Lepton-nucleon scattering

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IFIC

Outline

- General Introduction/Motivation
- Neutrino-nucleon interactions:
  - Inclusive cross section
  - Quasielastic scattering
  - Resonance excitation

Introduction

- Neutrino interactions/cross sections are important for:
  - Oscillation experiments
    - ν detection, E_ν reconstruction, ν flux calibration, backgrounds
  - Hadronic physics
    - Nucleon and Nucleon-Resonance (N-Δ, N-N*) axial form factors
    - Strangeness content of the nucleon spin
  - Nuclear physics
    - Information about: nuclear correlations, MEC, spectral functions
  - Astrophysics
    - Dynamics of the core-collapse in supernovae
    - r-process nucleosynthesis
  - Beyond Standard Model
    - Non-standard ν interactions

Relevance for oscillation experiments

- (Kinematic) E_ν reconstruction:

$$E_\nu = \frac{2 m_n E_\mu - m_\mu^2 - m_n^2 + m_p^2}{2(m_n - E_\mu + p_\mu \cos\theta_\mu)}$$

  - Important for oscillations: $P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta_{23} \, \sin^2 \frac{\Delta m_{23}^2 L}{4 E_\nu}$

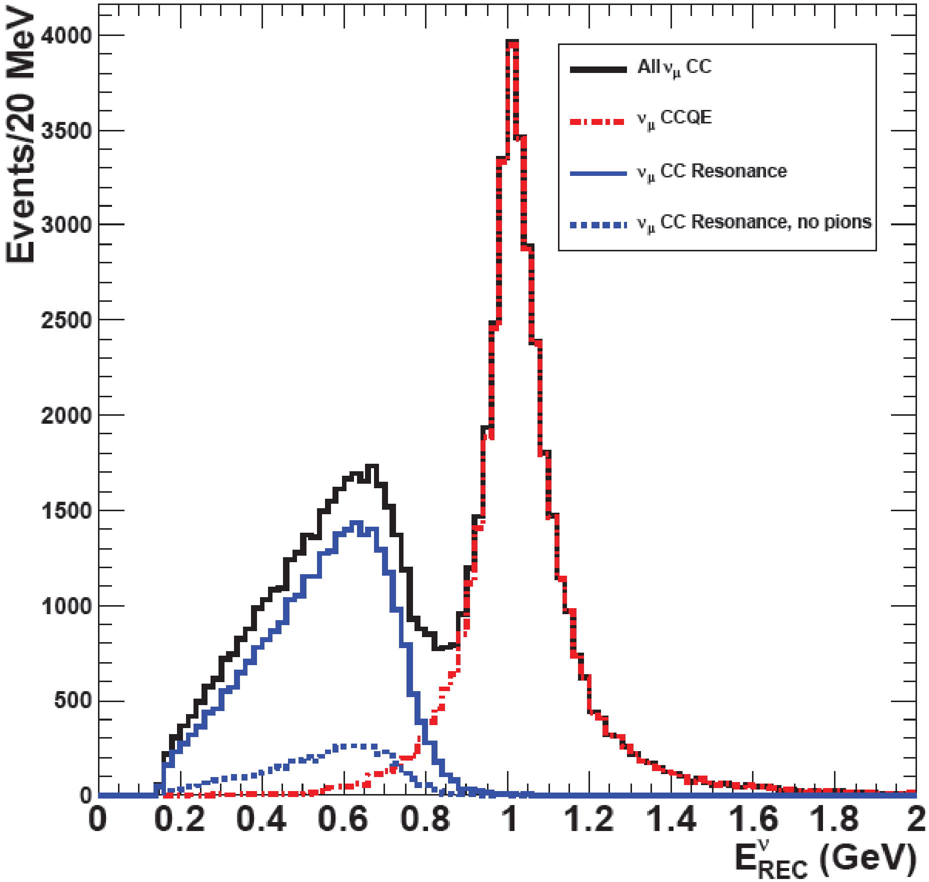
Relevance for oscillation experiments

- (Kinematic) E_ν reconstruction:

$$E_\nu = \frac{2m_n E_\mu - m_\mu^2 - m_n^2 + m_p^2}{2(m_n - E_\mu + p_\mu \cos\theta_\mu)}$$

  - Not exact on nuclear targets
  - CCQE-like events from
    - absorbed pions
    - 2p2h
    - ...

GENIE E_ν = 1 GeV



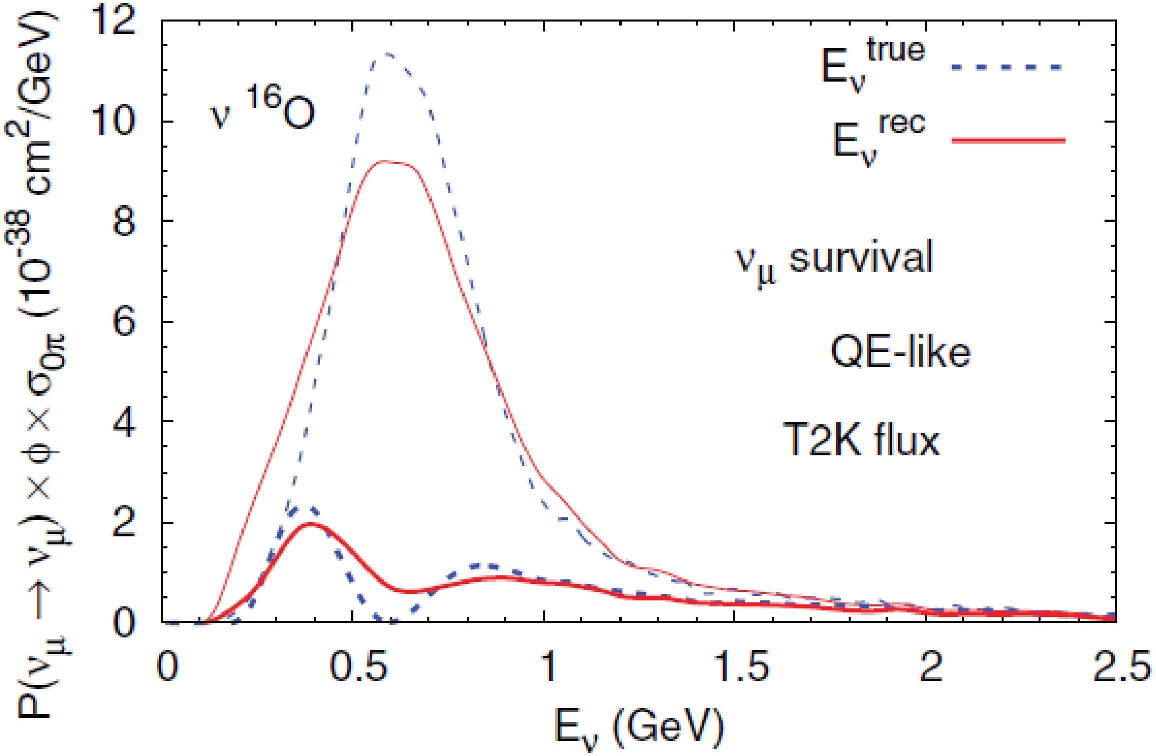
Relevance for oscillation experiments

- (Kinematic) E_ν reconstruction:

$$E_\nu = \frac{2m_n E_\mu - m_\mu^2 - m_n^2 + m_p^2}{2(m_n - E_\mu + p_\mu \cos\theta_\mu)}$$

- At T2K:

Lalakulich et al, PRC86 (2012) 054606



Relevance for oscillation experiments

- (Calorimetric) E_ν reconstruction (e.g MINOS)
  - $E_\nu = E_{lep} + E_{had}$

but

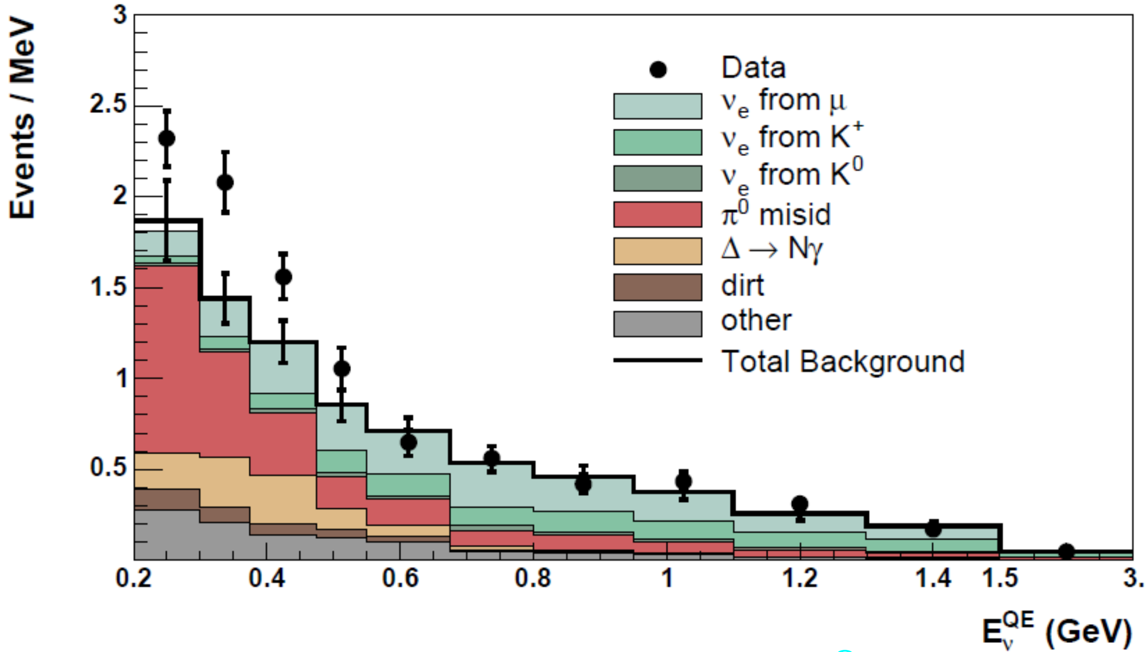
- There are invisible heavy fragments, neutrons or other undetected particles: $E_{vis} < E_{had}$

