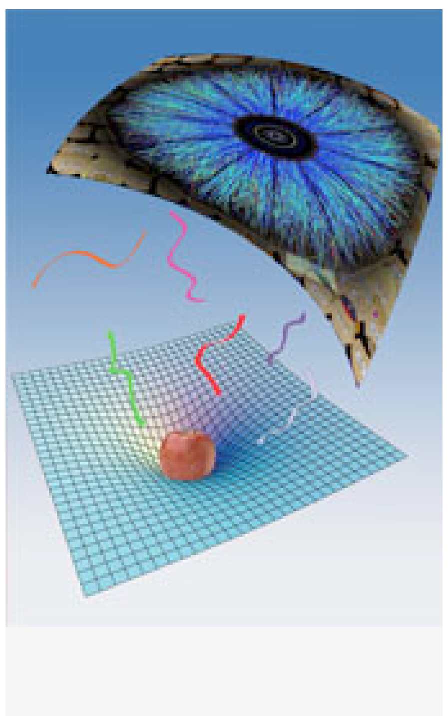
The Gauge/Gravity correspondence:
linking General Relativity and Quantum Field theory



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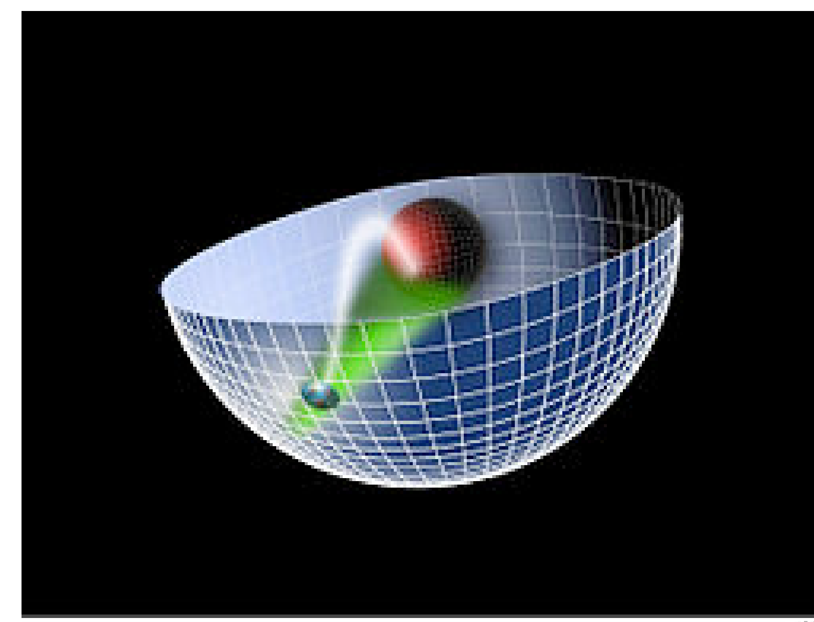


IFIC, Valencia, April 11, 2014

Main result: a duality relating QFT and gravity

Quantum Physics of strongly correlated many- body systems ⟷ Classical dynamics of black holes in one higher dimension

complicated — simple

Also known as:

- Holographic duality
- AdS/CFT correspondence

CFT: conformal field theory

AdS: anti-de Sitter space

In the spirit of condensed matter physics :

At strong coupling new weakly-coupled degrees of freedom emerge dynamically

New feature:

The emergent fields live in a space with one extra dimension → holography

The extra dimension is related to the energy scale

The duality was obtained in the context of string theory

It has applications in:

- Strong coupling dynamics of gauge theories (QCD, integrability in QFT, electroweak symmetry breaking and LHC physics, string phenomenology,...)
- Condensed matter physics (holographic superconductors, quantum phase transitions, cold atoms, topological insulators,...)
- Black hole physics and quantum gravity
- Entanglement and quantum information theory
- Relativistic hydrodynamics

⋮

Here, we will concentrate on some concrete topics

Outline:

- Motivation & formulation of the correspondence
- String theory basics
- Applications
- Some recent results

Motivation from the renormalization group

Non-gravitational field theory for a lattice with lattice spacing a

$$H = \sum_{x,i} J_i(x)\ \mathcal{O}^i(x)$$

$x \longrightarrow$ sites in the lattice

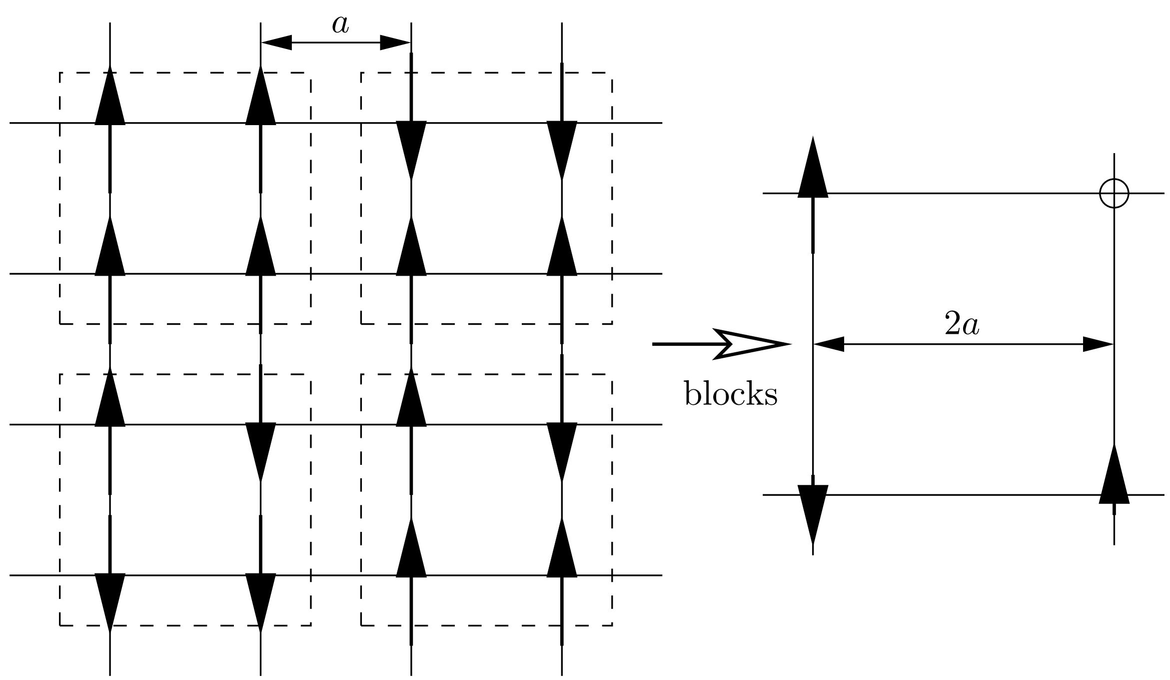
$i \longrightarrow$ operators

$J_i(x) \rightarrow$ coupling constant (source) for the operator $\mathcal{O}^i(x)$

Kadanoff-Wilson Renormalization group

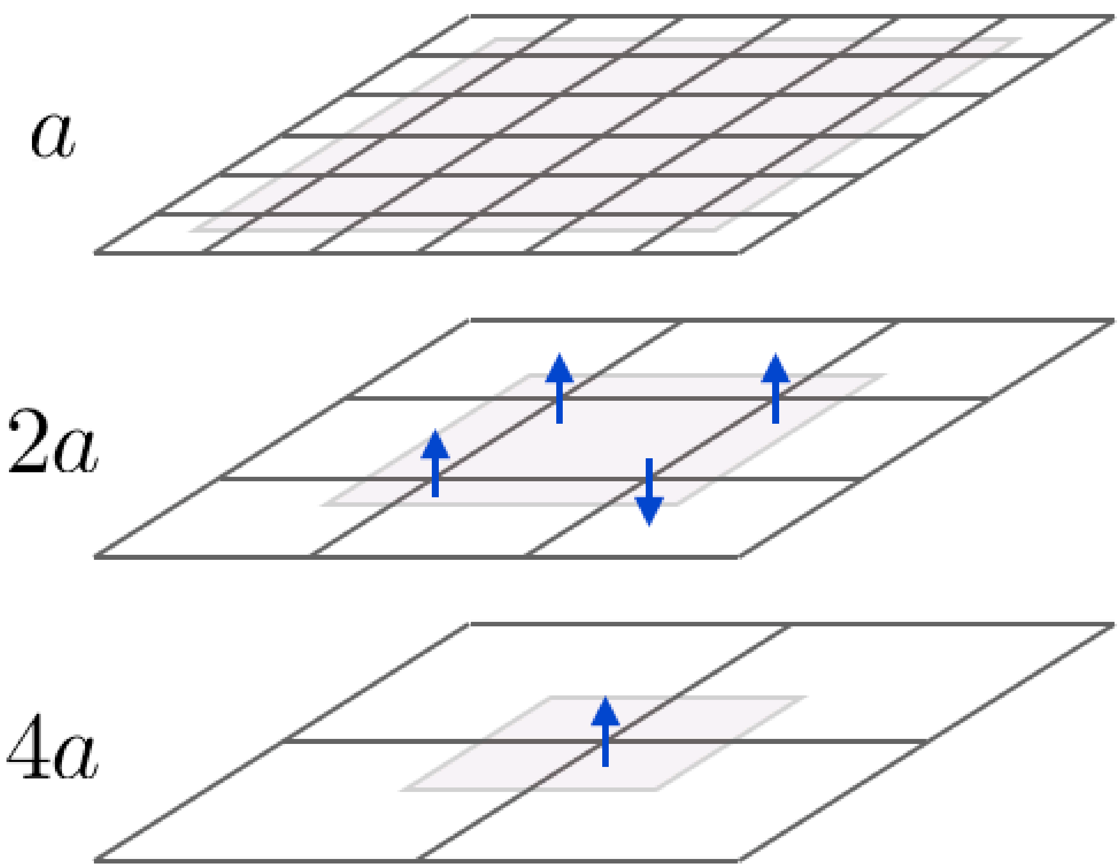
$\rightarrow$ Coarse grain the lattice by increasing the lattice spacing

