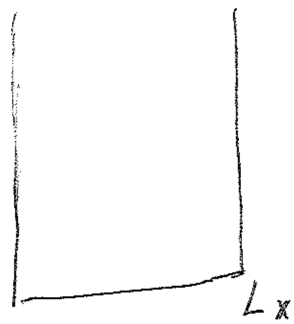
Brief introduction to many body Physics for description of nuclear reactions:

- Non interacting Fermi sea of protons and neutrons

large box of sides $L_x = L_y = L_z = L$



$$\psi(x) = A \sin K_x x$$

$$\sin K_x L_x = 0$$

$$K_x L = n\pi \quad ; n = 1, 2, 3$$

In 3 dimensions

$$\vec{K} L = \vec{n}\pi \quad ; \quad \begin{array}{l} n_x = \cancel{0}, \cancel{1}, 2, 3 \ldots \\ n_y = 0, 1, 2, 3 \ldots \\ n_z = 0, 1, 2, 3 \end{array}$$

To count the number of states we can put

$$\vec{K} L = 2\pi \vec{n} \quad ; \quad n_i = 0, \pm 1, \pm 2, \pm 2$$

(periodical boundary conditions)

N protons $\equiv$ All states are occupied up to a maximum momentum, K_F, Fermi momentum

$$N_{protons} = \underset{\substack{\downarrow \\ spin}}{2} \sum_{n_i} \rightarrow 2 \frac{\int d^3K \, L^3}{(2\pi)^3}$$

$\int d^3K \, L^3 \equiv$ phase space : $(2\pi)^3$ volume of unit cell in P.S.

$$N_p = 2 \, \frac{4}{3} \pi K_F^3 \, V$$

↓ volume

$$\frac{N_p}{V} = \rho = 2 \, \frac{4}{3} \pi K_F^3$$

↓ density of protons

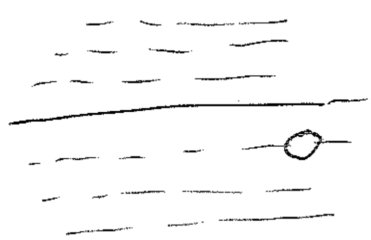
— Fields : Definition

$$\hat{\psi}(\bar{x}) = \sum_K \varphi_K(\bar{x}) \, a_K$$

$$\hat{\psi}^+(x) = \sum \varphi_K^*(x) \, a^+_K$$

$$\{ a_K , a^+_{K'} \} = \delta_{\vec{K},\vec{K}'}$$

– Particles and holes



K_F

destroys particle $a_K \equiv a_K$ $\qquad |\vec{K}| > K_F$

creates particle $a^+_K \equiv a^+_K$ $\qquad |K| > K_F$

$$a_K \equiv b^+_K \qquad |\vec{K}| \leq K_F$$

$$a^+_K \equiv b_K \qquad |\vec{K}| \leq K_F$$

↓ hole state

Destroy particle below Fermi sea $\equiv$

create a hole in the Fermi sea.

The differentiation is important because

$$a_K \, |\phi_0\rangle = 0 \qquad \text{(no "particles" in the Fermi sea)}$$

↓ ground state

$$b_K \, |\phi_0\rangle = 0 \qquad \text{(no holes in the Fermi sea)}$$

This allows to calculate the <u>Green's function</u> or propagator

$$i\, G(\vec{x}t, \vec{x}'t') = \frac{\langle \Psi_0 | T[\hat{\varphi}_H(x,t) \cdot \hat{\varphi}_H^+(\vec{x}',t')] |\Psi_0\rangle}{\langle \Psi_0 | \Psi_0\rangle}$$

$\Psi_0 \equiv$ true ground state

T : time ordering operator

$\hat{\varphi}_H$: fieds in Heisenberg representation

For non interacting Fermi sea $\Psi_0 \equiv \phi_0$

Heisenberg and interacting picture coincide

One finds

$$i\, G^0(\vec{x}t, \vec{x}'t') \; = \sum_{|\vec{k}| > k_F} \varphi_k(\vec{x})\, \varphi_k^*(\vec{x}')\, e^{-i\omega_k(t-t')}, \qquad t > t'$$

$$= - \sum_{|\vec{k}| \leq k_F} \varphi_k(\vec{x})\, \varphi_k^*(\vec{x}')\, e^{-i\omega_k(t-t')}, \qquad t < t'$$

$$\omega_k = \frac{\vec{k}^2}{2m} \qquad \left(\sqrt{\vec{k}^2 + m^2}\ , \text{relativistic}\right)$$

