



Perturbative and Non Perturbative QCD effects in transverse momentum distributions

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based on:

arXive: 1402.0869, with M.G. Echevarría (NIKHEF), A. Idilbi (Penn U.),

arXive: 1406., with U. D' Alesio (Cagliari), M.G. Echevarría (NIKHEF), S. Melis (Torino)

and also PLB726(2013)795,
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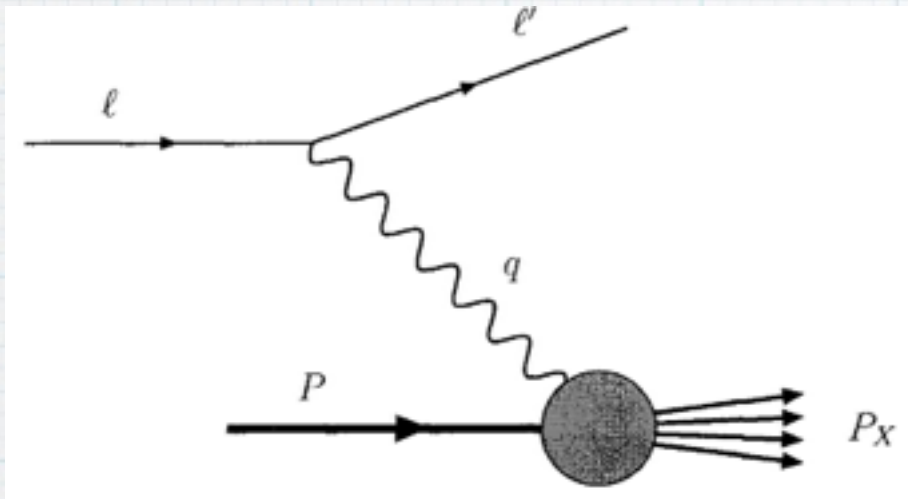
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Topics and outline

- * Transverse momentum distributions involve non-perturbative QCD effects which go beyond the usual PDF formalism. New factorization theorems are required.
- * Spin dependent observables and transverse momentum dependent observables need factorization theorems with TMD's
- * TMD's are the fundamental **non-perturbative** objects to be used in factorization theorems in (un-)polarized Drell-Yan, SIDIS, e^+e^- to 2 jets (multi-jets?)
- * Properties of TMD's:
 - 1) The evolution of all TMD's is universal (alike PDF and FF it is process independent)
 - 2) The evolution of all TMD's is spin independent and it is the same for TMDPDF and TMDFF
- * Extraction of unpolarized TMDPDF from Drell-Yan and Z-boson production using completely resummed TMD's at NNLL

Preliminary

Outline of Factorization theorem



SIDIS as a study case:
both PDF and FF

$$q^2 \gg q_T^2$$

Fact. scale

$$l(k) + N(P) \rightarrow l'(k') + h(P_h) + X(P_X)$$

$$W^{\mu\nu} = H(Q^2/\mu^2) \frac{2}{N_c} \sum_q e_q \int d^2 k_{n\perp} d^2 k_{\bar{n}\perp} \delta^{(2)}(\mathbf{q}_\perp + \mathbf{k}_{n\perp} - \mathbf{k}_{\bar{n}\perp})$$

$$\times \text{Tr} [F(x, \mathbf{k}_{n\perp}, S; Q^2/\alpha, \mu^2) \gamma^\mu D(z, \hat{P}_{h\perp}, S_h; Q^2/\alpha, \mu^2) \gamma^\nu]$$

Hard coeff.

$$\mathbf{k}_{\bar{n}\perp} = -\hat{\mathbf{P}}_{h\perp}/z$$

TMDPDF

TMDFF

Soft splitting
number

$$\zeta_F = Q^2/\alpha$$

$$\zeta_D = \alpha Q^2$$

A lot of TMD's

Mulders-Tangerman '96,
Boer Mulders ' 98
Mulders, 2001 (gluons)
Boer, Mulders, Collins
Mulders, Buffing, Mukherjee 2013
....

TMDPDF

Nucleon
Polarization

Quark Polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

Similar structures for
Gluons as initial states: in
Higgs production both f_1^g and $h_1^{\perp g}$

$$\begin{aligned}\tilde{F}_{f/N}^{[\gamma^+]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) &= \tilde{f}_1 - \frac{\epsilon_\perp^{ij} \mathbf{b}_\perp^i \mathbf{S}_\perp^j}{ib_T M_N} \tilde{f}_{1T}^{\perp(1)}, \\ \tilde{F}_{f/N}^{[\gamma^+ \gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) &= \lambda \tilde{g}_{1L} + \frac{(\mathbf{b}_\perp \cdot \mathbf{S}_\perp)}{ib_T M_N} \tilde{g}_{1T}^{(1)}, \\ \tilde{F}_{f/N}^{[i\sigma^{i+} \gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) &= \mathbf{S}_\perp^i \tilde{h}_1 + \frac{\lambda \mathbf{b}_\perp^i}{ib_T M_N} \tilde{h}_{1L}^{\perp(1)} \\ &\quad - \frac{(\mathbf{b}_\perp^i \mathbf{b}_\perp^j + \frac{1}{2} b_T^2 g_\perp^{ij}) \mathbf{S}_\perp^j}{(i)^2 b_T^2 M_N^2} \tilde{h}_{1T}^{\perp(2)} - \frac{\epsilon_\perp^{ij} \mathbf{b}_\perp^j}{ib_T M_N} \tilde{h}_1^{\perp(1)}\end{aligned}$$

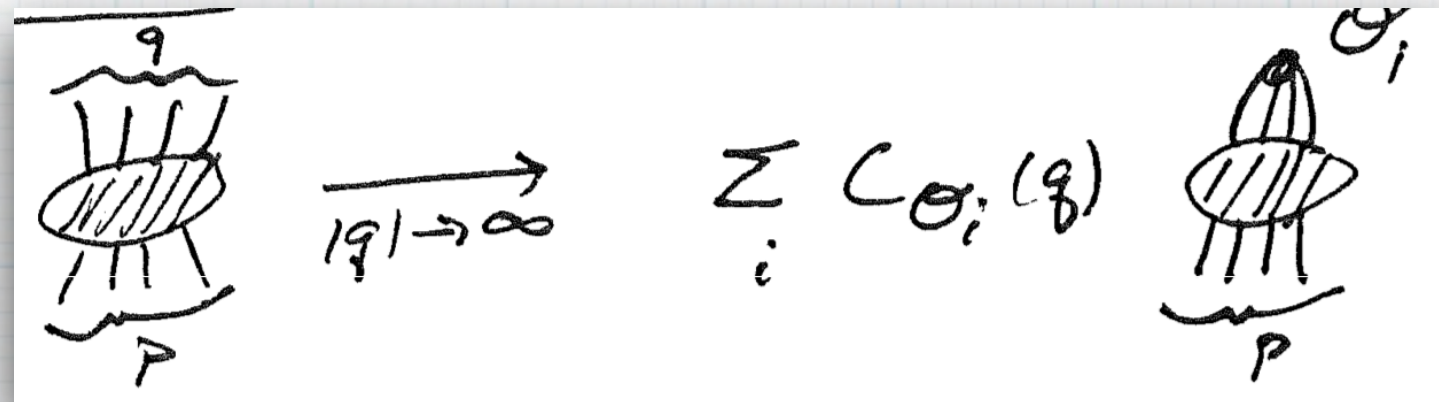
T-odd distributions

$$f_{1T,DIS}^\perp = -f_{1T,DY}^\perp$$

TMDFF

$$\begin{aligned}\tilde{D}_{h/f}^{[\gamma^-]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) &= \tilde{D}_1 - \frac{\epsilon_\perp^{ij} \mathbf{b}_\perp^i \mathbf{S}_{h\perp}^j}{(-ib_T) M_h} \tilde{D}_{1T}^{\perp(1)}, \\ \tilde{D}_{h/f}^{[\gamma^- \gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) &= \lambda \tilde{G}_{1L} + \frac{(\mathbf{b}_\perp \cdot \mathbf{S}_{h\perp})}{(-ib_T) M_h} \tilde{G}_{1T}^{(1)}, \\ \tilde{D}_{h/f}^{[i\sigma^{i-} \gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) &= \mathbf{S}_{h\perp}^i \tilde{H}_1 + \frac{\lambda \mathbf{b}_\perp^i}{(-ib_T) M_h} \tilde{H}_{1L}^{\perp(1)} \\ &\quad - \frac{(\mathbf{b}_\perp^i \mathbf{b}_\perp^j + \frac{1}{2} b_T^2 g_\perp^{ij}) \mathbf{S}_{h\perp}^j}{(-ib_T)^2 M_h^2} \tilde{H}_{1T}^{\perp(2)} - \frac{\epsilon_\perp^{ij} \mathbf{b}_\perp^j}{(-ib_T) M_h} \tilde{H}_1^{\perp(1)}\end{aligned}$$