$\rightarrow$ Replace multiple sites by a single site with the average value of the lattice variables



The couplings $J_i(x) \rightarrow$ change with the different steps:

$$J_i(x, a) \rightarrow J_i(x, 2a) \rightarrow J_i(x, 4a) \rightarrow \cdots$$



$$H = \sum_i J_i(x, a)\, \mathcal{O}^i(x)$$

$$H = \sum_i J_i(x, 2a)\, \mathcal{O}^i(x)$$

$$H = \sum_i J_i(x, 4a)\, \mathcal{O}^i(x)$$

The couplings are scale-dependent

$$J_i(x) \to J_i(x,u)$$

$u \to$ length scale at which we probe the system

$$u = (a, 2a, 4a, \cdots)$$

Coupling flow

$$u\frac{\partial}{\partial u} J_i(x,u) = \beta_i\Big(J_j(x,u), u\Big)$$

Weak coupling → β_i determined from perturbation theory

Strong coupling picture: → Think of u as an extra dimension

The multiple layers of lattices build up a new higher dimensional lattice
Regard the sources as fields in a space with one extra dimension

$$J_i(x,u) = \phi_i(x,r) \to \text{governed by some action}$$

AdS/CFT proposal:

The dynamics of the sources $\phi_i(x, r)$ in the bulk is determined by gravity (i.e. some metric)

It is a geometrization of the quantum dynamics encoded by the renormalization group

The sources must have the same tensor structure as the dual operators

$$\phi_i \, \mathcal{O}^i \text{ is a scalar}$$

-scalar field $\phi \rightarrow$ dual to a scalar operator $\mathcal{O}$

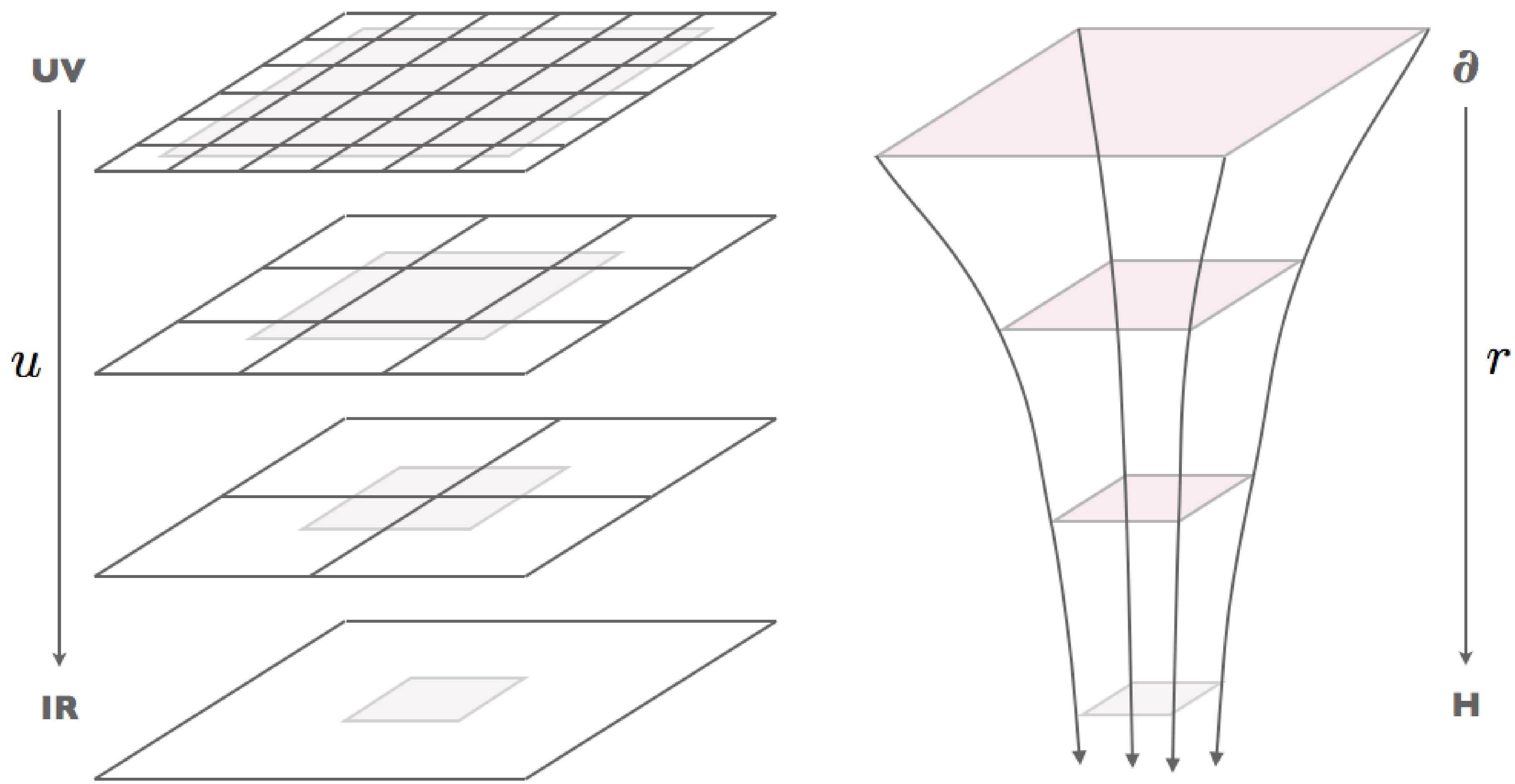
-vector field $A_\mu \rightarrow$ dual to a current J^μ

-spin-two field $g_{\mu\nu} \rightarrow$ dual to the energy-momentum tensor $T^{\mu\nu}$

Microscopic coupling in the UV → field of the gravity theory at the boundary

The field theory lives at the boundary of the higher dimensional space

$$J_i|_{UV} \quad = \quad \Phi_i|_\partial$$



Conceptual issue: how a higher-dimensional theory can have the same number of degrees of freedom as its lower-dimensional dual?

Matching of the degrees of freedom → measured by the entropy

QFT side: The entropy is extensive

$R_d \rightarrow d$-dimensional region

$S_{QFT} \propto \text{Vol}(R_d) \rightarrow$ proportional to the volume in d dimensions

Gravity side: The entropy in gravity is subextensive!

Entropy in a volume $\leq$ entropy of a black hole inside the volume

Bekenstein-Hawking formula:

$$S_{BH} = \frac{A_H}{4G_N}$$

$G_N \rightarrow$ Newton constant

J. Bekenstein

S. Hawking

$R_{d+1} \rightarrow$ region in $(d+1)$-dimensions

$R_d = \partial R_{d+1}$

$S_{GR}(R_{d+1}) \propto \text{Area}(R_d) \propto \text{Vol}(R_d)$

The dual higher-dimensional theory must be a gravity theory

Geometry at a fixed point ➡ vanishing β function ➡ CFT

Poincare invariant metric

$$ds^2 = \Omega^2(z)\,(-dt^2 + d\vec{x}^2 + dz^2)$$

$\vec{x} = (x^1, \cdots, x^{d-1})$

$z \longrightarrow$ extra dimension

Scale transformation

$$(t, \vec{x}) \longrightarrow \lambda(t, \vec{x}) \qquad z \longrightarrow \lambda z \qquad \lambda \longrightarrow \text{constant}$$

ds^2 invariant ➡ $\Omega(z) \to \lambda^{-1}\,\Omega(z)$ ➡ $\Omega(z) = \dfrac{L}{z}$

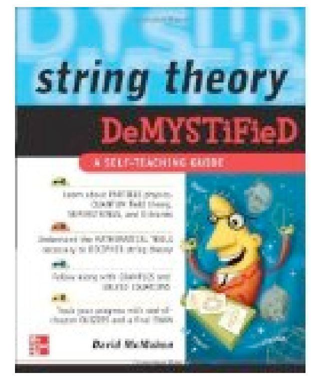
$$ds^2 = \frac{L^2}{z^2}\,(-dt^2 + d\vec{x}^2 + dz^2)$$

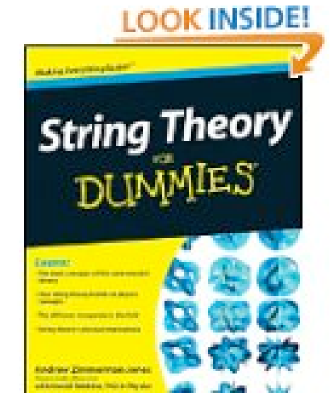
AdS boundary $\rightarrow z = 0$

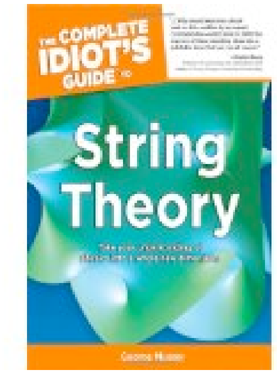
UV of the QFT

Line element of AdS_{d+1}

String theory user’s guide







What is string theory?

