Particle production at NLO in pA collisions: the wave function approach

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[T. A. , N. Armesto, G. Beuf, A. Kovner, M. Lublinsky, arXiv:1410.xxxx]

Outline

- Overview and motivation
- What is new in this approach?
- A little tasting of the calculation: the quark channel
- Results and conclusions

Overview

Single inclusive hadron production at forward rapidities in pA scattering:

1. **"k_T-factorized" approach** : Kovchegov & Tuchin
   - Both the projectile and the target are at very small-x (very high energy)$\Rightarrow$ Color Glass Condensate (CGC) is applicable to both!
2. **"Hybrid" formalism** : Dimitru, Hayashigaki & Jalilian-Marian
   - The wave function of the projectile proton is treated in the spirit of collinear factorization (an assembly of patrons with zero intrinsic transverse momenta)
   - Perturbative corrections to this wave function are provided by the usual QCD perturbative splitting processes.
   - Target is treated as distribution of strong color fields which during the scattering event transfer transverse momentum to the propagating partonic configuration. (CGC like treatment)

Hadroproduction at NLO within "Hybrid" formalism

T.A., A. Kovner - 2011

DOES LO "HYBRID" FORMULA TAKE INTO ACCOUNT ALL CONTRIBUTIONS AT HIGH $k_\perp$?

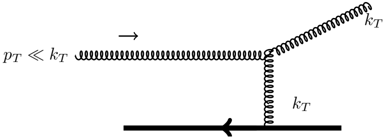
The single inclusive gluon spectrum :

$$\frac{dN}{d^2k d\eta d^2b} \propto \left[\frac{dN}{d^2k d\eta}\right]_{elastic} + \left[\frac{dN}{d^2k d\eta}\right]_{inelastic}$$

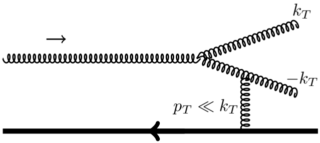
In the limit of large transverse momentum of the produced gluon $k \gg Q_s, \Lambda_{QCD}$

there are two dominant contributions:

"Elastic Scattering" (LO)

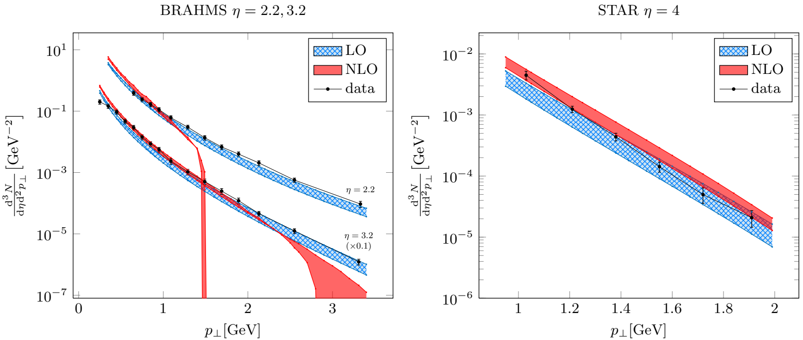


"Inelastic Scattering" (NLO)



Hadroproduction at NLO within "Hybrid" formalism

G.A. Chirilli, B.W. Xiao, F. Yuan - 2012
Full NLO calculation...
A.M.Stasto, B.W.Xiao, D. Zaslavsky, - 2013
Numerical analysis...



Comparison of BRAHMS (h^-) and STAR (π^0) yields in dAu collisions to results of the numerical calculation with rcBK gluon distribution, both at LO and with NLO corrections included.

What are the missing pieces of the puzzle?

1. **What scatters? The Yoffe Time Restriction**

   A consistent treatment of the limitation on the phase space of emissions due to finite life time of the low-x fluctuations.

2. **The choice of the simplest frame...**

   It is convenient to work in the frame where most energy of the process is carried by the target. In this frame :

   - The target moves fast and carries almost all the energy of the process.
   - The projectile moves fast enough to be able to accommodate partons with momentum fraction x_p but not so fast that it develops a large low x tail.

