Photon propagation in a cold axion background and strong magnetic fields

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Outline

- Motivation
- Equations of motion in an axion background and a magnetic field
- Forbidden wavelengths
- Proper modes and frequency difference
- Ellipticity and rotation

The strong CP problem

- θ-term in QCD:

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G_a^{\mu\nu} \tilde{G}_{a\mu\nu}, \quad \left(\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta} \right).$$

- It violates CP and induces a neutron electric dipole moment

$$d_n \simeq \frac{e\theta m_q}{m_N^2}.$$

- Experimental bound: $|d_n| < 2.9 \cdot 10^{-26} e$ cm.

- The bound on d_n translates to $\theta < 10^{-9}$.

The Peccei-Quinn solution and the axion

- Additional $U(1)_{PQ}$ symmetry spontaneously broken by the v.e.v. of a scalar.
- This effectively trades θ for a field: the axion

$$\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G_a^{\mu\nu}\tilde{G}_{a\mu\nu}.$$

- The axion mixes with the pion and gets a mass

$$m_a \simeq \frac{m_\pi f_\pi}{f_a}$$

- Axions couple to photons through

$$\mathcal{L}_{a\gamma\gamma} = g_{a\gamma\gamma}\frac{\alpha}{2\pi}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

- Axion-like particles.

Axions and dark matter. Bounds

- Cold axion background (CAB) field oscillates:

$$a_b(t) = a_0 \sin(m_a t), \quad \rho = \frac{1}{2} a_0^2 m_a^2.$$

- Bounds on the axion parameters:
  - Astrophysics: star energy loss due to $\gamma e \to ae$:

$$m_a < 10^{-2} \text{ eV}.$$

  - Cosmology: WMAP bound on cold dark matter ($\Omega h^2 < 0.12$):

$$m_a > 10^{-6} \text{ eV}.$$

- The dark matter density in the galactic halo is

$$\rho = 10^{-4} \text{ eV}^4.$$

Lagrangian

- Photon propagation in a CAB and a magnetic field.

- $\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g}{4}aF_{\mu\nu}\tilde{F}^{\mu\nu}$.
    - Dimensional coupling constant:

$$g = g_{a\gamma\gamma}\frac{2\alpha}{\pi f_a} = 10^{-18} - 10^{-22}\ \text{eV}^{-1}.$$

- Two backgrounds: constant magnetic field $\vec{B}$ and cold axion background $a_b(t)$.

Equations of motion

- The equations of motion are

$$(\Box + m_a^2)a + b^i \partial_t A_i = 0,$$

$$\Box A^i + b^i \partial_t a + \eta(t)\epsilon^{ijk}\partial_j A_k = 0,$$

where $\vec{b} = g\vec{B}$ and $\eta(t) = g\partial_t a_b(t) = \eta_0 \frac{\pi}{2}\cos(m_a t)$.

- b mixes the axion with the photon component parallel to the magnetic field

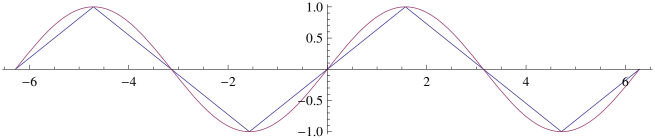
$$b = 10^{-14} - 10^{-18}\ \text{eV} \quad (\text{for } B = 10\text{T}).$$

- η mixes both photon components

$$\eta_0 = 10^{-20} - 10^{-24}\ \text{eV}.$$

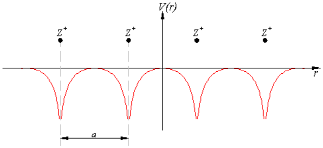
A simplification

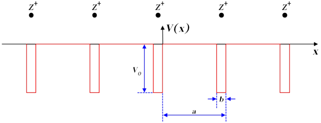
- Because of the cosine in $\eta(t)$, the solutions involve Mathieu functions.
- Instead, replace the sinusoidal axion background by a triangle wave:



- In this simplification, $\eta(t)$ is a square wave.

- Because of the time periodicity of $\eta(t)$ there are momentum gaps.
- The same thing happens in a semiconductor (Kronig-Penney model).