Origin of factorization and EFT



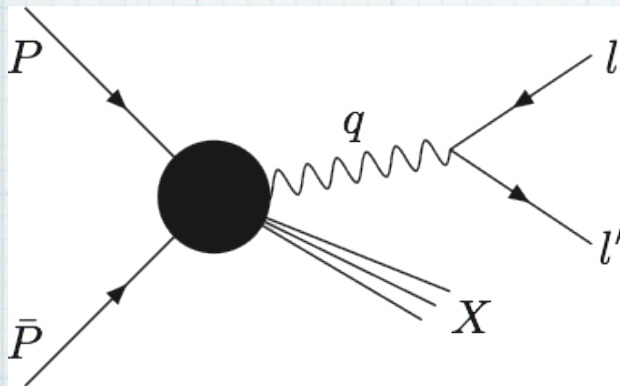
Drawing from J. Preskill lectures

In Green function, when the momenta " q " of some fields are much larger than the others we can "factor out" the hard momenta onto Wilson coefficient and effective matrix elements and use RGE on coefficients to resum large logs (Wilson, Zimmerman, Callan, Symanzyk '70-'72)

Wilson coefficients and hard factors depend only on the high scale " q " and the factorization scale " μ ": no other (IR-sensitive) scale.

Same philosophy in the construction of EFT: identification of modes, effective field theory Lagrangians, factorized cross sections, Wilson coefficients, ... 5

Inclusive DY case



$$q^2 = Q^2$$

Collins, Soper '82

$$\frac{d\sigma}{dQ^2} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 dx_2 \mathcal{H}_{ij}(x_1, x_2, Q^2, \mu^2) f_{i/P}(x_1, \mu^2) f_{j/\bar{P}}(x_2, \mu^2)$$

Short Distance

Long Distance

$$f_{i/P}(0^+, y^-, 0_\perp) = \frac{1}{2} \sum_{\sigma} \langle P, \sigma | [\bar{\xi}_n W_n](0^+, y^-, 0_\perp) \frac{\overleftarrow{\not{y}}}{2} [W_n^\dagger \xi_n](0) | P, \sigma \rangle$$

- All non-perturbative information is encoded in the PDF
- We want to get more information on the nucleon structure exploring transverse momentum dependent cross section
- Consider $\frac{d\sigma}{dQ^2 dq_T^2}$, can we play the same game?

Naive extension of TMDPDF

The naive extension of the TMD does not work

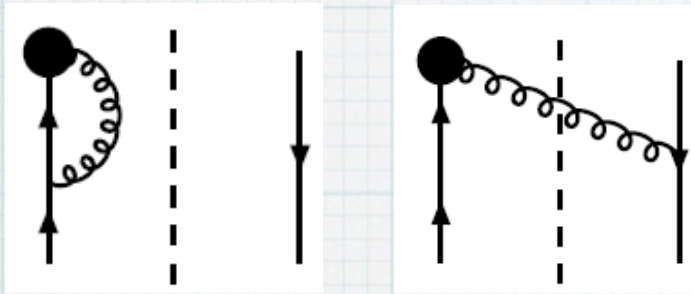
$$F_n^{naive}(0^+, y^-, \vec{y}_\perp) = \frac{1}{2} \sum_\sigma \langle P, \sigma | [\bar{\xi}_n W_n] (0^+, y^-, \vec{y}_\perp) \frac{\not{n}}{2} [W_n^\dagger \xi_n] (0) | P, \sigma \rangle$$

Transverse gauge links should be included [1], but the core problem are rapidity divergences: at one loop

$$\begin{aligned} \tilde{F}_n^{naive} = & \delta(1-x) + \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-x) \left[\frac{2}{\epsilon_{UV}} \ln \frac{\Delta^+}{Q^2} + \frac{3}{2\epsilon_{UV}} \right. \right. \\ & \left. \left. - \frac{1}{4} + \frac{3}{2} L_T + 2 L_T \ln \frac{\Delta^+}{Q^2} \right] \right. \\ & \left. - (1-x) \ln(1-x) - \mathcal{P}_{q/q} \ln \frac{\Delta^-}{\mu^2} - L_T \mathcal{P}_{q/q} \right\} \end{aligned}$$

This quantity cannot be renormalized

Naive extensions of TMDPDF



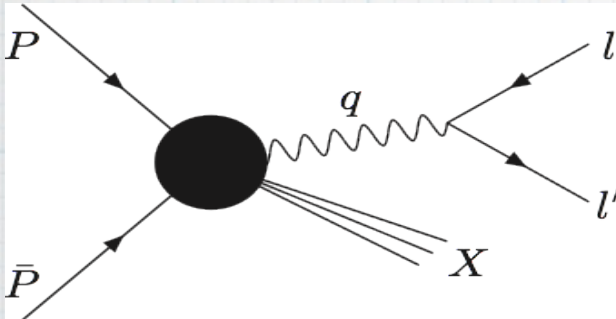
The problems appear when gluons and quarks are collinear

$$\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{[k^+ - i\epsilon][(p+k)^2 + i\epsilon][k^2 + i0]} \implies \frac{1}{\epsilon_{UV}} \times \int_0^1 \frac{1}{t}$$

In the PDF the problem is solved combining virtual and real gluon emission. However this cancellation does not happen now for unintegrated distributions

This cancellation must hold in QCD because there are no rapidity divergences in the hadronic tensor

Energy scales: DY example



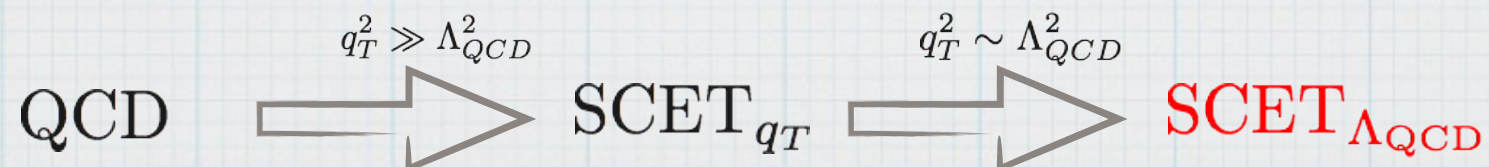
$$q^2 = Q^2 \gg q_T^2$$

$$q_T^2 \sim \Lambda_{QCD}^2 \quad \longrightarrow \quad \tilde{M} = H(Q^2/\mu^2) \tilde{F}_n(x_n, b; Q^2, \mu^2) \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

$$q_T^2 \gg \Lambda_{QCD}^2 \quad \longrightarrow \quad \tilde{M} = H(Q^2/\mu^2) \tilde{C}_n(b^2\mu^2, Q^2/\mu^2) \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2) f_n(x_n; \mu^2) f_{\bar{n}}(x_{\bar{n}}; \mu^2)$$

The IR has to be regulated consistently in the theories above and below every matching scale in order to properly extract the matching (Wilson) coefficients.

Problems with different energy scales are more easily treated with EFT



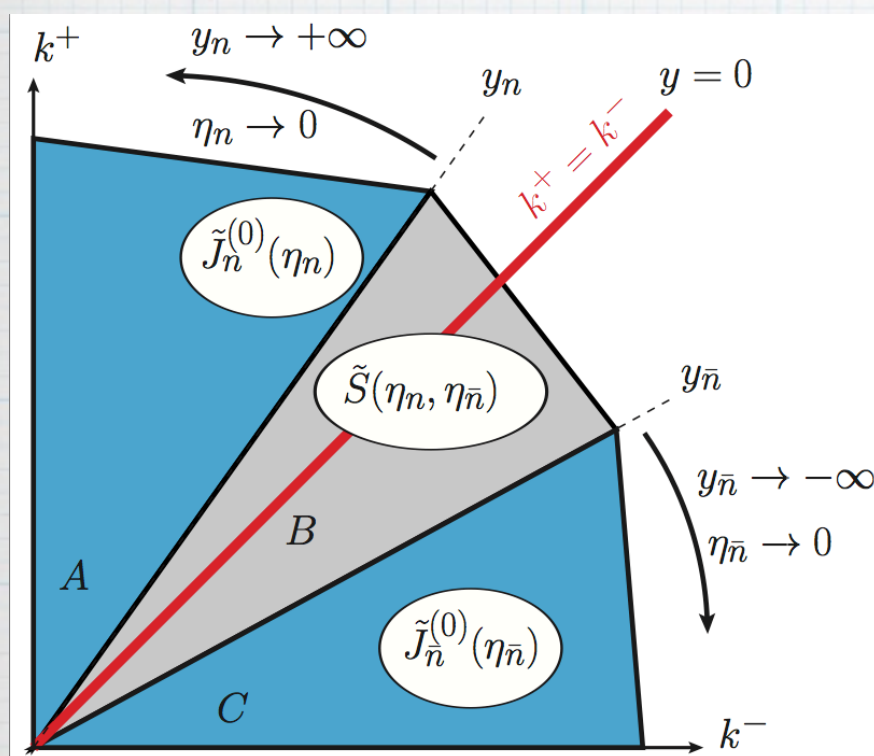
Modes in EFT

Using power counting we have collinear, anti-collinear, and soft sectors

$$H(Q^2) \tilde{J}_n^{(0)}(\eta_n) \tilde{S}(\eta_n, \eta_{\bar{n}}) \tilde{J}_{\bar{n}}^{(0)}(\eta_{\bar{n}})$$

In EFT each mode belongs to a Hilbert space separate from the others.

