Minimal Flavour Violation in *Two BEH Doublet* models

Another approach

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Collaboration

- Part of this work has been done with G.C. Branco, A. Carmona, M. Nebot, L. Pedro and M.N. Rebelo: arXiv:1401.6147, arXiv:1210.8163, arXiv:1102.0520, **arXiv:0911.1753**.

The flavour structure of the SM I

- It is controlled by the Yukawa matrices: Γ, Δ and Π

$$L_Y = -\overline{Q}_L\ \Gamma\Phi d_R - \overline{Q}_L\Delta\tilde{\Phi}u_R - \overline{L}_L\ \Pi\Phi l_R + h.c$$

- In the absence of Yukawa coupling the SM has a large global $U(3)^5$ symmetry $SU(3)_q^3 \otimes SU(3)_l^2 \otimes U(1)^5$ that correspond to $Q_L \longrightarrow W_L Q_L;\ u_R \longrightarrow W_R^u u_R;\ d_R \longrightarrow W_R^d d_R;\ L_L \longrightarrow W_L^L L_L;\ l_R \longrightarrow W_R^l l_R$
- The gauge and pure Higgs Lagrangians are invariant and the Yukawa couplings break this flavour symmetries. So the entire SM is not invariant under these weak basis (WB) transformations.
- Physical observable should be independent of the weak basis we chose to formulate the theory. So two theories whose Yukawa are related by

$$\begin{aligned} \Gamma &\longrightarrow W_L^\dagger \Gamma W_R^d \\ \Delta &\longrightarrow W_L^\dagger \Delta W_R^u \end{aligned}$$

should be identical.

The flavour structure of the SM II

- Therefore in general Physical Observables should be made out of

$$
\begin{aligned}
H_d &= M_d M_d^\dagger = \frac{v^2}{2}\Gamma\Gamma^\dagger \\
H_u &= M_u M_u^\dagger = \frac{v^2}{2}\Delta\Delta^\dagger
\end{aligned}
$$

- transforming as

$$
\begin{aligned}
H_d &\to W_L^\dagger H_d W_L \\
H_u &\to W_L^\dagger H_u W_L
\end{aligned}
$$

- And they will have the general form

$$
Tr\left[(H_u)^\alpha (H_d)^\beta (H_u)^\gamma (H_d)^\delta \ldots.\right]
$$

The flavour structure of the SM III

- The diagonalization of the mass matrices $M_d = v\Gamma/\sqrt{2}$ and $M_u = v\Delta/\sqrt{2}$ are done with

$$U_L^{d\dagger} M_d U_R^d = D_d = diag(m_d, m_s, m_b) = \frac{v}{\sqrt{2}} diag(y_d, y_s, y_b)$$
$$U_L^{u\dagger} M_u U_R^u = D_u = diag(m_u, m_c, m_t) = \frac{v}{\sqrt{2}} diag(y_u, y_c, y_t)$$

- Note that this is not a WBT. Without loose of generality, sometime is useful to choose a special WBT

$$W_L = U_L^d \quad ; \quad W_R^u = U_R^u \quad ; \quad W_R^d = U_R^d$$

such that

$$\Gamma = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \quad ; \quad \Delta = V^\dagger \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

Y^d and Y^u are the diagonal Yukawa coupling and all the changes of flavour are encoded in the CKM matrix $V = U_L^{u\dagger} U_L^d$.

Minimal Flavour Violation I

- The legacy of B factories can be summarized by:

Flavour violation and CP violation in flavour changing processes are dominated by the CKM mechanism.

- But even more, if to the SM Lagrangian we add for $K^0 - \overline{K}^0$, $B^0 - \overline{B}^0$, $B_s^0 - \overline{B}_s^0$ mixing the New Physics (NP) Lagrangian

$$\mathcal{L}_{NP} = \sum_{i \neq j} \frac{C_{ij}^2}{\Lambda^2} \left(\overline{Q}_{L_i} \gamma^\mu Q_{L_j} \right)^2$$

