BPS Skyrmions, a unified approach to nuclear matter: from nuclei to neutron stars

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VI CPAN Days

October 20th, 2014 - Sevilla, Spain

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[based on work in collaboration with C. Adam, J. Sanchez-Guillen, R. Vazquez and A. Wereszczynski]

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Skyrme Model

T.H.R. Skyrme: Low-energy field theory of QCD.

- Primary fields are mesons (pions).
- Baryons and nuclei are described by collective nonlinear excitations of the fundamental degrees of freedom.
- Topological charge = baryon number.

Simplest case (two flavors): target space = SU(2), Skyrme field U:

$$x^\mu \to U(x): \quad \mathbb{R}^3 \times \mathbb{R} \to SU(2)$$

Original Skyrme Model

Lagrangian of the Standard Skyrme Model:

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4$$

- Sigma model term:

$$\mathcal{L}_2 = -\frac{f_\pi^2}{4} \, \mathrm{Tr} \left(U^\dagger \partial_\mu U \, U^\dagger \partial^\mu U \right)$$

  Pion kinetic term

- Quartic Skyrme term:

$$\mathcal{L}_4 = \frac{1}{32e^2} \, \mathrm{Tr} \left([U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \right)$$

  Sufficient to circumvent Derrick stability. No BPS saturated solutions.

Generalized Skyrme Model

Effective field theory $\sim$ derivative expansion $\Rightarrow$ higher powers of derivatives expected.

$$\mathscr{L} = \mathscr{L}_2 + \mathscr{L}_4 + \mathscr{L}_6 + \mathscr{L}_0$$

- Sextic Term:

$$\mathscr{L}_6 = -\lambda^2 \pi^4 \mathcal{B}_\mu^2$$

where

$$\mathcal{B}^\mu = \frac{1}{24\pi^2} \mathrm{Tr}(\epsilon^{\mu\nu\rho\sigma} U^\dagger \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U).$$

(Square of the topological current) Quadratic in time derivatives $\Rightarrow$ standard hamiltonian formulation

- Potential term

$$\mathscr{L}_0 = -\mu^2 \mathcal{U}$$

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The model

Limit of generalized Skyrme Model:

$$\mathcal{L}_{06} = \mathcal{L}_6 + \mathcal{L}_0$$

- Parametrization for U

$$U = e^{i\xi\vec{n}\cdot\vec{\sigma}} = \cos\xi + i\sin\xi\vec{n}\cdot\vec{\sigma} \qquad \vec{n}^2 = 1$$

and stereographic projection

$$\vec{n} = \frac{1}{1+|u|^2}\left(u+\bar{u}, -i(u-\bar{u}), 1-|u|^2\right)$$

$$\Rightarrow \quad \mathcal{L}_{06} = \frac{\lambda^2 \sin^4\xi}{(1+|u|^2)^4}\left(\epsilon^{\mu\nu\rho\sigma}\xi_\nu u_\rho \bar{u}_\sigma\right)^2 - \mu^2\mathcal{U}(\xi)$$

The model

- Symmetries: Area-preserving diffeomorphisms on target space S^2 spanned by u:

$$\xi \to \xi\,, \quad u \to \tilde{u}(u, \bar{u}, \xi)$$

$$(1+|\tilde{u}|^2)^{-2} d\xi d\tilde{u} d\bar{\tilde{u}} = (1+|u|^2)^{-2} d\xi u d\bar{u}$$

- BPS (Bogomolny) bound

$$\begin{aligned} E &= \int d^3x \left(\frac{\lambda^2 \sin^4 \xi}{(1+|u|^2)^4} (\epsilon^{mnl} i \xi_m u_n \bar{u}_l)^2 + \mu^2 \mathcal{U}(\xi) \right) \\ &\geq 2\lambda\mu\pi^2 |B| \frac{1}{2\pi^2} \int_{S^3} d\Omega \sqrt{\mathcal{U}(\xi)} \equiv 2\lambda\mu\pi^2 < \sqrt{\mathcal{U}} >_{S^3} |B| \end{aligned}$$

- BPS equation:

$$\frac{\lambda \sin^2 \xi}{(1+|u|^2)^2} \epsilon^{mnl} i \xi_m u_n \bar{u}_l = \mp \mu \sqrt{\mathcal{U}}.$$

Example: Skyrme potential

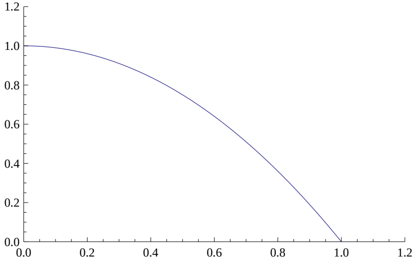
We assume the standard Skyrme potential

$$\mathcal{U} = \frac{1}{2}\mathrm{Tr}(1 - U) \;\rightarrow\; \mathcal{U}(\xi) = 1 - \cos\xi$$

With the ansatz: $\xi = \xi(r), \quad u(\theta,\phi) = \tan\frac{\theta}{2} e^{in\phi}$

