

High Scale Mixing Unification with Dirac Neutrinos

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May 22, 2014

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References: *arXiv*: 1312.7384

Outline

- ① Open questions in neutrino physics
- ② Neutrino mass terms
- ③ Neutrino mixing matrix
- ④ Neutrino mass spectrum
- ⑤ The absolute neutrino mass scale
- ⑥ CP violation
- ⑦ High scale mixing unification
- ⑧ Summary

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Open questions in neutrino physics

- **What is the absolute neutrino mass scale?**
Is the lightest neutrino massless? Hierarchical or degenerate?
- **What is the neutrino mass ordering?**
Normal or inverted ?
- **What is the origin of neutrino masses and flavor mixing?**
See saw mechanisms, flavour symmetries, ...
- **Is there CP violation in the lepton sector?**
What is the value of the Dirac CP-violating phase δ ?
- **What is the nature of neutrinos? Dirac or Majorana ?**
Lepton number violation, neutrino-less double beta decays.

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Majorana mass term

- A Majorana mass term of the form

$$\frac{1}{2} \overline{\nu_L} m_\nu \nu_L^c + H.c. \quad (1)$$

can be constructed without adding any new degrees of freedom to the SM.

- This term, however, breaks gauge invariance unless it is generated by spontaneous symmetry breaking from a gauge invariant term like

$$\frac{1}{2} \overline{l_L} \tilde{\Phi} f \tilde{\Phi}^T l_L^c + H.c., \quad (2)$$

where f is some flavour matrix of dimension $1/\text{mass}$, [Weinberg 79](#).

Dirac mass term

- If neutrinos are Dirac particles, the existence of ν_R is directly required to construct the mass term

$$\bar{\nu}_L m_D \nu_R + h.c.. \quad (3)$$

- Though this means adding new degrees of freedom to the SM at low energies, only three massive neutrinos are observed. One could say that there are no new particles in the strict sense, but just additional spin states for neutrinos.

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Neutrino mixing matrix

In the basis where charged Yukawa couplings are diagonal, the mixing matrix is identical to the Pontecorvo-Maki-Nakagawa-Sakata matrix and can be parametrized as

$$U_{pmns} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}(e^{i\frac{-\phi_1}{2}}, e^{i\frac{-\phi_2}{2}}, 1), \quad (4)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, the angles $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CP-violating phase and ϕ_1 and ϕ_2 are two Majorana CP-violation phases.

Neutrino mixing matrix

Quantity	Best Fit $\pm 1\text{-}\sigma$	3- σ Range
$\Delta m_{21}^2 = (m_2^2 - m_1^2) (10^{-5} \text{ eV}^2)$	$7.50^{+0.18}_{-0.19}$	7.00 – 8.09
$\Delta m_{31}^2 = (m_3^2 - m_1^2) (10^{-3} \text{ eV}^2)$	$2.473^{+0.070}_{-0.067}$	2.276 – 2.695
$\theta_{12}/^\circ$	$33.36^{+0.81}_{-0.78}$	31.09– 35.89
$\theta_{23}/^\circ$	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.3}_{-1.3}$	35.8 – 54.8
$\theta_{13}/^\circ$	$8.66^{+0.44}_{-0.46}$	7.19 – 9.96

Table: The global fits for neutrino oscillation parameters .

Gonzalez-Garcia et al. 2012

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Neutrino mass spectrum

- The sign of Δm_{31}^2 cannot be determined from the present data.
- The two possibilities, $\Delta m_{31(32)}^2 > 0$ or $\Delta m_{31(32)}^2 < 0$ correspond to two different types of ν -mass spectrum:
 - with *normal hierarchy* $m_1 < m_2 < m_3$, $\Delta m_{31}^2 > 0$, and
 - with *inverted hierarchy* $m_3 < m_1 < m_2$, $\Delta m_{32}^2 < 0$.
- Depending on the sign of Δm_{31}^2 , and the value of the lightest neutrino mass, the ν -mass spectrum can be
 - *Normal Hierarchical*: $m_1 \ll m_2 \ll m_3$, $m_2 \cong (\Delta m_{21}^2)^{\frac{1}{2}} \sim 8.7 \times 10^{-3}$ eV, $m_3 \cong |\Delta m_{31}^2|^{\frac{1}{2}} \sim 0.05$ eV;
 - *Inverted Hierarchical*: $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong |\Delta m_{32}^2|^{\frac{1}{2}} \sim 0.049$ eV;
 - *Quasi-Degenerate*: $m_1 \cong m_2 \cong m_3 \cong m_0$, $m_j^2 \gg |\Delta m_{31}^2|$, $m_0 \gtrsim 0.10$ eV.