- A theory of hadrons
- A theory of quantum gravity
- A theory of everything
- A theory of nothing

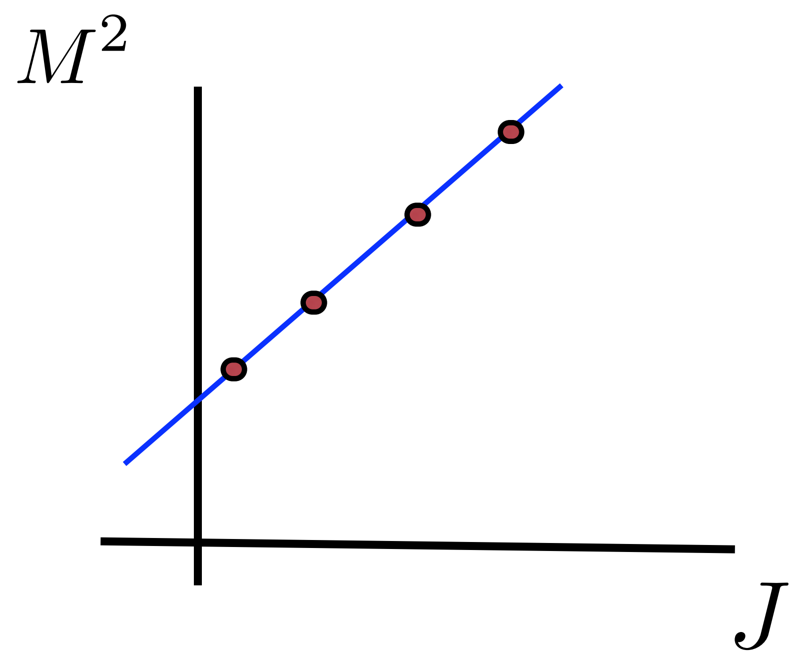
Greenhouse effect

> String theory is a greenhouse where new ideas grow to be transplanted elsewhere

String theory basics

Historical origin → Description of hadronic resonances of high spin (60's)

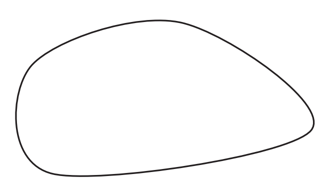
Regge trajectories →



→ $M^2 \sim J$

Basic objects extended along some characteristic distance l_s

open

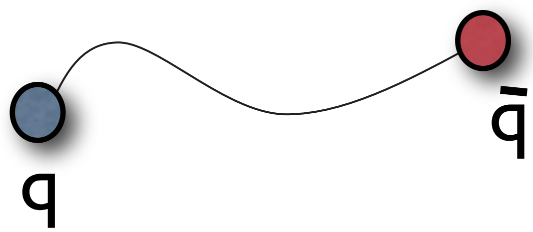


closed

→ non-local theory

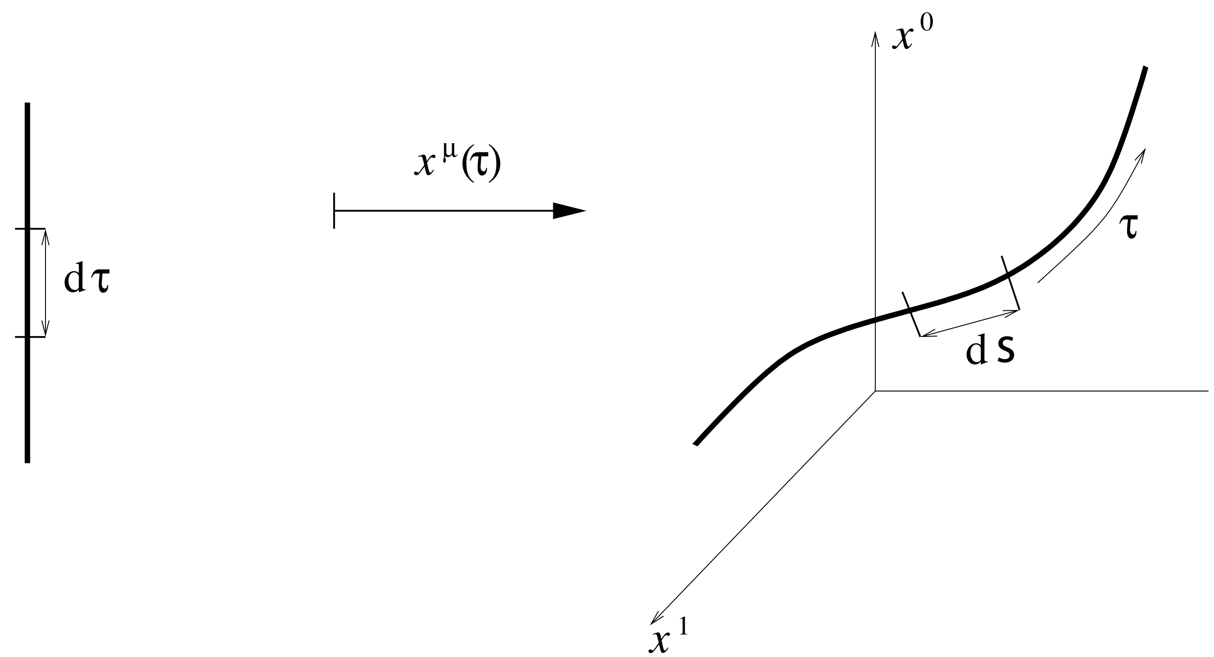
The rotational degree of freedom gives rise to high spins and Regge trajectories

In modern language a meson is a quark-antiquark pair joined by a string



$\longrightarrow$ $E \sim L$ $\longrightarrow$ confinement!

Classical relativistic particle



$\longrightarrow$ $x^\mu = x^\mu(\tau)$

$x^\mu \rightarrow$ parametrizes the space in which the point particle is moving

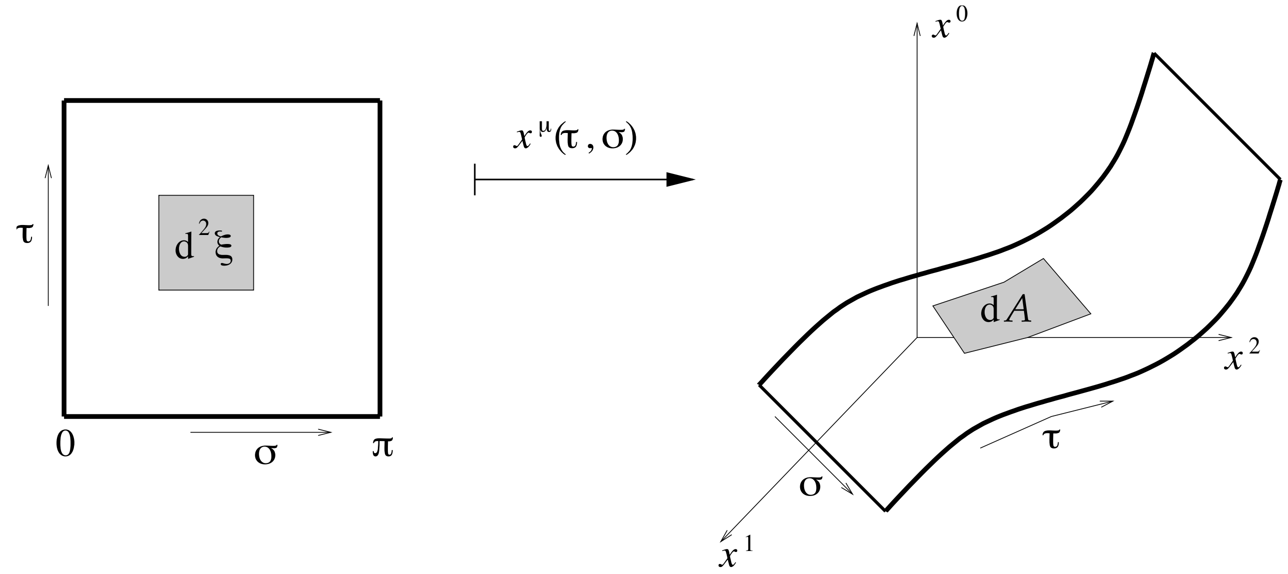
$\tau \rightarrow$ world-line coordinate