- E_{vis} -> E_{had} relies on the simulation

Relevance for oscillation experiments

- Backgrounds
  - E.g. in the MiniBooNE $\nu_\mu \to \nu_e$ search

Aguilar-Arevalo et al., PRL102 (2009) 101802



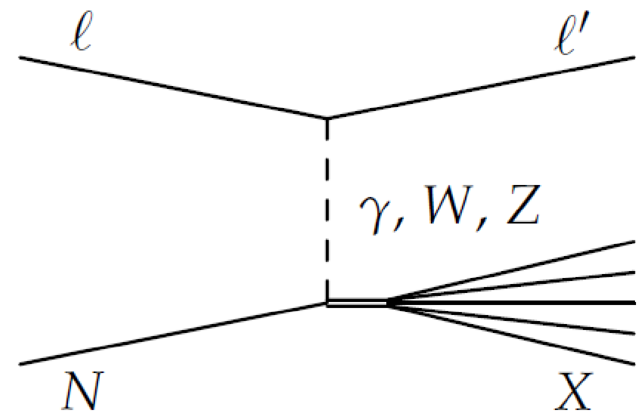
  - NC backgrounds: $\nu_l\, N \to \nu_l\, \pi^0\, N'$

$$\nu_l\, N \to \nu_l\, \gamma\, N'$$

  - Also important for $\nu_\mu \to \nu_e$ measurements at T2K

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



$$k = (k_0, \vec{k}) \qquad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}) \qquad p' = (E', \vec{p'})$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \qquad q^2 = -Q^2 < 0$$

$$\mathcal{L}_{\rm EW} = -eJ^\mu_{em} A_\mu - \frac{g}{2\cos\theta_W} J^\mu_{nc} Z_\mu - \frac{g}{2\sqrt{2}} J^\mu_{cc} W^+_\mu + h.c.$$

$$\sin\theta_W = \frac{e}{g} \qquad \cos\theta_W = \frac{M_W}{M_Z} \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

in the leptonic sector:

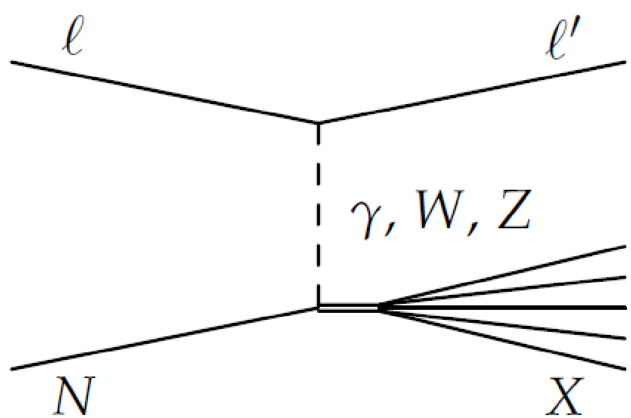
$$J^\mu_{em} = \bar{l}_i \gamma^\mu l_i + \bar{\nu}_i \gamma^\mu \nu_i \qquad i = e, \mu, \tau$$

$$J^\mu_{cc} = \bar{\nu}_i \gamma^\mu (1 - \gamma_5) l_i$$

$$J^\mu_{nc} = \bar{l}_i \gamma^\mu (g_V - g_A \gamma_5) l_i + \frac{1}{2} \bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_i \qquad g_V = -\frac{1}{2} + 2\sin^2\theta_W, \; g_A = -\frac{1}{2}$$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



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In Lab: $p = (M, \vec{0})$

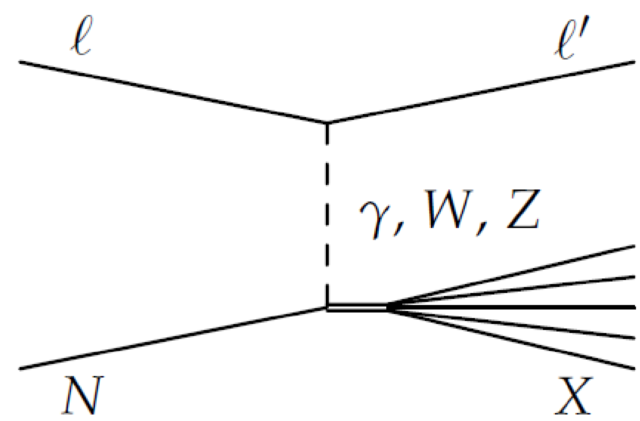
$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} L_{\mu\nu} W^{\mu\nu}$$

$$L_{\mu\nu} = k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta$$

$$W^{\mu\nu} = \frac{1}{2M} \overline{\sum_{\text{polar.}}} \sum_i \left(\int \frac{d^3 p_i}{2E'_i (2\pi)^3} \right) (2\pi)^3 \delta^4(k' + p' - k - p) \langle X | J^\mu | N \rangle \langle X | J^\nu | N \rangle^*$$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



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General structure of the hadronic tensor:

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2}$$
$$+ W_3 i\epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2M^2} + W_6 \frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}$$

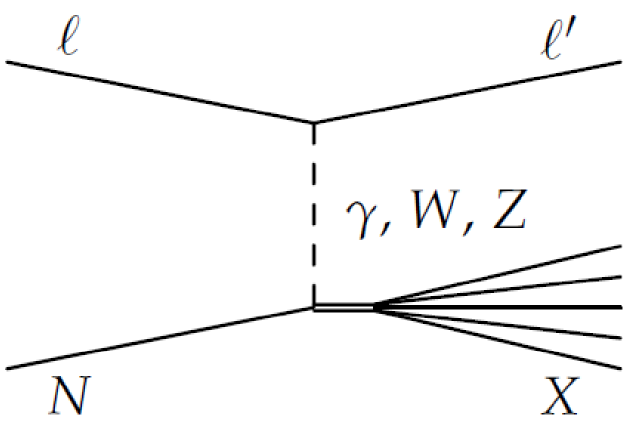
Structure functions: $W_i = W_i(\omega, q^2)$

For EM interactions: $q_\mu J^\mu = 0 \Rightarrow q_\mu W^{\mu\nu}_{em}$

$$W^{\mu\nu}_{em} = W_1 \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) + \frac{W_2}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)$$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



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In Lab: $p = (M, \vec{0})$

$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} \{W_1 2k \cdot k' + W_2 (2k'_0 k_0 - k \cdot k') + 2\frac{m_l^2}{M^2} [W_4 k \cdot k' - W_5 M k_0] + \frac{W_3}{M} [(k_0 + k'_0) k \cdot k' - k_0 m_l^2] \}$$

$m_l \to 0$

$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{2\pi^2} (k'_0)^2 \left[W_1 2 \sin^2 \frac{\theta'}{2} + W_2 \cos^2 \frac{\theta'}{2} \pm W_3 \frac{(k_0 + k'_0)}{M} \sin^2 \frac{\theta'}{2} \right]$$

Inclusive cross section

- Example: EM scattering on a point-like particle

$$\langle X | \, J^\mu \, | N \rangle \to \bar{u}(p')\gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3p'}{2E'} \delta^4(k' + p' - k - p) 4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p \cdot p' - M^2)$$

Using that:

$$\delta(E' + k_0' - M - k_0) = \frac{E'}{M} \delta \left(k_0' - k_0 - \frac{q^2}{2M} \right)$$

$$q^2 = (p' - p)^2 = 2M^2 - 2p \cdot p' \;\Rightarrow\; p \cdot p' - M^2 = -\frac{q^2}{2}$$

$$\frac{p \cdot q}{q^2} = \frac{M\omega}{q^2} = -\frac{1}{2}$$

Inclusive cross section

- Example: EM scattering on a point-like particle

$$\langle X|\, J^\mu \,|N\rangle \to \bar{u}(p')\gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M}\int \frac{d^3p'}{2E'}\delta^4(k'+p'-k-p)4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p\cdot p' - M^2)$$

one finds:

$$\begin{aligned} W_1 &= -\frac{q^2}{4M^2\omega}\delta\left(1+\frac{q^2}{2M\omega}\right) \\ W_2 &= \frac{1}{\omega}\delta\left(1+\frac{q^2}{2M\omega}\right) \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} 2MW_1 &\equiv F_1 = x\delta(1-x) = F_1(x) \\ \omega W_2 &\equiv \delta(1-x) = F_2(x) \end{aligned}$$

$$x = -\frac{q^2}{2M\omega}$$

Inclusive cross section

- Example: EM scattering on a point-like particle

$$\langle X|\, J^\mu \,|N\rangle \rightarrow \bar{u}(p')\gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M}\int \frac{d^3p'}{2E'}\delta^4(k'+p'-k-p)4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p\cdot p' - M^2)$$

