Spontaneous parity breaking in the Nambu–Jona-Lasinio model

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Based on

- A. A. Andrianov, D. Espriu & X. Planells, Eur. Phys. J. C 74 (2014) 2776.

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Outline

- Motivation of local parity breaking (LPB)
- The Nambu–Jona-Lasinio model with μ and μ_5
- Search for stable configurations
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  - Parity-breaking phase
  - Transition to the P-breaking phase
- Conclusions

Motivation of local parity breaking

Parity is one of the well established global symmetries of strong interactions. Yet there are reasons to believe that it may be broken since no fundamental principle forbids spontaneous parity breaking for $\mu \neq 0$.

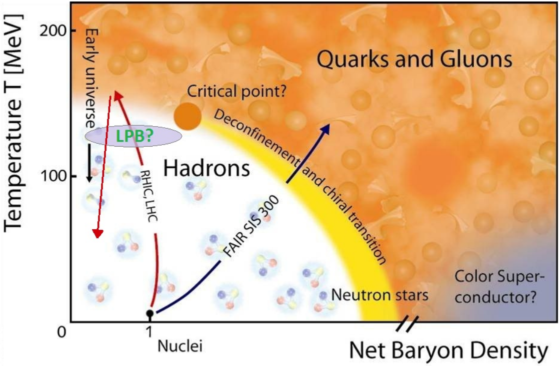
- *P- and CP-odd condensates = "pion" condensates*
  - A. Vilenkin, Phys. Rev. D22, 3080 (1980);
  - A.B. Migdal, Zh. Eksp. Teor. Fiz. 61 (1971);
  - T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974);
  - A. A. Andrianov & D. Espriu, Phys. Lett. B 663 (2008) 450;
  - A. A. Andrianov, V. A. Andrianov & D. Espriu, Phys. Lett. B 678 (2009) 416.
- *Topological fluctuations (finite volume)*
  - D. Kharzeev, R. D. Pisarski & M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998);
  - K. Buckley, T. Fugleberg, & A. Zhitnitsky, Phys. Rev. Lett. 84 (2000) 4814;
  - D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008);
  - A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Lett. B 710 (2012) 230.

Motivation of local parity breaking

Local large fluctuations in the topological charge probably exist in a hot environment.

For *peripheral* heavy ion collisions they may lead to the Chiral Magnetic Effect (CME): Large $\vec{B} \Rightarrow$ large $\vec{E} \Rightarrow$ charge separation.

For *central* collisions (and light quarks) they correspond to an ephemeral phase with axial chemical potential $\mu_5 \neq 0$.



Motivation of local parity breaking

Axial baryon charge and axial chemical potential

Topological charge T_5 may arise in a finite volume due to quantum fluctuations in a hot medium due to sphaleron transitions [Manton, McLerran, Rubakov, Shaposhnikov]

$$T_5 = \frac{1}{8\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} \text{Tr} \left(G^j \partial^k G^l - i \frac{2}{3} G^j G^k G^l \right)$$

and survive for a sizeable lifetime in a heavy-ion fireball. One can control the value of $\langle \Delta T_5 \rangle$ introducing into the QCD Lagrangian a topological chemical potential μ_θ via $\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5$, where

$$\Delta T_5 = T_5(t_f) - T_5(t_i) = \frac{1}{8\pi^2} \int_{t_i}^{t_f} dt \int_{\text{vol.}} d^3x \text{Tr} \left(G^{\mu\nu} \widetilde{G}_{\mu\nu} \right) .$$

The PCAC (broken by gluon anomaly) predicts the induced axial charge to be conserved during $\tau_{fireball}$ (in the chiral limit):

$$\frac{d}{dt} \left(Q_5^q - 2N_f T_5 \right) \simeq 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \bar{q} \gamma_0 \gamma_5 q = \langle N_L - N_R \rangle$$

The characteristic left-right oscillation time is governed by inverse quark masses.

- For u, d quarks $1/m_q \sim 1/5\ \mathrm{MeV}^{-1} \sim 40\ \mathrm{fm} \gg \tau_{\text{fireball}}$ and the left-right quark mixing can be neglected.
- For s quark $1/m_s \sim 1/150\ \mathrm{MeV}^{-1} \sim 1\ \mathrm{fm} \ll \tau_{\text{fireball}}$ and $\langle Q_5^s \rangle \simeq 0$ due to left-right oscillations.

For u, d quarks QCD with a background topological charge leads to the generation of an axial chemical potential μ_5, conjugate to Q_5^q

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle \iff \mu_5 \simeq \frac{1}{2N_f} \mu_\theta,$$

$$\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5 \iff \Delta \mathcal{L}_q = \mu_5 Q_5^q$$

Checking LPB in QCD with a finite μ is difficult:

- Lattice simulations with $\mu \neq 0$ present serious difficulties.
- QCD cannot be easily treated with μ. Non-equilibrium effects are difficult to study non-perturbatively.

Simpler models reproducing the main features of the theory may be useful.