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The absolute neutrino mass scale

- *The absolute neutrino mass scale* - the mass of the lightest neutrino is unknown.
- The most stringent upper bound on the $\overline{\nu_e}$ is

$$m_{\overline{\nu_e}} < 2.05\text{eV} \quad \text{at } 95 \% C.L. \quad (5)$$

Aseev et. al. 2011

- The KATRIN experiment is planned to reach sensitivity of $\overline{\nu_e} \sim 0.20 \text{ eV}$.
Eitel et. al. 2005, Drexlin et al 2013

The absolute neutrino mass scale

- The CMB data of the WMAP, combined with supernovae data and data on galaxy clustering provides upper limit on the sum of neutrino masses

$$\sum_i m_i \lesssim (0.3 - 1.3) \text{eV} \quad \text{at } 95 \% \text{C.L.} \quad (6)$$

[Abazajian et. al. 2011](#)

- The Planck collaboration combine their data on the CMB temperature power spectrum with the WMAP polarisation low and high multiple CMB data and the Baryon Acoustic Oscillation

$$\sum_i m_i < 0.23 \text{eV} \quad \text{at } 95 \% \text{C.L.} \quad (7)$$

- The sum of neutrino masses range from 0.23 eV to 1.08 eV, depending on values chosen for priors. [Ade et. al. 2013](#)

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Dirac CP violation

- *The CP-violating phase* - the Dirac phase δ is the analogue to the CKM phase. There are hints about δ from the data.
- The present best fit value is $\delta \cong 270^\circ$.
[Capozzi et. al. 2013](#), [Gonzalez-Garcia et. al. 2012](#)
- The CP conserving values $\delta = 0$ and π are disfavoured at 1.6σ to 2.6σ for $\Delta m_{31(32)}^2 > 0$.

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High scale mixing unification

- The central idea of this hypothesis is that the mixing angles of the quark sector become identical to those of neutrino sector at some unification scale, which may be the Grand Unified Theory (GUT) scale.
[Mohapatra et. al. 2003, 2005](#)
- A model for Majorana neutrinos, with the seesaw mechanism, exists where a quasi-degenerate neutrino spectrum and unification of mixing angles can arise.
[Mohapatra et. al. 2003](#)
- The Majorana neutrinos are studied later on in
[Agarwalla et. al. 2007, Abbas et. al. 2014](#) .
- The main requirement at GUT is
 $\theta_{12}^q = \theta_{12}, \theta_{13}^q = \theta_{13}, \theta_{23}^q = \theta_{23}.$

High scale mixing unification

Motivation

- In addition to the unification of forces of SM, GUT also unify quarks and leptons in a joint representation of the GUT symmetry group.
- As a consequence, flavor structures of quark and lepton sectors are no longer disconnected.
- This can lead to the relations between quark and lepton mixing angles.
[Antusch and Maurer 2011](#), [Marzocca et. al. 2011](#) .
- The HSMU may be a footprint of such an underlying GUT.
- In the absence of any symmetry, HSMU can be an accidental phenomenon.

High scale mixing unification

Implementation

- Bottom-up running of quark sector
($SM \rightarrow MSSM$)
- Top-down running of neutrino mixing parameters.
($MSSM \rightarrow SM$)

Dirac neutrinos

Motivation

- It is not known whether neutrinos are Majorana or Dirac particle.
- Only experiments can confirm the nature of neutrinos.
[GERDA](#)
- Dirac neutrinos can also explain the smallness of neutrino masses in some models using extra heavy degrees of freedom, from Kähler potential of supergravity, from GUT or compactification scales etc.
[Mohapatra and Valle 1986](#), [Arkani-Hamed et. al. 2001](#), [Borzumati and Nomura 2001](#), [Kitan 2002](#), [Abe et. al. 2005](#)
- A theoretical model with Dirac neutrinos which can give rise to HSMU and quasi degenerate mass spectrum is a challenging task.