Detail : Non interaction

$$\hat{\psi}_I(\bar{x},t) = \hat{\psi}_H(\bar{x},t) = \sum_{k>k_F} \varphi_k(\bar{x})\, e^{-i\omega_k t}\, a_k$$

$$+ \sum_{k\leq k_F} \varphi_k(\bar{x})\, e^{-i\omega_k t}\, b_k^+$$

$t > t'$

$$\langle \phi_0 | \left(\sum_{k>k_F} \varphi_k(\bar{x})\, e^{-i\omega_k t} a_k + \sum_{k\leq k_F} \varphi_k(\bar{x})\, e^{-i\omega_k t}\, b_k^+ \right)$$

$$\left(\sum_{k'>k_F} \varphi_{k'}^*(\bar{x}')\, e^{i\omega_{k'} t'} a_{k'}^+ + \sum_{k'\leq k_F} \varphi_{k'}(\bar{x}')\, e^{i\omega_{k'} t'} b_{k'} \right) |\phi_0\rangle$$

but $b_{k'}|\phi_0\rangle = 0$

$$= \langle \phi_0 | \left(\sum_{k>k_F} \varphi_k(x)\, e^{-i\omega_k t} a_k \right) \left(\sum_{k'>k_F} \varphi^*(\bar{x}')\, e^{i\omega_{k'} t'} a_{k'}^+ | \phi_0 \rangle \right.$$

$$\langle \phi_0 | a_k a_{k'}^+ | \phi_0 \rangle = \langle \phi_0 | a_k a_{k'}^+ + a_{k'}^+ a_k - a_{k'}^+ a_k | \phi_0 \rangle$$

$$= \langle \phi_0 | \{ a_k a_{k'}^+ \} | \phi_0 \rangle = \langle \phi_0 | \delta_{\bar{k}\bar{k}'} | \phi_0 \rangle = \delta_{\bar{k}\bar{k}'} \qquad 0$$

$$= \sum_{k>k_F} \varphi_k(\bar{x})\, \varphi_k^*(\bar{x}')\, e^{-i\omega_k (t-t')}$$

\# $t < t'$

$$(-)\langle\phi_0|\Big(\sum_{k'>k_F}\varphi^*_{k'}(\vec{x}')e^{+i\omega_{k'}t'}a^+_{k'}+\sum_{k'\le k_F}\varphi^*_{k'}(\vec{x}')e^{+i\omega_{k'}t'}b_{k'}\Big)$$

$$\Big(\sum_{k>k_F}\varphi_k(\vec{x})e^{-i\omega_k t}a_k+\sum_{k\le k_F}\varphi_k(\vec{x})e^{-i\omega_k t}b^+_k\Big)|\phi_0\rangle$$

$$a_k|\phi_0\rangle=0$$

$$=-\langle\phi_0|\Big(\sum_{k'\le k_F}\varphi^*_{k'}(\vec{x}')e^{i\omega_{k'}t'}b_{k'}\Big)\Big(\sum_{k\le k_F}\varphi_k(\vec{x})e^{-i\omega_k t}b^+_k\Big)|\phi_0\rangle$$

$$\langle\phi_0|b_{k'}b^+_k|\phi_0\rangle=\langle\phi_0|b_{k'}b^+_k+b^+_k b_{k'}-\underbrace{b^+_k b_{k'}|\phi_0\rangle}_{0}$$

$$=\langle\phi_0|\{b_{k'},b^+_k\}|\phi_0\rangle=\langle\phi_0|\delta_{\vec{k},\vec{k}'}|\phi_0\rangle=\delta_{\vec{k}\vec{k}'}$$

$$=-\sum_{k\le k_F}\varphi_k(\vec{x})\varphi^*_k(\vec{x}')e^{-i\omega_k(t-t')}$$

If we have translational invariance (infinite nuclear matter)

$$\varphi_k(\vec{x}) = \frac{1}{\sqrt{V}} e^{i\vec{k}\vec{x}}$$

$$\varphi_k(\vec{x})\,\varphi_k^*(\vec{x}') = \frac{1}{V} e^{i\vec{k}(\vec{x}-\vec{x}')}$$