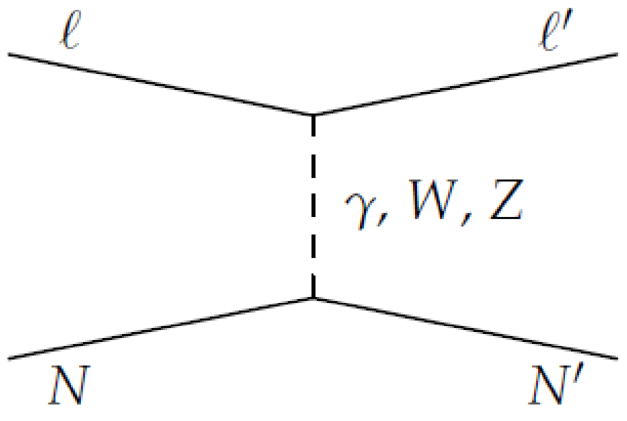
for real nucleons, at low $\vec{q}^{\,2}$

$$W_1 = -\frac{q^2}{4M^2\omega}\delta\left(1+\frac{q^2}{2M\omega}\right) \rightarrow -\frac{q^2}{4M^2\omega}G^2(q^2)\delta\left(1+\frac{q^2}{2M\omega}\right)$$

$$W_2 = \frac{1}{\omega}\delta\left(1+\frac{q^2}{2M\omega}\right) \rightarrow \frac{1}{\omega}G^2(q^2)\delta\left(1+\frac{q^2}{2M\omega}\right)$$

QE scattering on the nucleon

$$\begin{aligned}
\text{EM}: l^{\pm}(k) + N(p) &\rightarrow l^{\pm}(k') + N(p') \\
\text{CC}: \nu(k) + n(p) &\rightarrow l^{-}(k') + p(p') \\
\bar{\nu}(k) + p(p) &\rightarrow l^{+}(k') + n(p') \\
\text{NC}: \nu(k) + N(p) &\rightarrow \nu(k') + N(p') \\
\bar{\nu}(k) + N(p) &\rightarrow \bar{\nu}(k') + N(p')
\end{aligned}$$



$$\langle N' | \, J^\mu \, | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\mathcal{V}^\mu = \bar{u}(p') \left[\gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_S \right] u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p') \left[\gamma^\mu \gamma_5 F_A + \frac{i}{2M} \sigma^{\mu\nu} q_\nu \gamma_5 F_T + \frac{q^\mu}{M} \gamma_5 F_P \right] u(p)$$

- T-inv. $\Rightarrow F_i \in$ Reals
- T-inv. + C-sym. $\Rightarrow F_S = F_T = 0 \Leftrightarrow$ absence of 2nd class currents
- $F_i = F_i\,(q^2) \Leftrightarrow 2\, p \cdot q + q^2 = 0$

QE scattering on the nucleon

$$\mathcal{V}^\mu = \bar{u}(p')\left[\gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2\right]u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p')\left[\gamma^\mu\gamma_5 F_A + \frac{q^\mu}{M}\gamma_5 F_P\right]u(p)$$

- Sachs form factors: $G_E = F_1 + \frac{q^2}{2m_N}F_2$

$$G_M = F_1 + F_2$$

- In the Breit frame: $\vec{p} = -\vec{q}/2,\ \vec{p'} = \vec{q}/2,\ q^2 = -\vec{q}^2$

$$\langle N'_{s'}|\,\mathcal{V}^0\,|N_s\rangle = G_E(\vec{q}^2)\delta_{ss'}$$

$$\langle N'_{s'}|\,\vec{\mathcal{V}}\,|N_s\rangle = G_M(\vec{q}^2)i\chi_{s'}(\vec{\sigma}\times\vec{q})\chi_s$$

$$\langle N'_{s'}|\,\mathcal{A}^0\,|N_s\rangle = 0$$

$$\langle N'_{s'}|\,\vec{\mathcal{A}}\,|N_s\rangle = F_A(\vec{q}^2)(E+M)\left[\vec{\sigma} - \frac{(\vec{\sigma}\cdot\vec{q})\vec{\sigma}(\vec{\sigma}\cdot\vec{q})}{(E+M)^2}\right] + F_P(\vec{q}^2)\vec{q}\frac{(\vec{\sigma}\cdot\vec{q})}{M}$$

QE scattering on the nucleon

- (Flavor) structure of the quark currents:

$$\mathcal{L}_{\mathrm{EW}} = -eJ^\mu_{em}A_\mu - \frac{g}{2\cos\theta_W}J^\mu_{nc}Z_\mu - \frac{g}{2\sqrt{2}}J^\mu_{cc}W^+_\mu + h.c.$$

$$\begin{aligned} J^\mu_{em} &= \frac{2}{3}\bar{q}_u\gamma^\mu q_u - \frac{1}{3}(\bar{q}_d\gamma^\mu q_d + \bar{q}_s\gamma^\mu q_s) \qquad q_\mu J^\mu_{em} = 0 \leftarrow \text{CVC} \\ &= \frac{1}{2}\bar{q}\gamma^\mu\frac{\lambda_8}{\sqrt{3}}q + \bar{q}\gamma^\mu\frac{\lambda_3}{2}q \qquad \leftarrow SU(3)_{\text{flavor}} \\ &= \frac{1}{2}V^\mu_Y + V^\mu_3 \end{aligned}$$

$$J^\mu_{cc} = \bar{q}_u\gamma^\mu(1-\gamma_5)(q_d\cos\theta_C + q_s\sin\theta_C)$$

$$V^\mu_+ = \bar{q}_u\gamma^\mu q_d = \bar{q}_u\gamma^\mu\frac{\lambda_1 + i\lambda_2}{2}q_d = V^\mu_1 + iV^\mu_2$$

QE scattering on the nucleon

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$$V^\mu_+ = \bar{q}_u \gamma^\mu q_d = \bar{q}_u \gamma^\mu \frac{\lambda_1 + i\lambda_2}{2} q_d = V^\mu_1 + i V^\mu_2$$

$V_{1,2,3}$ are components of the same conserved flavor SU(3/2) vector current

$$V_a = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \quad \leftarrow SU(3)_{\rm flavor}$$

$$V_a = \bar{q}\gamma^\mu \frac{\tau_a}{2} q \quad \leftarrow SU(2)_{\rm flavor}$$

QE scattering on the nucleon

- (Flavor) structure of the quark currents:

$$\mathcal{L}_{\rm EW} = -e J^\mu_{em} A_\mu - \frac{g}{2\cos\theta_W} J^\mu_{nc} Z_\mu - \frac{g}{2\sqrt{2}} J^\mu_{cc} W^+_\mu + h.c.$$

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$$V^\mu_+ = \bar{q}_u \gamma^\mu q_d = \bar{q}_u \gamma^\mu \frac{\lambda_1 + i\lambda_2}{2} q_d = V^\mu_1 + i V^\mu_2$$

$$J^\mu_{nc} = \bar{q}_u \gamma^\mu \left[\frac{1}{2} - \left(\frac{2}{3}\right) 2\sin^2\theta_W - \frac{1}{2}\gamma_5\right] q_u + \bar{q}_d \gamma^\mu \left[-\frac{1}{2} - \left(-\frac{1}{3}\right) 2\sin^2\theta_W + \frac{1}{2}\gamma_5\right] q_d + (d \to s)$$

$$V^\mu_{nc} = (1 - 2\sin^2\theta_W) V^\mu_3 - 2\sin^2\theta_W \frac{1}{2} V^\mu_Y - \frac{1}{2}\bar{q}_s \gamma^\mu q_s$$

QE scattering on the nucleon

- Vector and EM form factors:

$\vec{V}^\alpha = \mathcal{V}^\alpha \frac{\vec{\tau}}{2} \leftarrow$ isovector current $\quad V_Y^\alpha = \mathcal{V}_Y^\alpha I \leftarrow$ hypercharge (isoscalar) current

$$\langle p | V_{\mathrm{EM}}^\alpha | p \rangle = \langle p | V_3^\alpha + \frac{1}{2} V_Y^\alpha | p \rangle = \frac{\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_p^\alpha$$

$$\langle n | V_{\mathrm{EM}}^\alpha | n \rangle = \langle n | V_3^\alpha + \frac{1}{2} V_Y^\alpha | n \rangle = \frac{-\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_n^\alpha$$