Dirac neutrinos

RG equations for mixing parameters

$$\dot{\theta}_{12} = \frac{-C y_\tau^2}{32 \pi^2} \frac{m_1^2 + m_2^2}{m_2^2 - m_1^2} \sin(2 \theta_{12}) \sin^2 \theta_{23} + \mathcal{O}(\theta_{13}) , \quad (8)$$

$$\begin{aligned} \dot{\theta}_{13} = & \frac{-C y_\tau^2}{32 \pi^2} \frac{1}{(m_3^2 - m_1^2)(m_3^2 - m_2^2)} \{ (m_2^2 - m_1^2) m_3^2 \cos \delta \cos \theta_{13} \sin(2 \theta_{12}) \sin(2 \theta_{23}) \\ & + [m_3^4 - (m_2^2 - m_1^2) m_3^2 \cos(2 \theta_{12}) - m_1^2 m_2^2] \cos^2 \theta_{23} \sin(2 \theta_{13}) \} , \end{aligned} \quad (9)$$

$$\dot{\theta}_{23} = \frac{-C y_\tau^2}{32 \pi^2} \frac{[m_3^4 - m_1^2 m_2^2 + (m_2^2 - m_1^2) m_3^2 \cos(2 \theta_{12})]}{(m_3^2 - m_1^2)(m_3^2 - m_2^2)} \sin(2 \theta_{23}) + \mathcal{O}(\theta_{13}) , \quad (10)$$

where the dot indicates the logarithmic derivative w.r.t. the renormalization scale μ and

$$C = \begin{cases} 1 , & \text{(MSSM)} , \\ -3/2 , & \text{(SM)} . \end{cases} \quad (11)$$

[Lindner et. al. 2005](#)

Dirac neutrinos

RG equations for mixing parameters

- Since $\dot{\theta}_{12} \propto -\frac{m_1^2 + m_2^2}{m_2^2 - m_1^2}$, θ_{12} always increases during top down running, most sensitive to radiative corrections than the other two angles.
- It does not depend the CP violation phase.
- $\dot{\theta}_{13}, \dot{\theta}_{23} \propto -\frac{1}{m_3^2 - m_2^2}$
- The values of θ_{23} and θ_{13} can either increase or decrease, depending on the sign of $(m_3^2 - m_2^2)$.
- While the evolution of θ_{13} depends on CP violation phase, that of θ_{23} does not.

Dirac neutrinos

RG equations for mass eigenvalues

The evolution of the mass eigenvalues is given by

$$16\pi^2 \dot{m}_1 = \left\{ C y_\tau^2 \left[\cos^2 \theta_{12} \cos^2 \theta_{23} \sin^2 \theta_{13} + \sin^2 \theta_{12} \sin^2 \theta_{23} - \frac{1}{2} \cos \delta \sin \theta_{13} \sin(2\theta_{12}) \sin(2\theta_{23}) \right] + \alpha_\nu \right\} m_1, \quad (12a)$$

$$16\pi^2 \dot{m}_2 = \left\{ C y_\tau^2 \left[\sin^2 \theta_{12} \cos^2 \theta_{23} \sin^2 \theta_{13} + \cos^2 \theta_{12} \sin^2 \theta_{23} + \frac{1}{2} \cos \delta \sin \theta_{13} \sin(2\theta_{12}) \sin(2\theta_{23}) \right] + \alpha_\nu \right\} m_2, \quad (12b)$$

$$16\pi^2 \dot{m}_3 = \{ C y_\tau^2 \cos^2 \theta_{13} \cos^2 \theta_{23} + \alpha_\nu \} m_3. \quad (12c)$$

α_ν represents the flavor-independent part of the RGE.

$$\alpha_\nu = \begin{cases} -\frac{9}{20} g_1^2 - \frac{9}{4} g_2^2 + T_{\text{SM}}, & (\text{SM}) \\ \text{Tr}(Y_\nu^\dagger Y_\nu) + 3 \text{Tr}(Y_u^\dagger Y_u) - \frac{3}{5} g_1^2 - 3 g_2^2, & (\text{MSSM}). \end{cases} \quad (13a)$$

Here, we define $T_{\text{SM}} \equiv \text{Tr} [Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3 Y_d^\dagger Y_d + 3 Y_u^\dagger Y_u]$.

Dirac neutrinos

RG equations for mass eigenvalues

- The evolution of masses is governed by α_ν through a common rescaling for small $\tan\beta$.
- For large $\tan\beta$, there are corrections specific to the individual masses.

The chosen inputs for the analysis

- Normal hierarchy and quasi-degenerate pattern.
- The quark mixing angles
 $\theta_{12}^q = 13.02^\circ = \theta_{12}$, $\theta_{13}^q = 0.173^\circ = \theta_{13}$, $\theta_{23}^q = 2.03^\circ = \theta_{23}$ and
 $\delta_{CP}^q = 68.93^\circ = \delta_{CP}$.
- $\tan \beta = 55$
- The unification scale = 2×10^{16} GeV
- The masses of neutrinos at unification scale
 $m_1 = 0.19014 - 0.19124$ eV, $m_2 = 0.191 - 0.192$ eV, $m_3 = 0.20504 - 0.20629$ eV.
- SUSY scale = 2 TeV.