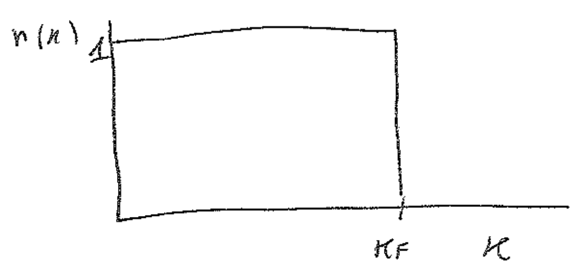
Then G^0 depends on $t-t'$, $\vec{x}-\vec{x}'$)

We define $G(\vec{k},\omega)$ as

$$G(\vec{k},\omega) = \int d(t-t')\, d^3(\vec{x}-\vec{x}')\; G(\vec{x}\,t, \vec{x}'t')$$

and then

$$G^0(\vec{k},\omega) \equiv \frac{1-n(\vec{k})}{\omega-\omega_k+i\epsilon} + \frac{n(\vec{k})}{\omega-\omega_k-i\epsilon}$$



$$n(\vec{k}) = \begin{cases} 1 & |\vec{k}| \le k_F \\ 0 & |\vec{k}| > k_F \end{cases}$$

occupation number

This is the base for many body calculations

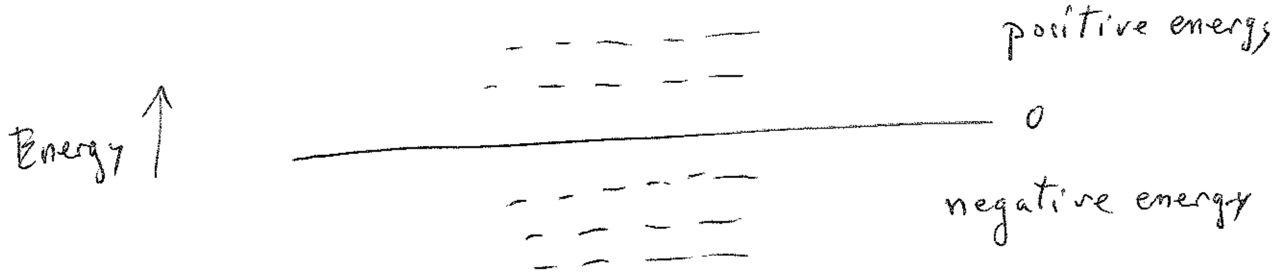
— For Those who Know Field Theory

Fermion Propagator $\dfrac{\cancel{K}+m}{K^2-m^2} \equiv$

$$\equiv \frac{m}{\omega_K}\left\{ \frac{\sum_r u_r(\vec{K})\,\bar{u}_r(\vec{K})}{K^0-\omega(K)+i\epsilon} + \frac{\sum_r v_r(-\vec{K})\,\bar{v}_r(-\vec{K})}{K^0+\omega(K)-i\epsilon} \right\}$$

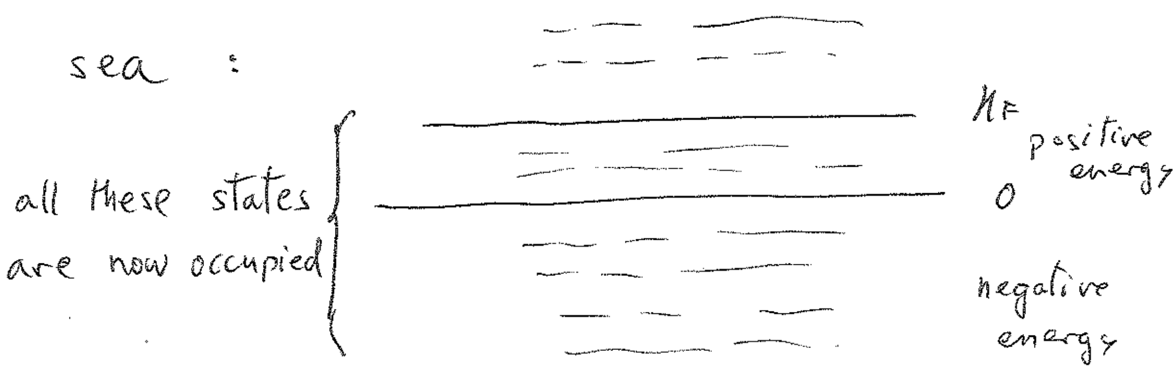
↓ states of negative energy (antiparticle)

Dirac picture :