To each mode correspond a different Lagrangian
Boosts mix soft and collinear modes (same invariant mass)



(+, -perp)

$$\begin{aligned} k_n &\sim Q(1, \lambda^2, \lambda) \rightarrow y \gg 0 \\ k_{\bar{n}} &\sim Q(\lambda^2, 1, \lambda) \rightarrow y \ll 0 \\ k_s &\sim Q(\lambda, \lambda, \lambda) \rightarrow y \approx 0 \\ \lambda &\sim \frac{q_T}{Q} \end{aligned}$$

$$\begin{aligned} n &= (1, 0, 0, 1) \\ \bar{n} &= (1, 0, 0, -1) \end{aligned}$$

$$\chi_n = W_n^\dagger \xi_n$$

$$J_n^{(0)}(0^+, y^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_1(P, \sigma_1) | \bar{\chi}_n(0^+, y^-, y_\perp) \not{n} \chi_n(0) | N_1(P, \sigma_1) \rangle$$

$$J_{\bar{n}}^{(0)}(y^+, 0^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_2(\bar{P}, \sigma_2) | \bar{\chi}_{\bar{n}}(0) \not{\bar{n}} \chi_{\bar{n}}(y^+, 0^-, y_\perp) | N_2(\bar{P}, \sigma_2) \rangle$$

$$S(0^+, 0^-, y_\perp) = \langle 0 | \text{Tr} \bar{T} [S_n^{T\dagger} S_{\bar{n}}^T](0^+, 0^-, y_\perp) T [S_{\bar{n}}^{T\dagger} S_n^T](0) | 0 \rangle, \quad \chi = W^{T\dagger} \xi$$

multipole expansion fixes arguments

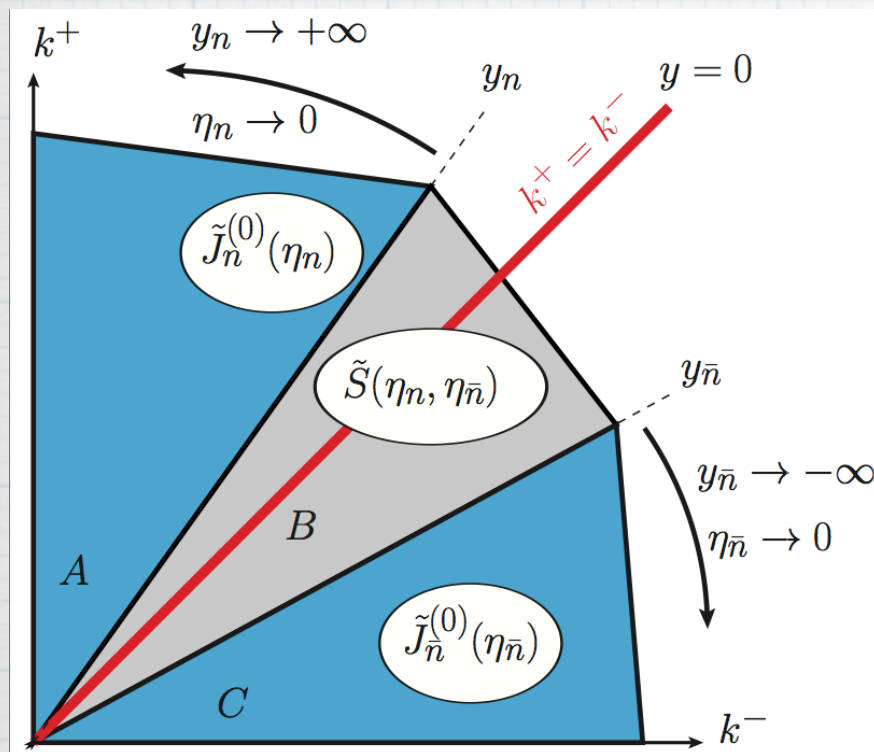
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(+, -, perp)

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$$k_s \sim Q(\lambda, \lambda, \lambda) \rightarrow y \approx 0$$

$$\lambda \sim \frac{q_T}{Q}$$

$$J_n^{(0)}(0^+, y^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_1(P, \sigma_1) | \bar{\chi}_n(0^+, y^-, y_\perp) \not{n} \chi_n(0) | N_1(P, \sigma_1) \rangle$$

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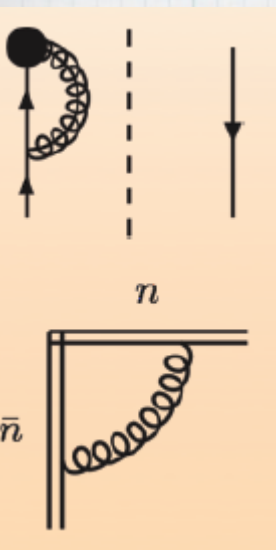
$$S(0^+, 0^-, y_\perp) = \langle 0 | \text{Tr } \bar{T} [S_n^{T\dagger} S_{\bar{n}}^T](0^+, 0^-, y_\perp) T [S_{\bar{n}}^{T\dagger} S_n^T](0) | 0 \rangle, \quad \chi = W^{T\dagger} \xi$$

None of these sectors is well defined:
rapidity divergences

multipole expansion fixes arguments

Double counting and collinear matrix elements

The soft limit of collinear graphs overlaps with soft matrix elements



$$= -2ig^2 C_F \delta(1-x) \delta^{(2)}(\vec{k}_{n\perp}) \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{[k^+ - i0^+][(p+k)^2 + i0^-][k^2 + i0]}$$



$$= -2ig^2 C_F \delta^{(2)}(\vec{k}_{n\perp}) \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^+ - i0^+][k^- + i0^-][k^2 + i0]}$$

In order to get the pure collinear matrix elements one has to subtract the overlapping part: the subtraction is regulator dependent.

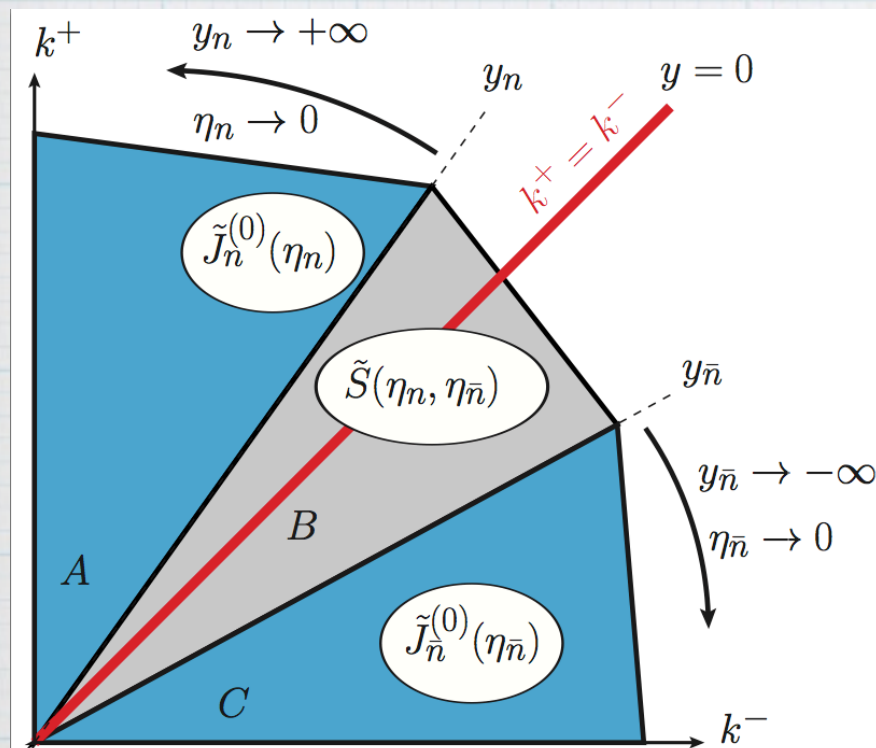
The factorization theorem must be expressed in terms of PURE COLLINEAR objects