Action $\longrightarrow$

$$S = -m \int ds = -m \int_{\tau_0}^{\tau_1} d\tau \, \sqrt{-\eta_{\mu\nu} \, \dot{x}^\mu \, \dot{x}^\nu}$$

Classical relativistic string

It describes a surface in spacetime ⟹ worldsheet



Nambu-Goto action

$$S_{NG} = -T \int dA \qquad T = \frac{1}{2\pi\, l_s^2} \qquad l_s \rightarrow \text{string length}$$

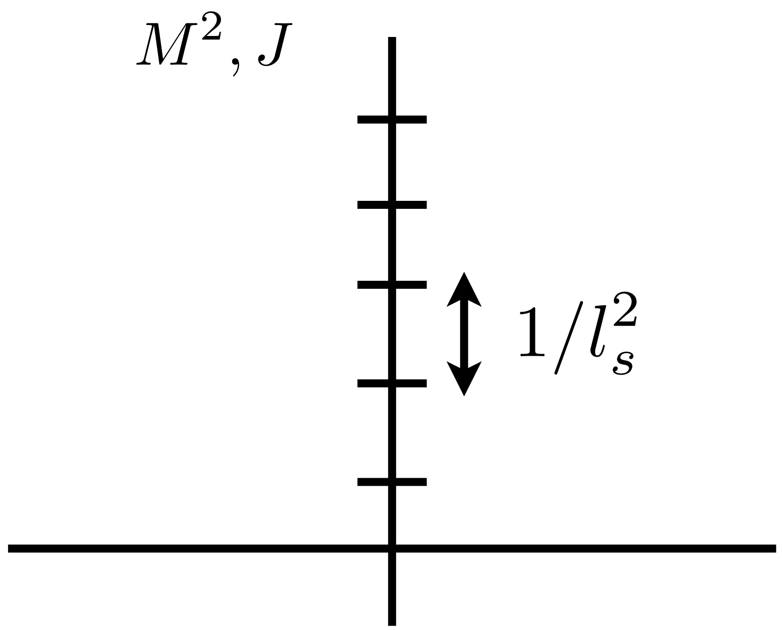
The classical eom's for the relativistic string can be solved in general for different boundary conditions (Neumann and Dirichlet)

String quantization

It opens Pandora's box!!

-Oscillation modes can be interpreted as particles

-Spectrum with infinite tower of particles with growing masses and spins



$$\text{mass gap} \sim 1/l_s$$

Consistency requires:

-Supersymmetry (symmetry between bosons and fermions)

If not there are tachyons in the spectrum (particles with $m^2 < 0$)

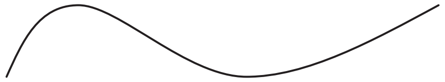
-The number of spacetime dimensions must be D=10

The extra dimensions should be regarded as defining a configuration space (as the phase space in classical mechanics)

T. Kaluza

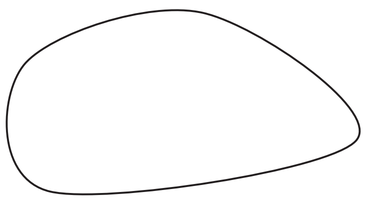
O. Klein

Massless modes of the open string



Contains massless particles of spin one with the couplings needed to have gauge symmetry → gauge bosons (photons, gluons, ...)

Massless modes of the closed string



It contains a particle of spin 2 and zero mass which can be interpreted as the graviton (the quantum of gravity)

Moreover

Quantum consistency implies Einstein equations in 10d plus corrections:

$$R_{\mu\nu} + \cdots = 0 \qquad \text{(Ricci flow)}$$

String theory is a theory of quantum gravity!!

Thus $l_s \sim l_P$ (and not of the order of the hadronic scale ~ 1 fm)

Elementary strings with zero thickness were born for the wrong purpose

Non-perturbative structure of string theory

String theories have solitons → extended objects

Dp-branes

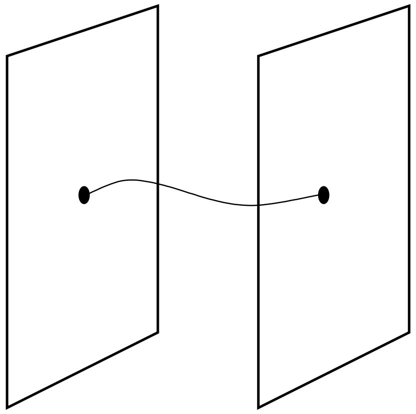
J. Polchinski

E. Witten

Extended in $p+1$ directions (p spatial + time)

-Defined as hypersurfaces where strings end

-Obtained by quantizing the string with fixed ends along hyperplanes (Dirichlet boundary conditions). →



The D-branes are dynamical objects

Excitations of a D-brane

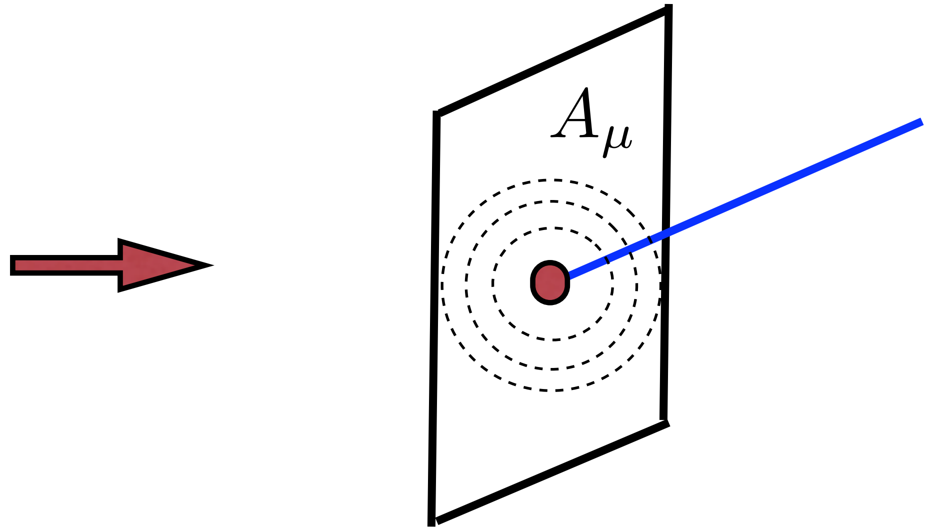
-Deformation of shape and rigid motion

$$\text{Parametrized by } 9-p \text{ coordinates} \rightarrow \phi^i \;\; (i = 1, \cdots, 9-p)$$

They are scalar fields on the worldvolume

-Internal excitations

The endpoint of the string is a charge that sources a gauge field on the worldvolume



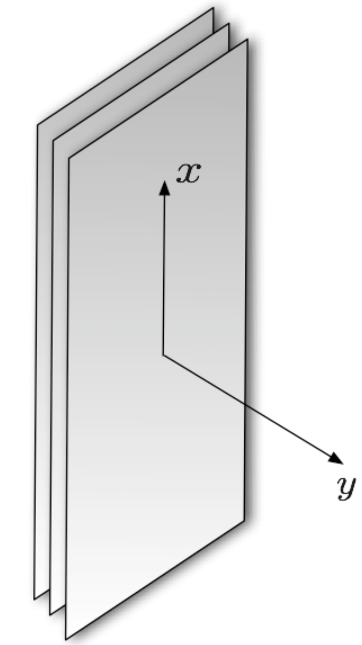
Dirac-Born-Infeld action

$$S_{DBI} = -T_{Dp} \int d^{p+1}x \sqrt{-\det(g_{\mu\nu} + 2\pi l_s^2 F_{\mu\nu})}$$

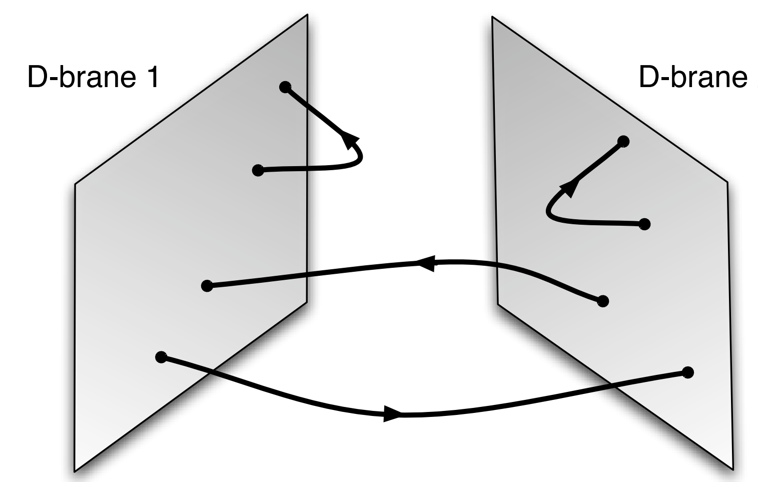
Multiple branes realize non-abelian gauge symmetry

N paralell coincident branes realize $U(N)$ YM

$A_\mu, \phi^i \rightarrow$ adjoints of $U(N)$



The non-abelian nature comes from strings stretched between different branes



The $U(1)$ can be decoupled

stack of N Dp-branes $\Rightarrow$ $SU(N)$ gauge theory in $p+1$ dimensions!