In this Dirac picture the states of negative energy are occupied

Fermi sea :

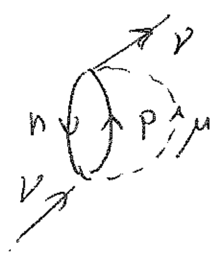


$$\text{New Propagator: } \frac{m}{\omega_K}\left\{ \underbrace{\frac{\sum_r u_r(\vec{K})\,\bar{u}_r(\vec{K})}{K^0-\omega(K)+i\epsilon}}_{\text{above Fermi sea}} + \underbrace{\frac{\sum_r u_r(\vec{K})\,\bar{u}_r(\vec{K})}{K^0-\omega(K)-i\epsilon}}_{\text{below Fermi sea}} + \frac{\sum_r v_r(-\vec{K})\,\bar{v}_r(-\vec{K})}{K^0+\omega(K)-i\epsilon} \right\}$$

— Application to neutrino scattering

$$\nu_\mu(k) + n(p) \to \mu^-(k') + p(p')$$

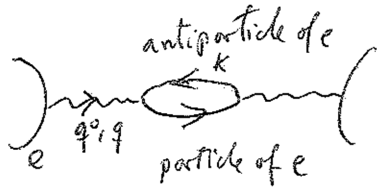
The process can be visualized as



create a hole in Fermi sea of neutrons

create a particle of protons <u>on</u> <u>top</u> of the Fermi sea

Simil in Field Theory : vaccum polarization



$$\Pi_\gamma(q^0,q) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^0 + \omega(k) - i\epsilon} \frac{1}{q^0 + k^0 - \omega(\vec{k}+\vec{q}) + i\epsilon}$$

....

$$k^0$$

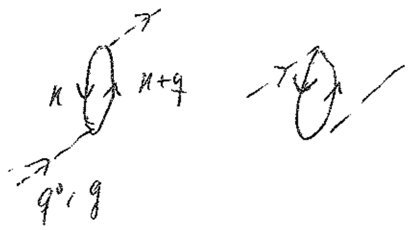
$$-\omega(k) \qquad \omega(k) - q^0$$

Integral of q^0 via Residues

$$-2\pi i \frac{1}{\omega(\vec{k}+\vec{q}) - q^0 + \omega(k) - i\epsilon}$$

$$\Pi_\gamma = +i \int \frac{d^3k}{(2\pi)^3} \frac{1}{q^0 - \omega(k) - \omega(\vec{k}+\vec{q}) + i\epsilon} \cdots\cdots$$

- particle - hole excitation in the nuclear medium



$$\int \frac{d^4k}{(2\pi)^4} \left[\frac{1-n(\vec k)}{k^0-\omega(k)+i\epsilon} + \frac{n(\vec k)}{k^0-\omega(k)-i\epsilon} \right]$$
$$\left[\frac{1-n(\vec k+\vec q)}{k^0+q^0-\omega(\vec k+\vec q)+i\epsilon} + \frac{n(\vec k+\vec q)}{k^0+q^0-\omega(\vec k+\vec q)-i\epsilon} \right]$$

perform the k^0 integration

$$U_N = 4 \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{n(\vec k)\,[1-n(\vec k+\vec q)]}{q^0-\varepsilon(\vec k+\vec q)+\varepsilon(\vec k)+i\epsilon} + \frac{n(\vec k+\vec q)\,[1-n(\vec k)]}{-q^0-\varepsilon(\vec k)+\varepsilon(\vec k+\vec q)+i\epsilon} \right\}$$

↓

for symmetric nuclear matter { 2 spin, 2 neutrons + protons)

For $\nu_\mu + n \rightarrow p + \mu$ only first term and the sum only over $\underline{\underline{n}}$

$$\bar U = 2 \int \frac{d^3k}{(2\pi)^3} \, \frac{n(\vec k)\,[1-n(\vec k+\vec q)]}{q^0-\varepsilon(\vec k+\vec q)+\varepsilon(\vec k)+i\epsilon}$$

and this has an easy analytical form

$$\mathrm{Re}\, \bar U(q^0,q) = \frac{3}{2}\rho_0 \frac{M}{q\,k_F} \left\{ z + \frac{1}{2}(1-z^2) \ln \frac{|z+1|}{|z-1|} \right\}$$

$$\mathrm{Im}\, \bar U(q^0,q) = -\pi \frac{3}{4}\rho_0 \frac{M}{q\,k_F} \left\{ (1-z^2)\,\theta(1-|z|) \right\}$$

$$z = \left(q^0 - \frac{\vec q^{\,2}}{2M} \right) \frac{M}{q\,k_F}$$

M = nucleon mass

- Evaluation of $\nu_\mu + n \to p + \mu^-$ in nuclei.