Then: $\langle p | V_{\mathrm{CC}}^\alpha | n \rangle = \langle p | V_1^\alpha + i V_2^\alpha | n \rangle = \mathcal{V}^\alpha = \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha$

$$\begin{aligned} \langle p | V_{\mathrm{NC}}^\alpha | p \rangle &= \langle p | (1 - 2\sin^2\theta_W) V_3^\alpha - \sin^2\theta_W V_Y^\alpha | p \rangle \\ &= \left(\frac{1}{2} - \sin^2\theta_W \right) \mathcal{V}^\alpha + \sin^2\theta_W \mathcal{V}_Y^\alpha \\ &= \left(\frac{1}{2} - 2\sin^2\theta_W \right) \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha \end{aligned}$$

- Vector CC and NC form factors can be expressed in terms of EM ones

QE scattering on the nucleon

- (Flavor) structure of the quark currents:

$$\mathcal{L}_{\mathrm{EW}} = -eJ^{\mu}_{em}A_{\mu} - \frac{g}{2\cos\theta_W}J^{\mu}_{nc}Z_{\mu} - \frac{g}{2\sqrt{2}}J^{\mu}_{cc}W^{+}_{\mu} + h.c.$$

$$J^{\mu}_{cc} = \bar{q}_u\gamma^{\mu}(1-\gamma_5)(q_d\cos\theta_C + q_s\sin\theta_C)$$

$$A^{\mu}_{+} = \bar{q}_u\gamma^{\mu}\gamma_5 q_d = \bar{q}_u\gamma^{\mu}\gamma_5\frac{\lambda_1 + i\lambda_2}{2}q_d = A^{\mu}_1 + iA^{\mu}_2$$

$$J^{\mu}_{nc} = \bar{q}_u\gamma^{\mu}\left[\frac{1}{2} - \left(\frac{2}{3}\right)2\sin^2\theta_W - \frac{1}{2}\gamma_5\right]q_u + \bar{q}_d\gamma^{\mu}\left[-\frac{1}{2} - \left(-\frac{1}{3}\right)2\sin^2\theta_W + \frac{1}{2}\gamma_5\right]q_d + (d\to s)$$

$$A^{\mu}_{nc} = A^{\mu}_3 + \frac{1}{2}\bar{q}_s\gamma^{\mu}\gamma_5 q_s$$

$A_{1,2,3}$ are components of the same partially conserved flavor SU(3/2) axial current

$$A_a = \bar{q}\gamma^{\mu}\gamma_5\frac{\lambda_a}{2}q \leftarrow SU(3)_{\mathrm{flavor}}$$

$$A_a = \bar{q}\gamma^{\mu}\gamma_5\frac{\tau_a}{2}q \leftarrow SU(2)_{\mathrm{flavor}}$$

QE scattering on the nucleon

- (Flavor) structure of the quark currents:

$$A_a = \bar{q}\gamma^\mu\gamma_5\frac{\lambda_a}{2}q$$

- PCAC: $\partial_\mu A_a^\mu = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q \qquad m = diag(m_u, m_d, m_s)$

- The axial current in conserved in the chiral (m $\to$ 0) limit
- In terms of (effective) hadronic degrees of freedom

$$A_a^\mu = -f_\pi\partial^\mu\pi_a + ... \Leftrightarrow \partial_\mu A_a^\mu = f_\pi m_\pi^2\pi_a + ...$$

- Consequences:

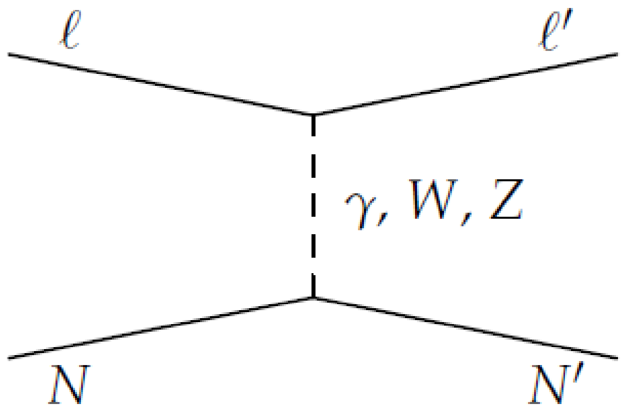
$$F_P(Q^2) = \frac{2M^2}{Q^2+m_\pi^2}F_A(Q^2)$$

$F_A(0) \equiv g_A = 2g_{NN\pi}$ $\leftarrow$ Goldberger-Treiman relation

$$\mathcal{L}_{NN\pi} = -\frac{g_{NN\pi}}{f_\pi}\bar{N}\gamma_\mu\gamma_5(\partial^\mu\vec{\pi})\vec{\tau}N$$

QE scattering on the nucleon

- Cross section:



$$\frac{d\sigma}{dk_0' d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} L_{\mu\nu} W^{\mu\nu}$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3p'}{2E'} \delta^4(k' + p' - k - p) H^{\nu\mu}$$

$$H^{\alpha\beta} = \mathrm{Tr}\left[(\not{p} + M)\, \gamma^0 \,(\Gamma^\alpha)^\dagger\, \gamma^0 \,(\not{p}' + M)\, \Gamma^\beta \right]$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 - \gamma^\mu \gamma_5 F_A - \frac{q^\mu}{M} \gamma_5 F_P$$

QE scattering on the nucleon

- Cross section:
  - As an expansion in small variables $q^2, m_l^2 \ll M^2, E_\nu^2$

$$\frac{d\sigma}{dq^2} = \frac{1}{2\pi} G^2 c_{\mathrm{EW}}^2 \left[R - \frac{m_l^2}{4E_\nu^2} S + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

  - CC: $c_{\mathrm{CC}} = \cos\theta_C$

$$\begin{aligned} R_{\mathrm{CC}} &= 1 + g_A^2 \\ S_{\mathrm{CC}} &= \frac{2E_\nu + M}{M} + g_A^2 \frac{2E_\nu - M}{M} \\ T_{\mathrm{CC}} &= 1 - g_A^2 + 2\frac{E_\nu}{M}(1 \mp g_A)^2 \mp 4\frac{E_\nu}{M} g_A \kappa^{\mathrm{V}} - \left(\frac{E_\nu}{M}\kappa^{\mathrm{V}}\right)^2 \\ &+ 4E_\nu^2 \left[\frac{1}{3}\left(\langle r_p^2 \rangle - \langle r_n^2 \rangle + g_A^2 \langle r_A^2 \rangle\right) - \frac{1}{2M^2}\kappa^{\mathrm{V}} \right] \end{aligned}$$

$$\kappa^{\mathrm{V}} = \mu_p - \mu_n - 1$$

QE scattering on the nucleon

- Cross section:
  - As an expansion in small variables $q^2, m_l^2 \ll M^2, E_\nu^2$

$$\frac{d\sigma}{dq^2} = \frac{1}{2\pi} G^2 c_{\mathrm{EW}}^2 \left[R - \frac{m_l^2}{4E_\nu^2} S + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

  - CC: $c_{\mathrm{CC}} = \cos\theta_C$
  - Large fraction of the CCQE cross section depends on a small number of nucleon properties
    - Charges, magnetic moments, mean squared radii

$$\langle r_p^2 \rangle = \frac{6}{G_E^{(p)}(0)} \frac{dG_E^{(p)}(q^2)}{dq^2}\bigg|_{q^2=0}, \qquad \langle r_n^2 \rangle = 6 \frac{dG_E^{(n)}(q^2)}{dq^2}\bigg|_{q^2=0}$$