- Compact solution with radius proportional to $n^{1/3}$.
- Energy linear in the baryon number:

$$E = \frac{64\sqrt{2}\pi}{15}\mu\lambda|n|$$



C.Adam, J. Sanchez-Guillen, A. Wereszczynski, Phys. Lett. B **691,** 105 (2010)

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BPS Skyrme model coupled to gravity

In general, the corresponding action is

$$S_{06} = \int d^4x |g|^{\frac{1}{2}} \left(-\lambda^2\pi^4 |g|^{-1} g_{\rho\sigma}\mathcal{B}^\rho\mathcal{B}^\sigma - \mu^2\mathcal{U}\right)$$

- Energy-momentum tensor (perfect fluid)

$$T^{\rho\sigma} = -2|g|^{-\frac{1}{2}}\frac{\delta}{\delta g_{\rho\sigma}}S_{06} = (p+\rho)u^\rho u^\sigma - p g^{\rho\sigma}$$

  where 4-velocity

$$u^\rho = \mathcal{B}^\rho / \sqrt{g_{\sigma\pi}\mathcal{B}^\sigma\mathcal{B}^\pi}$$

- Energy density and pressure

$$\begin{aligned} \rho &= \lambda^2\pi^4 |g|^{-1} g_{\rho\sigma}\mathcal{B}^\rho\mathcal{B}^\sigma + \mu^2\mathcal{U} \\ p &= \lambda^2\pi^4 |g|^{-1} g_{\rho\sigma}\mathcal{B}^\rho\mathcal{B}^\sigma - \mu^2\mathcal{U} \end{aligned}$$

BPS Skyrme model coupled to gravity

- Axially symmetric ansatz for Skyrme field

$$\xi = \xi(r), \quad \vec{n} = (\sin\theta\cos B\phi, \sin\theta\sin B\phi, \cos\theta)$$

- Metric (Schwarzschild coordinates)

$$ds^2 = \mathbf{A}(r)dt^2 - \mathbf{B}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- Compatible with Einstein eq. $G_{\rho\sigma} = \frac{\kappa^2}{2} T_{\rho\sigma} \;\; \Rightarrow$

$$\begin{aligned}
\frac{1}{r}\frac{\mathbf{B}'}{\mathbf{B}} &= -\frac{1}{r^2}(\mathbf{B}-1) + \frac{\kappa^2}{2}\mathbf{B}\rho \\
r\,(\mathbf{B}p)' &= \frac{1}{2}(1-\mathbf{B})\mathbf{B}(\rho+3p) + \frac{\kappa^2}{2}\mu^2 r^2 \mathbf{B}^2 \mathcal{U}(h)p \\
\frac{\mathbf{A}'}{\mathbf{A}} &= \frac{1}{r}(\mathbf{B}-1) + \frac{\kappa^2}{2} r\mathbf{B}p
\end{aligned}$$

where ($h \equiv (1/2)(1-\cos\xi)$)

$$\rho = \frac{4B^2\lambda^2}{\mathbf{B}r^4} h(1-h)h'^2 + \mu^2\mathcal{U}(h), \quad p = \rho - 2\mu^2\mathcal{U}(h)$$

How to proceed with the calculations

- Fit λ, μ to nucleon mass $m_{\mathrm{N}} = m_{\mathrm{He}}/4 = 931.75\ \mathrm{MeV}$, nucleon radius $r_{\mathrm{N}} = 1.25\ \mathrm{fm}$

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- Integrate via shooting from the center:
  at $r = 0$: $h(0) = 1$ (anti-vacuum), $\mathbf{B}(0) = 1$ (no enclosed matter), free constant $\rho(0)$
  till R where $h(R) = 0$ (vacuum) and $p'(R) \equiv 0$

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  free constant $\rho(0)$
  till R where $h(R) = 0$ (vacuum) and $p'(R) \equiv 0$
- Concretely, we will assume two different potentials:

$$\mathcal{U} = \mathcal{U}_\pi = 1 - \cos\xi,$$

$$\mathcal{U} = \mathcal{U}_\pi^2 = (1 - \cos\xi)^2,$$

Solving te system

Solutions exist up to maximum value of B or $n = B/B_\odot$ where

$$\mathcal{U}_\pi : \quad n_{\max} = 5.005, \quad M_{\max} = 3.734 M_\odot, \quad R_{\max} = 18.458\ \text{km},$$
$$\mathcal{U}_\pi^2 : \quad n_{\max} = 3.271, \quad M_{\max} = 2.4388 M_\odot, \quad R_{\max} = 16.44\ \text{km}$$

Recent observations: $M_{\max} \sim 2M_\odot$ established, indications for $M_{\max} \sim 2.5M_\odot$
10 km $< R_{\max} <$ 20 km

Neutron star mass vs baryon number

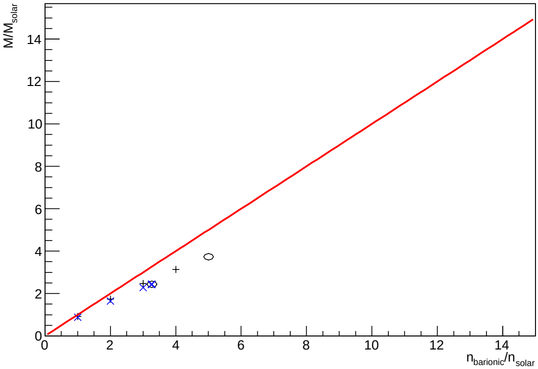

Figure: Neutron star mass as function of baryon number, both in solar units. Symbol plus ($+$): potential $\mathcal{U}_\pi$. Symbol cross ($\times$): potential $\mathcal{U}_\pi^2$.

