



# A Comprehensive Amplitude Analysis of Photon-Photon Collision

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# Outlines

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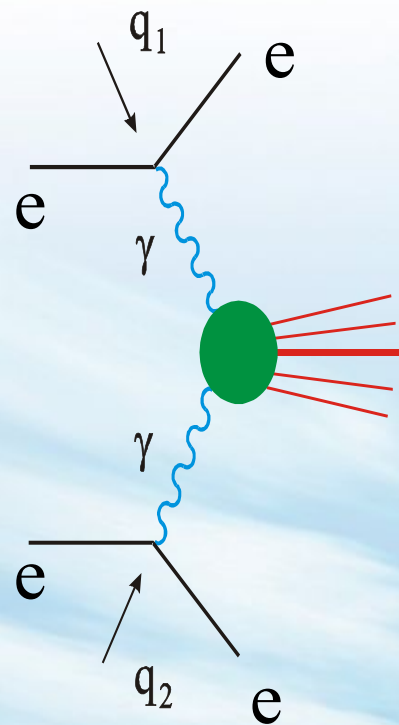
**photon couplings**

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**Summary**

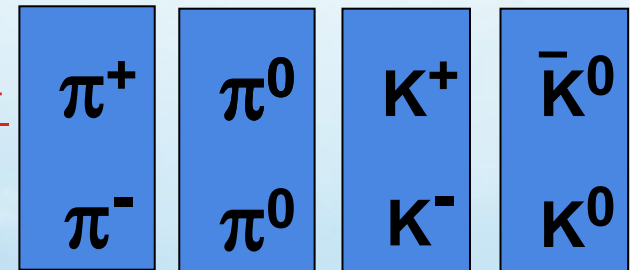
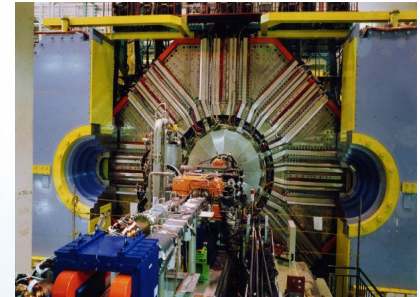
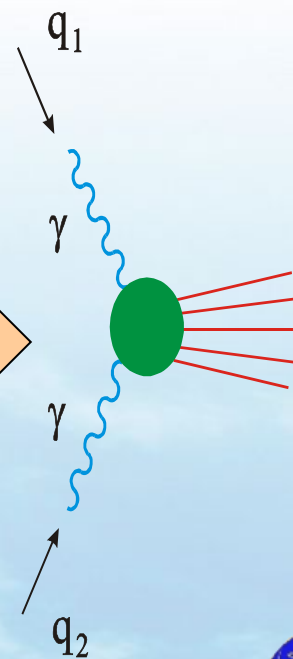
# 1. Introduction

- $\gamma\gamma \rightarrow \text{MM}$  has the cleanest background to probe hadron structure.