    - axial coupling and axial radius

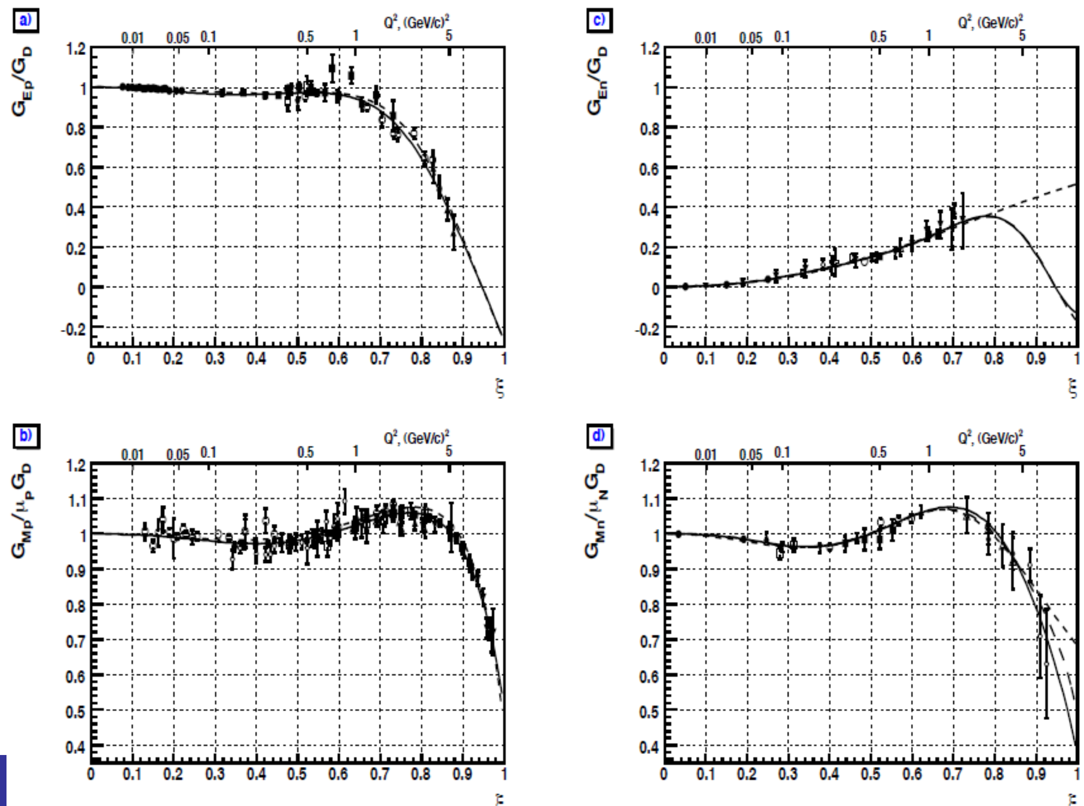
$$\langle r_A^2 \rangle = \frac{6}{F_A(0)} \frac{dF_A(q^2)}{dq^2}\bigg|_{q^2=0}$$

ν QE scattering on the nucleon

- Measurement of the axial radius:
  - CCQE on H and D (BNL, ANL)

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{M_A^2}\right)^{-2} \qquad \langle r_A^2 \rangle = \frac{12}{{M_A}^2}$$

  - M_A= 1.016 $\pm$ 0.026 GeV   Bodek et al., EPJC 53 (2008)
  - Using:



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    - M_A= 1.016 $\pm$ 0.026 GeV   Bodek et al., EPJC 53 (2008)

  - From π electroproduction on p:

$$6 \left. \frac{dE_{0+}^{(-)}}{dq^2} \right|_{q^2=0} = \langle r_A^2 \rangle + \frac{3}{M}\left(\kappa^{\mathrm{V}} + \frac{1}{2}\right) + \frac{3}{64 f_\pi^2}\left(1 - \frac{12}{\pi^2}\right)$$

    - M_A= 1.014 $\pm$ 0.016 GeV   Liesenfeld et al., PLB 468 (1999) 20

QE scattering on the nucleon

- Cross section:
  - As an expansion in small variables $q^2, m_l^2 \ll M^2, E_\nu^2$

$$\frac{d\sigma}{dq^2} = \frac{1}{2\pi} G^2 c_{\rm EW}^2 \left[R - \frac{m_l^2}{4E_\nu^2} S + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

  - NC: $c_{\rm NC} = 1/4$

$$R_{\rm NC}^{(p)} = \alpha_{\rm V}^2 + (g_A - \Delta s)^2$$

$$T_{\rm NC}^{(p)} = \alpha_{\rm V}^2 - (g_A - \Delta s)^2 + 2\frac{E_\nu}{M}\left[\alpha_{\rm V} \mp (g_A - \Delta s)\right]^2 \mp 4\frac{E_\nu}{M}(g_A - \Delta s)\kappa_{\rm NC}^{(p)} - \left(\frac{E_\nu}{M}\kappa_{\rm NC}^{(p)}\right)^2$$
$$+ 4E_\nu^2 \left\{ \alpha_{\rm V} \left[\frac{1}{3}\left(\alpha_{\rm V}\langle r_p^2\rangle - \langle r_n^2\rangle - \langle r_s^2\rangle\right) - \frac{1}{2M^2}\kappa_{\rm NC}^{(p)} \right] + \frac{1}{3}(g_A - \Delta s)\left(g_A\langle r_A^2\rangle - \Delta s\langle r_{As}^2\rangle\right) \right\}$$

$$R_{\rm NC}^{(n)} = 1 + (g_A + \Delta s)^2$$

$$T_{\rm NC}^{(n)} = 1 - (g_A + \Delta s)^2 + 2\frac{E_\nu}{M}\left[1 \mp (g_A + \Delta s)\right]^2 \pm 4\frac{E_\nu}{M}(g_A + \Delta s)\kappa_{\rm NC}^{(n)} - \left(\frac{E_\nu}{M}\kappa_{\rm NC}^{(n)}\right)^2$$
$$+ 4E_\nu^2 \left\{ -\frac{1}{3}\left(\alpha_{\rm V}\langle r_n^2\rangle - \langle r_p^2\rangle - \langle r_s^2\rangle\right) + \frac{1}{2M^2}\kappa_{\rm NC}^{(n)} + \frac{1}{3}(g_A + \Delta s)\left(g_A\langle r_A^2\rangle + \Delta s\langle r_{As}^2\rangle\right) \right\}$$

$$\kappa_{\rm NC}^{(p)} = \alpha_{\rm V}(\mu_p - 1) - \mu_n - \mu_s \qquad \kappa_{\rm NC}^{(n)} = 1 - \mu_p + \alpha_{\rm V}\mu_n - \mu_s \qquad \alpha_{\rm V} = 1 - 4\sin^2\theta_W$$

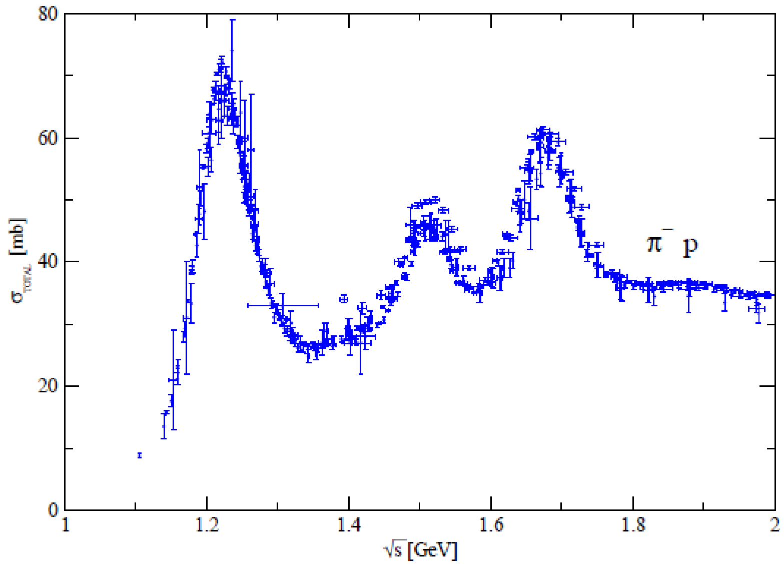
QE scattering on the nucleon

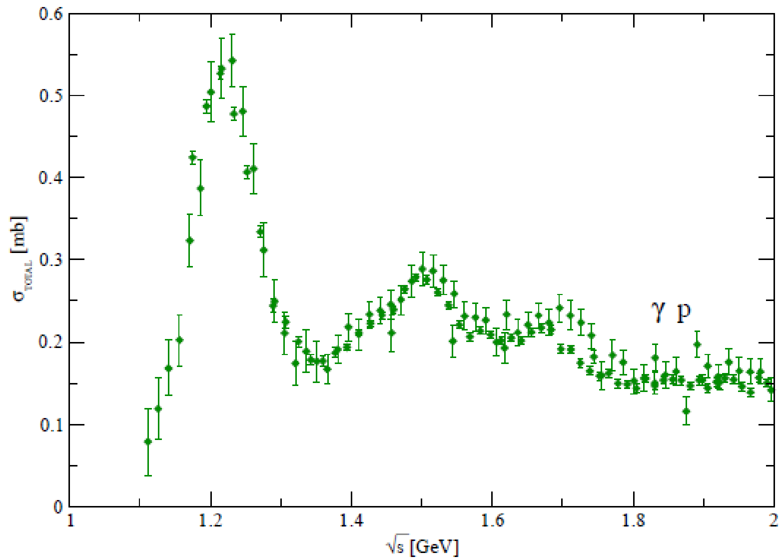
- Strangeness content of the nucleon:
  - $\langle r_s^2 \rangle, \; \mu_s, \; \langle r_{As}^2 \rangle$ $\leftarrow$ insignificant
  - Δs strange axial coupling $\Leftrightarrow$ strange quark contribution to the spin

$$\left. \frac{d\sigma_{\rm NC}^{(p)}/dq^2}{d\sigma_{\rm NC}^{(n)}/dq^2} \right|_{q^2=0} = \frac{\alpha_{\rm V}^2 + (g_A - \Delta s)^2}{1 + (g_A + \Delta s)^2} \approx \frac{(g_A - \Delta s)^2}{1 + (g_A + \Delta s)^2} \approx \begin{cases} 0.62 & \text{if } \Delta s = 0 \\ 1.27 & \text{if } \Delta s = -0.3 \end{cases}$$