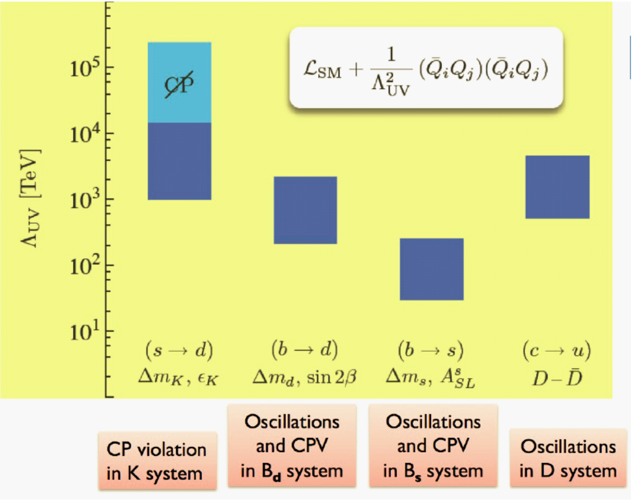
The condition

$$|A_{NP}| < |A_{SM}|$$

Implies

$$\Lambda > \begin{cases} \left(10^4 \text{ TeV}\right) \left(C_{sd}\right) \\ \left(10^2 \text{ TeV}\right) \left(C_{bq}\right) \end{cases}$$

Minimal Flavour Violation II



Minimal Flavour Violation III

If one insist, for example, that the scale of NP is at the TeV, we need to suppress very strongly c_{ij}.they cannot be order 1, the NP flavour structure is highly **non generic**.

- A popular way to implement this non generic structure of the NP operators is the so called Minimal Flavour Violation (MFV) hypothesis. It consists of two ingredients:

1. A flavour symmetry: $SU\left(3\right)_{Q_L} \otimes SU\left(3\right)_{u_R} \otimes SU\left(3\right)_{d_R}$
2. A set of symmetry breaking flavour terms: The Yukawa Couplings Γ and Δ are the unique sources of flavour symmetry breaking in the NP model we are considering.

Minimal Flavour Violation IV

In our previous example we have that C should transform as $\mathcal{O} \to W_L^\dagger \mathcal{O} W_L$

$$C = \left(aI + b\Gamma\Gamma^\dagger + c\Delta\Delta^\dagger + d\Gamma\Gamma^\dagger\Gamma\Gamma^\dagger + e\Delta\Delta^\dagger\Delta\Delta^\dagger + \cdots\cdots\cdots\cdots \right)$$

$$\Gamma = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \quad ; \quad \Delta = V^\dagger \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

Taking into account the differences of the eigenvalues of the Yukawa couplings, neglecting all of them but the top, we get that the unique flavour changing relevant structure is:

$$\begin{aligned} \left(\Delta\Delta^\dagger\right)^n_{i\neq j} &\propto \left(V^\dagger y_t^2 P_3 V\right)^n_{i\neq j} = y_t^{2n} V_{3i}^* V_{3j} \\ C_{i\neq j} &\propto f\left(y_t^2\right) V_{3i}^* V_{3j} \end{aligned}$$

Minimal Flavour Violation V

- So in theories of NP with MFV one gets:

$$\mathcal{A}\left(d_j \longrightarrow d_i\right)_{MFV} = \left(V_{ti}^* V_{tj}\right) \mathcal{A}_{SM}^{(\Delta F=1))} \left(1 + a_1 \frac{16\pi^2 M_W^2}{\Lambda^2}\right)$$

$$\mathcal{A}\left(M_{ij} \longrightarrow \overline{M}_{ij}\right)_{MFV} = \left(V_{ti}^* V_{tj}\right)^2 \mathcal{A}_{SM}^{(\Delta F=2))} \left(1 + a_2 \frac{16\pi^2 M_W^2}{\Lambda^2}\right)$$

that essentially implies the same relative correction for $b \longrightarrow s$, $b \longrightarrow d$ and $s \longrightarrow d$ transitions. Some important predictions are

$$\frac{\Gamma\left(B_d \longrightarrow l^+ l^-\right)}{\Gamma\left(B_s \longrightarrow l^+ l^-\right)} \approx \frac{f_{B_d}^2 m_{B_d}}{f_{B_s}^2 m_{B_s}} \frac{\left|V_{td}\right|^2}{\left|V_{ts}\right|^2}$$

$$\frac{\Delta M_{B_d}}{\Delta M_{B_s}} = \frac{f_{B_d}^2 m_{B_d} B_{B_d}}{f_{B_s}^2 m_{B_s} B_{B_s}} \frac{\left|V_{td}\right|^2}{\left|V_{ts}\right|^2}$$

MFV IN 2HDM I

- In the two Higgs doublet model, the flavour structure is much more involved ($\tilde{\Phi}_j = i\sigma_2 \Phi_j^*$)

$$L_Y = -\overline{Q}_L \left(\Gamma_1 \Phi_1 + \Gamma_2 \Phi_2\right) d_R - \overline{Q}_L \left(\Delta_1 \tilde{\Phi}_1 + \Delta_2 \tilde{\Phi}_2\right) u_R + h.c.$$