3. **The rapidity to which eikonal scattering amplitudes have to be evolved??**

   - $Y_g = \ln\frac{1}{x_g}$ & $x_g = e^{-\eta}\frac{p_\perp}{\sqrt{2s}}$ X
   - $Y_T = \ln\frac{s}{s_0}$ ✓

The quark channel

The parton level production cross section at LO :

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = \frac{1}{(2\pi)^2}\int d^2x d^2y \; e^{ip_\perp(x-y)} s_{Y_T}(x,y)$$

fundamental dipole scattering amplitude

$$\boxed{s(x,y) = \frac{1}{N_c} tr \; [S^F(x) S^{F\dagger}(y)]}$$

At NLO the quark splits in the projectile wave function with probability of order α_s into a quark-gluon configuration.

The dressed quark state :

$$|(q)\, x_B P^+, k_\perp, \alpha, s\rangle_D = \int d^2x e^{ik_\perp x}\Big\{ A^q |(q)\, x_B P^+, x, \alpha, s\rangle$$

$$+g\int \frac{dp^+}{2\pi} d^2y\, d^2z\, F_{(qg)}(x_B P^+, \xi, y-x, z-x)_{s\bar{s};j}\; t^a_{\alpha\beta}$$

$$|(q)\; y, p^+ = (1-\xi)x_B P^+, \beta, \bar{s}; (g)\; z, q^+ = \xi x_B P^+, a, j\rangle\Big\}$$

- A^q is of order g^2 and needed to preserve the normalisation of the state at order α_s.
- $F_{(qg)}$ is the function that defines the splitting of a quark into a quark-gluon pair.

The quark channel

The dressed quark scatters on the target and produces final state particles.

Within "hybrid" formalism, the scattering of the qg pair is treated as a completely coherent process $\Rightarrow$ each parton picks an eikonal phase during the interaction with the target.

THIS IS ONLY POSSIBLE if the coherence time (Yoffe Time) of the configuration is greater than the propagation time through the target.

$$\boxed{\text{coherent scattering} \Rightarrow \frac{2(1-\xi)\xi x_B P^+}{k_\perp^2} > \tau}$$

$\tau \equiv$ a fixed time scale determined by the longitudinal size of the target.

It enters to calculation via initial energy $P^+/\tau = s_0$.

The Yoffe time restriction can be written also in terms of initial energy!

The pairs that do not exist long enough are not resolved. Those pairs:

- have large $k_\perp$ and have small transverse size.
- scattering and particle production form those pairs are indistinguishable from single parent quark.

The quark channel

The standard eikonal paradigm for propagation of the initial dressed quark with vanishing transverse momentum through the target leads to the final state

$$
\begin{aligned}
|\mathrm{out}, \alpha, s\rangle = \int_x \Big\{ & S^F_{\alpha\beta}(x)\, |(q)x, \beta, s\rangle_D \\
+ \frac{g^2}{2\pi} & \int d\left[\xi x_p P^+\right] \int_{y,z} \left[t^a_{\alpha\beta} S^F_{\beta\gamma}(y) S^A_{ab}(z) - S^F_{\alpha\beta}(x) t^b_{\beta\gamma}\right] \bar{F}^2_{(qg)}(\xi, x_p, y-x, z-x) \\
& t^b_{\gamma\delta}\, |(q)\; x,\; \delta, s\rangle_D \\
+ \frac{g}{2\pi} & \int d\left[\frac{x_p P^+}{1-\xi}\right] \int_{y,z} F_{(qg)}(\xi, x_p, y-x, z-x)_{s,\bar{s},i} \left[t^a_{\alpha\beta} S^F_{\beta\gamma}(y) S^A_{ab}(z) - S^F_{\alpha\beta}(x) t^b_{\beta\gamma}\right] \\
& |(q)\; y, (1-\xi), \gamma, \bar{s}; (g)\; z, \xi, b, i\rangle_D \Big\}
\end{aligned}
$$