Rapidity divergences

- * Modes can be distinguished only by their rapidity, so need a rapidity regulator (Manohar, Stewart, 2006)
- * All properties of TMD are regulator independent
- * We performed our calculation on-the-light cone and using delta-regulator (Chiu, Fuhrer, Hoang Manohar, 2009). Checks with other regulators agree (Collins 2011, Chiu, Jain, Neill, Rothstein 2012, ..)

$$\frac{i(\not{p} + \not{k})}{(p+k)^2 + i\Delta^-} \rightarrow \frac{1}{k^- + i\delta^-}, \delta^- = \frac{\Delta^-}{p^+}$$

$$\frac{i(\not{\bar{p}} - \not{k})}{(\bar{p}-k)^2 + i\Delta^+} \rightarrow \frac{1}{-k^+ + i\delta^+}, \delta^+ = \frac{\Delta^+}{\bar{p}^-}$$



Rapidity divergences at one loop:

$$\tilde{J}_{n1}^{(0)}(\Delta^-) = \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-x) \left[\frac{2}{\epsilon_{UV}^2} - \frac{2}{\epsilon_{UV}} \ln \frac{\Delta^-}{\mu^2} + \frac{3}{2\epsilon_{UV}} - \frac{1}{4} - \frac{2\pi^2}{12} - L_T^2 \right] + \frac{3}{2} L_T - 2L_T \ln \frac{\Delta^-}{\mu^2} \right\} - (1-x) \ln(1-x) - \mathcal{P}_{q/q} \ln \frac{\Delta^-}{\mu^2} - L_T \mathcal{P}_{q/q}$$

Pure collinear matrix elements are “per se” ill defined

- A well-defined TMDPDF should:
 1. Be compatible with a factorization theorem.
 2. Have no mixed UV/nUV divergencies, i.e., be renormalizable
 3. Have a matching coefficient onto PDFs independent of nUV regulators.

Definition of TMD's

Positive and negative rapidity quanta can be collected into 2 different TMDs because of the splitting of the soft function

$$\tilde{S}(b_T; \frac{Q^2 \mu^2}{\Delta^+ \Delta^-}, \mu^2) = \tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-) \tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+),$$

$$\tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-) = \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \alpha \frac{\Delta^-}{\bar{p}^-}\right)},$$

$$\tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+) = \sqrt{\tilde{S}\left(\frac{1}{\alpha} \frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

$$\zeta_F = Q^2 / \alpha$$

$$\zeta_D = \alpha Q^2$$

Pure collinear

TMDPDF

$$\ln F_{ij}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2; \Delta^-) = \ln \tilde{\Phi}_{ij}^{(0)}(x, \mathbf{b}, S; \mu^2; \Delta^-) + \ln \tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-)$$

TMDFF

$$\ln D_{ij}(x, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2; \Delta^+) = \ln \tilde{\Delta}_{ij}^{(0)}(x, \mathbf{b}, S_h; \mu^2; \Delta^+) + \ln \tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+)$$

Soft function: structure and properties

In the high q_T limit: $Q \gg q_T \gg \Lambda_{QCD}$ the hadronic tensor is

$$\tilde{M} = H(Q^2/\mu^2) \tilde{C}(x_n, z_{\bar{n}}, L_{\perp}, Q^2/\mu^2) f_n(x_n, \Delta^-/\mu^2) d_{\bar{n}}(z_{\bar{n}}, \Delta^+/\mu^2)$$

and PDF, f_n , and FF, d_n , have single log dependence on UV/IR cutoff (Korchensky, Radyushkin 1987)

$$\ln f_n = \mathcal{R}_{f1}(x_n, \alpha_s) + \mathcal{R}_{f2}(x_n, \alpha_s) \ln \frac{\Delta^-}{\mu^2}$$
$$\ln d_{\bar{n}} = \mathcal{R}_{f1}(z_{\bar{n}}, \alpha_s) + \mathcal{R}_{f2}(z_{\bar{n}}, \alpha_s) \ln \frac{\Delta^+}{\mu^2}$$

Soft function: structure and properties

All this implies that each **pure** collinear and soft sectors are of the form

$$\ln \tilde{J}_n^{(0)} = \mathcal{R}_{n1}(x_n, \alpha_s, L_\perp) + \mathcal{R}_{n2}(x_n, \alpha_s, L_\perp) \ln \frac{\Delta^-}{\mu^2},$$

$$\ln \tilde{J}_{\bar{n}}^{(0)} = \mathcal{R}_{\bar{n}1}(x_{\bar{n}}, \alpha_s, L_\perp) + \mathcal{R}_{\bar{n}2}(x_{\bar{n}}, \alpha_s, L_\perp) \ln \frac{\Delta^+}{\mu^2},$$



$$\ln \tilde{S} = \mathcal{R}_{s1}(\alpha_s, L_\perp) + 2D(\alpha_s, L_\perp) \ln \frac{\Delta^- \Delta^+}{Q^2 \mu^2}.$$

- * Each sector is linear in logs of the rapidity cut
- * Each collinear sector depends just on 1 IR/rapidity regulator
- * Each collinear sector depends solely on the corresponding collinear momentum. It is not possible that a Q dependence arise in a pure collinear sector, Q dependence arises only in the soft sector.
- * The soft function is linear in the logs of rapidity regulator to cancel the corresponding logs in collinear sectors (In QCD there are no rapidity divergences)
- * The soft function is Hermitian, so it is the same for DIS, DY and ee to 2 jets

Splitting of the soft function

$$\begin{aligned}\ln \tilde{S} &= \mathcal{R}_s(b_T, \alpha_s) + 2D(b_T, \alpha_s) \ln \left(\frac{\Delta^+ \Delta^-}{Q^2 \mu^2} \right) \\ \ln \tilde{S}_- &= \frac{1}{2} \mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln \left(\frac{(\Delta^-)^2}{\zeta_F \mu^2} \right), \\ \ln \tilde{S}_+ &= \frac{1}{2} \mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln \left(\frac{(\Delta^+)^2}{\zeta_D \mu^2} \right)\end{aligned}$$

Using single log dependence

Q-dependence of TMD's

$$\begin{aligned}\zeta_F &= Q^2 / \alpha \\ \zeta_D &= \alpha Q^2\end{aligned}$$

$$\begin{aligned}\frac{d}{d \ln \zeta_F} \ln \tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) &= -D(b_T; \mu^2), \\ \frac{d}{d \ln \zeta_D} \ln \tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) &= -D(b_T; \mu^2).\end{aligned}$$

The Q-dependence of the TMD is dictated by the soft function:
spin independent

One loop results for TMDPDF (DY case)

$$\tilde{M} = H(Q^2 / \mu^2) \tilde{F}_n(x_n, b^2; Q^2, \mu^2) \tilde{F}_{\bar{n}}(x_{\bar{n}}, b^2; Q^2, \mu^2)$$

$$\begin{aligned} \tilde{F}_n &= \tilde{J}_n^{(0)}(\Delta^-) \sqrt{\tilde{S}(\Delta^-, \Delta^-)} \\ \tilde{F}_{\bar{n}} &= \tilde{J}_{\bar{n}}^{(0)}(\Delta^+) \sqrt{\tilde{S}(\Delta^+, \Delta^+)} \end{aligned}$$

$$\begin{aligned} \tilde{F}_{n1} &= \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-x) \left[\frac{1}{\epsilon_{UV}^2} - \frac{1}{\epsilon_{UV}} \ln \frac{Q^2}{\mu^2} + \frac{3}{2\epsilon_{UV}} \right. \right. && \rightarrow \text{NO Mixed divergences} \\ &- \frac{1}{2} L_T^2 + \frac{3}{2} L_T - L_T \ln \frac{Q^2}{\mu^2} - \frac{\pi^2}{12} + (1-x) - L_T P_{q/q} && \rightarrow \text{Matching coeff. to PDF} \\ &\left. \left. - P_{q/q} \ln \frac{\Delta^-}{\mu^2} - \frac{1}{4} \delta(1-x) - (1-x)[1 + \ln(1-x)] \right] \right\} && \rightarrow \text{PDF} \end{aligned}$$

