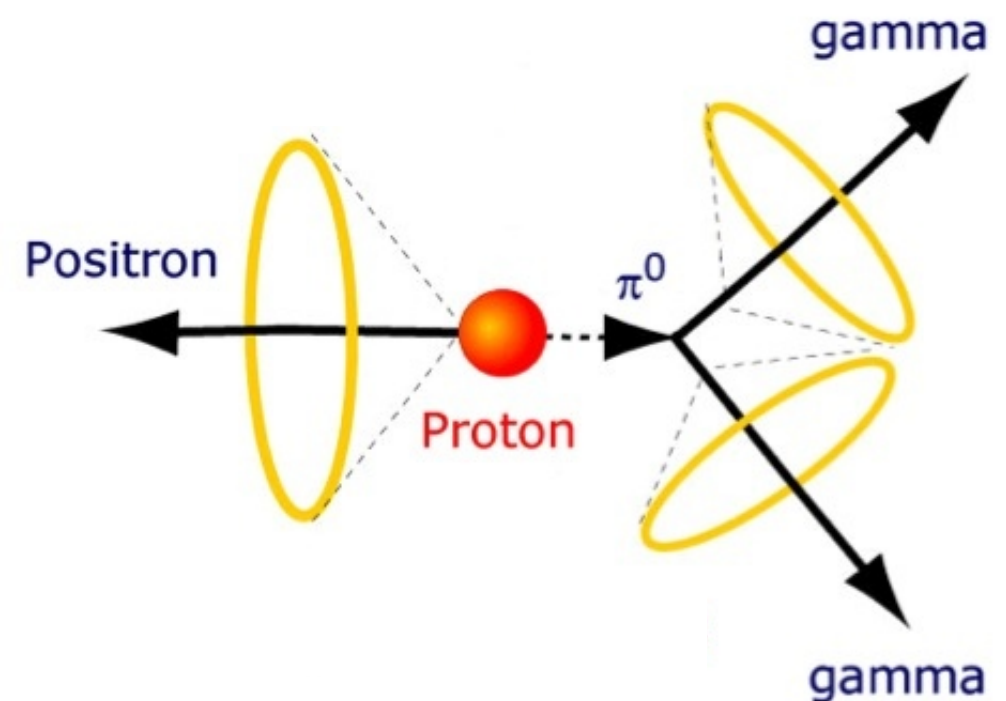
$$p_\pi = p_e = 459 \text{ MeV}$$

$$p_{\gamma/\pi R} = 68 \text{ MeV}$$

Main background: $\nu N \rightarrow N e^+ + \# \pi$ inelastic CC scattering of atmospheric neutrinos

Other complication - nuclear effects

- majority of nucleons in oxygen
- Fermi motion
- pion charge exchange
- absorption



Proton decay in water

“Golden channel”: $p \rightarrow \pi^0 e^+$
 $\pi^0 \rightarrow 2\gamma$

$$p_\pi = p_e = 459 \text{ MeV}$$

$$p_{\gamma/\pi R} = 68 \text{ MeV}$$

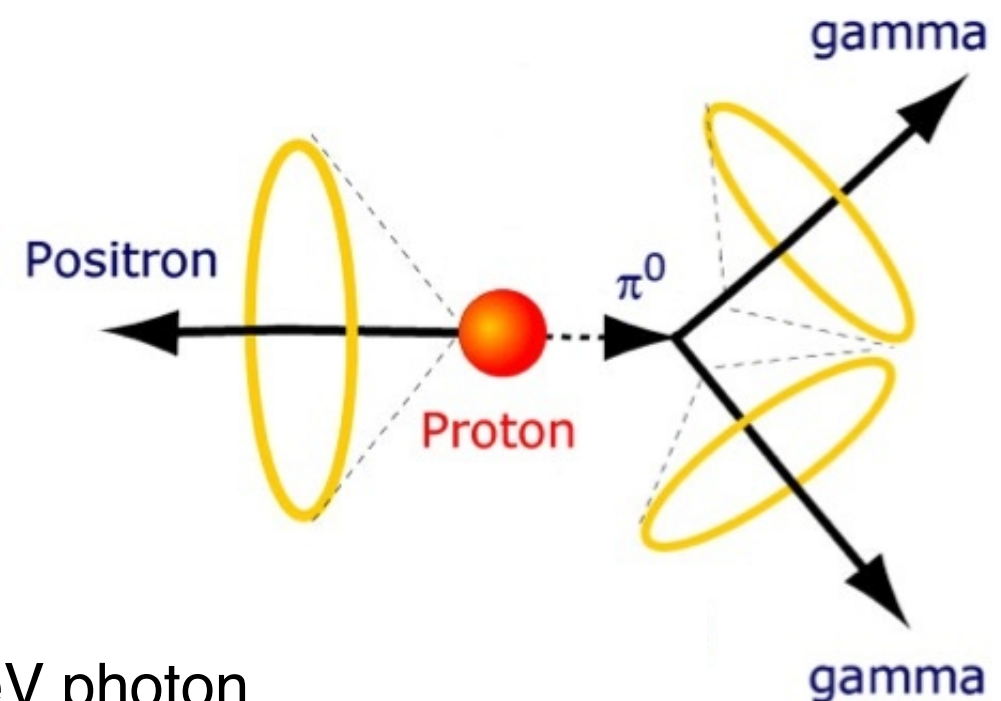
Main background: $\nu N \rightarrow N e^+ + \# \pi$ inelastic CC scattering of atmospheric neutrinos

Other complication - nuclear effects

- majority of nucleons in oxygen
- Fermi motion
- pion charge exchange
- absorption

Other signals

- nuclear recombination - extra 6.3 MeV photon
- neutron capture at a dope (Gd, ...)



Monopoles

No way to produce in lab, only cosmics + Callan-Rubakov effect

Monopoles

No way to produce in lab, only cosmics + Callan-Rubakov effect

- galactic magnetic field depletion
- pulsar stability
- proton stability

Freese, Turner

Monopoles

No way to produce in lab, only cosmics + Callan-Rubakov effect

- galactic magnetic field depletion
- pulsar stability
- proton stability

Freese, Turner

Upper limits on the flux density around Earth

Theory: $\Phi_M(\text{Earth})_{\text{theory}} \lesssim 10^{-22} \sim 10^{-27} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}$

Experiment: $\Phi_M(\text{Earth})_{\text{exp.}} \lesssim 10^{-16} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}$ MACRO 2001 (Gran Sasso)

Monopoles

No way to produce in lab, only cosmics + Callan-Rubakov effect

- galactic magnetic field depletion
- pulsar stability
- proton stability

Freese, Turner

Upper limits on the flux density around Earth

Theory: $\Phi_M(\text{Earth})_{\text{theory}} \lesssim 10^{-22} \sim 10^{-27} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}$

Experiment: $\Phi_M(\text{Earth})_{\text{exp.}} \lesssim 10^{-16} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}$ MACRO 2001 (Gran Sasso)

N.B. early (fake) monopole-like events Price et al., 1975 PRL August 25