Weyl invariance and quantum corrections in the Jordan frame

based on: E. Álvarez, M H-V and C. P. Martín, arXiv:1404.0806 [hep-th] & work in progress

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Motivation

Conformal invariance is relevant for high energy physics. When gravity is present, the corresponding symmetry is Weyl invariance, which is indeed a gauge symmetry if gravity is dynamical

$$g_{\mu\nu}(x) \to \Omega^2 g_{\mu\nu}(x)$$
$$\phi \to \Omega^\lambda \phi$$

The presence of such a symmetry implies that there are not physical scales in the theory.

All scales are thus dynamically generated when the solutions for the classical fields break scale invariance (e.g. introducing a non-vanishing vev)

Motivation

This kind of models with spontaneous symmetry breaking of the Weyl invariance have been introduced for many physical applications

- Inflationary models

  Shaposhnikov, Salvio & Strumia, Linde & Kallosh

- Resolution of cosmological singularities

  Bars

- Providing a good UV behavior for Quantum gravity

  Wetterich, Percacci

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The questions we ask are

- How to construct the most general Weyl invariant theory?
- What happens when quantum corrections enter into this scheme?
- Is it necessary to upgrade the concept of Quantum Effective Action?

Ricci gauging

There is a geometrical way to construct a Weyl invariant action including gravity and matter fields

Iorio, O'Raifeartaigh, Sachs & Wiesendanger

- Ricci gauging

$$W_\mu \to W_\mu - \Omega^{-1}\nabla_\mu \Omega$$

- Thus inducing a geometrical structure defined by the following non-metric connection and covariant derivative

$$\Gamma(W)^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \delta^\mu_\nu W_\rho + \delta^\mu_\rho W_\nu - g_{\nu\rho} W^\mu$$
$$D_\mu \mathcal{T} = \nabla_\mu \mathcal{T} - \lambda_{\mathcal{T}} W_\mu \mathcal{T}$$

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- For example

$$R(W) = R + 2(n-1)\nabla_\mu W^\mu - (n-2)(n-1) W_\mu W^\mu$$

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If the theory is not scale invariant

- Introduce $W_\mu = \nabla_\mu \phi$ so that $F_{\mu\nu} = 0$
- Substitute any physical scale by the appropiate power of ϕ
- This happens to be indeed the Stueckelberg trick

$$g^{'}_{\mu\nu} = e^{2\phi} g_{\mu\nu}$$

Conformal Dilaton Gravity

By using this method, the most simple action one can construct is indeed the well-known action for a scalar field non-minimally coupled to gravity

$$S = -\int d^n x \sqrt{|g|} \left(\frac{(n-2)}{8(n-1)} \phi^2 R + \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + \lambda \phi 4 \right)$$

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There are generalizations:

$$S = -\int d^n x \sqrt{|g|} \left\{ \sum_i (-1)^i \left(\frac{(n-2)}{8(n-1)} \phi_i^2 R + \frac{1}{2} \nabla_\mu \phi_i \nabla^\mu \phi_i \right) + \sum_{i+j=4} \lambda_{ij} \phi_1^i \phi_2^j \right\}$$

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$$S_{CG} = \int d^n x \sqrt{|g|} \left(\alpha W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} + \beta R^*_{\mu\nu\rho\sigma} R^{*\mu\nu\rho\sigma} \right)$$

Getting away from the Jordan frame

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$$g'_{\mu\nu} = \frac{1}{M_p^{n-2}} \left(\frac{(n-2)}{8(n-1)} \right)^{\frac{2}{n-2}} \phi^{\frac{4}{n-2}} g_{\mu\nu}$$

- For example, the CDG action is just the conformal transformation of Einstein-Hilbert action with cosmological constant
- The model with two scalars is equivalent to

$$S = -\int d^n x \sqrt{|g|} \left(M_p^{n-2}(R + 2\lambda) - \frac{1}{2} \nabla_\mu \phi^2 \nabla_\mu \phi^2 - \frac{(n-2)}{8(n-1)} R\phi^2 + V(\phi) \right)$$

Getting away from the Jordan frame

It is quite surprising that the field redefinition corresponds to taking a gauge in the classical way

$$\phi = M_p^{\frac{n-2}{2}}$$

therefore, the non-linear transformation seems to be just a contrive way to express a gauge fixing.

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There are some ambiguities in the definition of the physical scale for the decoupling of gravity

- Compute quantum corrections in the Jordan frame

QC in the Jordan frame

Let us take the simplest Weyl invariant action with this properties: Conformal Dilaton Gravity

$$S = -\int d^n x \sqrt{|g|} \left(\frac{(n-2)}{8(n-1)} \phi^2 R + \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + \lambda \phi 4 \right)$$

We wish to compute QC in the Jordan frame

- In curved space
- For general ϕ and $g_{\mu\nu}$

We introduce then the Heat Kernel by using a zeta function regularization in the quantum effective action

$$W = -\frac{1}{2} Log(det(\mathcal{D}))$$

where D is a differential operator governing the one-loop dynamics of quantum fluctuations

$$S_2 = \int d^n x \sqrt{|g|} \Psi^A \mathcal{D} \Psi_A$$

Heat Kernel

We rewrite the logarithm as

$$Log(det(\mathcal{D})) = \int \frac{dt}{t} Tr\left(e^{-t\mathcal{D}}\right)$$

and introduce the Heat Kernel of the operator $\mathcal{D}$ as

$$K(t,\mathcal{D}) = Tr\left(e^{-t\mathcal{D}}\right) \quad \rightarrow \quad W = -\frac{1}{2}\int \frac{dt}{t^{1-s}} K(t,\mathcal{D})$$

which enjoys a short-time expansion

$$K(t,\mathcal{D}) \sim t^{-\frac{n}{2}} \sum_{i=0} t^i a_i(\mathcal{D})$$

where the $a_i(\mathcal{D})$ are computable in terms of local invariants of the manifold.