The function $F_{(qg)}$ is written as

$$
F_{(qg)} = \frac{i}{\sqrt{2\xi x_B P^+}} \left\{ \delta_{s\bar{s}} \delta_{ij} (2-\xi) - i\epsilon_{ij} \sigma^3_{s\bar{s}} \xi \right\} \delta^2\Big(x - [(1-\xi)y + \xi z]\Big) A^i_{\xi, x_B}(y-z)
$$

Modified Weiszacker-Williams field

The quark channel

The modified Weizsacker-Williams field is defined as

$$A^i_{\xi,x_B}(y-z) = -i\int_{l_\perp^2<2\xi(1-\xi)x_B\frac{P^+}{\tau}} \frac{d^2l_\perp}{(2\pi)^2}\frac{l^i_\perp}{l^2_\perp}e^{il_\perp(y-z)}$$

with transverse momentum $l_\perp$ is

$$\boxed{l_\perp = \xi p_\perp - (1-\xi)q_\perp}$$

- The Yoffe time constraint is implemented on the phase space $\{k_\perp, \xi\}$ in the definition of $F_{(qg)}(y-x, z-x)$ rather than in the integral over ξ.
- Neglecting the Yoffe time constraint on $l_\perp$, one gets for the Fourier tranform with respect to $l_\perp$ the standard Weizsacker-Williams field at point z.
- With the Yoffe time constraint, the relative contribution of short distances are suppressed. $F_{(qg)}$ at small $z-x$ gives smaller contribution.

The quark channel

The quark production cross section is given by the expectation value of the dressed quark number in the outgoing state, multiplied by the number of dressed quarks in the incoming wave function:

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = x_p f^D_{p_\perp}(x_p)\langle \text{out}|D^\dagger(k_\perp, x)D(k_\perp, x)|\text{out}\rangle$$

For the quark production we find

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = \underbrace{\frac{1}{(2\pi)^2} x_p f^D_{p_\perp}(x_p) \int d^2x d^2y e^{ip_\perp(x-y)} s_{Y_T}(x, y)}_{\text{LO}} + \underbrace{\frac{d\sigma^q_1}{d^2p_\perp d\eta}}_{\text{NLO}}$$

The quark production cross section at NLO :

$$\frac{d\sigma^q_1}{d^2p_\perp d\eta} = p^+ \frac{d\sigma^q_1}{d^2p_\perp dp^+} = p^+ \frac{d\sigma^{q\to q,\mathrm{r}}_1}{d^2p_\perp dp^+} + p^+ \frac{d\sigma^{q\to q,\mathrm{v}}_1}{d^2p_\perp dp^+}$$

The quark channel

The NLO cross section contain collinear divergences.
The dipole scattering matrix vanishes when the size of the dipole is larger than the inverse saturation momentum $\Rightarrow s(x,z)_{x-z\to\infty} = 0$.
This behavior cuts off the large z integration region except in three terms :

$$I_1^r = \frac{g^2}{(2\pi)^3} C_F \int dx_B \, f_{p_\perp}^{D,q}(x_B) \int d\xi \, \frac{x_p}{1-\xi} \, \delta\left(x_B - \frac{x_p}{1-\xi}\right) \left[\frac{1+(1-\xi)^2}{\xi}\right] C_{p_\perp^2}(\xi, x_B) \times \int_{y\bar{y}} e^{ip_\perp(y-\bar{y})s[y,\bar{y}]}$$

$$I_2^r = \frac{g^2}{(2\pi)^3} C_F \int dx_B \, f_{p_\perp}^{D,q}(x_B) \int d\xi \, \frac{x_p}{1-\xi} \, \delta\left(x_B - \frac{x_p}{1-\xi}\right) \left[\frac{1+(1-\xi)^2}{\xi}\right] (1-\xi)^2 C_{p_\perp^2}(\xi, x_B) \times \int_{y\bar{y}} e^{ip_\perp(y-\bar{y})} s\left[(1-\xi)y, (1-\xi)\bar{y}\right]$$

$$I^v = -(1+1)\frac{g^2}{(2\pi)^3} C_F \int dx_B \, f_{p_\perp}^{D,q}(x_B) \, x_p \, \delta(x_B - x_p) \int d\xi \left[\frac{1+(1-\xi)^2}{\xi}\right] C_{p_\perp^2}(\xi, x_B) \times \int_{y\bar{y}} e^{ip_\perp(y-\bar{y})} s[y,\bar{y}]$$

where the integral over z up to "factorization scale" μ can be defined for example as

$$C_{\mu^2}(\xi, x_B) = \int_{z^2 > 1/\mu^2} d^2z A^i_{\xi,x_B}(z) A^i_{\xi,x_B}(z)$$