- We explore the Nambu–Jona-Lasinio model (NJL) where the gluon interactions among fermions are replaced by effective 4-fermion couplings.

The Nambu–Jona-Lasinio model with μ and μ_5

Why NJL?

NJL vs QCD

- ✓ Appearance of chiral symmetry breaking (CSB)
- ✓ Global symmetries can be arranged to be identical
- × Confinement is absent in NJL
- × CS restoration in NJL

NJL $\neq$ QCD and we cannot attempt to draw definite conclusions on QCD; just to point out possible phases requiring further analysis.

Previous studies of NJL + μ or NJL + μ_5, but not including both. We incorporate both chemical potentials in order to find the different stable phases.

The inclusion of μ_5 changes radically the phase structure of the model (while μ is not a key player).

NJL Lagrangian for 2 flavours and N colours (in Euclidean conventions)

$$\mathcal{L} = \bar{\psi}(\partial\!\!\!/ + m - \mu\gamma_0 - \mu_5\gamma_0\gamma_5)\psi$$
$$- \frac{G_1}{N}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \frac{G_2}{N}[(\bar{\psi}\vec{\tau}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2],$$

with $U(2)_L \times U(2)_R$ $(SU(2)_L \times SU(2)_R \times U(1)_V)$ in the case that $G_1 = G_2$ $(G_1 \neq G_2)$.

2 doublets are introduced: $\{\sigma, \vec{\pi}\}$ and $\{\eta \equiv \eta_q, \vec{a} \equiv \vec{a}_0(980)\}$ and shift in the fields is performed in order to remove the 4-fermion interactions

$$\mathcal{L} = \bar{\psi}[\partial\!\!\!/ + m - \mu\gamma_0 - \mu_5\gamma_0\gamma_5 + (\sigma + i\gamma_5\vec{\tau}\vec{\pi}) + (i\gamma_5\eta + \vec{\tau}\vec{a})]\psi$$
$$+ \frac{N}{4G_1}(\sigma^2 + \vec{\pi}^2) + \frac{N}{4G_2}(\eta^2 + \vec{a}^2)$$

Fermions are integrated out and fluctuations are neglected (mean field approximation) [$\sigma = \langle\sigma\rangle$, etc]

$$V_{\text{eff}} = \frac{N}{4G_1}(\sigma^2 + \vec{\pi}^2) + \frac{N}{4G_2}(\eta^2 + \vec{a}^2) - \text{Tr}\log\mathcal{M}(\mu,\mu_5),$$

where the trace is in the isospin and Dirac spaces in addition to a 4-momentum integration.

The fermion operator is defined as

$$\mathcal{M}(\mu,\mu_5) = \not{\partial} + (M + \vec{\tau}\vec{a}) - \mu\gamma_0 - \mu_5\gamma_0\gamma_5 + i\gamma_5(\vec{\tau}\vec{\pi} + \eta),$$

with the constituent quark mass $M \equiv m + \sigma$.

In the search for stable configurations, we will need the derivatives of the fermion determinant, which are divergent in the UV. We use DR and a 3-dimensional cut-off both at $T = 0$.

Search for stable configurations

We explore the phases allowed by the previous potential. For simplicity, we assume $a = \langle \bar{\psi} \tau^3 \psi \rangle = 0$.

Stable phases:

- Chirally symmetric phase (in the chiral limit with $m = 0$ and $\mu = \mu_5 = 0$): $\sigma = 0$ [not discussed]
- Chirally broken phase (CSB): $\sigma \neq 0$
- Parity-breaking phase with CSB: $\sigma, \eta \neq 0$

QCD- inspired theories find a "pion" condensation. However, this possibility is not found in NJL (with or without CSB).

The rest of the possible phases with $\langle \bar{\psi} \tau^3 \psi \rangle = 0$ require $m = 0$; they are not true minima. In particular, there is no phase with parity breaking and $\sigma = 0$.

Chirally broken phase

In the CSB phase we find the following mass spectrum relations for any non-trivial μ and μ_5:

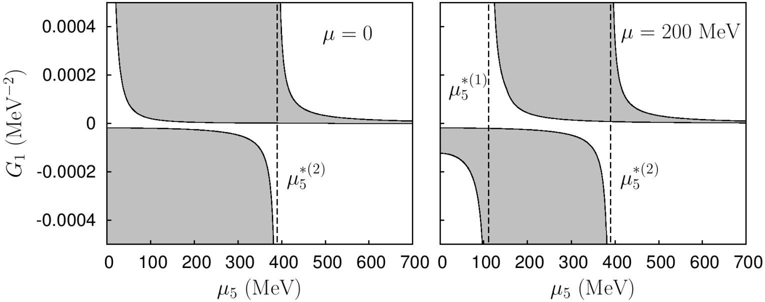
$$m_\sigma^2 - m_\pi^2 = m_a^2 - m_\eta^2 > 0,$$

$$m_a^2 - m_\sigma^2 = m_\eta^2 - m_\pi^2 = \frac{N}{2}\left(\frac{1}{G_2} - \frac{1}{G_1}\right).$$

The second equation implies that $\frac{1}{G_2} - \frac{1}{G_1} > 0$ if we want to make an analogy with QCD.