Equivalent photon approximation

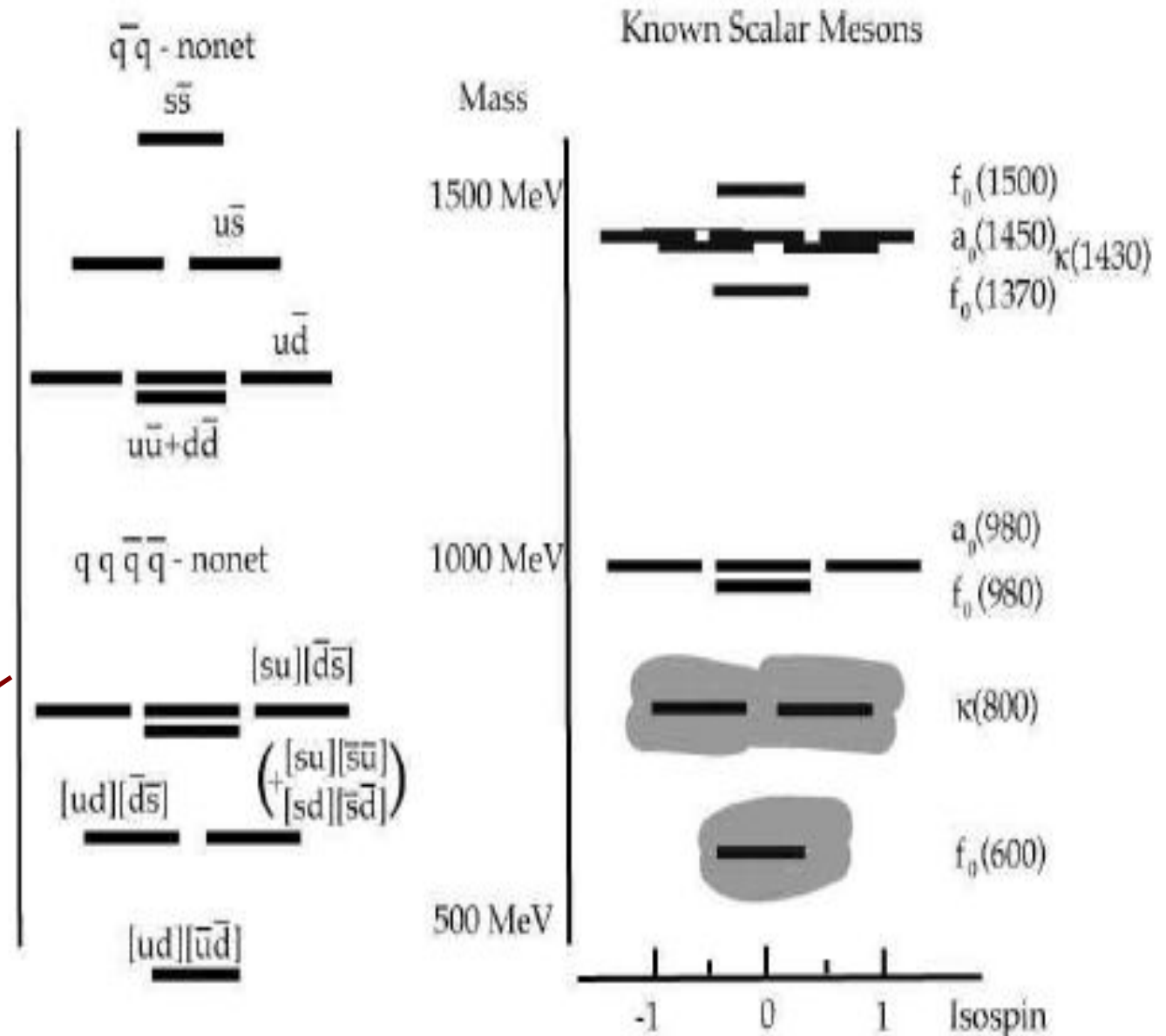
$$q_1^2 \simeq q_2^2 \simeq 0$$



# Introduction

- The scalars and tensors appear in such processes are rather interesting.

Jaffe  
Phys. Rept. 409 (2005) 1



# Tensors

- They are  $q\bar{q}$ , or  $q\bar{q} + gg$ . Their photon couplings are interesting as the connection to scalars. If sigma is the companion of  $f_2(1270)$  with  $(\bar{u}u + \bar{d}d)/\sqrt{2}$  composition, in quark model it has

$$\frac{\Gamma(\sigma \rightarrow \gamma\gamma)}{\Gamma(f_2 \rightarrow \gamma\gamma)} = \frac{15}{4} \left( \frac{m_\sigma}{m_{f_2}} \right)^n$$