Gauge fixing

$$\eta^s = \begin{pmatrix} \eta^\mu \\ c \end{pmatrix} \qquad \bar{\eta}^s = \begin{pmatrix} \bar{\eta}^\mu & \bar{c} \end{pmatrix}$$

$$S = S_{CDG} + s\left(X_D + X_W\right),$$

where

$$X_D = \int d^n x \sqrt{|\bar{g}|}\, \bar{\eta}_\mu \left(-\frac{4(n-1)}{n-2} B^\mu + F_D^\mu\right)$$
$$X_W = \int d^n x \sqrt{|g|}\, g^{\mu\nu} \partial_\mu \bar{c} \partial_\nu \left(f - \alpha\phi\right)$$

and

$$F_D^\nu = (1-\gamma)\left(\bar{\nabla}^\mu k_\mu^\nu - \frac{1}{2}\bar{\nabla}^\nu k\right) + \gamma\,\bar{\phi}\left(\bar{\nabla}^\mu h_\mu^\nu - \frac{1}{2}\bar{\nabla}^\nu h\right) - 2\bar{\nabla}^\nu \phi.$$

Heat Kernel

After some work, the action can be written as

$$S_2 = \int d^n x \sqrt{|g|} \Psi^A \left(G_{AB} \nabla^2 + N^{\mu}_{AB} \nabla_{\mu} + M_{AB} \right) \Psi^B$$

So we can use the short-time expansion of the Heat Kernel to compute the UV divergences in 4 dimensions

$$W^{\infty} = \lim_{n \to 4} \frac{1}{n-4} \frac{1}{16\pi^2} \int d^n x \sqrt{|g|}\, a_4$$

where

$$a_4 = \frac{1}{360} \left[-60 R\, Tr(M) + 180\, Tr(M^2) + 30\, Tr(F_{\mu\nu} F^{\mu\nu}) + \right.$$
$$\left. + Tr\mathbb{I} \left(5R^2 - 2R_{\mu\nu} R^{\mu\nu} + 2R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right) \right]$$

The counterterm

So we find a counterterm in the Jordan frame

$$a_4 = \frac{53}{45} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - \frac{361}{90} R_{\mu\nu} R^{\mu\nu} + \frac{43}{36} R^2 + \alpha_1 \left(\frac{1}{\phi} \frac{\delta S}{\delta \phi} \right)^2 + \alpha_2 \left(\frac{g^{\mu\nu}}{\phi^2} \frac{\delta S}{\delta g^{\mu\nu}} \right)^2$$

where α_1 and α_2 are gauge dependent.
The terms proportional to the equations of motion generate non-analytic operators such as

$$\frac{\nabla_\mu \phi \nabla^\mu \phi \nabla^\nu \phi \nabla_\nu \phi}{\phi^4}$$

We also find something like a conformal anomaly...

Answering the question

Is this equivalent to compute QC in the Einstein frame? Our UV divergences are

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while the 't Hooft-Veltmann counterterm for Einstein's gravity in a particular gauge is

$$a_4 = \frac{149}{180} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + \frac{7}{20} R_{\mu\nu} R^{\mu\nu} + \frac{3}{8} R^2$$

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- They only coincide on-shell
- We cannot generate the new operators by going back to Jordan frame
- What happens if we have an extra field with a potential?? Consequences for inflationary models??

Two-loops

At two-loops, if we trust the on-shell equivalence between EH gravity adn CDG, then there is an anomaly given by the transformation of the Gorov-Sagnotti counterterm

$$W_0^{\infty}[\bar{g_{\mu\nu}}, \bar{\phi}] = \frac{1}{n-4}\frac{M_p^{(n-6)}}{(4\pi)^4}\frac{209}{2880}\int d^4x\sqrt{|g|}\ \bar{W}^6 \tag{1}$$

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This "anomaly" could be cancelled by introducing a logarithmic dependence on the scalar field

$$W_0^{nh}[\bar{g_{\mu\nu}}, \bar{\phi}] = \frac{12}{(4\pi)^4}\frac{209}{2880}\int d^4x\sqrt{|g|}\;\frac{\log(\bar{\phi}^2)}{\bar{\phi}^2}\bar{W}^6$$

- This is another way of expressing the, now fashion, Weyl-invariant regularization scheme of Englert

$$\mu \to \phi$$

- The issue of if these non-localities are accepted to define a suitable QFT is an open question for the moment

Conclusions

- The strategy of going away of the Jordan frame seems to be equivalent to a classical gauge fixing
- Quantum corrections are frame dependent. By computing in the Einstein frame, one does not capture all the UV divergences
- It would be interesting to study the behavior of the model with two scalar fields in the Jordan frame
- Unique effective action??

Steinwachs & Kamenshchik, Moss