Results

Running of masses and angles

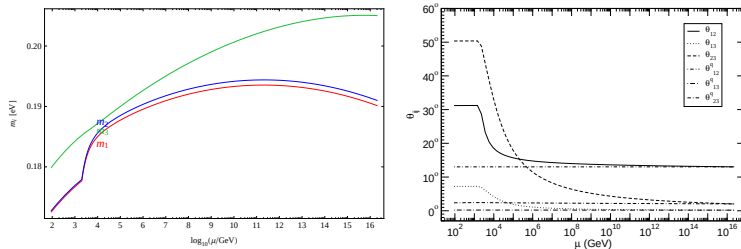


Figure: The RG evolution of the neutrino masses and mixing angles

Results

Correlation between θ_{13} and θ_{23}

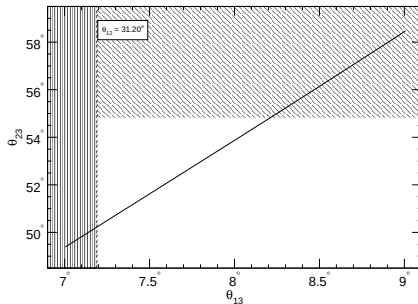


Figure: The behaviour of leptonic mixing angle θ_{13} with θ_{23} . The shaded regions lie outside the 3σ range. θ_{12} at low scale is held fixed at 31.20°

Results

Correlation between θ_{13} and θ_{23}

- The allowed range of θ_{13} is $7.19^\circ - 8.21^\circ$.
- The allowed range of θ_{23} is $50.25^\circ - 54.80^\circ$.
- θ_{23} is non-maximal and always lies in the second octant.
- The lightest neutrino mass is $0.17254 - 0.17390$ eV.
- $\Delta m_{sol}^2 = (7.34055 - 7.88577) \times 10^{-5} \text{ eV}^2$ and
 $\Delta m_{atm}^2 = (2.38329 - 2.51761) \times 10^{-3} \text{ eV}^2$.

Results

Correlation between θ_{13} and θ_{23}

- The sum of neutrino masses at low scale, corresponding to the above mentioned values, turns out to be $\sum m_i = 0.52517 - 0.52887$ eV, where $i = 1, 2, 3$.
- The recent cosmological upper limit, from Planck collaboration, on the sum of neutrino masses range from 0.23 eV to 1.08 eV, depending on values chosen for priors.
- The “averaged electron neutrino mass” obtained from our analysis is $m_e = 0.17274 - 0.17407$ eV which is slightly below the present reach of KATRIN experiment
- The Dirac CP violating phase $\delta_{CP} = (26.24 - 28.16)^\circ$.
- Full parameter scan under preparation.

Results

Variation of the SUSY scale

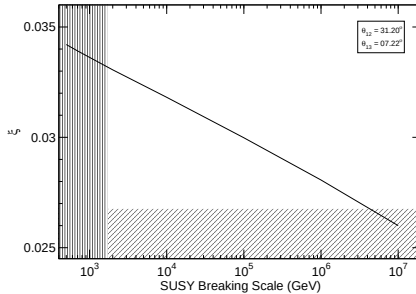


Figure: The variation of ξ with SUSY breaking scale. The shaded region lie outside the allowed parameter range. The vertically shaded region is disallowed by LHC SUSY searches whereas the horizontal one lies outside the 3σ range of ξ

$$\xi = \Delta m_{sol}^2 / \Delta m_{atm}^2$$

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Summary

- It is an open question whether neutrinos are Majorana or Dirac particle.
- Only experiment can confirm their nature.
- The quark-lepton unification is one of the attractive features of the GUT models.
- The HSMU hypothesis could be a manifestation of the quark-lepton unification.
- If the neutrinos are Dirac type, we show that HSMU has testable predictions.
- The large mixing angles in neutrino sector is explained through radiative magnification.
- The non zero value of the mixing angle θ_{13} naturally obtained.
- The mixing angles θ_{13} and θ_{23} have a strong correlation.
- The novel prediction is that θ_{23} is non-maximal and lies in the second octant.
- The full parameter scan and CP violation studies are under progress.