'2' by F.E. Close *et. al*,  
PRD43 (1991) 2161

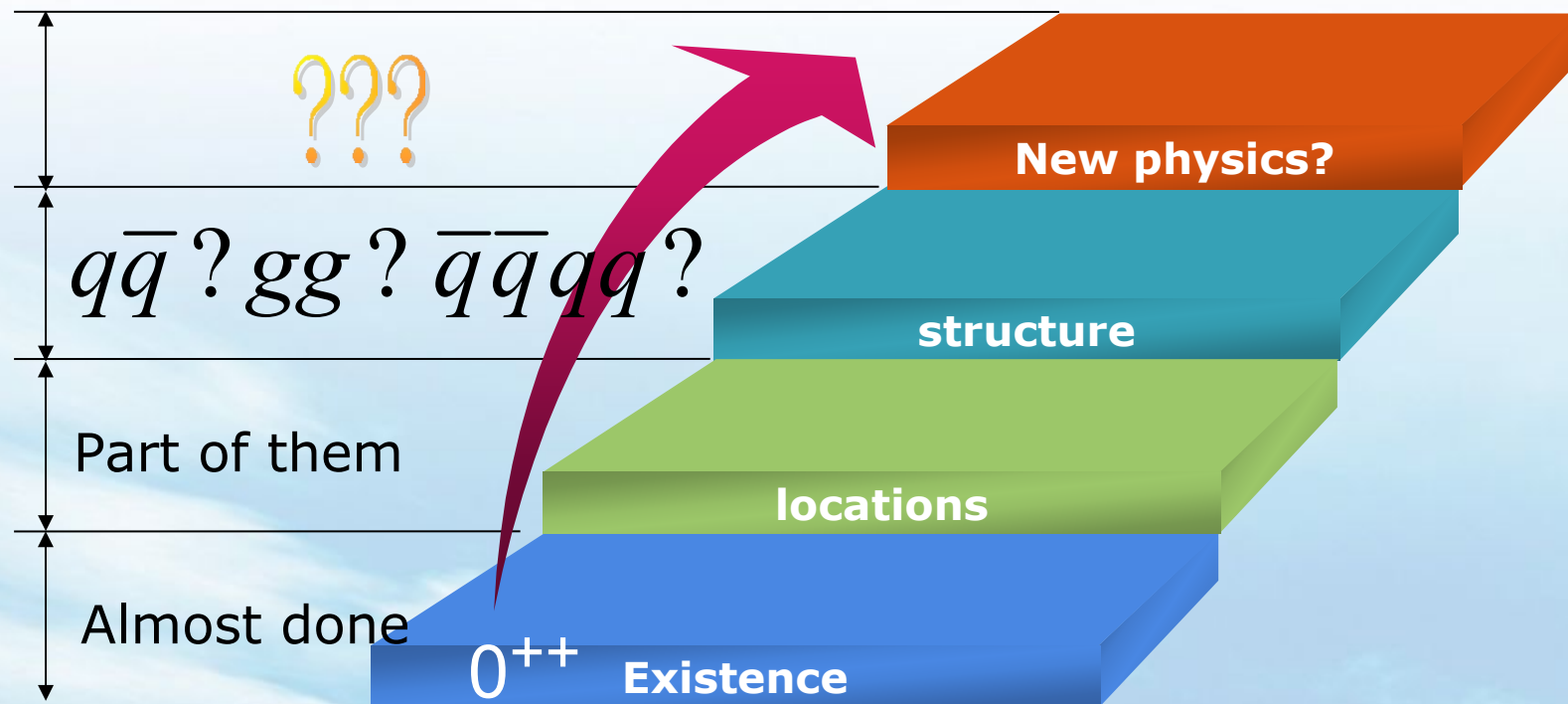
$n=3$ , Coulomb potential

?