Universality of unpolarized TMDPDFs: one loop results

Universality of the soft function

$$S_1^{r,DIS} = S_1^{r,DY} + \frac{\alpha_s C_F}{2\pi} \delta^{(2)}(\vec{k}_T) \pi^2; \quad S_1^{v,DIS} = S_1^{v,DY} - \frac{\alpha_s C_F}{2\pi} \delta^{(2)}(\vec{k}_T) \pi^2;$$

Universality of the pure collinear function

$$\hat{j}_{n1}^{r,DIS} = \hat{j}_{n1}^{r,DY} - \frac{\alpha_s C_F}{2\pi} \delta^{(2)}(\vec{k}_T) \pi^2; \quad \hat{j}_{n1}^{v,DIS} = \hat{j}_{n1}^{v,DY} + \frac{\alpha_s C_F}{2\pi} \delta^{(2)}(\vec{k}_T) \pi^2;$$

Both Naive Collinear And Soft ME Are Universal!
The TMDPDF is Universal

Evolution kernel for TMD's

$$\begin{aligned}\tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_{F,f}, \mu_f^2) &= \tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_{F,i}, \mu_i^2) \tilde{R}(b_T; \zeta_{F,i}, \mu_i^2, \zeta_{F,f}, \mu_f^2) , \\ \tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_{D,f}, \mu_f^2) &= \tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_{D,i}, \mu_i^2) \tilde{R}(b_T; \zeta_{D,i}, \mu_i^2, \zeta_{D,f}, \mu_f^2) , \\ \tilde{R}(b; \zeta_i, \mu_i^2, \zeta_f, \mu_f^2) &= \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\bar{\mu}}{\bar{\mu}} \gamma \left(\alpha_s(\bar{\mu}), \ln \frac{\zeta_f}{\bar{\mu}^2} \right) \right\} \left(\frac{\zeta_f}{\zeta_i} \right)^{-D(b_T; \mu_i)} ,\end{aligned}$$

For consistency the A.D. of the TMD is the opposite of the one of the hard coefficient

$$\begin{aligned}\gamma_H &= -\gamma_F \left(\alpha_s(\mu), \ln \frac{\zeta_F}{\mu^2} \right) - \gamma_D \left(\alpha_s(\mu), \ln \frac{\zeta_D}{\mu^2} \right) \\ \gamma_{F,D} \left(\alpha_s(\mu), \ln \frac{\zeta_{F,D}}{\mu^2} \right) &= -\Gamma_{\text{cusp}}(\alpha_s(\mu)) \ln \frac{\zeta_{F,D}}{\mu^2} - \gamma^V(\alpha_s(\mu))\end{aligned}$$

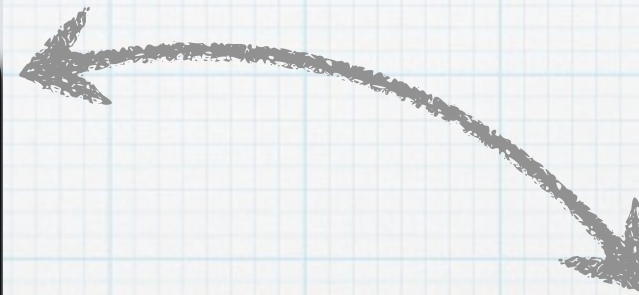
$$\frac{dD}{d \ln \mu} = \Gamma_{\text{cusp}}$$

$\mathcal{D}$ -resummation

$$\frac{dD(b; \mu)}{d \ln \mu} = \Gamma_{cusp}(\alpha_s)$$

$$D(b; \mu) = \sum_{n=1}^{\infty} d_n(L_{\perp}) \left(\frac{\alpha_s}{4\pi} \right)^n$$

	LL	NLL	NNLL
$d_1(L_{\perp}) =$	$d_1^{(1)} L_{\perp}$	$+ d_1^{(0)}$	
$d_2(L_{\perp}) =$	$d_2^{(2)} L_{\perp}^2$	$+ d_2^{(1)} L_{\perp}$	$+ d_2^{(0)}$
$d_3(L_{\perp}) =$	$d_3^{(3)} L_{\perp}^3$	$+ d_3^{(2)} L_{\perp}^2$	$+ d_3^{(1)} L_{\perp} + d_3^{(0)}$
$d_4(L_{\perp}) =$	$d_4^{(4)} L_{\perp}^4$	$+ d_4^{(3)} L_{\perp}^3$	$+ d_4^{(2)} L_{\perp}^2 + d_4^{(1)} L_{\perp} + d_4^{(0)}$
$d_5(L_{\perp}) =$	$\dots$		



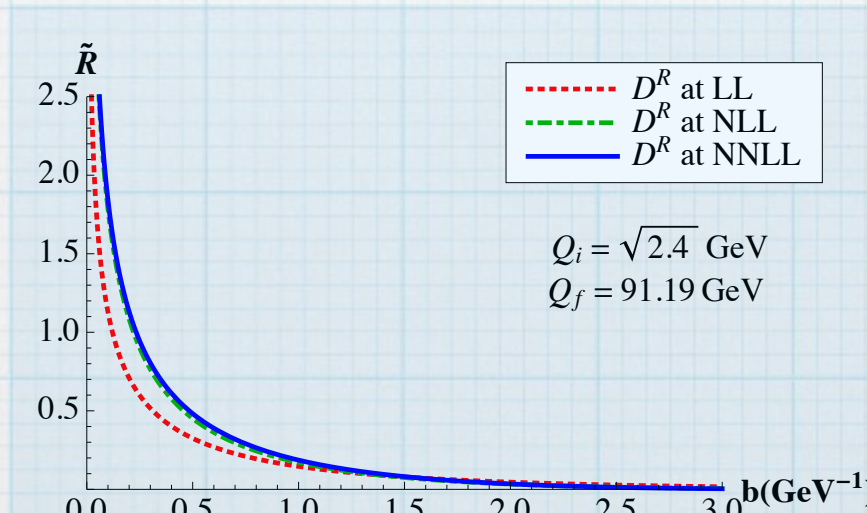
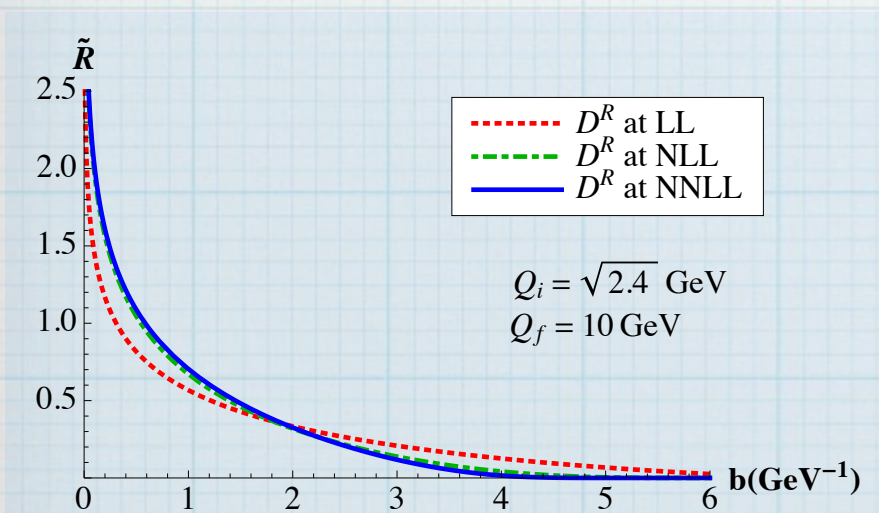
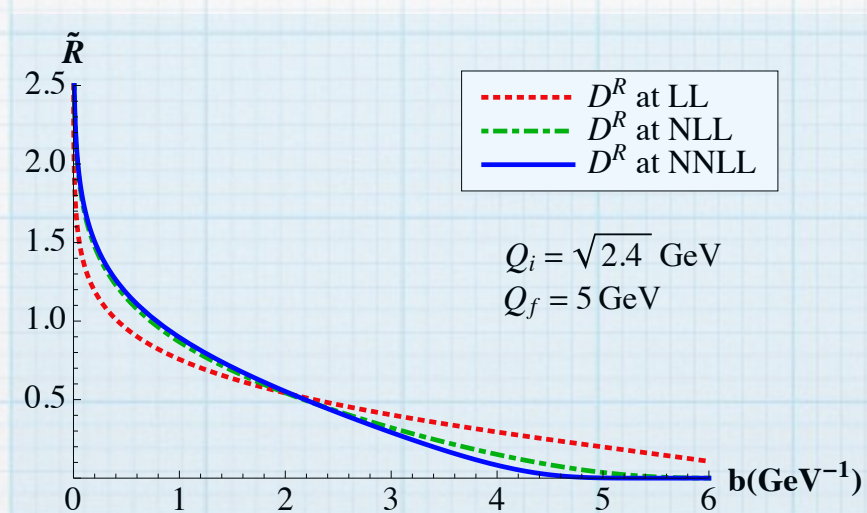
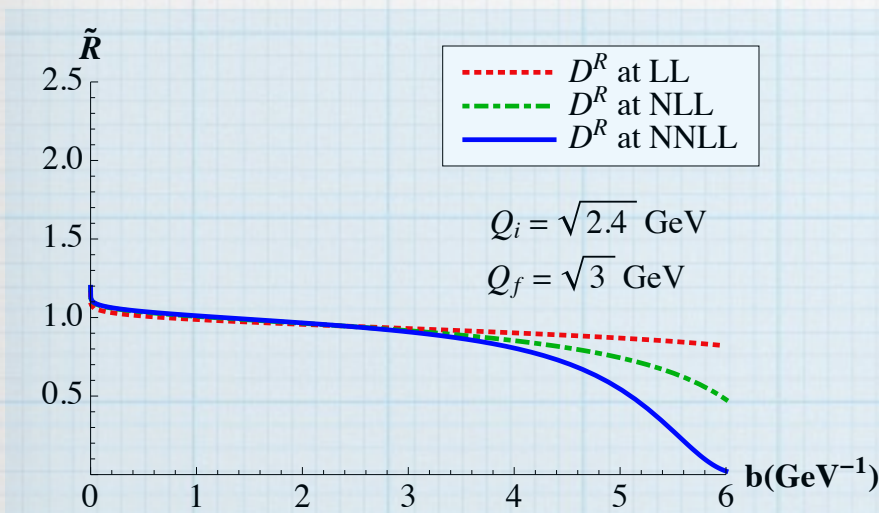
$$D(b; Q_i) = D(b; \mu_b) + \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma_{cusp}; \quad \mu_b = 2e^{-\gamma_E}/b$$