$$\nu_\mu(k) + n(p) \to \mu^-(k') + p(p')$$

$$\sigma = \frac{1}{V_{rel}} \int \frac{d^3k'}{(2\pi)^3} \int \frac{d^3p'}{(2\pi)^3} \prod_i \left(\frac{M_i}{E_i}\right) \bar{\Sigma} \Sigma |T|^2 (2\pi)^4 \delta^4(k+p-k'-p')$$

$$= \frac{1}{V_{rel}} \int \frac{d^3k'}{(2\pi)^3} \prod_i \left(\frac{M_i}{E_i}\right) \bar{\Sigma}\Sigma |T|^2 \, 2\pi \, \delta(E_\nu + E_n - E_\mu - E_p)$$

T is the transition matrix for this reaction

$$T = \frac{G}{\sqrt{2}} \cos\theta_c \, l^\mu J_\mu$$

In a nucleus we would multiply this cross section by N (number of neutrons)

$$\sigma(\nu A) = N \sigma_{\nu, n}$$

Let $\rho_n(\vec{r})$ be the actual density of neutrons in a nucleus

$$\sigma(\nu A) = \int d^3r \, \rho_n(r) \int \frac{d^3k'}{(2\pi)^3} \cdots\cdots$$

But

$$\rho_n(r) = 2 \int \frac{d^3p}{(2\pi)^3} n(p, \vec{r})$$

$n \equiv$ occupation number for a local Fermi sea $k_F(r)$

$$k_F(r) \equiv [3\pi^2 \rho_n(r)]^{1/3}$$

$$n(p, \vec{r}) \equiv \begin{cases} 1 & |\vec{p}| \le k_F(r) \\ 0 & |\vec{p}| > k_F(r) \end{cases}$$

Then

$$\sigma = \int d^3r \quad 2\int \frac{d^3p}{(2\pi)^3}\, n(p,\vec r) \int \frac{d^3k'}{(2\pi)^3} \prod_i \left(\frac{M_i}{E_i}\right) \bar\Sigma \Sigma |T|^2$$
$$\cdot\, 2\pi\, \delta(E_\nu + E_n - E_\mu - E_p)$$

But we did not take into account the Pauli exclusion principle of protons

$\Rightarrow$ Protons must be excited on top of the local Fermi sea.

$$\Rightarrow \ast\ 1 - n_p(\vec p + \vec k - \vec k')$$

So we find the combination ($\frac{M}{E} \sim 1$ for nucleons)

$$2\int \frac{d^3p}{(2\pi)^3}\, n_n(\vec p)\, [1 - n_p(\vec p + \underset{\vec k - \vec k'}{\vec q})]\, 2\pi\, \delta(\underbrace{E_\nu(k) - E_\mu(k')}_{q^0} + E_n(p) - E_p(\vec p + \vec q))$$

But this is $-2\, \mathrm{Im}\, \bar U(q^0, q)$

$$\sigma = \int d^3r \int \frac{d^3k'}{(2\pi)^3} \prod_{\mu,\nu} \frac{M_i}{E_i} (-2)\, \mathrm{Im}\, \bar U(k_\nu - k_\mu, \rho(r))\, \bar\Sigma \Sigma |T|^2$$

At the end, with the expense of an extra r integration we account for
- Fermi motion of the neutrons
- Pauli blocking of the protons

consistent with the <u>realistic</u> $\rho(r)$ density of the nucleus

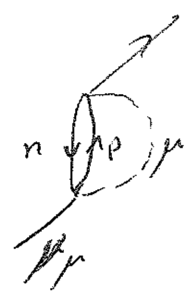
— Application to the inverse process : muon capture

$$\mu^- \, p \rightarrow n \, \nu_\mu$$

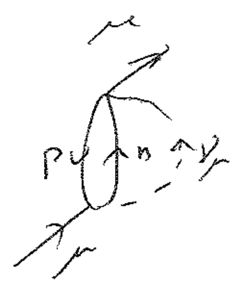
but here μ^- is in a 1s atomic orbit

$$\Gamma = \int d^3r \, |\phi_\mu(\vec{r})|^2 \int \frac{d^3 p_\nu}{(2\pi)^3} \prod_{\mu\nu} \frac{M_i}{E_i} \bar{\Sigma} \Sigma |T|^2 (-2) \, \mathrm{Im}\, U(p_\mu - p_\nu)$$