  - A recent global fit: Pate, Trujillo, arXiv:1308.5694

Baryon resonances





From PDG database

Baryon resonances

- Resonance properties
  - Originally from π N $\to$ π N
    - Quantum numbers
    - Breit-Wigner mass, width, branching ratios
    - Cleaner: pole position and residues
  - γ N $\to$ π N, γ^* N $\to$ π N
    - Electromagnetic properties: helicity amplitudes

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 1/2 \left| \epsilon^+_\mu J^\mu_{\mathrm{EM}} \right| N, J_z = -1/2 \right\rangle \zeta$$

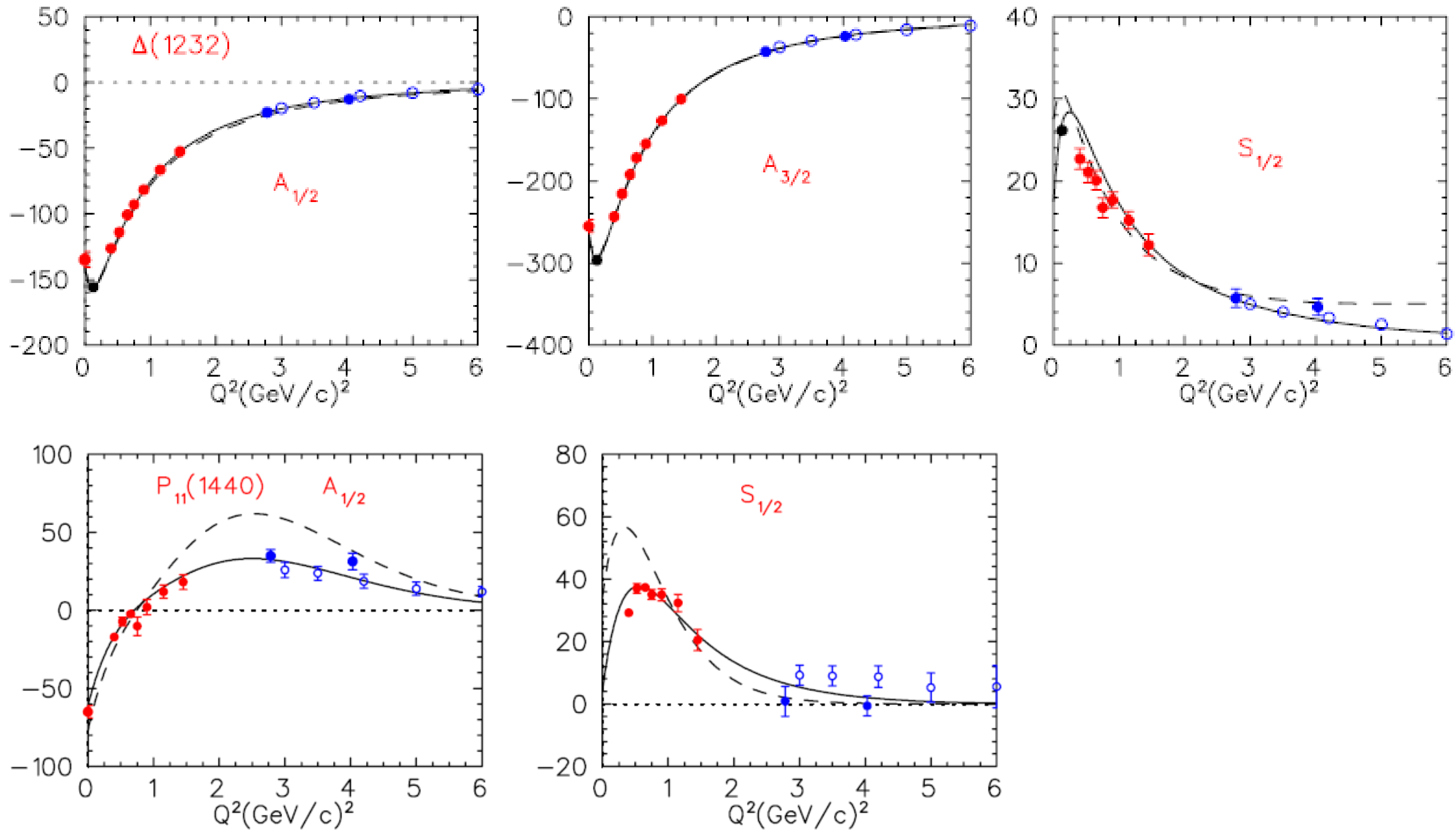
$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 3/2 \left| \epsilon^+_\mu J^\mu_{\mathrm{EM}} \right| N, J_z = 1/2 \right\rangle \zeta$$

$$S_{1/2} = -\sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\mathbf{q}|}{\sqrt{Q^2}} \left\langle R, J_z = 1/2 \left| \epsilon^0_\mu J^\mu_{\mathrm{EM}} \right| N, J_z = 1/2 \right\rangle \zeta$$

MAID

- Transition N-R e.m. helicity amplitudes extracted for all 4-star resonaces with W < 1.8 GeV
- For example:

Tiator et al., EPJ Special Topics 198 (2011)



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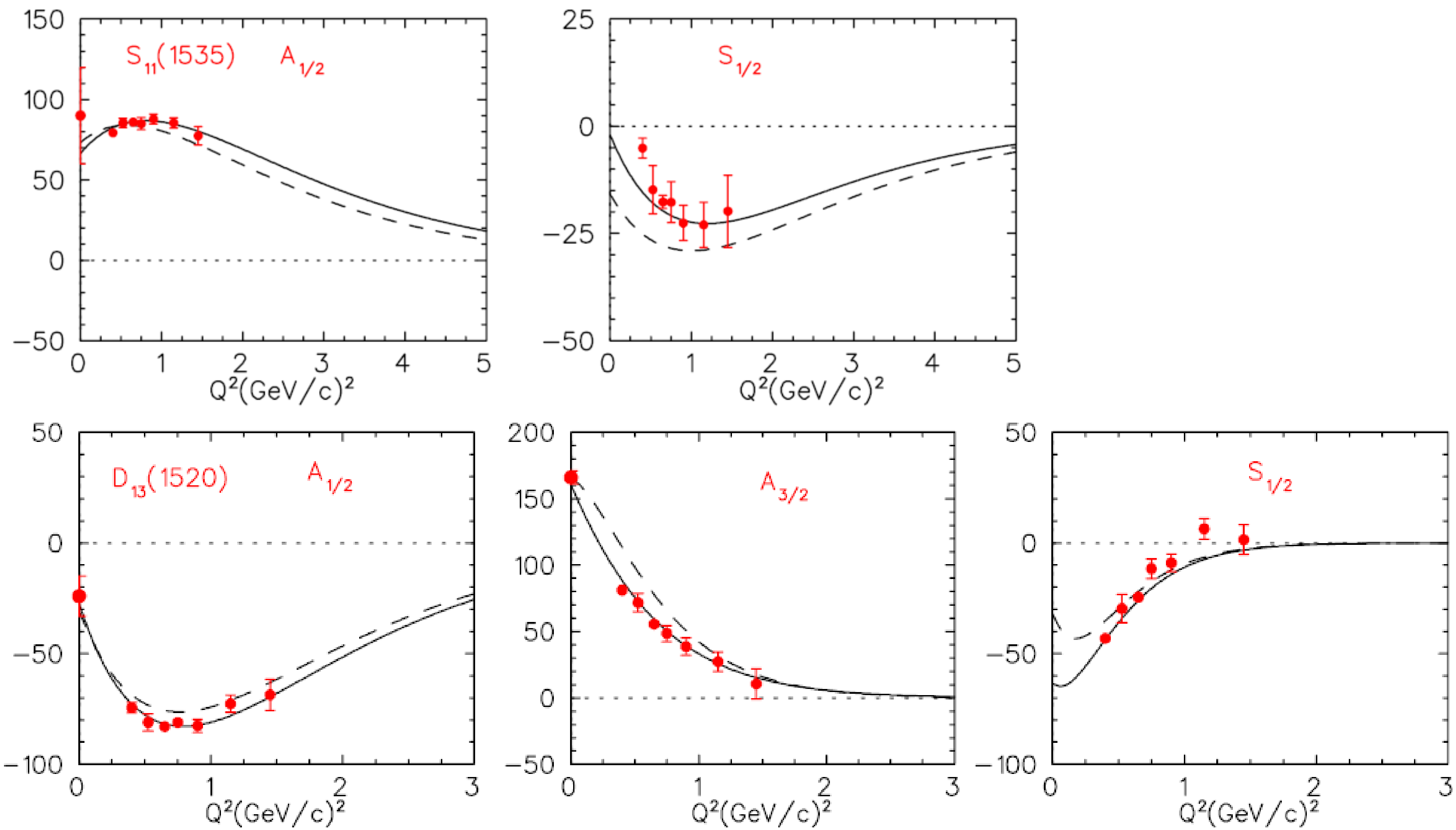