$$D(b; Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln \frac{\alpha_s(Q_i)}{\alpha_s(\mu_b)} \longrightarrow D(b; Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln(1 - X)$$

$$\alpha_s(\mu_b) = \alpha_s(Q)/(1 - X) \quad \text{Landau pole}$$

The perturbative expansion of the $\mathcal{D}$ is valid in limited (but large, using **resummation**) portion of Impact Parameter Space

Plots for evolution kernel



* Very good convergence up to $b=4-5/\text{GeV}$ in all cases

* The region sensitive to the Landau pole is strongly suppressed $b > 5/\text{GeV}$

* For $Q_f = M_Z$ we are sensitive only to $b < 1.5/\text{GeV}$ region

* For $Q_f = 3-5 \text{ GeV}$ we are sensitive only to $b < 4/\text{GeV}$ region

* For $Q_f < 2 \text{ GeV}$ we can be sensitive to the Landau pole region

- * Studying processes at different energies one explores different regions in IPS
- * The Landau pole problems appear there where also the Factorization Hyp. fails

Unpolarized TMDPF: construction and fits

* Basic test, preliminary to all spin dependent analysis, many ingredients as in standard perturbative QCD.

* More or less standard recipe for TMD construction (CSS, ...):

○ take the asymptotic limit of the TMDPDF

$$\tilde{F}_{q/N}(x, \vec{b}, Q_i, \mu) = \left(\frac{Q_i^2 b^2}{4e^{2\gamma_E}} \right)^{-D_R(b, \mu)} \sum_j \tilde{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu) \otimes f_{j/N}(x; \mu) \otimes M_q(x, \vec{b}, Q_i)$$

OPE to PDF, valid for $q_T \gg \Lambda_{QCD}$

PDF

Process independent
Non-perturbative correction

Common to all analysis:
Florence (Catani et al.), Zurich (Gehrmann. et al)

○ Exponentiation of part of the coefficient and **complete resummation of the logs in the exponent**

(Kodaira, Trentadue 1982, Becher, Neubert Wilhelm 2011)

$$\tilde{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu) \equiv \exp(h_\Gamma - h_\gamma) \hat{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu)$$

$$\frac{dh_\Gamma}{d \ln \mu} = \Gamma_{cusp} L_\perp$$

$$\frac{dh_\gamma}{d \ln \mu} = \gamma^V$$

$$h_\Gamma^R(b, \mu) = \int_{\alpha_s(1/\hat{b})}^{\alpha_s(\mu)} d\alpha' \frac{\Gamma_{cusp}^F(\alpha')}{\beta(\alpha')} \int_{\alpha_s(1/\hat{b})}^{\alpha'} \frac{d\alpha}{\beta(\alpha)} \cdot$$

and finally write $a(1/b)$ in terms of $a(\mu)$ and fix $\mu=Q_i$.
Logs are minimized with the choice $Q_i=Q_0+q_T$

Experimental Data

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
points	32	16	41	9
$\sqrt{s}$	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
σ	248 ± 11 pb	221 ± 11.2 pb	256 ± 15.2 pb	255.8 ± 16.7 pb

Z, run I: Becher, Neubert, Wilhelm 2011
Catani et al. 2009

	E288 200	E288 300	E288 400	R209
points	35	35	49	6
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	62 GeV
E_{beam}	200 GeV	300 GeV	400 GeV	-
Beam/Target	p Cu	p Cu	p Cu	p p
M range used	4-9 GeV	4-9 GeV	5-9 and 10.5-14 GeV	5-8 and 11-25 GeV
Other kin. var	$y=0.4$	$y=0.21$	$y=0.03$	
Observable	$Ed^3\sigma/d^3p$	$Ed^3\sigma/d^3p$	$Ed^3\sigma/d^3p$	$d\sigma/dq_T^2$

Expected to be insensitive to Landau pole region
Factorization hypothesis hold

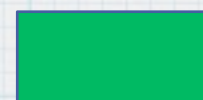
Theoretical settings

- * Matching scale of TMDPDF to PDF at $Q_i=2 \text{ GeV}+q_T$
- * Hard coefficient with π^2 resummation (Ahrens, Becher, Lin Yang, Neubert '08)
- * Checked both NLL and NNLL
- * Several sets of PDF checked (MSTW, CTEQ)
- * Checked several form of non-perturbative models: gaussian, exponential, Q-dependence, ...
- * Non-perturbative input

$$M_q(x, \vec{b}, Q_i) = \exp[-\lambda_1 b](1 + b^2 \lambda_2 + \dots)$$

Order	γ	Γ_{cusp}	C	D
LL	-	α	tree	-
NLL	α	α^2	tree	α
NNLL	α^2	α^3	α	α^2
NNNLL	α^3	α^4	α^2	α^3