Neutron star mass vs. neutron star radius

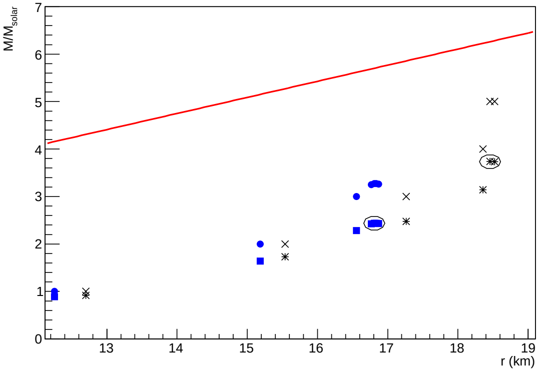

Figure: Neutron star mass as a function of the neutron star radius. Symbol asterisk ($*$): potential $\mathcal{U}_\pi$. Symbol square: potential $\mathcal{U}_\pi^2$. Maximum values are indicated by circles. Also the value of *n* is plotted. The cross ($\times$) corresponds to $\mathcal{U}_\pi$ and the circle to $\mathcal{U}_\pi^2$.

Equation of State for $n = 1$

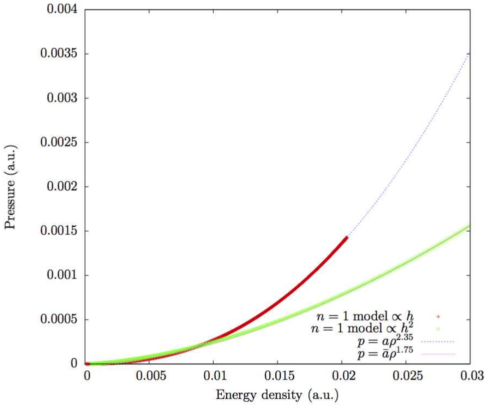

Figure: Symbol plus (+): potential $\mathcal{U}_\pi$. Symbol cross ($\times$): potential $\mathcal{U}_\pi^2$. Dotted lines: corresponding fit functions.

Equation of State for n_{max}

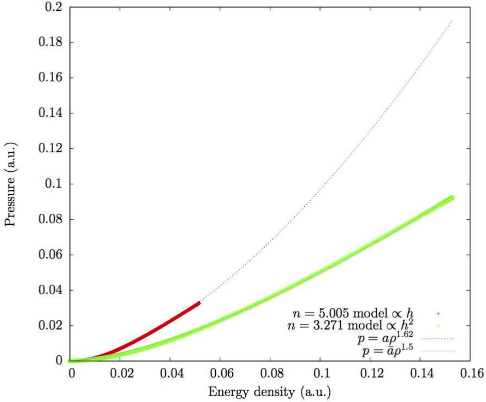

Figure: Symbol plus ($+$): potential $\mathcal{U}_\pi$. Symbol cross ($\times$): potential $\mathcal{U}_\pi^2$. Dotted lines: corresponding fit functions.

Results

- $M_{\rm max} \sim 2.5 - 3.5 M_{\odot}$ and $R_{\rm max} \sim 10 - 20$km precisely in right range

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EoS of polytropic star but $a = a(B)$, $b = b(B)$

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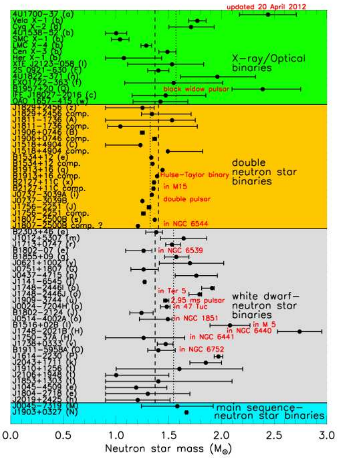
- Concretely EoS stiffer for larger B (larger M)

C.Adam, C. Naya, J. Sanchez-Guillen, R. Vazquez, A. Wereszczynski, arXiv:1407:3799

Comparing to Observed Neutron Star Masses

J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. 62 (2012) 485



Conclusions

- Novel Skyrme model as a limit of generalized ones with analytical solutions.
- Linear mass - baryon number relation (BPS property).
- BPS Skyrme model produces excellent results also for neutron stars (concretely $M_{\max}$ and $R_{\max}$).
- Extrapolation from $B = 1$ to $B \sim 10^{57}$ within one and the same model.
- Equation of state (EoS) NOT universal
$\Rightarrow M(R)$ is monotonously growing function.
- Completely compatible with (not very precise) observational data.
- Work in progress: Better crust corrections and new potentials.


Thank you!