— Particle-hole propagator in nuclei



or

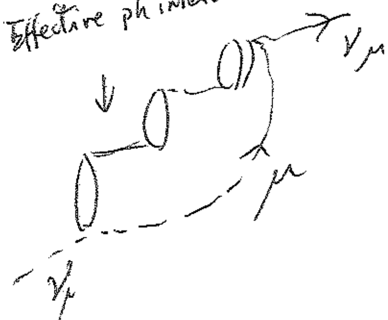


This is what has been calculated

ν scattering

μ capture

Yet, once a p-h excitation has been created it can propagate through the nucleus



RPA is a good approach to ph propagation and trivial to implement in this approach

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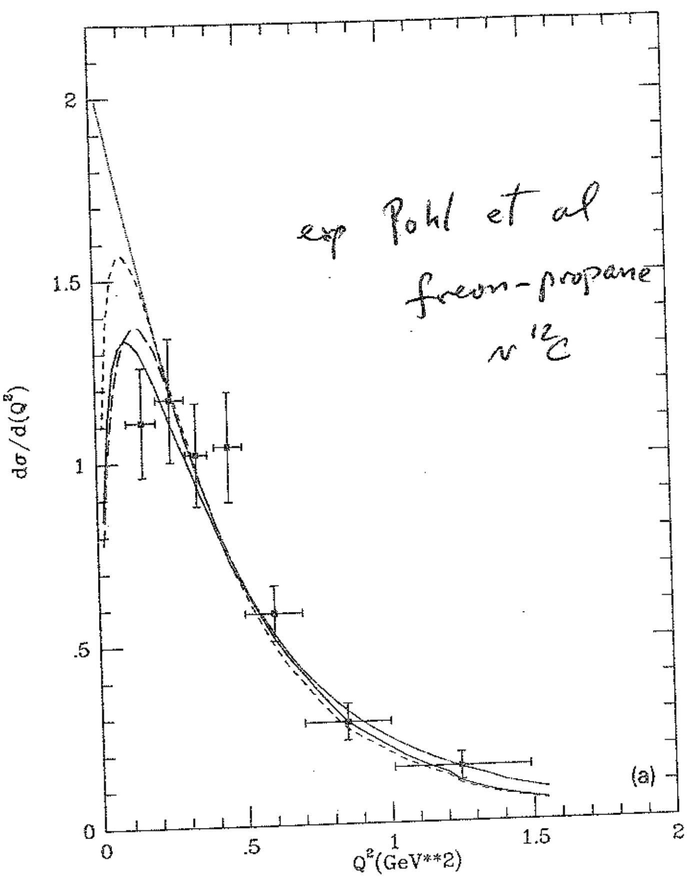
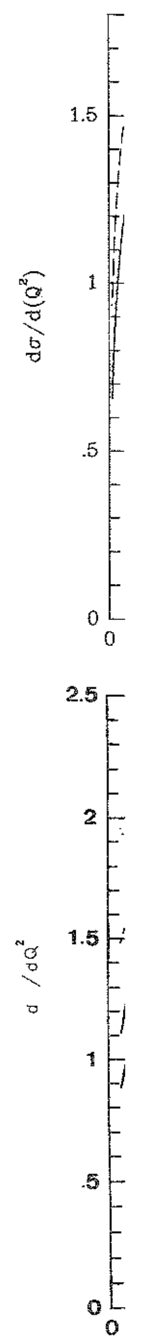


Fig. 4. $d\sigma/dq^2$ versus q^2 compared with neutrino experiments of (a) Pohl *et al* [19]), (b) Bonnetti *et al.* [15]) and (c) SKAT collaboration [24]). Fermi-gas model (long dash); present calculation without renormalization (short dash) and with renormalization (solid). Also shown in (a) and (c) are the results for free-nucleon case for comparision (dotted). Differential cross section in units of $[10^{-38}\ \mathrm{cm}^2/\mathrm{GeV}^2]$.

