# Heterotic NS5-branes and closed-string K-theory

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*Heterotic NS5-branes from closed string tachyon condensation*, I. García-Etxebarria, M. Montero, A. Uranga, hep-th/1405.0009

# Heterotic NS5 and closed-string tachyons

- String theory is not a theory fo strings!
- D branes in type I, II $=$ open string tachyon solitons Sen '98, Witten '98.
- Some understanding of NS5 branes from closed-string tachyons in type II Adams, Polchinski, Silverstein '01.
- We can describe the heterotic NS5-brane as a tachyon soliton using **supercritical heterotic strings**.

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# Heterotic supercritical theory $HO^{(+n)}$

Field content: Hellerman '04

- Left-movers: $32+n$ fermions $\lambda^a$ in the vector of $SO(32+n)$
- Right-movers: $10+n$ embedding functions $X^\mu$+superpartners $\psi^\mu$ $(\mu=1,\ldots 10)$ and $Y^m$, $\psi^m$ $(m=1\ldots n)$.

Extra ingredient: $Z_2$ orbifold acting as $(\lambda^a, Y^m, \psi^m) \to (-\lambda^a, -Y^m, -\psi^m)$.

Massless spectrum:

- $(10+n)$d fields: $\phi, g, B$ + $SO(32+n)$ gauge field+ $\mathcal{T}^a$ in the fundamental of the gauge field.
- 10d fields: Gravitino+fermions charged under $V\otimes V_{\text{rot.}}$, $V\equiv SO(32+n)$ bundle, $V_{\text{rot.}}\equiv SO(n)$ normal bundle.

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# Supercritical strings: How?

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- The price: **closed string tachyons**
- Bosonic string: Usual tachyon.
- Heterotic supercritical strings: A new set of tachyons in the adjoint.
- Type 0: A single tachyon from the ground state of NS-NS sector.

# Tachyon condensation: The idea

Hellerman & Swanson, '07

- The tachyon $\mathcal{T}^a$ couples to the worldsheet as a superpotential,

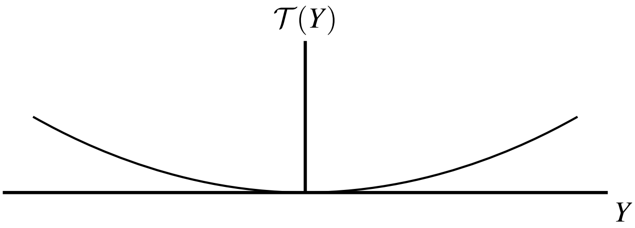
$$\Delta\mathcal{L} = -\frac{1}{2\pi}\int d\theta_+ \sum_a \lambda^a \mathcal{T}^a(X,Y).$$

After eliminating auxiliary fields,

$$\Delta\mathcal{L} = -\frac{1}{8\pi}\Big(\sum_a (T^a)^2\Big) + \frac{i}{2\pi}\sqrt{\frac{\alpha'}{2}}\left[\partial_\mu \mathcal{T}^a \lambda^a \psi^\mu + \partial_m \mathcal{T}^a \lambda^a \psi^m\right]$$

# Tachyon condensation: Dynamics

Orbifold action takes $\mathcal{T}^a(X,Y) \to \mathcal{T}^a(X,-Y)$. Locally, $\mathcal{T}^a \sim M^a_m \exp(\beta X^+) Y^m$. $X^+$-dependence to obey $\mathcal{T}^a$ e.o.m. Potential $\sim \exp(\beta X^+) Y^2$ grows without bound so dynamics is confined to $Y = 0$ slice.



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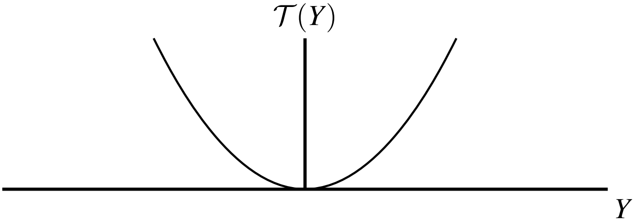

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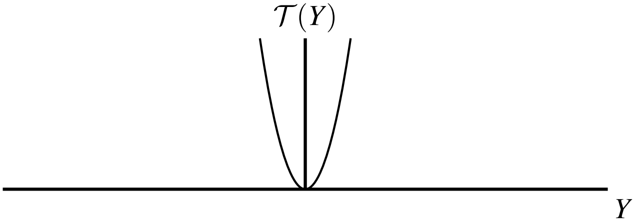

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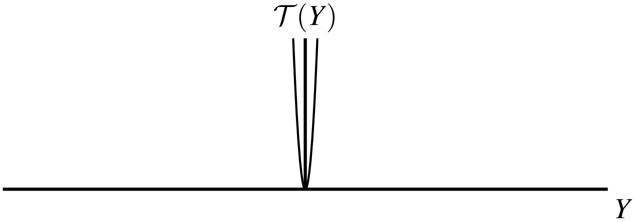

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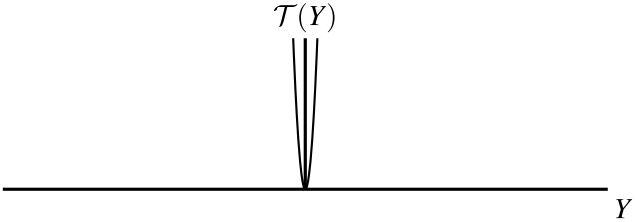


Fermions couple to $\partial_m \mathcal{T}^a$.