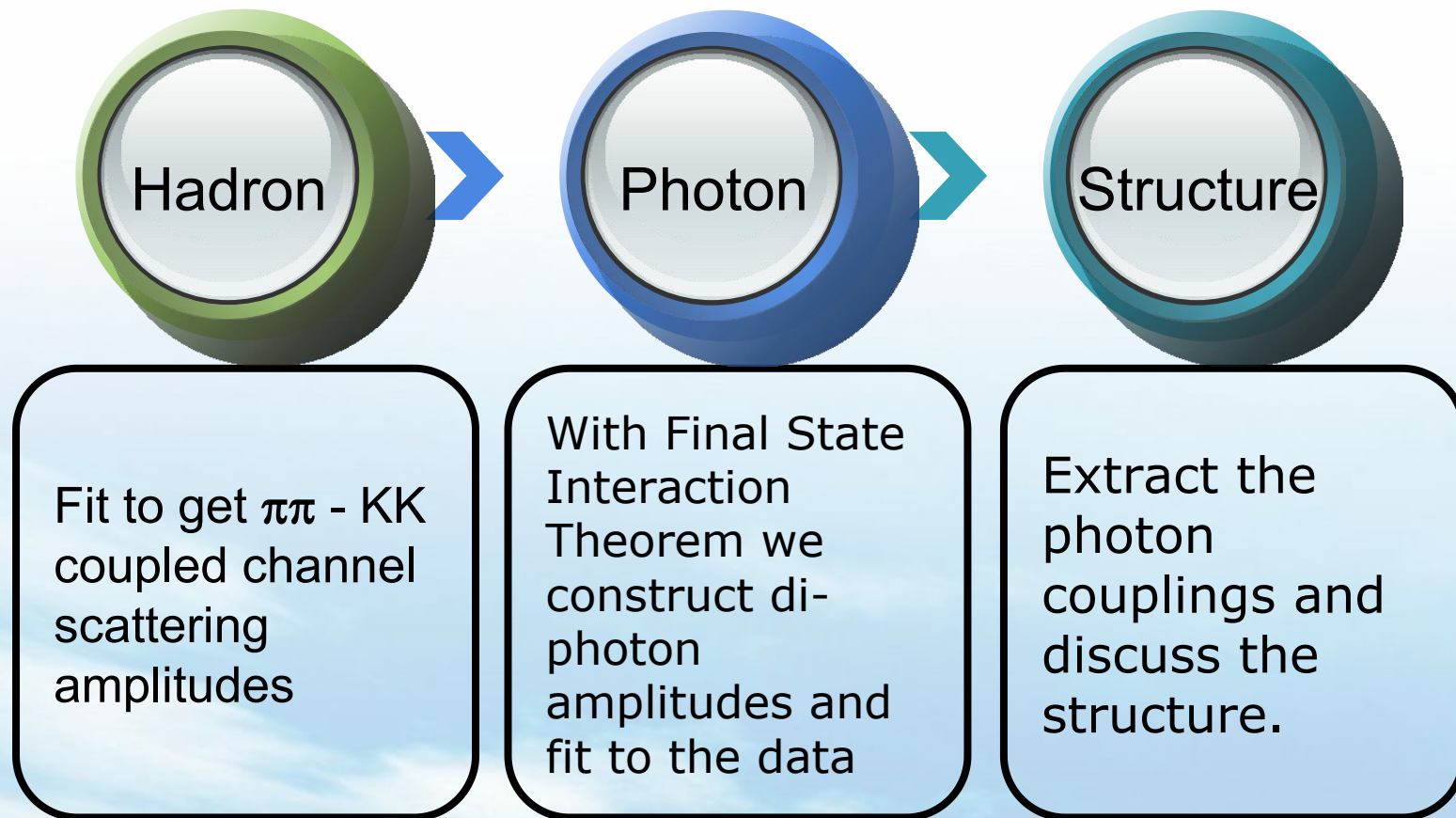
$n=0$ , linear confinement

# Scalars

- What is scalar? The same quantum number with QCD vacuum.



# Strategy



## 2. Hadronic scattering

- **$\pi\pi$ - $K\bar{K}$  coupled channel scattering amplitudes**

- We use K-matrix to represent S and D partial waves, with Isospin=0 .

$$T = \frac{K}{1 - i\rho K}$$

- For Isospin 2 partial waves we use the parametrization of Constraint Fits to Data IV (CFDIV).

→ Pelaez *et al.*  
PRD83 (2011) 074004

# Hadronic scattering

- $\pi\pi - K\bar{K}$  scattering inputs
  - Data on Phase shifts and inelasticities of  $\pi\pi - K\bar{K}$  coupled channel scattering.
  - Dispersion analysis based on symmetry and fit to data:
    - T-matrix of  $\pi\pi$  scattering by CFDIV) .
    - $\pi\pi \rightarrow K\bar{K}$  amplitudes given by Roy-Steiner Equation.