becomes negligible for $q^2 > 0.3\ \mathrm{GeV}/c^2$ (compare with free-nucleon case). The effect of strong renormalization due to ph and Δh correlations is quite large in this region where nuclear effects are important. When comparing the present results with the simple Fermi-gas model, we see that the differential cross sections in the Fermi-gas model are surprisingly close to our present results with strong renormalization taken into account. The strong suppression in the differential cross section due to nuclear effects seems to be quite well simulated by the simple Fermi-gas calculations. An improved Fermi-gas calculation with local density approximation will overestimate the differential cross section especially at lower q^2. In comparing these results with the experimental results of Bonnetti *et al.* and Pohl *et al.* we find a good agreement except at very low q^2, where the experiments show still stronger suppression. The effect of strong renormalization helps in improving the agreement, but the quality



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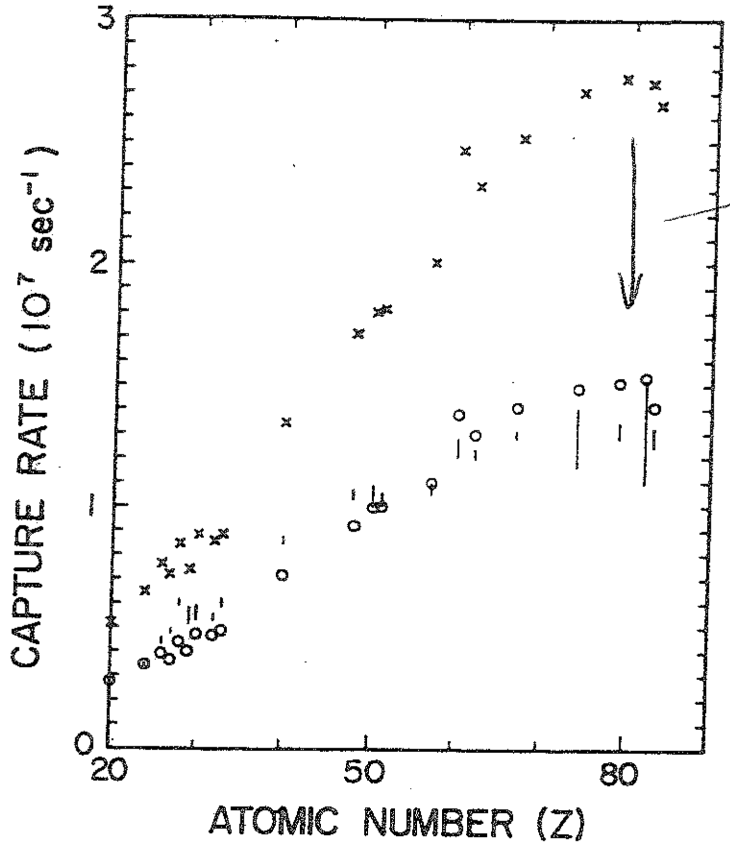
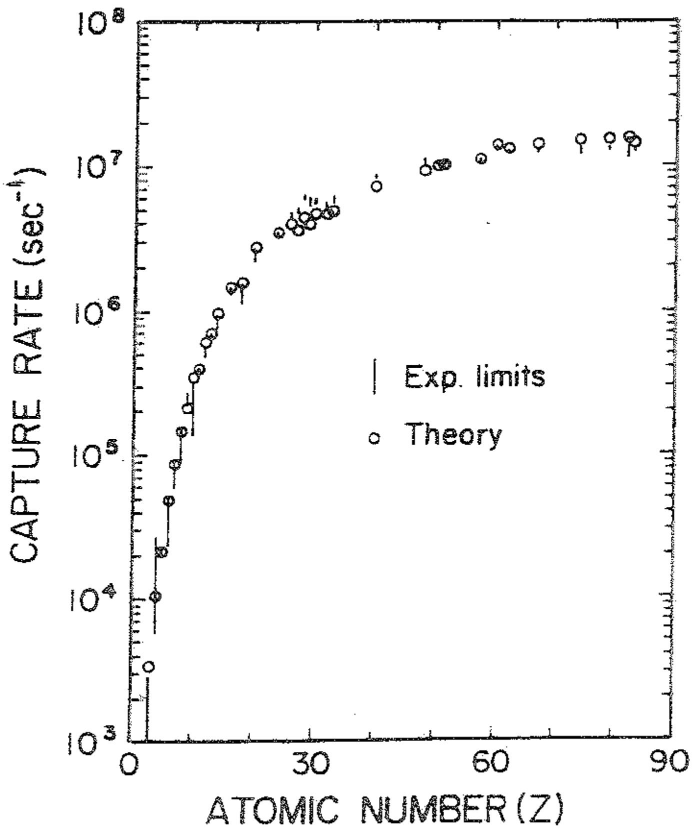


effect of ph propagation



for negative muons captured by the most stable isotopes. Circles are our theoretical xperimental limits from different groups are shown. Data are from ref. [26]).