- In the Higgs basis: $\langle H_1 \rangle^T = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} \end{pmatrix}, \langle H_2 \rangle^T = \begin{pmatrix} 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} e^{i\theta_1} \frac{v_1}{v} & e^{i\theta_1} \frac{v_2}{v} \\ e^{i\theta_2} \frac{v_2}{v} & -e^{i\theta_2} \frac{v_1}{v} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

$$L_Y = -\overline{Q}_L \frac{\sqrt{2}}{v} \left[M_d H_1 + N_d H_2\right] d_R - \overline{Q}_L \frac{\sqrt{2}}{v} \left[M_u \widetilde{H}_1 + N_u \widetilde{H}_2\right] u_R + h.c.$$

$$\begin{aligned} M_d &= \frac{1}{\sqrt{2}} \left(\Gamma_1 v_1 e^{i\theta_1} + \Gamma_2 v_2 e^{i\theta_2}\right) \\ N_d &= \frac{1}{\sqrt{2}} \left(\Gamma_1 v_2 e^{i\theta_1} - \Gamma_2 v_1 e^{i\theta_2}\right) \\ M_u &= \frac{1}{\sqrt{2}} \left(\Delta_1 v_1 e^{-i\theta_1} + \Delta_2 v_2 e^{-i\theta_2}\right) \\ N_u &= \frac{1}{\sqrt{2}} \left(\Delta_1 v_2 e^{-i\theta_1} - \Delta_2 v_1 e^{-i\theta_2}\right) \end{aligned}$$

MFV IN 2HDM II

- In this framework the MFV hypothesis would be: **all the flavour symmetries are broken by only two independent structures transforming as**

$$\begin{aligned} O^d &\rightarrow W_L^\dagger O^d W_R^d \\ O^u &\rightarrow W_L^\dagger O^u W_R^u \end{aligned}$$

- One could choose $\left(\frac{\sqrt{2}}{v}\right) M_d$ and $\left(\frac{\sqrt{2}}{v}\right) M_u$. Traditionally it is used: Γ_1 and Δ_2. And the MFV expansion is now

$$\begin{aligned} \Gamma_2 &= \left[\epsilon_0 I + \epsilon_1 \Gamma_1 \Gamma_1^\dagger + \epsilon_2 \Delta_2 \Delta_2^\dagger + \epsilon_3 \Delta_2 \Delta_2^\dagger \Gamma_1 \Gamma_1^\dagger + \epsilon_4 \Gamma_1 \Gamma_1^\dagger \Delta_2 \Delta_2^\dagger + \cdot\cdot\right] \Gamma_1 \\ \Delta_1 &= \left[\epsilon'_0 I + \epsilon'_1 \Gamma_1 \Gamma_1^\dagger + \epsilon'_2 \Delta_2 \Delta_2^\dagger + \epsilon'_3 \Delta_2 \Delta_2^\dagger \Gamma_1 \Gamma_1^\dagger + \epsilon'_4 \Gamma_1 \Gamma_1^\dagger \Delta_2 \Delta_2^\dagger + \cdot\cdot\right] \Delta_2 \end{aligned}$$

MFV IN 2HDM III

- The presence of two vacuum expectation values invalidates the argument previously used to neglect the down Yukawa couplings. Therefore here one has to keep

$$\Gamma_1\Gamma_1^\dagger \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b^2 \end{pmatrix} \quad ; \quad \Delta_2\Delta_2^\dagger \sim V^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t^2 \end{pmatrix} V$$

MFV IN 2HDM: new approach I

- We can try again the MFV in the 2HDM but this time we will use M_d and M_u as the basic objects that brakes the flavour symmetries

$$L_Y = -\overline{Q}_L \frac{\sqrt{2}}{v} \left[M_d H_1 + N_d H_2 \right] d_R - \overline{Q}_L \frac{\sqrt{2}}{v} \left[M_u \widetilde{H}_1 + N_u \widetilde{H}_2 \right] u_R + h.c.$$

$$\begin{aligned} M_d &= \tfrac{1}{\sqrt{2}} \left(\Gamma_1 v_1 e^{i\theta_1} + \Gamma_2 v_2 e^{i\theta_2} \right) & M_u &= \tfrac{1}{\sqrt{2}} \left(\Delta_1 v_1 e^{-i\theta_1} + \Delta_2 v_2 e^{-i\theta_2} \right) \\ N_d &= \tfrac{1}{\sqrt{2}} \left(\Gamma_1 v_2 e^{i\theta_1} - \Gamma_2 v_1 e^{i\theta_2} \right) & N_u &= \tfrac{1}{\sqrt{2}} \left(\Delta_1 v_2 e^{-i\theta_1} - \Delta_2 v_1 e^{-i\theta_2} \right) \end{aligned}$$