N D3-branes $\rightarrow$ $\mathcal{N}=4,\ SU(N)$ SYM in $4d$

Exact CFT

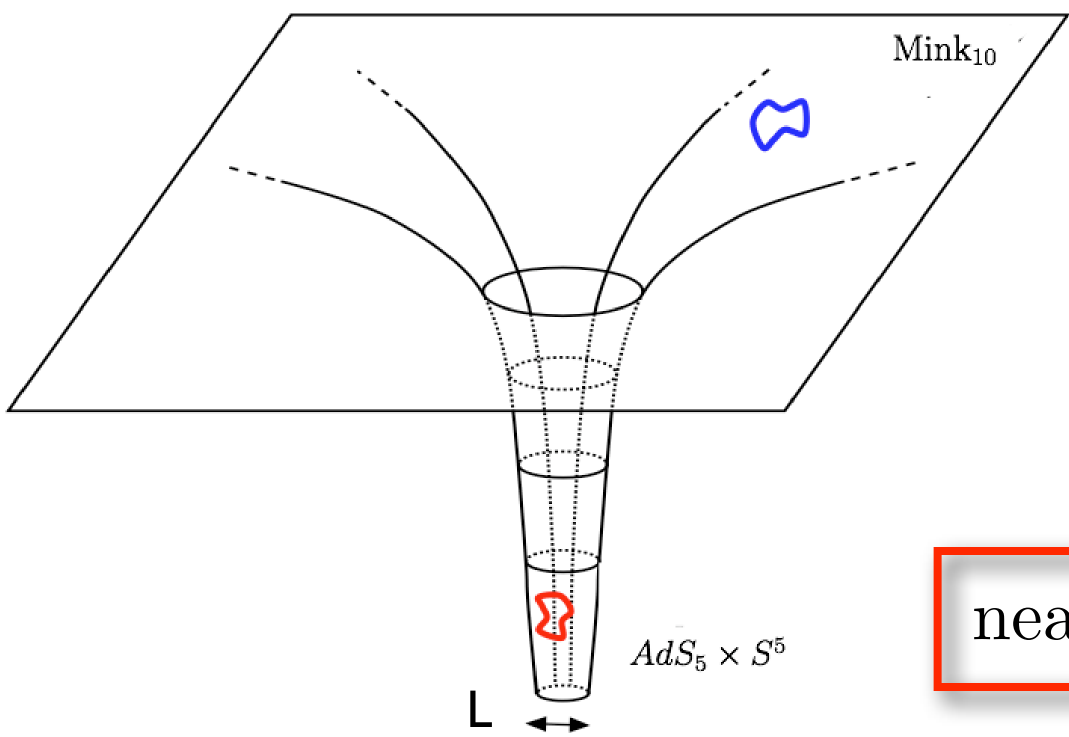
The D-branes provide a completely new perspective on gauge theories

One can move the branes, put then in different spaces, .. → dualities, less SUSY, different field content &vacua,..

Geometric insight on gauge dynamics → brane engineering

String theory is a gravity theory → any matter distorts the spacetime

D3-brane geometry $\rightarrow$ asymptotically 10d Minkowski with a infinite throat



near-horizon geometry $\rightarrow AdS_5 \times \mathbb{S}^5$ with radius L

Maldacena conjecture
(1997)

J. Maldacena

$$\mathcal{N}=4\ SU(N)\ \text{SYM theory equivalent to string theory in } AdS_5\times S^5$$

Relation of parameters

$$\left(\frac{L}{l_s}\right)^4 = N\,g_{YM}^2$$

't Hooft coupling → $\lambda = N\,g_{YM}^2$ → $\dfrac{l_s^2}{L^2} = \dfrac{1}{\sqrt{\lambda}}$

$$G = l_P^8 = \frac{\pi^4}{2}\,g_{YM}^4\,l_s^8 \quad \rightarrow \quad \left(\frac{l_p}{L}\right)^8 = \frac{\pi^4}{2N^2}$$

The dual theory is classical gravity if:

$\frac{l_p}{L} << 1 \rightarrow$ no quantum gravity corrections

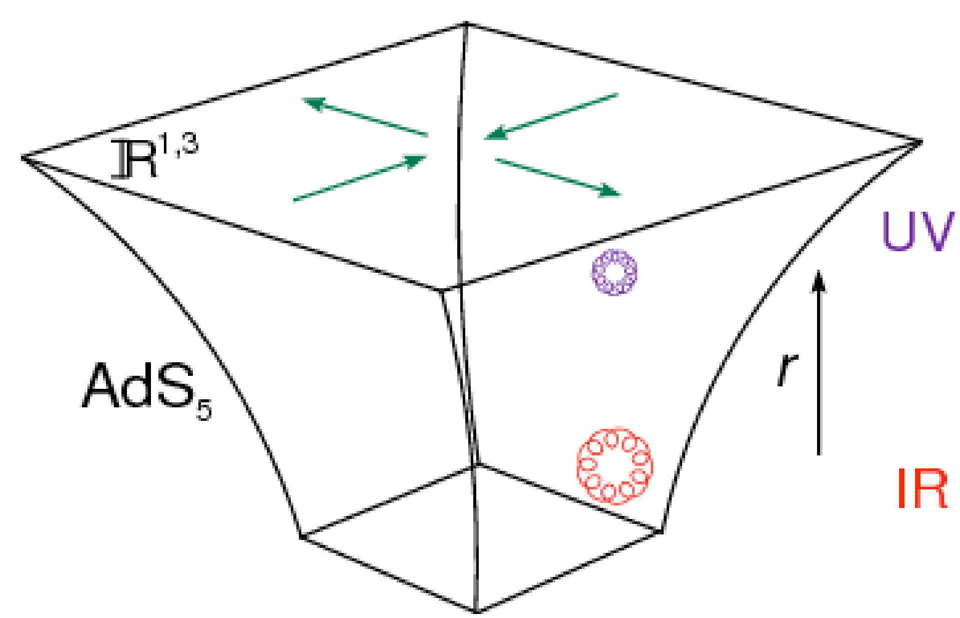
$\frac{l_s}{L} << 1 \rightarrow$ no stringy corrections

Equivalent to $N >> 1$ and $\lambda >> 1$ → **planar strongly coupled SYM**

Large N → Large number of degrees of freedom per unit volume

't Hooft limit

The ordinary Minkowski spacetime is identified with the boundary of AdS →

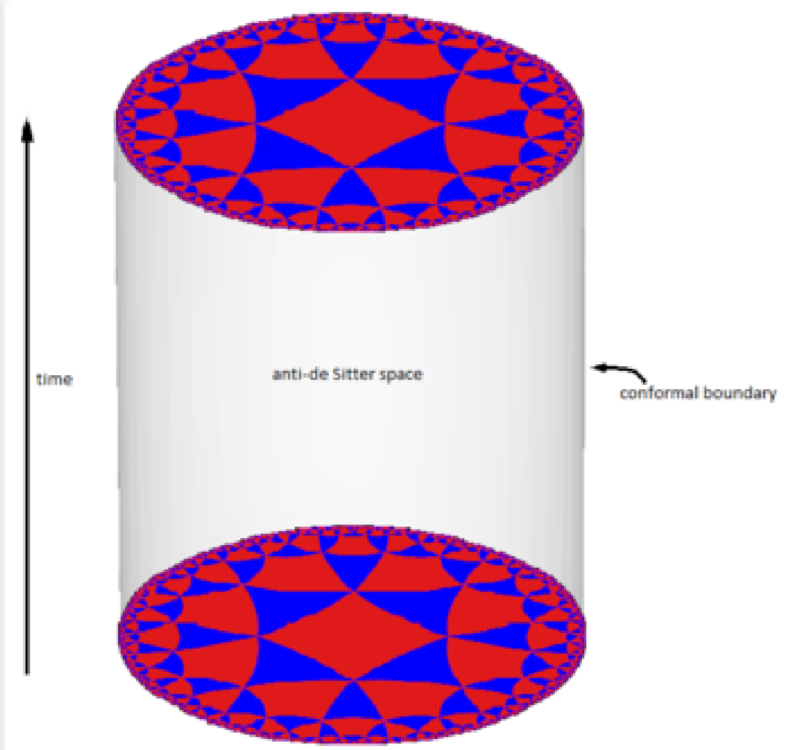


AdS/CFT prescription for the QFT generating function

$$Z_{QFT}[\phi_0] = \left\langle \exp\left[\int \phi_0\, \mathcal{O}\right] \right\rangle_{QFT} = Z_{gravity}[\phi \to \phi_0]$$