PDF's and Fragmentation functions

f^D that appears in the LO term is the number of "dressed quarks" in the proton. Part of the $O(\alpha_s)$ terms complete it to the NLO pdf of bare quarks.

$$\begin{aligned} f^q_{p_\perp}(x_p) &= f^D_{p_\perp}(x_p) + \frac{g^2 C_F}{2\pi}\int_0^{1-x_p}\frac{d\xi}{1-\xi} f^D_{p_\perp}\left(\frac{x_p}{1-\xi}\right)\frac{1+(1-\xi)^2}{\xi}\int_z C_{p_\perp^2}\left(\xi, \frac{x_p}{1-\xi}\right) \\ &- \frac{g^2 C_F}{2\pi} f^D_{p_\perp}(x_p)\int d\xi \frac{1+(1-\xi)^2}{\xi} C_{p_\perp^2}(\xi, x_p) \end{aligned}$$

The fragmentation function of the "dressed quark":

$$\begin{aligned} D^{D,q}_H(\zeta) = D^q_H(\zeta) &+ \frac{g^2}{2\pi} C_F D^q_H(\zeta)\int d\xi \frac{1+(1-\xi)^2}{\xi} C_{p_\perp^2}\left(\xi, \frac{x_p}{\zeta}\right) \\ &- \frac{g^2}{2\pi} C_F \int_0^{1-\zeta}\frac{d\xi}{1-\xi} D^q_H\left(\frac{\zeta}{1-\xi}\right)\frac{1+(1-\xi)^2}{\xi} C_{p_\perp^2}\left(\xi, \frac{x_p}{\zeta}\right) \end{aligned}$$

The Final Result

Calculating all the channels and following the same procedure as in the case of quark channel the final result reads

$$
\begin{aligned}
p^+ \frac{d\sigma^H}{d^2 p_\perp dp^+} &= \frac{1}{(2\pi)^2} \int \frac{d\zeta}{\zeta^2} D_H^q(\zeta) \frac{x_p}{\zeta} f_{p_\perp}^q \left(\frac{x_p}{\zeta}\right) \int_{y\bar{y}} e^{i\frac{p_\perp}{\zeta}(y-\bar{y})} s[y,\bar{y}] \\
&+ \int \frac{d\zeta}{\zeta^2} D_H^q(\zeta) \frac{d\bar{\sigma}^q}{d^2 p_\perp d\eta} \left(\frac{p_\perp}{\zeta}, \frac{x_p}{\zeta}\right) \\
&+ \frac{1}{(2\pi)^2} \int \frac{d\zeta}{\zeta^2} D_H^g(\zeta) \frac{x_p}{\zeta} f_{p_\perp}^g \left(\frac{x_p}{\zeta}\right) \int_{y\bar{y}} e^{i\frac{p_\perp}{\zeta}(y-\bar{y})} \tilde{s}[y,\bar{y}] \\
&+ \int \frac{d\zeta}{\zeta^2} D_H^g(\zeta) \frac{d\bar{\sigma}^g}{d^2 p_\perp d\eta} \left(\frac{p_\perp}{\zeta}, \frac{x_p}{\zeta}\right)
\end{aligned}
$$

NLO terms are completely collinear safe!

What about the evolution?