- So we need the MFV expansion of N_d and N_u. This time one would write ($H_d = M_d M_d^\dagger$, $H_u = M_u M_u^\dagger$)

$$\begin{aligned} N_d &= \left[\epsilon_0 I + \epsilon_1 H_d + \epsilon_2 H_u + \epsilon_3 H_u H_d + \epsilon_4 H_d H_u + \cdots \right] M_d \\ N_u &= \left[\epsilon'_0 I + \epsilon'_1 H_d + \epsilon'_2 H_u + \epsilon'_3 H_u H_d + \epsilon'_4 H_d H_u + \cdots \right] M_u \end{aligned}$$

MFV IN 2HDM: new approach II

- The key point for the new approach is that there are other matrices in addition to H_d and H_u that transform under a WB transformation as

$$\mathcal{O} \to W_L^\dagger \mathcal{O} W_L$$

- From

$$M_d = U_L^d D_d U_R^{d\dagger}$$

$$H_d = M_d M_d^\dagger = U_L^d D_d^2 U_L^{d\dagger} = U_L^d \sum_{i=1}^{3} m_{d_i}^2 P_i U_L^{d\dagger} = \sum_{i=1}^{3} m_{d_i}^2 \mathcal{P}_i^{d_L}$$

$$\boxed{\mathcal{P}_i^{d_L} = U_L^d P_i U_L^{d\dagger}} \quad ; \quad (P_i)_{jk} = \delta_{ij}\delta_{ik}$$

MFV IN 2HDM: new approach III

- are the projector operators over the lefthanded down quarks in an arbitrary weak basis.

$$\mathcal{P}_i^{d_L}\mathcal{P}_j^{d_L} = \delta_{ij}\mathcal{P}_i^{d_L}$$
$$\sum_{i=1}^{3}\mathcal{P}_i^{d_L} = I$$

- and obviously they transform as H_d :

$$\mathcal{P}_i^{d_L} \longrightarrow W_L^\dagger \mathcal{P}_i^{d_L} W_L$$

- In general we can define up, down, left and right projection operators:

$$\begin{array}{lcl} H_d = M_d M_d^\dagger = \sum_{i=1}^3 m_{d_i}^2 \mathcal{P}_i^{d_L} & ; & \mathcal{P}_i^{d_L} = U_L^d P_i U_L^{d\dagger} \\ H_u = M_u M_u^\dagger = \sum_{i=1}^3 m_{u_i}^2 \mathcal{P}_i^{u_L} & ; & \mathcal{P}_i^{u_L} = U_L^u P_i U_L^{u\dagger} \end{array}$$

MFV IN 2HDM: new approach IV

- That transform under a WB transformation as

$$\mathcal{P}_i^{d_L} \rightarrow W_L^\dagger \mathcal{P}_i^{d_L} W_L \quad ; \quad \mathcal{P}_i^{u_L} \rightarrow W_L^\dagger \mathcal{P}_i^{u_L} W_L$$

- Therefore we get **the most general MFV expansion in 2HDM**:

$$N_d = \left(a_0 I + a_{1j} \mathcal{P}_j^{d_L} + a_{2j} \mathcal{P}_j^{u_L} + a_{3ij} \mathcal{P}_i^{u_L} \mathcal{P}_j^{d_L} + a_{4ij} \mathcal{P}_i^{d_L} \mathcal{P}_j^{u_L} + \cdots \right) M_d$$
$$N_u = \left(a'_0 I + a'_{1j} \mathcal{P}_j^{d_L} + a'_{2j} \mathcal{P}_j^{u_L} + a'_{3ij} \mathcal{P}_i^{u_L} \mathcal{P}_j^{d_L} + a'_{4ij} \mathcal{P}_i^{d_L} \mathcal{P}_j^{u_L} + \cdots \right) M_u$$

- Remarkably enough it can be shown that **renormalizable models known long time ago and enforced by flavour symmetries** (Branco, Grimus, Lavoura) **realize the most simple MFV expansion with controlled FCYC.**
- For example one BGL model is enforced by

$$Q_{L_3} \rightarrow e^{i\alpha} Q_{L_3} \quad ; \quad u_{R_3} \rightarrow e^{i2\alpha} u_{R_3} \quad ; \quad \Phi_2 \rightarrow e^{i\alpha} \Phi_2$$