$$\phi_0(x) = \phi(z=0,x) = \phi|_{\partial AdS}(x)$$

$$Z_{gravity}[\phi \to \phi_0] = \sum_{\{\phi \to \phi_0\}} e^{S_{gravity}}$$

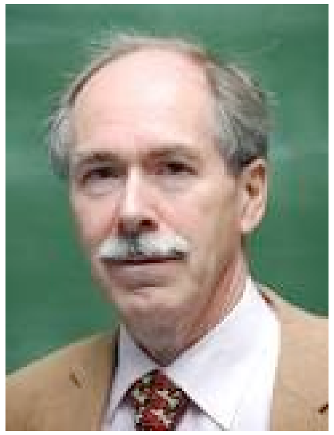


When classical gravity dominates

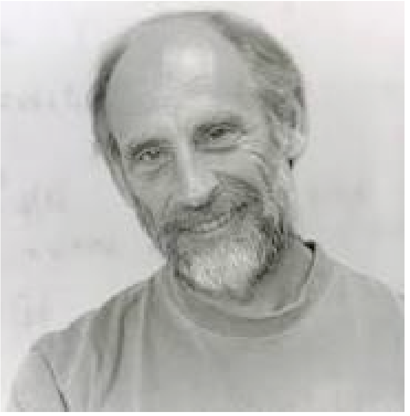
$$Z_{QFT}[\phi_0] = e^{S^{on-shell}_{gravity}[\phi \to \phi_0]}$$

→ typically divergent
(holographic renormalization)

AdS/CFT realizes the holographic principle



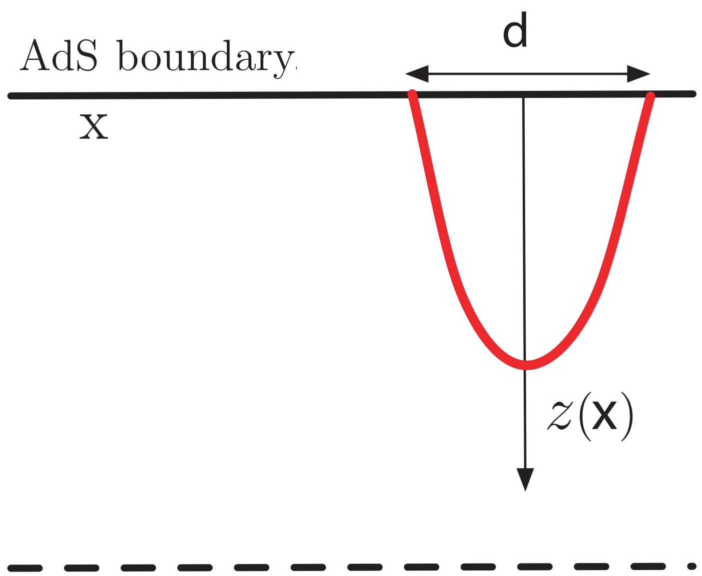
G. 't Hooft



L. Susskind

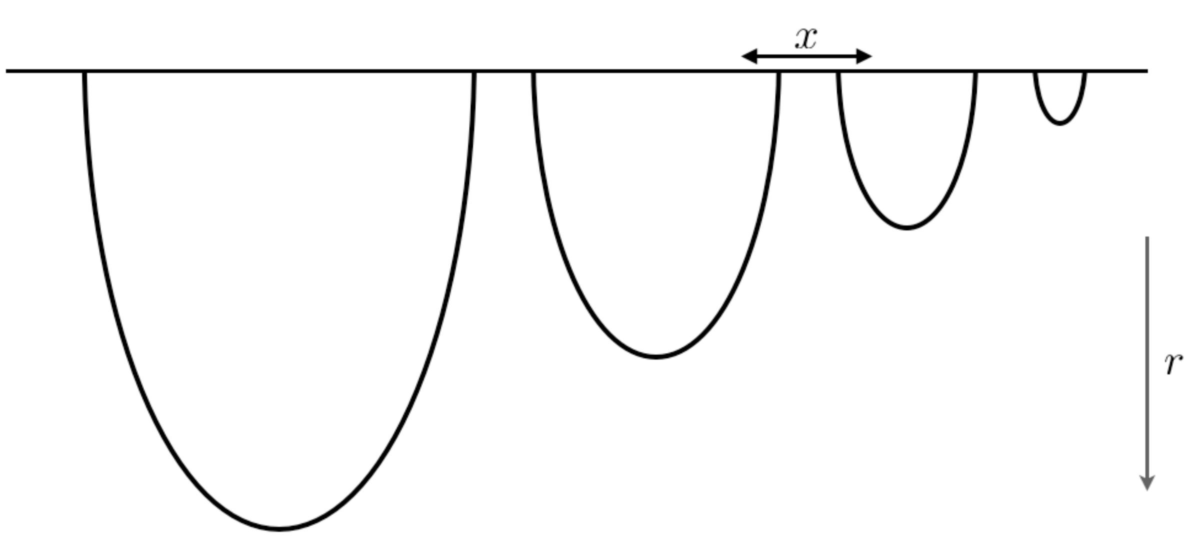
quark-antiquark force

hanging string extended in x in AdS_5



A problem in classical mechanics

long distances in $x \rightarrow$ deeper into the AdS bulk



potential → $$V_{q\bar{q}} = -\frac{4\pi^2\sqrt{\lambda}}{\left(\Gamma\left(\frac{1}{4}\right)\right)^4}\,\frac{1}{d}$$

→ Coulombic (conformal invariant)

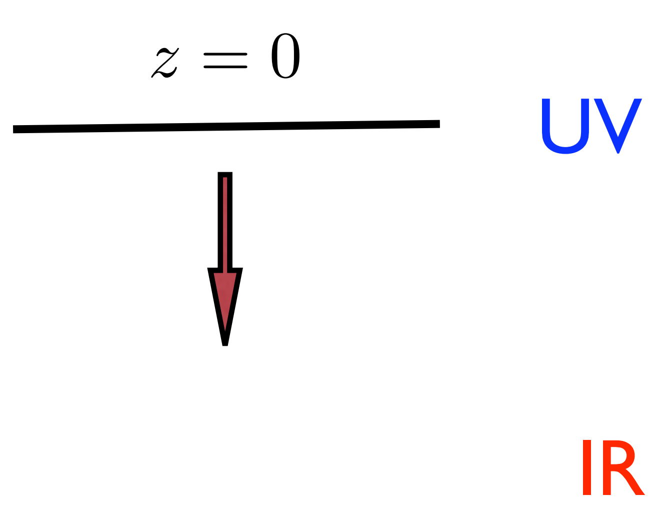
→ non-perturbative in λ

Perturbative result → $$V_{q\bar{q}}^{(per)} = -\frac{\pi\lambda}{d}$$

Non-perturbative QFT result from a classical mechanics calculation!!

CFT → excitations at arbitrary low energies

geometry with bottomless throat

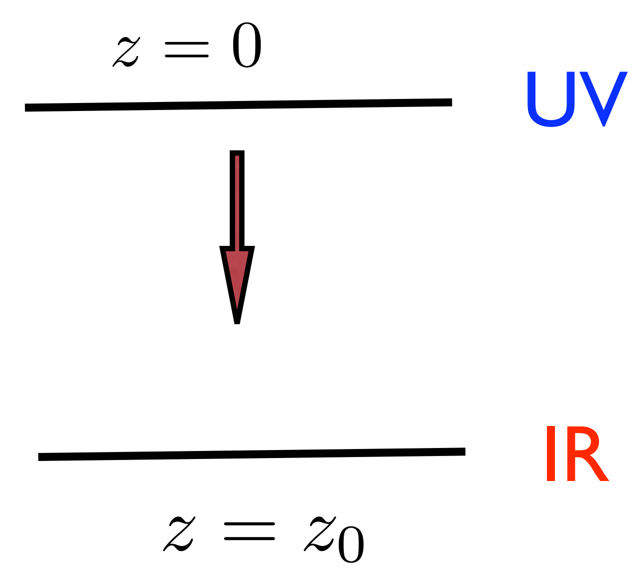


Generalization to non-conformal theories

non-conformal theory $\rightarrow$ minimal scale $\rightarrow$ geometry ends smoothly at some z_0