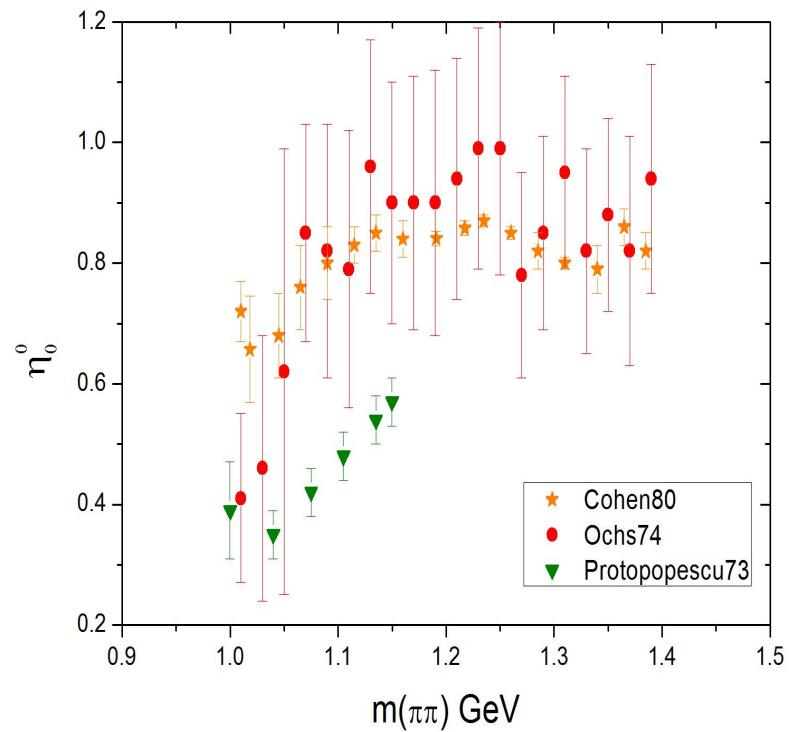
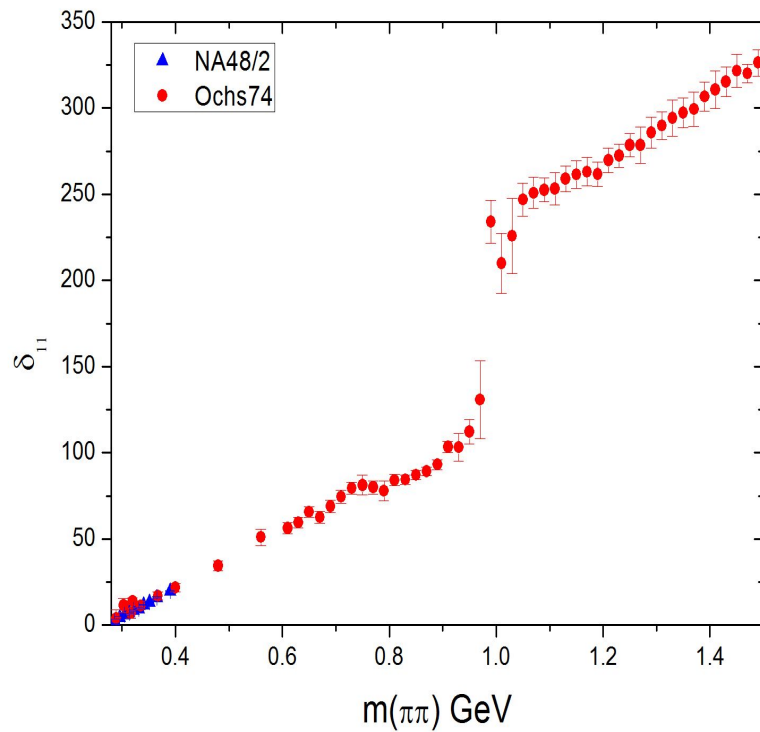
Descotes *et.al*  
EPJC33 (2004) 409

# Hadronic amplitudes

- **$\pi\pi$  scattering inputs**
  - BABAR's Dalitz plot analysis of  $D_s^+ \rightarrow (\pi^+\pi^-)\pi^+$  and  $D_s^+ \rightarrow (K^+K^-)\pi^+$  process.