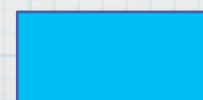
Naive attempts



Aybat, Collins , Qiu,
Rogers; Aybat, Rogers;
Anselmino, Boglione, Melis

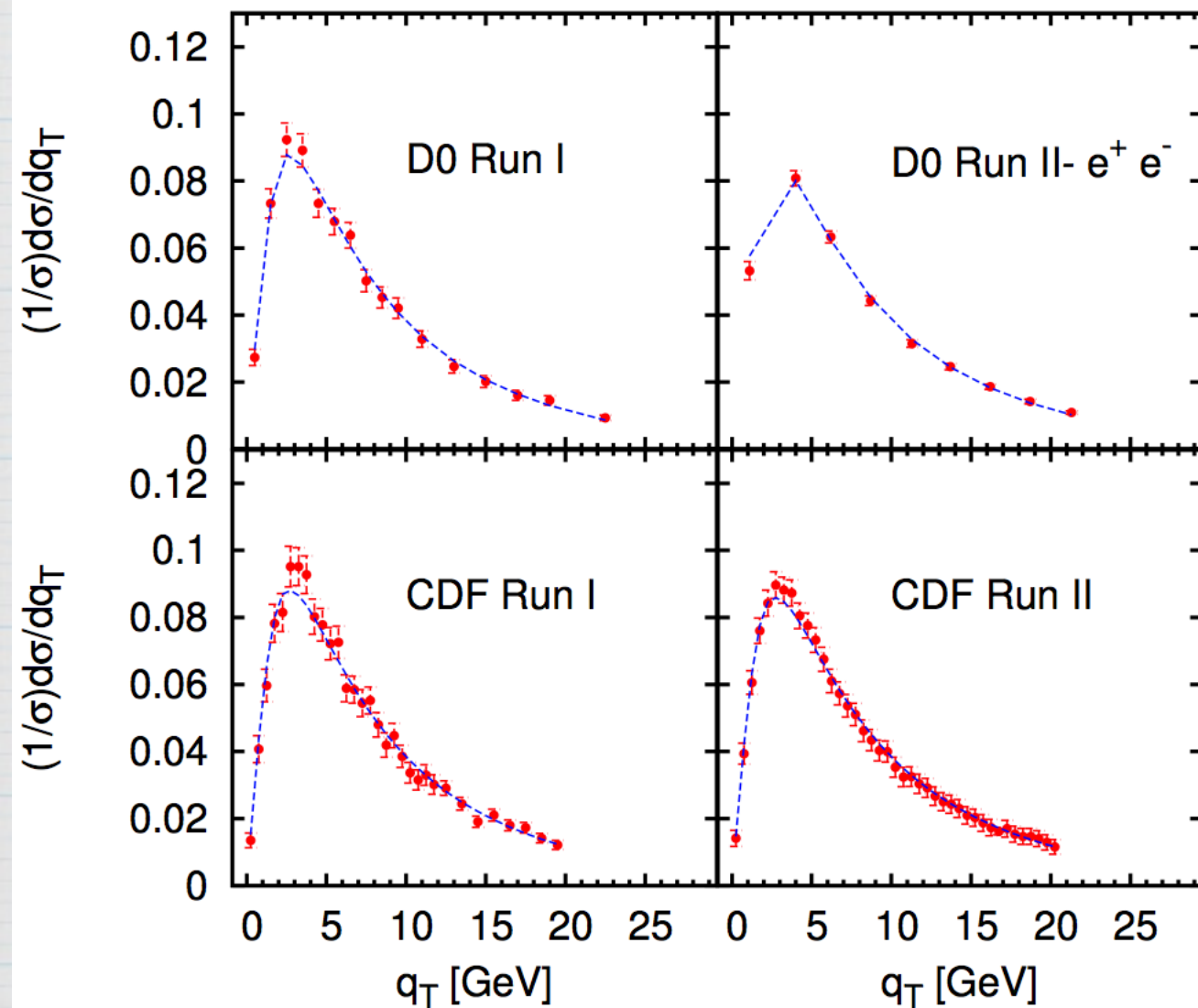


EIS



Known pieces: C for unpolarized TMD
from Catani et al. '12,
Gehrmann, Luebbert. Lin Yang '12, '14

Preliminary results at NNLL



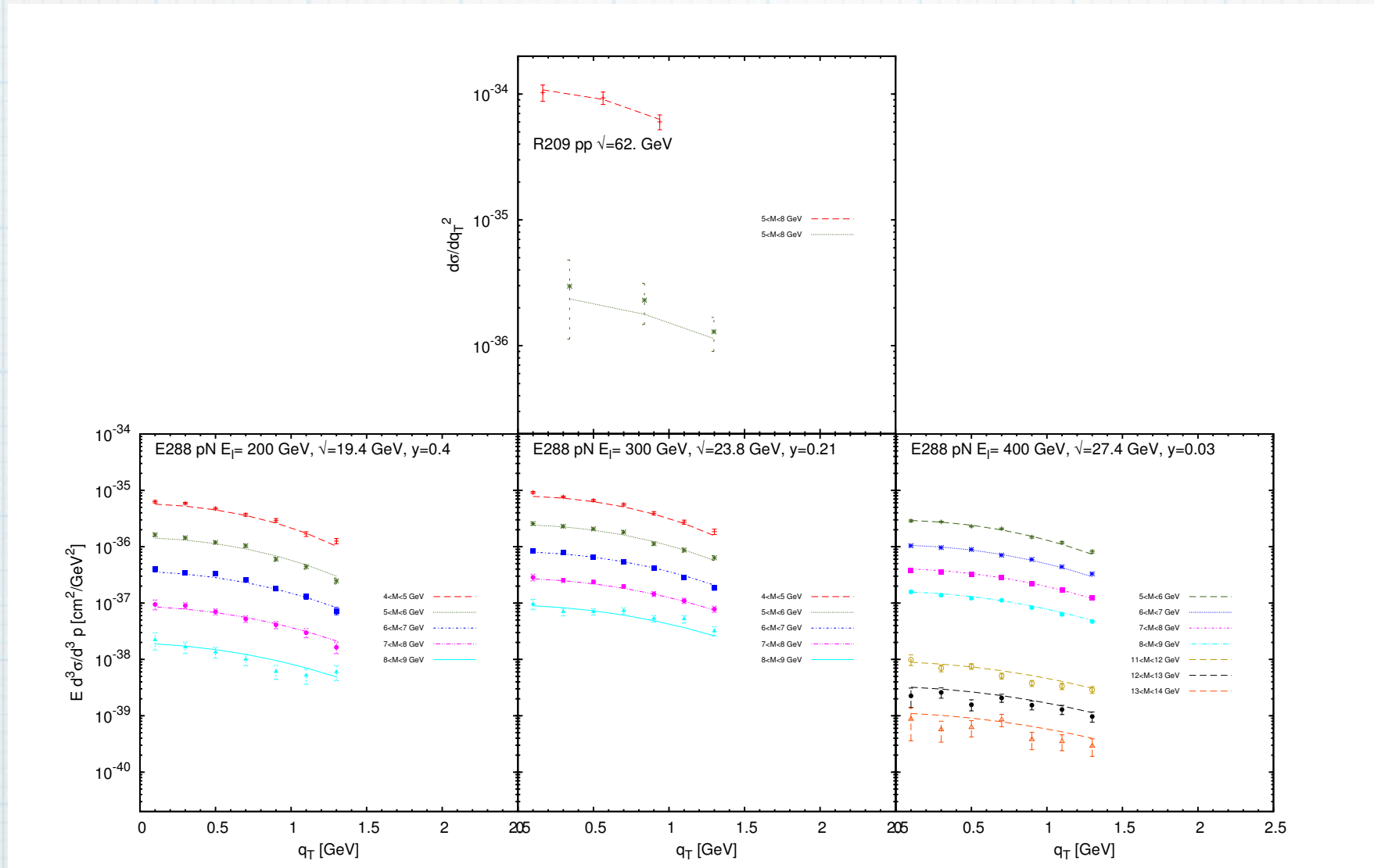
Z-boson data are (fairly) sensitive to functional non-perturbative form (gaussian vs exponential) and (poorly) sensitive just to λ_1 . In order to fix it we need the global fit

Data:
$$\frac{1}{\sigma_{exp}} \left(\frac{d\sigma}{dq_T} \right)_{exp}$$

Theory:
$$\frac{1}{\sigma_{th}} \left(\frac{d\sigma}{dq_T} \right)_{th}$$

DYNNLO: Catani, Grazzini '07, Catani, Cieri, Ferrera, de Florian, Grazzini '09

Preliminary results at NNLL



Exp. Normalization
NE288, NR209
deduced from the fit.