Confining theories with a mass gap m

$$z_0 \sim \frac{1}{m}$$



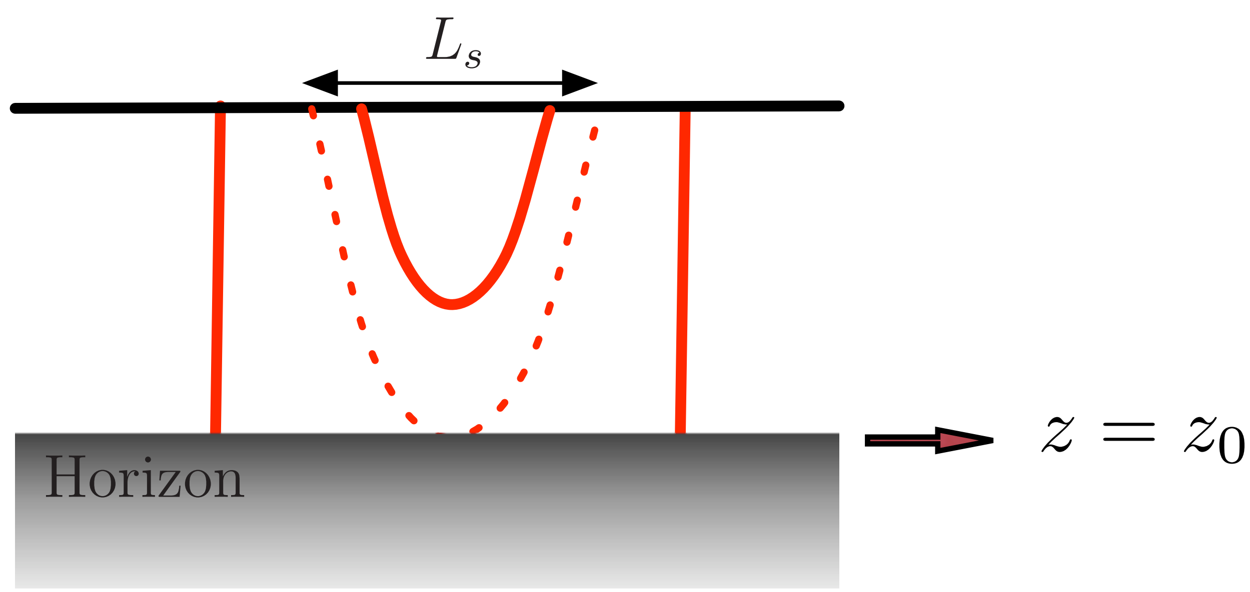
Finite temperature theories with temperature T

$$z_0 \sim \frac{1}{T}$$

The dual geometry is a black hole with event horizon

T is the Hawking temperature

Quark-antiquark potential at finite temperature

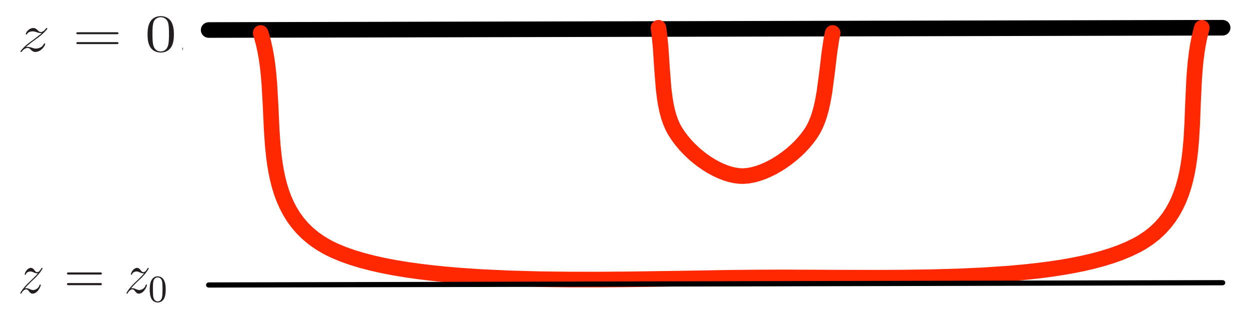


- There is a maximal value of d ($d_{max} \sim z_0$)
- At high $T \rightarrow$ disconnected configuration energetically favored

Models thermal screening in a plasma!

$q\bar{q}$ potential in a confining background

Qualitative picture ⟶



When $d \to \infty$ the profile is almost rectangular

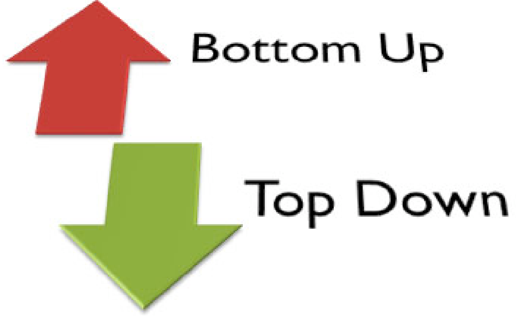
⟶ Vertical parts $\to$ masses of the static quarks

⟶ Horizontal part $\to$ $q\bar{q}$ potential

$$V = \sigma_s \, d \qquad \sigma_s \sim \sqrt{\lambda}\, M^2 \qquad M \sim \frac{1}{z_0}$$

Area law ⟶ Confinement

Two approaches



Top-down

Start with a brane setup, solve Einstein equations, find the geometry and identify the QFT dual

- Full control of the QFT-gravity dictionary
- Difficult to find geometries with the same properties as the realistic QFTs
- But one can model confinement, chiral symmetry breaking, anomalies, flavors, mesons, baryons,...

Bottom-up

Just consider metrics and apply the rules of the holographic duality

- Good phenomenological approach
- Suitable for condensed matter applications

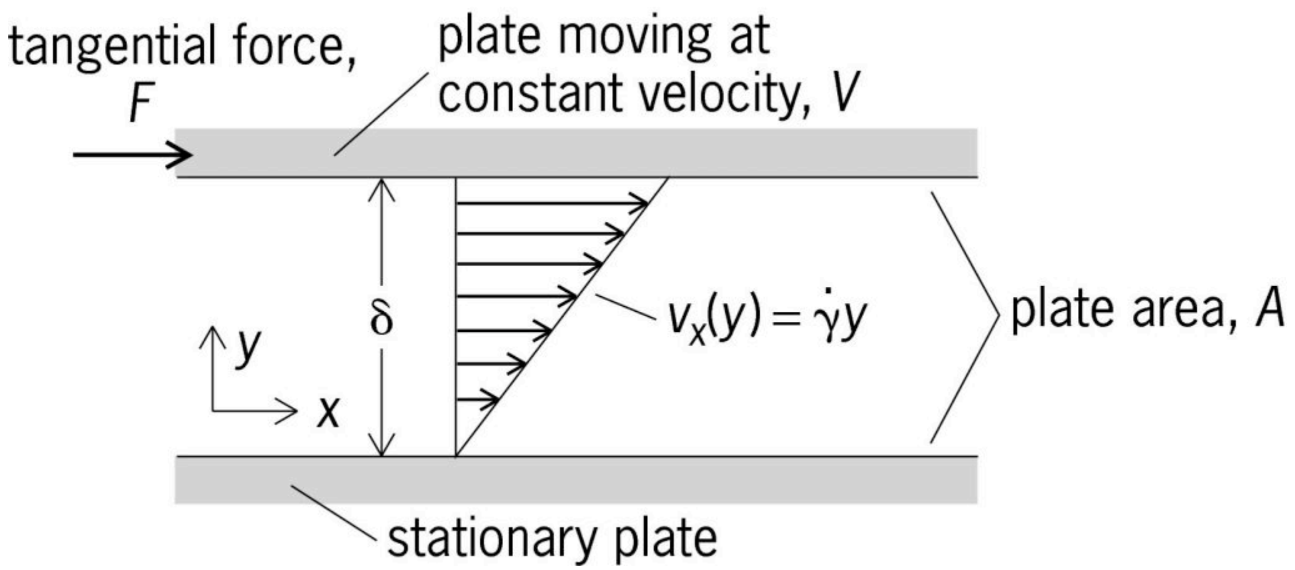
Applications in hydrodynamics

AdS/CFT allows to compute the shear viscosity for strongly coupled quantum systems

shear viscosity
η

→ Transport coefficient which measures the resistence of the system to flow under a shearing force

Determines the gradient of velocities of different layers of the fluid



η measures the propagation of disturbances

$$\text{Ideal liquids} \Longrightarrow \eta \approx 0 \text{ (strongly coupled)}$$
$$\text{Ideal gasses} \Longrightarrow \eta \to \infty \text{ (weakly coupled)}$$

How ideal can be a quantum liquid?