Weak Resonance excitation

- Resonances contribute to:
  - the inclusive $\nu_l \, N \to l \, X$ cross section
  - several exclusive channels: $\nu_l \, N \to l \, N' \, \pi$

    $\nu_l \, N \to l \, N' \, \gamma$

    $\nu_l \, N \to l \, N' \, \eta$

    $\nu_l \, N \to l \, \Lambda(\Sigma) \, \bar{K}$
  - At $E_\nu \sim 1$ GeV (MiniBooNE, SciBooNE, T2K,...) Δ(1232) is dominant

But

  - At E_ν>1 GeV (MINERνA) N* become important

Weak Resonance excitation

- CC N* excitation: $\nu_l(k)\, N(p) \rightarrow l^-(k')\, N^*(p')$

$$\frac{d\sigma}{dk_0' d\Omega'} = \frac{1}{32\pi^2} \frac{|\vec{k'}|}{k_0 M_N} \mathcal{A}(p') |\bar{\mathcal{M}}|^2$$

← Inclusive cross section

$$\mathcal{A}(p') = \frac{M^*}{\pi} \frac{\Gamma(p')}{(p'^2 - M^{*2})^2 + M^{*2}\Gamma^2(p')}$$

$\Gamma(p')$ ← total momentum dependent width

$$\mathcal{M} = \frac{G_F \cos\theta_C}{\sqrt{2}} l^\alpha J_\alpha$$

$$l^\alpha = \bar{u}(k')\gamma^\alpha(1-\gamma_5)u(k)$$

← leptonic current

$$J_\alpha = V_\alpha - A_\alpha$$

← hadronic current
can be parametrized in terms of
N-N* transition form factors

Weak Resonance excitation

- Second resonance peak: N*(1440), N*(1520), N*(1535)
  - N*(1440) $J^P=1/2^+$

$$J_\alpha = \bar{u}(p')\left[\frac{F_1^V}{(2M_N)^2}(\not{q}q_\alpha - q^2\gamma_\alpha) + i\frac{F_2^V}{2M_N}\sigma_{\alpha\beta}q^\beta - F_A\gamma_\alpha\gamma_5 - \frac{F_P}{M_N}\gamma_5 q_\alpha\right]u(p)$$

  - N*(1535) $J^P=1/2^-$

$$J_\alpha = \bar{u}(p')\left[\frac{F_1^V}{(2M_N)^2}(\not{q}q_\alpha - q^2\gamma_\alpha)\gamma_5 + i\frac{F_2^V}{2M_N}\sigma_{\alpha\beta}q^\beta\gamma_5 - F_A\gamma_\alpha - \frac{F_P}{M_N}q_\alpha\right]u(p)$$

  - N*(1520) $J^P=3/2^-$

$$J_\alpha = \bar{u}^\mu(p')\left[\frac{C_3^V}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^V}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2}(g_{\alpha\mu}q\cdot p - q_\alpha p_\mu)\right.$$
$$\left. + \left(\frac{C_3^A}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^A}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2}q_\alpha q_\mu\right)\gamma_5\right]u(p)$$

Weak Resonance excitation

- Vector CC and NC form factors can be expressed in terms of EM ones
  - CC: $F^V_{1,2} = F^p_{1,2} - F^n_{1,2}$
  - NC: $\tilde{F}^{p(n)}_{1,2} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F^{p(n)}_{1,2} - F^{n(p)}_{1,2}$
  - The same applies for $C^V_{1,2,3}$
- Helicity amplitudes from π photo- and electro-production data

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 1/2 \left| \epsilon^+_\mu J^\mu_{\mathrm{EM}} \right| N, J_z = -1/2 \right\rangle \zeta$$

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- Helicity amplitudes $\Rightarrow$ EM form factors

Weak Resonance excitation

- Axial transition form factors
  - Poorly known (if at all...)
  - PCAC: $q^\alpha A_\alpha \approx 0$
  - π-pole dominance of the pseudoscalar form factor: F_P, C_6^A

- N*(1440) J^P=1/2$^+$

PCAC $\Rightarrow$ $$F_P = -\frac{(M^* + M_N)M_N}{q^2 - m_\pi^2} F_A$$

Using $$\mathcal{L}_{N^*N\pi} = -\frac{g_{N^*N\pi}}{f_\pi} \bar{N}^* \gamma_\mu \gamma_5 (\partial^\mu \vec{\pi}) \vec{\tau} N$$

$g_{N^*N\pi} \Leftrightarrow \Gamma(N^* \to N\pi)$

$f_\pi \leftarrow \pi$ decay constant

π-pole dominance $\Rightarrow$ $$F_P = -2 g_{N^*N\pi} F(q^2) \frac{(M^* + M_N)M_N}{q^2 - m_\pi^2} \qquad F(0) = 1$$

Therefore $F_A(0) = 2 g_{N^*N\pi}$ $\leftarrow$ Goldberger-Treiman relation

Educated guess: $$F_A(q^2) = F_A(0) \left(1 - \frac{q^2}{{M_A}^2}\right)^{-2} \qquad M_A = 1 \text{ GeV}$$

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  - π-pole dominance of the pseudoscalar form factor: F_P, C_6^A

- N*(1520) $J^P=3/2^-$

PCAC $\Rightarrow$ $C_6^A = -\frac{M_N^2}{q^2 - m_\pi^2} C_5^A$

Using $\mathcal{L}_{N^* N\pi} = -\frac{g_{N^* N\pi}}{f_\pi} \bar{N}^*_\mu \gamma_5 (\partial^\mu \vec{\pi}) \vec{\tau} N$

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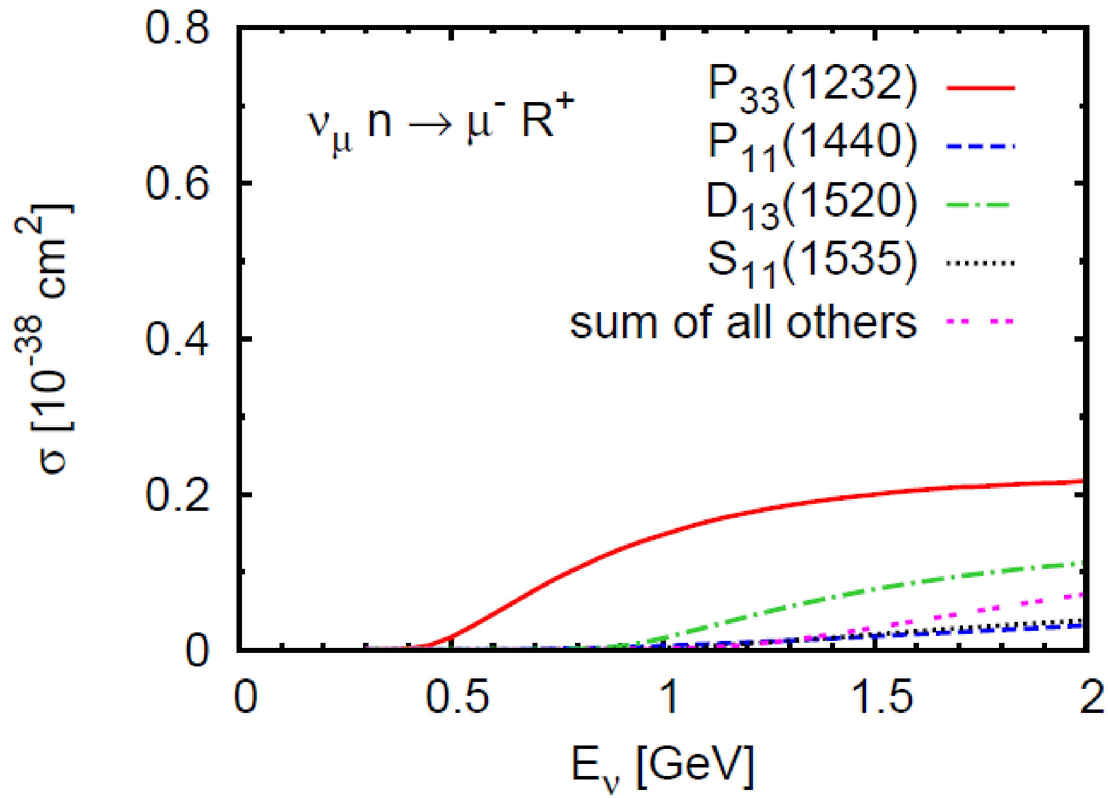
Therefore $C_5^A(0) = -2 g_{N^* N\pi}$ $\leftarrow$ Goldberger-Treiman relation

Educated guess: $C_5^A(q^2) = C_5^A(0) \left(1 - \frac{q^2}{{M_A}^2}\right)^{-2}$ $M_A = 1\ \mathrm{GeV}$ $\quad C_3^4 = C_4^A = 0$