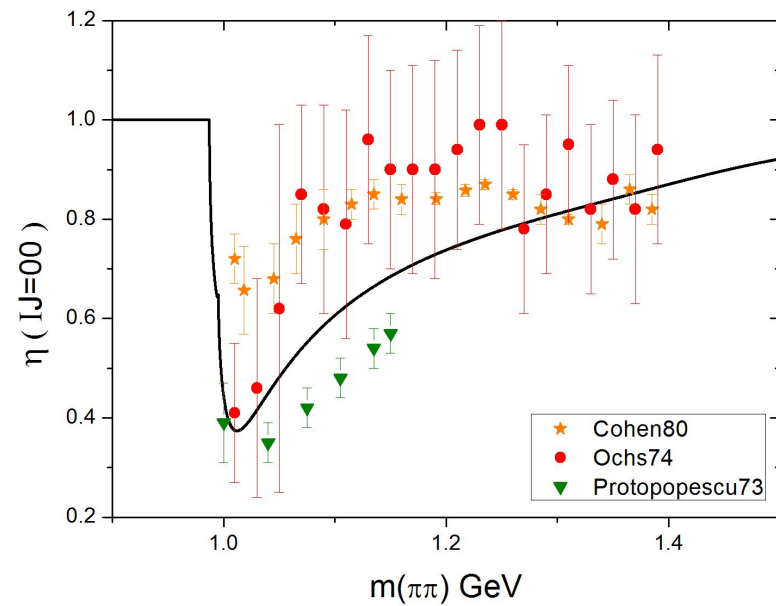
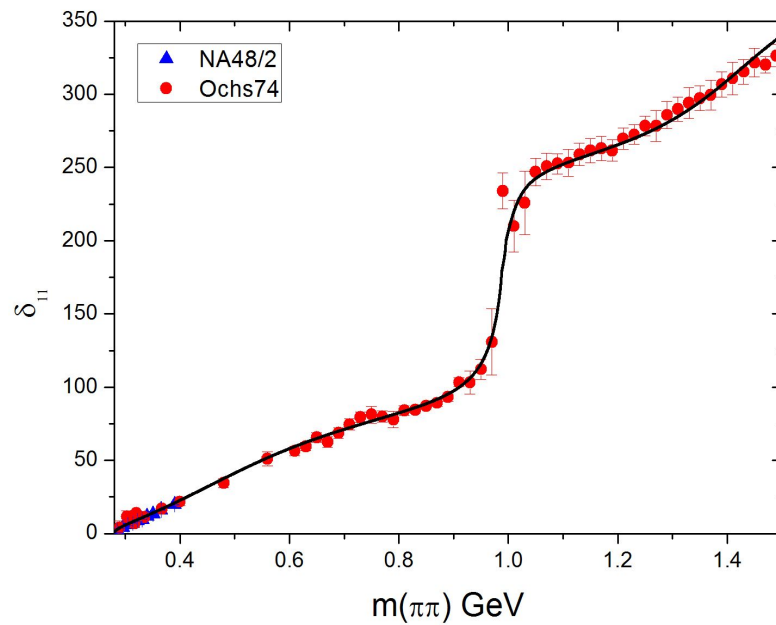
# Data: phase shift and inelasticity