# Tachyon condensation

Allow some dependence of $\mathcal{T}^a$ on $X$. At some particular points, it might be that $\partial_m \mathcal{T}^a = 0$.

To ensure it, protect it topologically: Nontrivial $V$, $V_{\text{rot.}}$

- Tachyon profile $\partial_m \mathcal{T}^a$ is a map $V_{\text{rot.}} \to V$, isomorphism at infinity.
- $(n=4)$ Nontrivial $\text{tr}(F^2_{SO(32+n)}) - \text{tr}(R^2_{\text{rot.}})$ results in $\partial_m \mathcal{T}^a = 0$ somewhere:

$$\partial_m \mathcal{T}^a \sim \vec{\Gamma}_{ma} \cdot \vec{X}$$

- This leads to a codimension 4 defect after tachyon condensation.

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# Tachyon condensation

We have found a 6d object located at a representative of $P.D\{\text{tr}(F^2_{SO(32+n)}) - \text{tr}(R^2_{\text{rot.}})\}$. For it to be an NS5, we should have

$$dH = \ldots + \text{tr}(F^2_{SO(32+n)}) - \text{tr}(R^2_{\text{rot.}})$$

We must find a modified Bianchi identity in the $HO^{(+n)}$ theory.

- Anomaly cancellation in 2d
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# Anomalies in 10d

- Heterotic anomaly cancellation is modified. The anomaly cancellation conditions are those of type I with $n$ extra $D9 - \overline{D9}$ pairs Sugimoto '99, Schwarz-Witten '01.
- Consistent with S-duality Hellerman '04.
- Anomaly polynomial factorizes as $I_4 I_8$, with

$$I_4 = \mathrm{tr}(F^2_{SO(32+n)}) - \mathrm{tr}(R^2) - \mathrm{tr}(R^2_{\mathrm{rot.}}).$$

Usual GS mechanism implies modified Bianchi id.

**The 6d heterotic soliton carries the $H$ charge of an NS5 brane**

# CFT analysis

To see if tachyon soliton really corresponds to NS5 brane, analyze CFT near the zero of the tachyon:

$$\mathcal{T}^a \sim B^a_{\mu m} X^\mu Y^m$$

For a particular choice, we get a $(0,4)$ singular CFT which arises in the small instanton limit of a standard sigma model description of instantons

Witten '95

**Tachyon condensation with nontrivial** $\text{tr}(F^2_{SO(32+n)}) - \text{tr}(R^2_{\text{rot.}})$ **yields an NS5 in the critical theory.**

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# Closed string K theory

NS5 brane charge is represented by the cohomology class $\mathrm{tr}(F^2_{SO(32+n)}) - \mathrm{tr}(R^2_{\text{rot.}})$. This suggests a lift to a K theory class $(V, V_{\text{rot.}})$.

- This would be S-dual to type I with extra $D9 - \overline{D9}$ pairs.
- NS5 charge is related to $KO(S^4) = \mathbb{Z}$.
- Other nontrivial K-theory groups:
  - $KO(S^8) = \mathbb{Z}$ Fundamental string
  - $KO(S^2) = \mathbb{Z}_2$ 7-brane [Berasaluce, M. M, Retolaza, Uranga '13]
  - $KO(S^9) = \mathbb{Z}_2$ Massive heterotic spinor
  - $KO(S^1) = \mathbb{Z}_2$ domain wall
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# Summary

- Supercritical heterotic strings allow for the construction of critical NS5 as a closed tachyon soliton.
- Modified Bianchi identity in $10 + n$ dimensions.
- NS5 brane charge unveils real K-theory charge classification in heterotic string theory.

Outlook: Type II!

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Outlook: Type II!

# Thank you very much!

# Type II preview

Supercritical type II theories can be analysed by $D$-brane probes. Supercritical $D$-branes contain a Chern-Simons coupling

$$\int_{\Sigma} ch(F) \sqrt{\frac{\hat{A}(N\Sigma)}{\hat{A}(T\Sigma)}}$$

- Tachyon condensation “erases” part of the normal bundle.
- Suggests that the final state in the critical theory is a gravitational instanton bot in IIA and IIB.

# Anomalies in 2d worldsheet

- $\lambda^a$ are a section of the $SO(32+n)$ gauge bundle.
$$I_4^{\lambda_a} = \mathrm{tr}(F^2_{SO(32+n)})$$
- $\psi^\mu$ are sections of the critical slice tangent bundle
$$I_4^{\psi^\mu} = -\mathrm{tr}(R^2)$$
- $\psi^m$ are sections of the normal bundle.
$$I_4^{\psi^m} = -\mathrm{tr}(R^2_{\mathrm{rot.}})$$

This anomaly is cancelled by inflow from a coupling

$$\int_{10+n} *H_3 \wedge Q_3$$

with $dQ_3 = I_4^{\lambda_a} + I_4^{\psi^\mu} + I_4^{\psi^m}$. This leads to a modified Bianchi identity

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