Total: 4 parameters

$$\lambda_1 = 0.33 \pm 0.05_{stat}$$

$$\lambda_2 = 0.13 \pm 0.03_{stat}$$

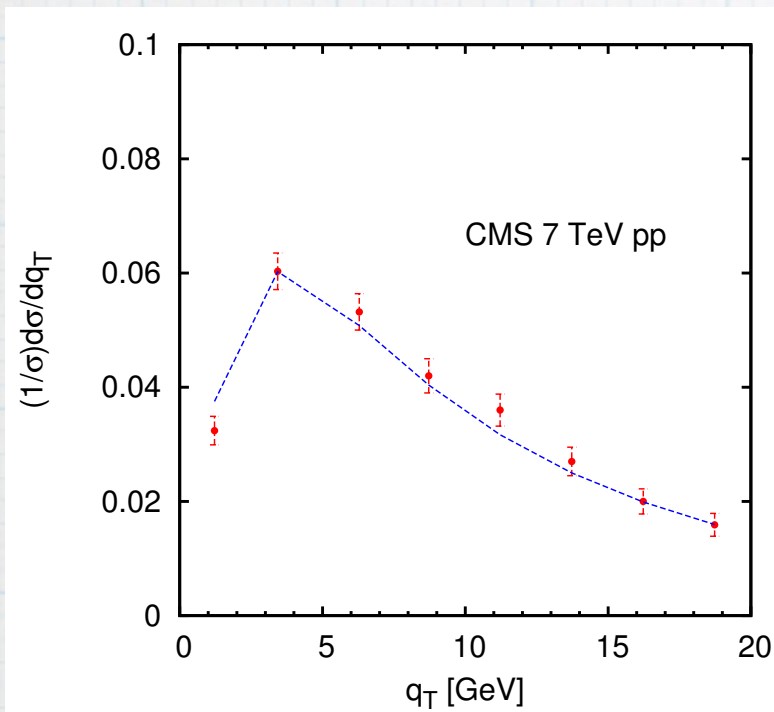
$$N_{E288} = 0.85 \pm 0.04_{stat}$$

$$N_{R209} = 1.5 \pm 0.02_{stat}$$

$$\frac{\chi^2}{points} \Big|_{\textcolor{red}{Z}} \simeq 0.6$$

$$\frac{\chi^2}{points} \Big|_{\textcolor{red}{global}} \simeq 1.1$$

Preliminary results at NNLL



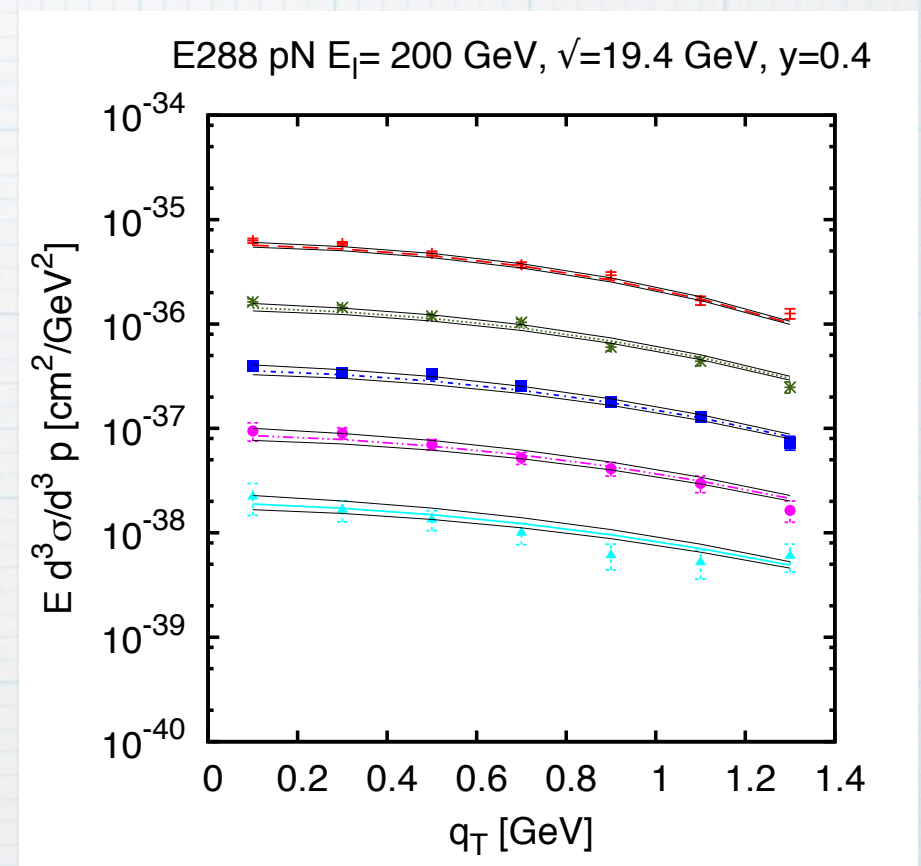
Prediction for Z production with CMS data.
Bin width not shown

$$\frac{\chi^2}{\text{points}} \simeq 0.9$$

Scale dependence

$$m_c \sim 1.3 \text{ GeV} < Q_0 < 2.7 \text{ GeV}$$

1st bin energy



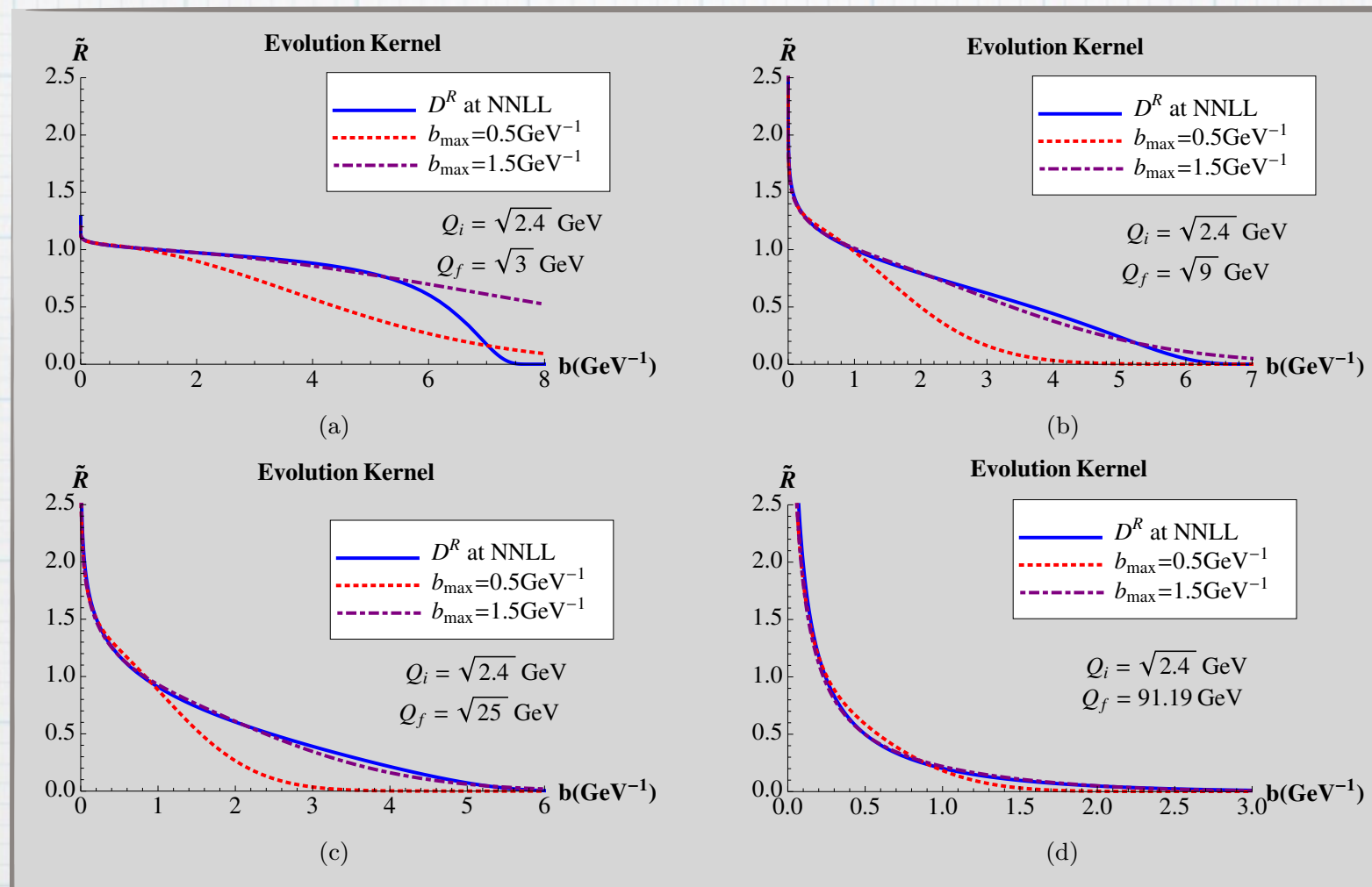
- * Data are not sensitive to: the Landau pole region in IPS, non-perturbative Q -dependence. The study of flavor dependence needs to include also W -production data....

Conclusions

- * The correct measurement of non-perturbative effects in transverse momentum dependent observables requires the definition of TMDs (We want to use TMDPDF in the same way as PDF)
- * Pure collinear and soft matrix elements are not well defined when the dependence on transverse momentum is included. The proper transverse momentum distributions must be defined as a combination of collinear and soft parts (rapidity divergence problem).
- * The soft matrix elements are responsible for Q -dependence of TMD's.
- * TMD's are universal (the same for DIS, DY, $ee \rightarrow 2j$).
- * The evolution of TMDPDF and TMDFF is the same and spin independent.
- * First fits for unpolarized TMDPDF in DY. Data with $4 < Q/\text{GeV} < 10$ can fix non-perturbative parameters, which have some impact on vector boson production and DY processes in LHC. More data required. SIDIS and $ee \rightarrow 2j$ analysis to be done.
- * TMD non-perturbative QCD effects should be included in high precision LHC observables.
- * Analysis of spin dependent observables including evolution is starting now. Data from Belle, Compass, JLab, LHC..

Back up

EISS vs CSS



CSS: The evolution is modeled with a b_{max} and a gaussian. In this way it is defined also BEYOND the Landau pole