- Position and width of the gaps

$$k_n = \frac{nm_a}{2}, \;\; n \in \mathbb{N}; \quad \Delta k \sim \begin{cases} \frac{\eta_0}{n\pi} & \text{for } n \text{ odd} \\ \frac{\eta_0^2}{2nm_a} & \text{for } n \text{ even} \end{cases}$$

Eigenvalues and eigenvectors

- In momentum space the e.o.m. are

$$\begin{pmatrix} -\omega^2 + k^2 + m_a^2 & \omega b & 0 \\ \omega b & -\omega^2 + k^2 & -\eta_0 k \\ 0 & -\eta_0 k & -\omega^2 + k^2 \end{pmatrix} \begin{pmatrix} a \\ iA_\parallel \\ A_\perp \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

- The eigenfrequencies are:

$$\begin{aligned} \omega_a^2 &\approx (k^2 + m_a^2)\left(1 + \frac{b^2}{m_a^2}\right), \\ \omega_{1,2}^2 &\approx k^2 \mp k\sqrt{\eta_0^2 + \left(\frac{b^2 k}{2m_a^2}\right)^2} - \frac{b^2 k^2}{2m_a^2}. \end{aligned}$$

Eigenvalues and eigenvectors

- ω_a corresponds to an axion state:

$$\begin{pmatrix} 1 \\ \dfrac{b\sqrt{k^2+m_a^2}}{m_a^2} \\ -\dfrac{\eta_0 bk\sqrt{k^2+m_a^2}}{m_a^4} \end{pmatrix}$$

- Small $A_{||}$ component and smaller $A_{\perp}$ component.

Eigenvalues and eigenvectors

- ω_1 and ω_2 correspond to photon modes:

$$\omega_1 : \begin{pmatrix} -\dfrac{bk}{m_a^2} \\ 1 \\ \varepsilon \end{pmatrix}, \qquad \omega_2 : \begin{pmatrix} \dfrac{bk}{m_a^2}\varepsilon \\ -\varepsilon \\ 1 \end{pmatrix},$$

$$\varepsilon = \frac{\eta_0}{\sqrt{\eta_0^2 + \left(\frac{b^2 k}{2m_a^2}\right)^2} + \frac{b^2 k}{2m_a^2}}.$$

$(0 \leq |\varepsilon| < 1)$

- Relative difference in photon frequencies

$$\frac{\Delta\omega_\gamma}{\omega_\gamma} \approx \sqrt{\left(\frac{\eta_0}{k}\right)^2 + \left(\frac{b^2}{2m_a^2}\right)^2}, \quad \frac{b}{m_a} = g_{a\gamma\gamma}\frac{4\alpha}{\pi}\frac{B}{f_a m_a}.$$

- For PQ axions,

$$\frac{b}{m_a} \approx 10^{-14}\left(\frac{B}{10\ \mathrm{T}}\right), \quad \frac{\Delta\omega_\gamma}{\omega_\gamma} < 10^{-20}.$$

- For other axion-like particles it could be as high as

$$\frac{\Delta\omega_\gamma}{\omega_\gamma} = 10^{-18}.$$

- Present laser interferometry can observe a relative difference of 10^{-17}, but 10^{-20} could be achievable.

Ellipticity and rotation

- Start with a linear polarization at an angle β with the magnetic field.
- After a distance x the angle of the polarization plane is

$$\alpha(x) \approx \beta - \frac{\eta_0 x}{2} - \frac{\epsilon}{2} \sin 2\beta$$

and an ellipticity appears: $e = \dfrac{1}{2} |\varphi \sin 2\beta|$

$$\epsilon \approx -\frac{\omega^2 b^2}{m_a^4}\left(1 - \cos \frac{m_a^2 x}{2\omega}\right), \quad \varphi \approx \frac{\omega^2 b^2}{m_a^4}\left(\frac{m_a^2 x}{2\omega} - \sin \frac{m_a^2 x}{2\omega}\right).$$

- The effects of $\vec{B}$ are proportional to $\sin 2\beta$.

Mirrors

- To achieve long distances in small volumes, use mirrors.
- The rotation due to $\vec{B}$ always increases the angle, so it can be accumulated when bouncing.
- The effect of the CAB is a net rotation independent of the initial angle. It tends to cancel when light bounces.
- If the distance betweeen mirrors is tuned to $L = \pi m_a^{-1}$, η changes sign when the light bounces.
- $m_a = 10^{-2} - 10^{-6}$ eV $\longrightarrow L = 0.1$ mm $- 1$ m.

Summary

- The axion is a dark matter candidate as well as a solution to the strong CP problem
- Some photon wavelengths are forbidden in a cold axion background
- Frequency differences between the two photon modes could be detected by interferometry
- Light traveling through these backgrounds acquires a rotation and ellipticity that could be measured