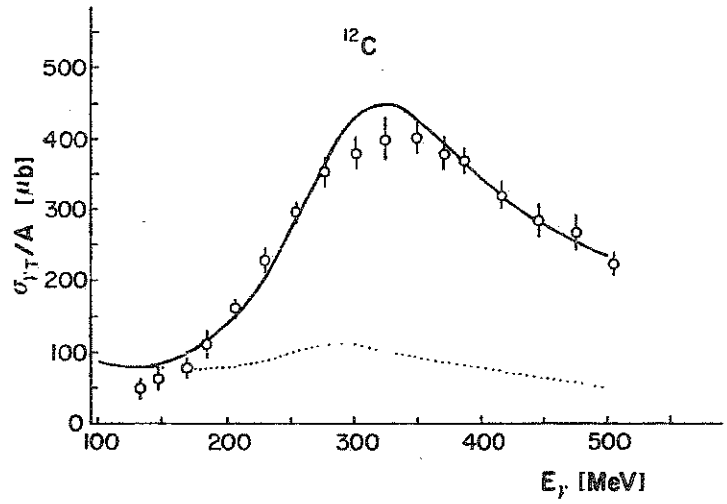


Fig. 45. Results for σ_A/A as a function of the photon energy for ^{12}C. Experiment from ref. [6]). The lower curve is the result for direct photon absorption.

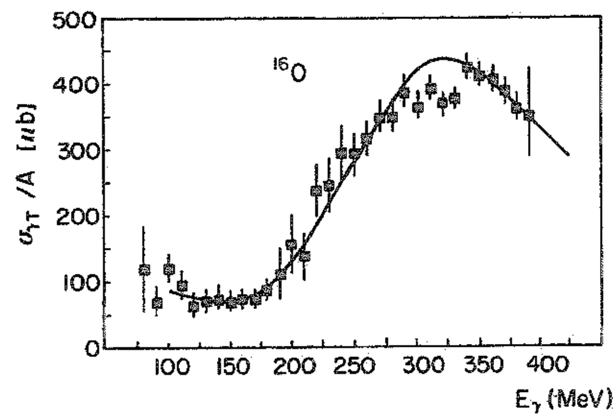


Fig. 46. Results for σ_A/A as a function of the photon energy for ^{16}O. Experiment from ref. [5]).

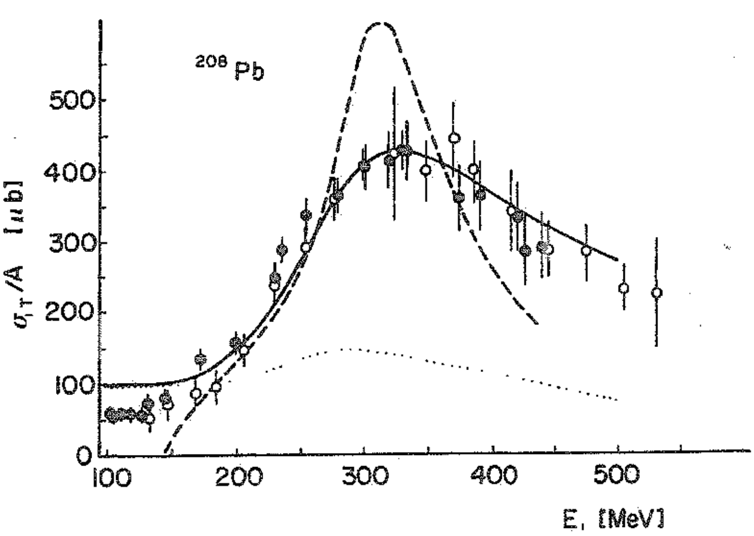


Fig. 47. Continuous line: results for σ_A/A as a function of the photon energy for ^{208}Pb. The dashed line shows the impulse approximation result $(Z\sigma_{\gamma p} + N\sigma_{\gamma n})/A$ for comparison. The dotted line is the result for direct photon absorption. Experimental data: dark dots from ref. [3]), while dots from ref. [6]).