AdS/CFT result at infinite coupling for a theory dual to Einstein gravity coupled to matter fields

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

$s \rightarrow$ entropy density

Numerically $\rightarrow$ $\frac{\eta}{s} = 0.07957$ $\rightarrow$ very small almost universal result at strong coupling

Finite coupling corrections $\rightarrow$

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\,\zeta(3)}{\lambda^{\frac{3}{2}}} + \cdots\right)$$

$\zeta(3) = 1.2020$

Weak coupling calculation

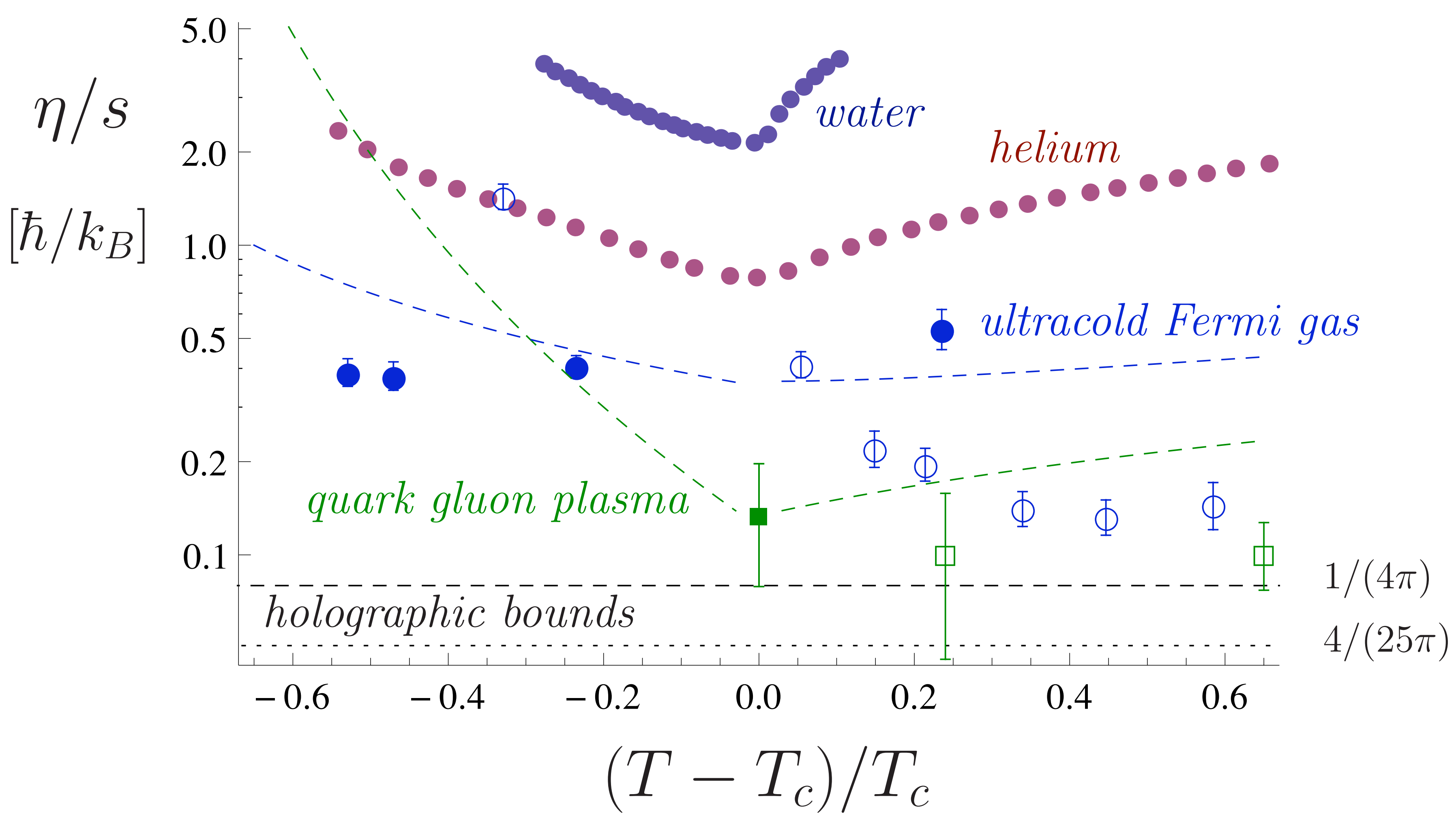
$$\frac{\eta}{s} = \frac{A}{\lambda^2 \log\left(\frac{B}{\sqrt{\lambda}}\right)}$$

$\eta/s \rightarrow \infty$ as $\lambda \rightarrow 0$

Kovtun, Son and Starinets (KSS) holographic bound $\rightarrow \eta/s \geq \frac{1}{4\pi}$

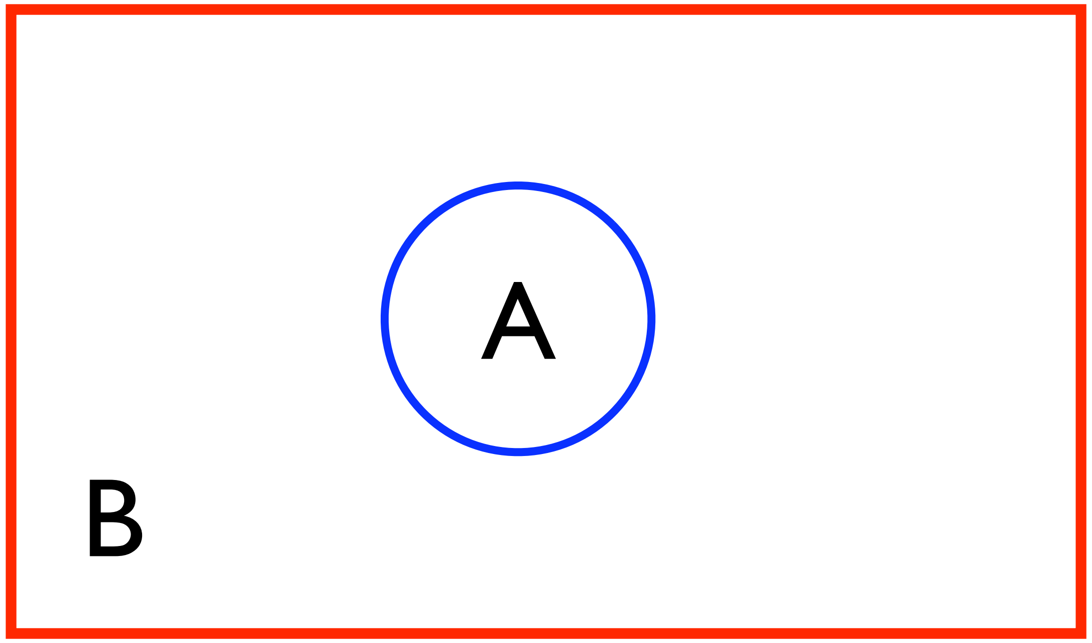
Lowest η/s in Nature

- Quark-Gluon plasma at RHIC
- Ultracold atomic Fermi gases at very low T



Entanglement entropy

Divide a quantum system in two parts A and B

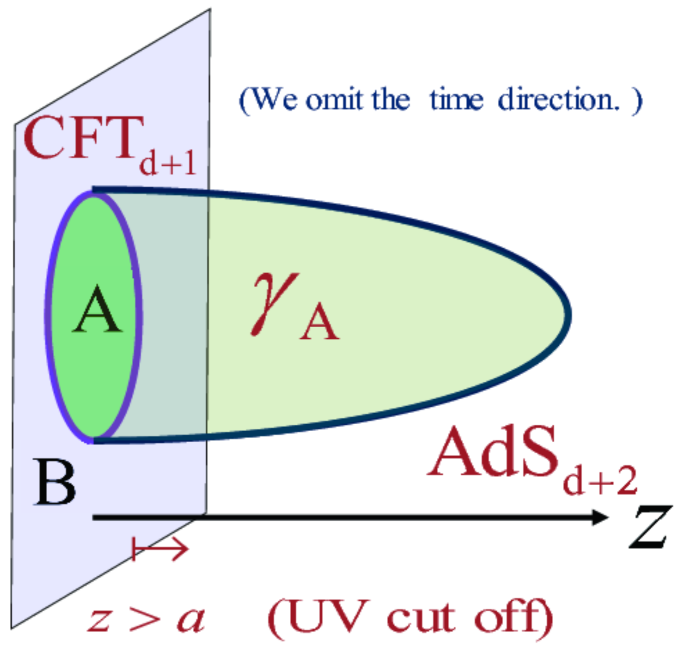


The entanglement entropy is the entropy that is only accesible to the subsystem A and not to B

- Measures the amount of quantum information (proportional to the number of degrees of freedom)
- Quantum order parameter (similar to Wilson loops for condended matter) for topological phase transitions.
- Order parameter for confinement/deconfinement phase transitions in gauge theories
- It has similarities with black hole entropy (lost information hidden in B)

AdS/CFT proposal (Ryu&Takayanagi)

$\gamma_A \rightarrow$ minimal area surface in the bulk such that $\partial A = \partial \gamma_A$



$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}$$

Inspired in Beckenstein-Hawking formula!

- Easy to calculate
- Connects condensed matter physics with quantum gravity !
- Thermalization after a quantum quench (sudden change of hamiltonian) related to BH formation and evaporation and to the information paradox

That's it!®

Thank you for your attention!