Inclusive resonance production



T. Leitner, O. Buss, LAR, U. Mosel, PRC 79 (2009)
T. Leitner, PhD Thesis, 2009

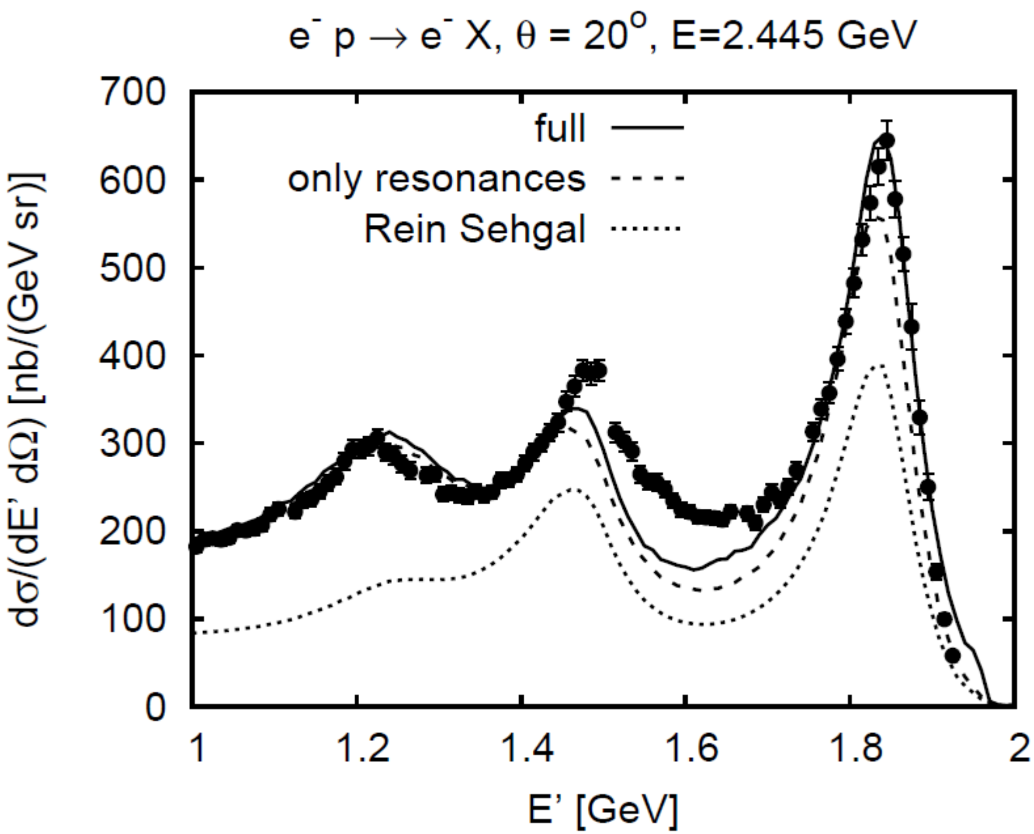
- At E_ν = 2 GeV, CCN*(1520)/CCΔ $\sim$ 0.5, CCN*(1440,1535)/CCΔ $\sim$ 0.22
- N*(1520) is important for $\nu_l \, N \to l \, N' \, \pi$

Resonances in ν generators

- Rein-Sehgal model: Rein, Sehgal, Ann. Phys. 133 (1981) 79.
  - Used by almost all MC generators
  - Relativistic quark model of Feynman-Kislinger-Ravndal with SU(6) spin-flavor symmetry
  - Helicity amplitudes for 18 baryon resonances
  - Lepton mass = 0
    - Corrections: Kuzmin et al., Mod. Phys. Lett. A19 (2004)
      Berger, Sehgal, PRD 76 (2007)
      Graczyk, Sobczyk, PRD 77 (2008)
    - Poor description of π electroproduction data on p

Resonances in ν generators

- Rein-Sehgal model: Rein, Sehgal, Ann. Phys. 133 (1981) 79.
  - Us
  - Re ... avndal with SU(6) sp
  - He
  - Le ... Leitner et al., POS NUFACT08
    - ... 2004)
    - ...)



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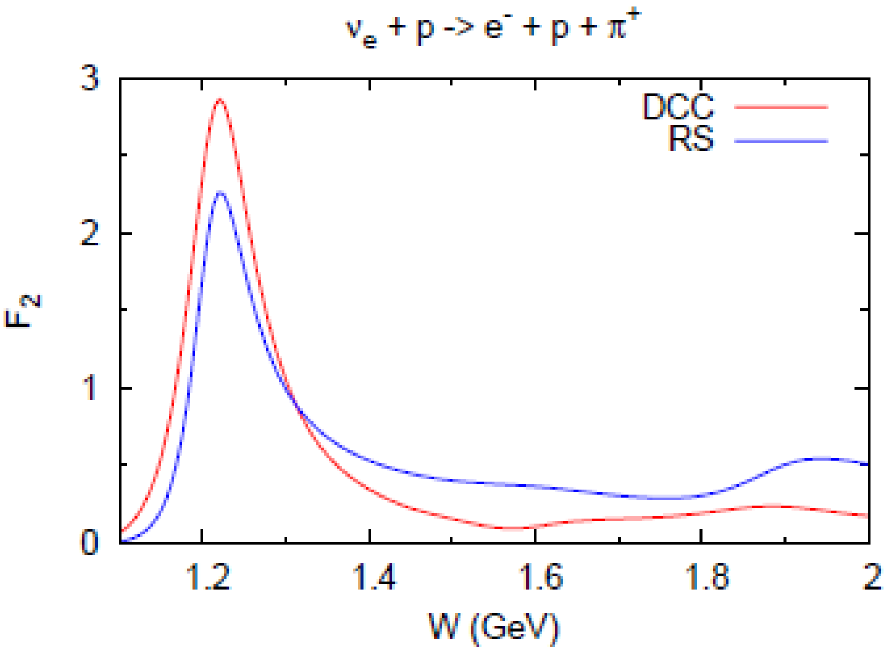

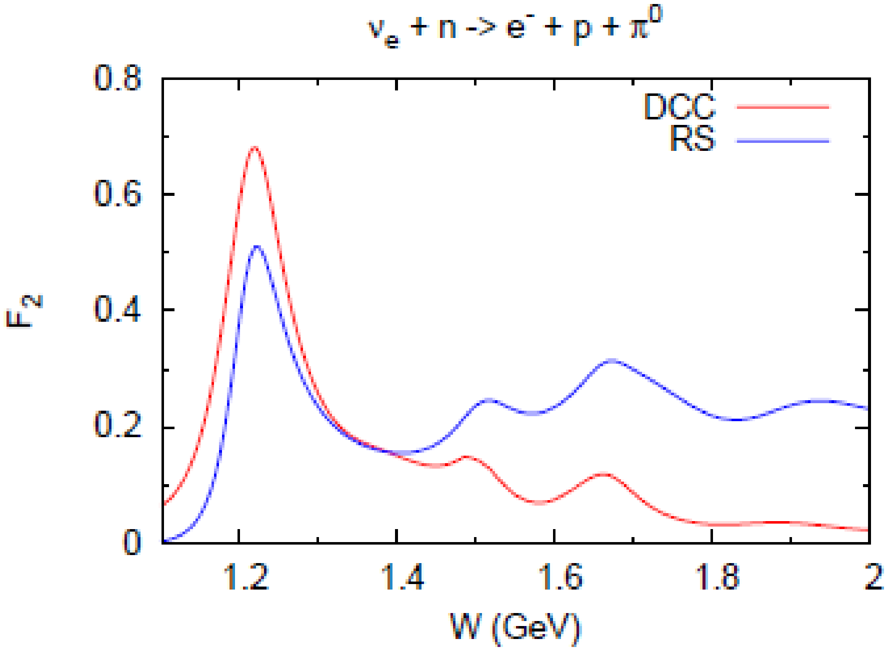

- Also unsatisfactory in the axial sector: Kamano et al., PRD86 (2012)

PCAC (at $Q^2 \to 0$): $\pi N \to X \Leftrightarrow F_2$

Summary

- A proper understanding of ν-nucleon interactions is the first step towards a realistic description of ν-nucleus scattering
- Inelastic scattering in the few-GeV region dominated by baryon resonance excitation
- Available experimental information from hadronic and electromagnetic reactions can be used to constrain ν-nucleon interactions
- New measurements on H and D targets are highly desirable

- Bibliography:

  - A. W. Thomas, W. Weise, The structure of the nucleon
  - F. Halzen, A. D. Martin, Quarks and leptons
  - J. D. Walecka, Electron Scattering for Nuclear and Nucleon Structure
  - T. J. Leitner, Neutrino-Nucleus interaction in a coupled-channel hadronic transport model (PhD thesis)
  - LAR, Y. Hayato, J. Nieves, Progress and open questions in the physics of neutrino cross sections, arXiv:1403.2673, New J. Phys.

NuSTEC School

- Fermilab, October 18-27, 2014
- Topics:
  - Electroweak interactions on the nucleon
  - Strong and electroweak interactions in nuclei
  - The nuclear physics of electron and neutrino scattering in nuclei in the quasielastic regime and beyond
  - Pion production on the nucleon
  - Description of exclusive channels and final state interactions
  - Inclusive electron and neutrino scattering in the deep inelastic regime
  - Impact of uncertainties in neutrino cross sections
  - Selected experimental illustrations