MFV IN 2HDM: new approach V

- It correspond to the model defined by the MFV expansion

$$\begin{aligned} N_d &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{u_L}\right] M_d \\ N_u &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{u_L}\right] M_u \end{aligned}$$

- This is the Up3 model and obviously there are other two Up1,2 models. These models have FCYC in the down sector controlled by quark masses and CKM matrix elements:

$$\begin{aligned} \widehat{N}_d &= U_L^{d\dagger} N_d U_R^d = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) V^\dagger P_3 V\right] D_d \\ \widehat{N}_u &= U_L^{u\dagger} N_u U_R^u = \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) P_3\right] D_u \end{aligned}$$

$$\begin{aligned} \left(\widehat{N}_d\right)_{ij} &= \left[\frac{v_2}{v_1} \delta_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) V_{3i}^* V_{3j}\right] m_{dj} \\ \left(\widehat{N}_u\right)_{ij} &= \left[\frac{v_2}{v_1} \delta_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \delta_{ij}\delta_{i3}\right] m_{u_j} \end{aligned}$$

MFV IN 2HDM: new approach VI

- In a similar way there are three Down models with FCYC in the up sector defined by

$$Q_{L_3} \longrightarrow e^{i\alpha} Q_{L_3} \quad ; \quad d_{R_3} \longrightarrow e^{i2\alpha} d_{R_3} \quad ; \quad \Phi_2 \longrightarrow e^{i\alpha} \Phi_2$$

$$\begin{aligned} N_d &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \mathcal{P}_3^{d_L} \right] M_d \\ N_u &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \mathcal{P}_3^{d_L} \right] M_u \end{aligned}$$

- Because two exact relations of the 2HDM are

$$\begin{aligned} N_d &= \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \frac{v_2}{\sqrt{2}} e^{i\theta_2} \Gamma_2 \\ N_u &= \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) \frac{v_2}{\sqrt{2}} e^{-i\theta_2} \Delta_2 \end{aligned}$$

MFV IN 2HDM: new approach VII

- These BGL models are fully defined by the relations (Up3)

$$\begin{aligned} \frac{v_2}{\sqrt{2}} e^{i\theta_2} \Gamma_2 &= \mathcal{P}_3^{u_L} M_d \\ \frac{v_2}{\sqrt{2}} e^{-i\theta_2} \Delta_2 &= \mathcal{P}_3^{u_L} M_u = M_u \mathcal{P}_3^{u_R} \end{aligned}$$

- So these models are also defined by

$$\begin{aligned} &\mathcal{P}_3^{u_L} \Gamma_2 = \Gamma_2 \text{ and } \mathcal{P}_3^{u_L} \Gamma_1 = 0 \\ &\mathcal{P}_3^{u_L} \Delta_2 = \Delta_2 \text{ and } \mathcal{P}_3^{u_L} \Delta_1 = 0 \quad ; \quad \Delta_2 \mathcal{P}_3^{u_R} = \Delta_2 \text{ and } \Delta_1 \mathcal{P}_3^{u_R} = 0 \end{aligned}$$

MFV IN 2HDM: new approach VIII

- In some basis this implies

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

$$\Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

$$Q_{L_3} \longrightarrow e^{i\alpha} Q_{L_3} \quad ; \quad u_{R_3} \longrightarrow e^{i2\alpha} u_{R_3} \quad ; \quad \Phi_2 \longrightarrow e^{i\alpha} \Phi_2$$

MFV IN 2HDM: new approach IX

- Other models like

$$\begin{aligned}
N_d &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{u_L}\right] M_d &\leftrightarrow\quad& \mathcal{P}_3^{u_L}\Gamma_2 = \Gamma_2,\ \mathcal{P}_3^{u_L}\Gamma_1 = 0 \\
N_u &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{d_L}\right] M_u &\leftrightarrow\quad& \mathcal{P}_3^{d_L}\Delta_2 = \Delta_2,\ \mathcal{P}_3^{d_L}\Delta_1 = 0
\end{aligned}$$

$$\begin{aligned}
N_d &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_3^{u_L}\right] M_d &\leftrightarrow\quad& \mathcal{P}_3^{u_L}\Gamma_2 = \Gamma_2,\ \mathcal{P}_3^{u_L}\Gamma_1 = 0 \\
N_u &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_2^{u_L}\right] M_u &\leftrightarrow\quad& \left\{\begin{array}{l} \mathcal{P}_2^{u_L}\Delta_2 = \Delta_2,\ \mathcal{P}_2^{u_L}\Delta_1 = 0 \\ \Delta_2\mathcal{P}_2^{u_R} = \Delta_2,\ \Delta_1\mathcal{P}_2^{u_R} = 0 \end{array}\right.
\end{aligned}$$

are not stable under RGE. There are other models stable under RGE that in one sense or another are trivially equivalent to BGL.