Chirally broken phase

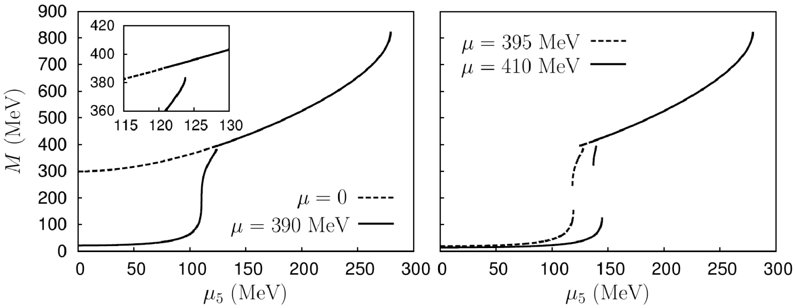
Allowed region of G_1 as a function of μ_5 with fixed μ and $m = 0$ for a stable CSB phase (dark region).



Non-trivial behaviour due to the presence of both chemical potentials.

Chirally broken phase

Evolution of the constituent quark mass M depending on μ_5 for different values of the chemical potential μ.



Certain values of μ_5 exhibit coexisting stable solutions (1st order phase transitions). NJL with external drivers has a complex phase diagram.

Parity-breaking phase

The only possibility for P-breaking is $\sigma \neq 0$ and $\eta \neq 0$. The main features are:

- Mixing among the $\sigma - \eta$ and $\pi - a$ states. No states with well defined parity.
- $U(1)_A$ is needed to be broken ($G_1 \neq G_2$) together with $m \neq 0$ (if not, stationary point but not true minimum).
- $M \simeq \sigma =$ ctant and the dependence on μ, μ_5 is absorbed in η.
- Stability requires $\frac{1}{G_2} - \frac{1}{G_1} < 0$!!

Recall that in the CSB phase:

$$m_a^2 - m_\sigma^2 = m_\eta^2 - m_\pi^2 = \frac{N}{2}\left(\frac{1}{G_2} - \frac{1}{G_1}\right)$$

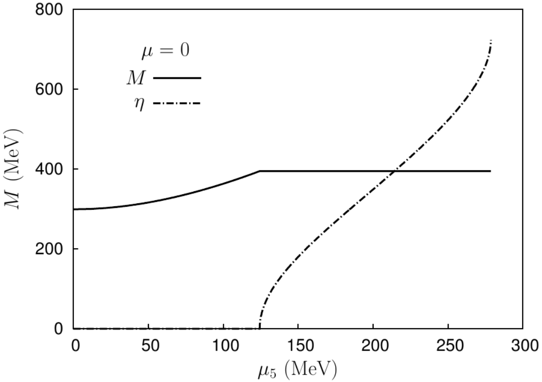
Stable P-breaking phase $\Longrightarrow$ ~~QCD~~

Transition to the P-breaking phase

Requiring stability to the CSB and the P-breaking phases implies:

$$\frac{1}{G_1}\left(1-\frac{m}{M_0}\right) < \frac{1}{G_2} < \frac{1}{G_1}$$

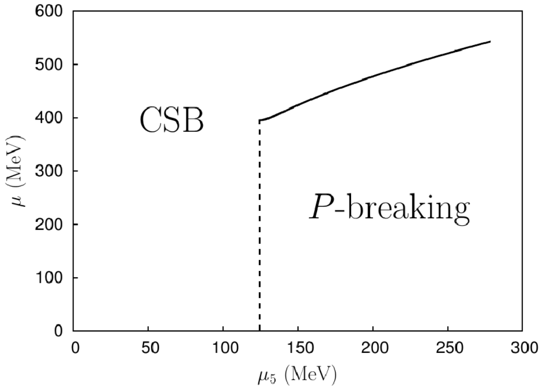
where $M_0 = M(G_1, \mu = \mu_5 = 0)$. If $m \to 0$ the inequalities break down! For $\mu < M^c$, we have a 2nd order phase transition.



Transition to the *P*-breaking phase

For $\mu > M^c$ the phase transition is a 1st order one. The behaviour of the condensates is similar to the previous case but with some complications.

Transition line from the CSB to the *P*-breaking phase.



The vertical dashed line is related to a 2nd order phase transition while the solid one corresponds to a 1st order one.

Conclusions

- LPB not forbidden by any physical principle in QCD at finite temperature/density.
- Topological fluctuations transmit their influence to hadronic physics via an axial chemical potential.
- Despite NJL is not QCD, this model may provide a first guidance to explore the effects of an axial charge in QCD.
- NJL with vector and axial chemical potentials shows a phase where parity is spontaneously broken (inverted mass spectrum). This phase may really exist in QCD.
- The order of magnitude of μ_5 reached in HIC should be similar to the one explored in NJL in order the new phase to be stable, this is $\mu_5 \simeq 100 - 300$ MeV.

Thank you for your attention!