- The way we set up the problem, the dipole scattering amplitude is evolved up to rapidity $Y_T = \ln \frac{s}{s_0}$ starting with an initial condition provided at Y_T^0.
- The final result should not care which s_0 we choose if we evolve the dipole cross section appropriately.
- The dependence on s_0 enters explicitly through the cutoff on the phase space and through the dependence of the scattering amplitude on the amount of evolution Y_T. Therefore

$$s_0 \frac{d}{ds_0}\left[\frac{d\sigma}{d^2p_\perp d\eta}\right] = \left[s_0 \frac{\partial}{\partial s_0} - \frac{ds_{Y_T}}{dY_T}\frac{\delta}{\delta s_{Y_T}}\right]\frac{d\sigma}{d^2p_\perp d\eta} = 0$$

and

$$s_0 \frac{\partial}{\partial s_0}\left[\frac{d\sigma}{d^2p_\perp d\eta}\right] = -\frac{\alpha_s N_c}{\pi} x_p f(x_p) \int_{y,\bar{y},z} \frac{1}{(2\pi)^3} e^{ip_\perp(y-\bar{y})} \frac{(y-\bar{y})^2}{(y-z)^2(\bar{y}-z)^2} \times \left[s(y,\bar{y}) - s(y,z)s(z,\bar{y})\right]$$

$\Rightarrow$ the dipole amplitude evolves according to the BK equation...

Summary

- By introducing the *Yoffe Time Restriction*, we have defined clearly the limits of coherent scattering and distinguish what will be resolved by the target and what not.
- We have unambiguously define the rapidity up to which the scattering amplitude has to be evolved.
- We have shown that how the Balitsky-Kovchegov evolution equation arises as the appropriate tool to evolve the leading order amplitude in this setup.
- Need numerical analysis to make sure that we have cured the original problem!!!

Back-up slides

The real contribution to quark production cross section:

$$
\begin{aligned}
p^+\frac{d\sigma_1^{q\to q,\mathrm{r}}}{d^2p_\perp dp^+} \;=\;& \frac{g^2}{(2\pi)^3}\int dx_B\, f^q_{p_\perp}(x_B)\int d\xi\,\frac{x_p}{1-\xi}\,\delta\left(x_B-\frac{x_p}{1-\xi}\right)\left[\frac{1+(1-\xi)^2}{\xi}\right]\int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})}\\
&\times A^i_{\xi,x_B}(y-z)A^i_{\xi,x_B}(\bar{y}-z)\Big\{C_F\, s[y,\bar{y}]+(1-\xi)^2\, C_F\, s\big[(1-\xi)y,(1-\xi)\bar{y}\big]\\
&-\frac{N_c}{2}\Big(s\big[y-\xi(y-z),z\big]\, s[z,\bar{y}]+s\big[z,\bar{y}-\xi(\bar{y}-z)\big]\, s[y,z]\Big)\\
&+\frac{1}{2N_c}\Big(s\big[y-\xi(y-z),\bar{y}\big]+s\big[y,\bar{y}-\xi(\bar{y}-z)\big]\Big)\Big\}
\end{aligned}
$$

Back-up slides

The collinear finite part:

$$
\begin{aligned}
p^+ \frac{d\bar{\sigma}_1^{q\to q,\mathrm{r}}}{d^2 p_\perp dp^+} \;=\;& \frac{g^2}{(2\pi)^3} C_F \int dx_B \; f_{p_\perp}^{D,q}(x_B) \int d\xi \; \frac{x_p}{1-\xi} \, \delta\left(x_B - \frac{x_p}{1-\xi}\right) \; \left[\frac{1+(1-\xi)^2}{\xi}\right] \int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})} \\
& \left[A^i_{\xi,x_B}(y-z) A^i_{\xi,x_B}(\bar{y}-z) - C_{p_\perp^2}(\xi, x_B)\right] \left\{ s[y,\bar{y}] + (1-\xi)^2 \, s\big[(1-\xi)y, (1-\xi)\bar{y}\big] \right\} \\
& + \frac{g^2}{(2\pi)^3} \int dx_B \; f_{p_\perp}^{D,q}(x_B) \int d\xi \; \frac{x_p}{1-\xi} \, \delta\left(x_B - \frac{x_p}{1-\xi}\right) \; \left[\frac{1+(1-\xi)^2}{\xi}\right] \int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})} \\
& \times A^i_{\xi,x_B}(y-z) A^i_{\xi,x_B}(\bar{y}-z) \left\{ -\frac{N_c}{2}\Big(s\big[y-\xi(y-z), z\big] \, s[z,\bar{y}] + s\big[z, \bar{y}-\xi(\bar{y}-z)\big] \, s[y,z] \Big) \right. \\
& \left. \qquad + \frac{1}{2N_c}\Big(s\big[y-\xi(y-z), \bar{y}\big] + s\big[y, \bar{y}-\xi(\bar{y}-z)\big] \Big) \right\}
\end{aligned}
$$