Including Leptons (Dirac Neutrinos) I

- In the framework where the SM is enlarged with three right handed neutrino ν_R but imposing at the same time total lepton number conservation we are in the case of Dirac Neutrinos. The Yukawa sector in this case is

$$\begin{aligned} L_Y &= -\overline{Q}_L \left(\Gamma_1 \Phi_1 + \Gamma_2 \Phi_2\right) d_R - \overline{Q}_L \left(\Delta_1 \tilde{\Phi}_1 + \Delta_2 \tilde{\Phi}_2\right) u_R + h.c. \\ &= -\overline{L}_L \left(\Pi_1 \Phi_1 + \Pi_2 \Phi_2\right) l_R - \overline{L}_L \left(\Sigma_1 \tilde{\Phi}_1 + \Sigma_2 \tilde{\Phi}_2\right) \nu_R + h.c. \end{aligned}$$

and the extension of BGL to leptonic sector is absolutely similar to the quark sector. The full model will be enforced by

$$\begin{array}{l} Q_{L_i} \longrightarrow e^{i\alpha} Q_{L_i} \quad ; \quad u_{R_i} \longrightarrow e^{i2\alpha} u_{R_i} \quad ; \quad \Phi_2 \longrightarrow e^{i\alpha} \Phi_2 \\ L_{Lj} \longrightarrow e^{i\alpha} L_{L_j} \quad ; \quad \nu_{R_j} \longrightarrow e^{i2\alpha} \nu_{R_j} \end{array}$$

Including Leptons (Dirac Neutrinos) II

- Here we have 3×3 models of BGL type with FCYC in the down quark sector and in the charged leptons sector. The 3×3 stands for the three i and j we can choose. In this case the model is fully defined by

$$\begin{aligned}
N_d &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_i^{u_L}\right] M_d &\leftrightarrow\quad & \mathcal{P}_i^{u_L}\Gamma_2 = \Gamma_2,\ \mathcal{P}_i^{u_L}\Gamma_1 = 0 \\
N_u &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_i^{u_L}\right] M_u &\leftrightarrow\quad & \mathcal{P}_i^{u_L}\Delta_2 = \Delta_2,\ \mathcal{P}_i^{u_L}\Delta_1 = 0 \\
N_l &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_j^{\nu_L}\right] M_l &\leftrightarrow\quad & \mathcal{P}_j^{\nu_L}\Pi_2 = \Pi_2,\ \mathcal{P}_j^{\nu_L}\Pi_1 = 0 \\
N_\nu &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_j^{\nu_L}\right] M_\nu &\leftrightarrow\quad & \mathcal{P}_j^{\nu_L}\Sigma_2 = \Sigma_2,\ \mathcal{P}_j^{\nu_L}\Sigma_1 = 0
\end{aligned}$$

- By changing

$$\begin{aligned}
u_{R_i} &\longrightarrow e^{i2\alpha} u_{R_i} &\quad \text{by} \quad & d_{R_i} \longrightarrow e^{i2\alpha} d_{R_i} \\
\nu_{R_j} &\longrightarrow e^{i2\alpha} \nu_{R_j} &\quad \text{by} \quad & l_{R_j} \longrightarrow e^{i2\alpha} l_{R_j}
\end{aligned}$$

We generate a total of $(2 \times 2) \times (3 \times 3)$ models with FCYC in different sectors and controlled by V_{CKM}.

The Majorana Case I

- Including a Majorana Mass term can be analyzed by adding an effective Majorana mass term for the three light neutrinos of the form

$$\mathcal{L}_{Majorana} = \frac{1}{2} {\nu_L^0}^T C^{-1} m_\nu \nu_L^0$$

which violates lepton number. Such a mass term is generated after spontaneous gauge symmetry breaking from an effective dimension five operator $\mathcal{O}$ which, in the two Higgs doublet model can be written as:

$$\mathcal{O} = \sum_{i,j=1}^{2} \sum_{\alpha,\beta=e,\mu,\tau} \kappa_{\alpha\beta}^{(ij)} \left(\overline{L^c_{L\alpha}} \widetilde{\phi}_i^* \right) \left(\widetilde{\phi}_j^\dagger L_{L\beta} \right)$$

The Majorana Case II

- So we have now 6 flavour structures: four $\kappa^{(ij)}$ and the two Π_i from $-\overline{L}_L\left(\Pi_1\Phi_1+\Pi_2\Phi_2\right)l_R$. A priori, it looks more difficult to implement MFV in the Majorana case. However, it is remarkable that it can be done by imposing the following Z_4 symmetry in the effective Lagrangian:

$$L^0_{Lj}\to\exp\left(i\alpha\right)\,L^0_{Lj},\qquad \Phi_2\to\exp\left(i\alpha\right)\Phi_2$$

with $\alpha=\pi/2$. It implies ($j=3$)

$$\kappa^{(12)}=\kappa^{(21)}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\quad;\quad m_\nu=v_1^2\kappa^{(11)}+v_2^2\kappa^{(22)}e^{2i\theta}$$

$$\kappa^{(11)}=\begin{pmatrix}\times&\times&0\\\times&\times&0\\0&0&0\end{pmatrix}\quad;\quad\kappa^{(22)}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&\times\end{pmatrix}$$

The Majorana Case III

$$\Pi_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} ; \qquad \Pi_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- The neutrino mass matrix m_ν is block diagonal with each block given by a different matrix. As a consequence, in the diagonalization of m_ν, the matrices $\kappa^{(11)}$ and $\kappa^{(22)}$ are diagonalized separately. Therefore, any linear combination of these two matrices will be simultaneously diagonalized. As a result the lepton number violating Weinberg $\mathcal{O}$ does not give rise to Higgs mediated FCNC in the neutrino sector. For the charged lepton sector we will have Higgs mediated FCNC.
- The projector conditions equivalent to this symmetry are:

$$\kappa^{(12)}{=}\kappa^{(21)} = 0 \quad ; \quad \kappa^{(11)}\mathcal{P}_3^\nu = 0 \quad ; \quad \kappa^{(22)}\mathcal{P}_3^\nu = \kappa^{(22)}$$

$$\mathcal{P}_3^\nu \Pi_1 = 0 \qquad ; \quad \mathcal{P}_3^\nu \Pi_2 = \Pi_2$$

CP violating invariants in the SM and BGL I

- It is well-known that the leading order invariant -without projecting over any quark- that violates CP in the SM is

$$J_{SM} = \mathrm{Im}\; Tr\left[M_u M_u^\dagger M_d M_d^\dagger \left(M_u M_u^\dagger\right)^2 \left(M_d M_d^\dagger\right)^2\right] =$$
$$\prod_{i<j}^{3}\left(m_{u_j}^2 - m_{u_i}^2\right)\prod_{k<l}^{3}\left(m_{d_l}^2 - m_{d_k}^2\right)\mathrm{Im}\left[V_{22}V_{33}V_{23}^*V_{32}^*\right]$$

  that appears at order twelve in Yukawa couplings

- In BGL it turns out that the leading order invariant -without projecting over any quark- that violates CP appears at order eight in Yukawa couplings and is for the model (BGL up 3)

$$J_{BGL}^{u3} = \mathrm{Im}\; Tr\left[M_d N_d^\dagger M_d M_d^\dagger M_u M_u^\dagger M_d M_d^\dagger\right] =$$
$$\left(\frac{v_2}{v_1}+\frac{v_1}{v_2}\right)\left(m_c^2 - m_u^2\right)\prod_{k<l}^{3}\left(m_{d_l}^2 - m_{d_k}^2\right)\mathrm{Im}\left[V_{22}V_{33}V_{23}^*V_{32}^*\right]$$

CP violating invariants in the SM and BGL II

- This result is in agreement with the MFV character of BGL models, namely, all flavour changing and CP violation are controlled by V, therefore this CP violating quantity must be proportional to the imaginary part of rephasing invariant quartets of V as in the SM.
- Another important result is that J^{u3}_{BGL} is different from zero even if $m_t = m_c$ or $m_t = m_u$. In fact the discrete symmetry leading to this specific BGL model singles out the top quark.
- It is important to emphasize that this invariant is defined in such a way that the trace involves the sum over all quarks, therefore it can be related to the baryon asymmetry generated at the electroweak phase transition

CP violating invariants in the SM and BGL III

- If we consider a model of the type (BGL down 1), where

$$\begin{aligned} N_d &= \left[\tfrac{v_2}{v_1} I - \left(\tfrac{v_2}{v_1} + \tfrac{v_1}{v_2}\right) \mathcal{P}_1^{d_L}\right] M_d \\ N_u &= \left[\tfrac{v_2}{v_1} I - \left(\tfrac{v_2}{v_1} + \tfrac{v_1}{v_2}\right) \mathcal{P}_1^{d_L}\right] M_u \end{aligned}$$

  we can get an enhancement in the CP violating contribution to the baryon asymmetry of order:

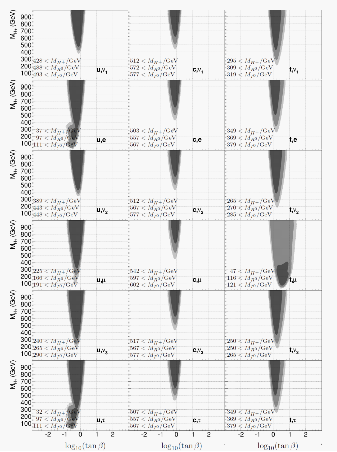
$$\frac{J^{d1}_{BGL}}{J_{SM}} \left(\frac{E^{12}}{E^8}\right) \simeq \frac{J^{d2}_{BGL}}{J_{SM}} \left(\frac{E^{12}}{E^8}\right) \simeq \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \frac{E^4}{m_b^2 m_s^2}$$

  where E is the scale relevant for baryogenesis at the electroweak phase transition. For $E \sim 100 GeV$ we get an enhancement of 10^{10}.

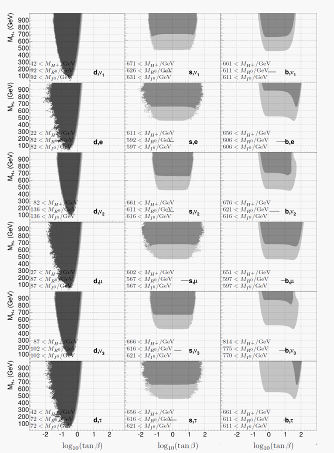
Phenomenology I

- We have analysis the 36 models and we have included
- Processes mediated by charged scalars at tree level: universality in leptonic processes ($\tau \rightarrow l\nu\overline{\nu}$) and semileptonic ($B \rightarrow \tau\nu$, $B \rightarrow D^{(*)}\tau\nu$, $D_s^+ \rightarrow \tau^+\nu$).
- Processes mediated by neutral scalar at tree level: pure leptonic ($\mu \rightarrow 3e$...), neutral meson mixing contributions ($B^0 - \overline{B}^0$...) including CP violation, helicity suppressed rare decays ($B_s^0 \rightarrow l^+l^-$....).
- Loop induced decays, mainly: $B \rightarrow X_s\gamma$ (also $l_i \rightarrow l_j\gamma$).
- Electroweak precision data: $Z \rightarrow b\overline{b}$, S, T...
- We have work in the following scenario: The Higgs with non zero vacuum expectation is the one discovered and does not mix with the other scalar.

Phenomenology II



Phenomenology III



Conclusions I

- We have present a new and general approach to the MFV expansion.
- In the 2HDM the most general MFV expansion is

$$\boxed{\begin{aligned} N_d &= \left(a_0 I + a_{1j}\mathcal{P}_j^{d_L} + a_{2j}\mathcal{P}_j^{u_L} + a_{3ij}\mathcal{P}_i^{u_L}\mathcal{P}_j^{d_L} + a_{4ij}\mathcal{P}_i^{d_L}\mathcal{P}_j^{u_L} + \cdot\right) M_d \\ N_u &= \left(a'_0 I + a'_{1j}\mathcal{P}_j^{d_L} + a'_{2j}\mathcal{P}_j^{u_L} + a'_{3ij}\mathcal{P}_i^{u_L}\mathcal{P}_j^{d_L} + a'_{4ij}\mathcal{P}_i^{d_L}\mathcal{P}_j^{u_L} + \cdot\right) M_u \end{aligned}}$$

- In these sense we have find that the first models that implement the MFV ansantz in a fully renormalizable theory: The Branco Grimus Lavoura models with the flavour symmetries

$$Q_{L_j} \longrightarrow e^{i\alpha} Q_{L_j} \quad ; \quad u_{R_j} \longrightarrow e^{i2\alpha} u_{R_j} \quad ; \quad \Phi_2 \longrightarrow e^{i\alpha}\Phi_2$$

Conclusions II

meet this MFV expansion in a very simple way

$$
\begin{aligned}
N_d &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_j^{u_L}\right] M_d \\
N_u &= \left[\frac{v_2}{v_1} I - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_j^{u_L}\right] M_u
\end{aligned}
$$

- The extension to the leptonic sector is possible both for Dirac and Majorana neutrinos, generating essentially 36 models
- In some of these models there can be a substantial enhancement in the CP violation contribution to the baryon asymmetry generated at the electroweak phase transition.
- The flavour and electroweak precision constraints allow the new scalars with masses reachable at the next round of experiments