Back-up slides

The virtual contribution to quark production cross section:

$$
\begin{aligned}
p^+ \frac{d\sigma_1^{q\to q,\mathrm{v}}}{d^2p_\perp dp^+} \;=\; & \frac{g^2}{(2\pi)^3}\int \frac{dx_B}{x_B P^+}\, x_p P^+\, f^q_{p_\perp}(x_B)\, \delta\left(x_B P^+ - x_p P^+\right)\int d\left[\xi x_p P^+\right]\left[\frac{1+(1-\xi)^2}{\xi}\right]\int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})} \\
& \times\Bigg\{ A^i_{\xi,x_B}(y-z)A^i_{\xi,x_B}(y-z)\Big[-C_F\; s[y,\bar{y}] + \frac{N_c}{2}\, s\Big[y+\xi(y-z), z+\xi(y-z)\Big]\, s\Big[z+\xi(y-z),\bar{y}\Big] \\
& -\frac{1}{2N_c}\, s\Big[y+\xi(y-z),\bar{y}\Big]\Big] \\
& +A^i_{\xi,x_B}(\bar{y}-z)A^i_{\xi,x_B}(\bar{y}-z)\Big[-C_F\; s[y,\bar{y}] + \frac{N_c}{2}\, s\Big[z+\xi(\bar{y}-z), \bar{y}+\xi(\bar{y}-z)\Big]\, s\Big[y, z+\xi(\bar{y}-z)\Big] \\
& -\frac{1}{2N_c}\, s\Big[y,\bar{y}+\xi(\bar{y}-z)\Big]\Big]\Bigg\}
\end{aligned}
$$

Back-up slides

The collinear finite part:

$$
\begin{aligned}
p^+\frac{d\bar{\sigma}_1^{q\to q,\mathrm{v}}}{d^2p_\perp dp^+} \;=\; & -\frac{g^2}{(2\pi)^3}C_F\int dx_B\; f^{D,q}_{p_\perp}(x_B)\; x_p\; \delta\left(x_B-x_p\right)\int d\xi\left[\frac{1+(1-\xi)^2}{\xi}\right]\int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})}s[y,\bar{y}]\\
& \times\Big\{A^i_{\xi,x_B}(y-z)A^i_{\xi,x_B}(y-z)+A^i_{\xi,x_B}(\bar{y}-z)A^i_{\xi,x_B}(\bar{y}-z)-2C_{p_\perp^2}(\xi,x_B)\Big\}\\
& +\frac{g^2}{(2\pi)^3}C_F\int dx_B\; f^{D,q}_{p_\perp}(x_B)\; x_p\; \delta\left(x_B-x_p\right)\int d\xi\left[\frac{1+(1-\xi)^2}{\xi}\right]\int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})}\\
& \times\Big\{A^i_{\xi,x_B}(y-z)A^i_{\xi,x_B}(y-z)\Big[\frac{N_c}{2}\,s\Big[y+\xi(y-z),z+\xi(y-z)\Big]\,s\Big[z+\xi(y-z),\bar{y}\Big]\\
& -\frac{1}{2N_c}\,s\Big[y+\xi(y-z),\bar{y}\Big]\Big]\\
& +A^i_{\xi,x_B}(\bar{y}-z)A^i_{\xi,x_B}(\bar{y}-z)\Big[\frac{N_c}{2}\,s\Big[z+\xi(\bar{y}-z),\bar{y}+\xi(\bar{y}-z)\Big]\,s\Big[y,z+\xi(\bar{y}-z)\Big]\\
& -\frac{1}{2N_c}\,s\Big[y,\bar{y}+\xi(\bar{y}-z)\Big]\Big]\Big\}
\end{aligned}
$$