## ■ $\pi\pi$ phase shift and inelasticity



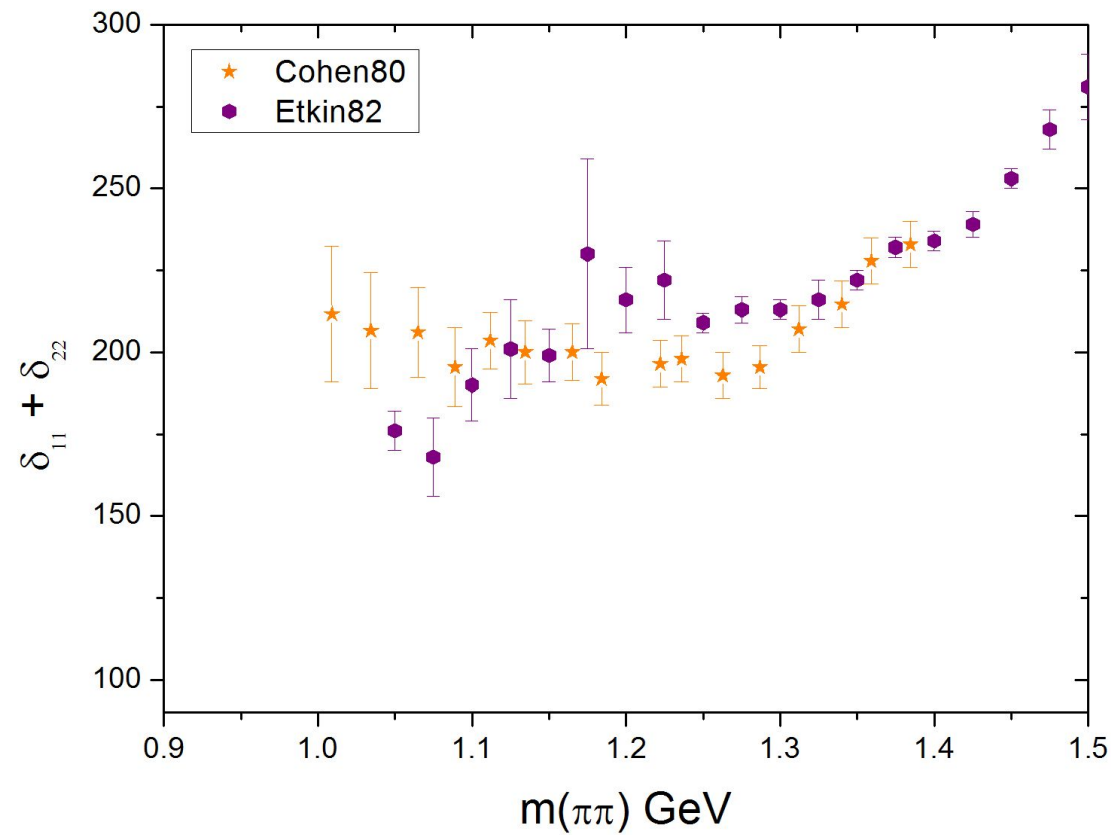
# Data: phase shift and inelasticity

## ■ Our fit



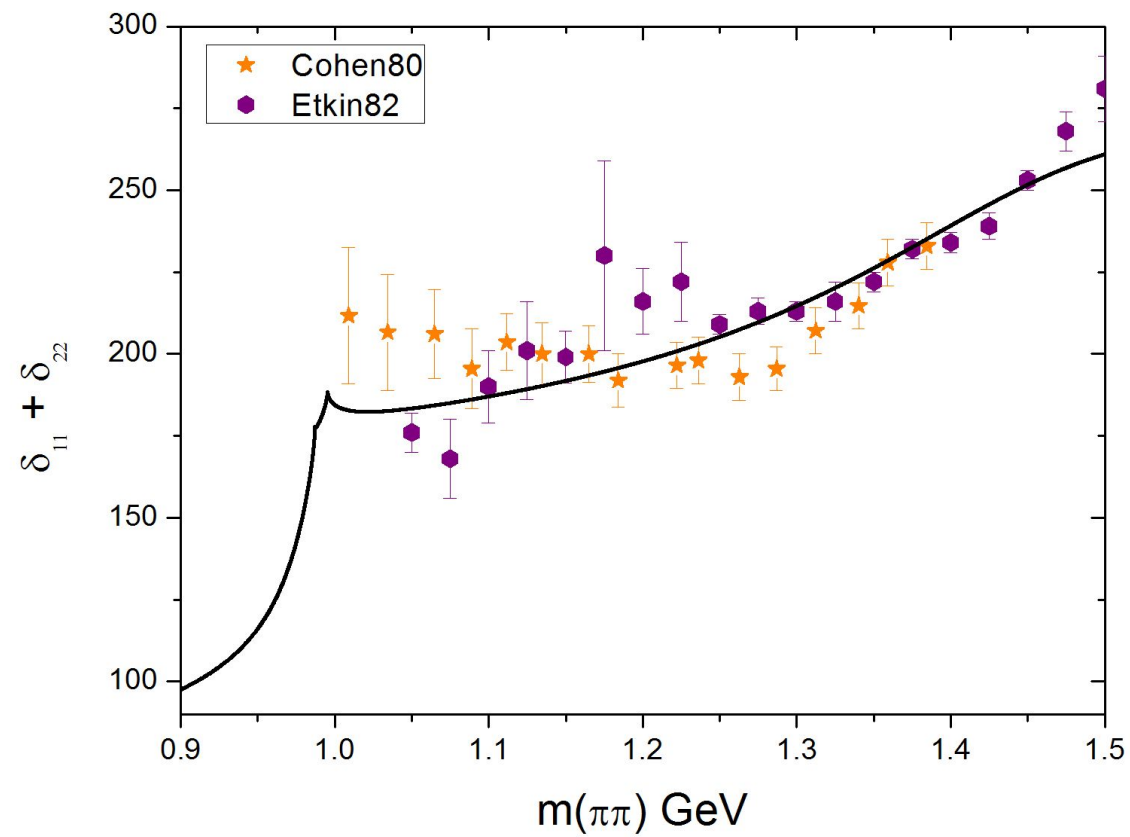
## Data: phase shift and inelasticity

- $\pi\pi \rightarrow K\bar{K}$  phase shift



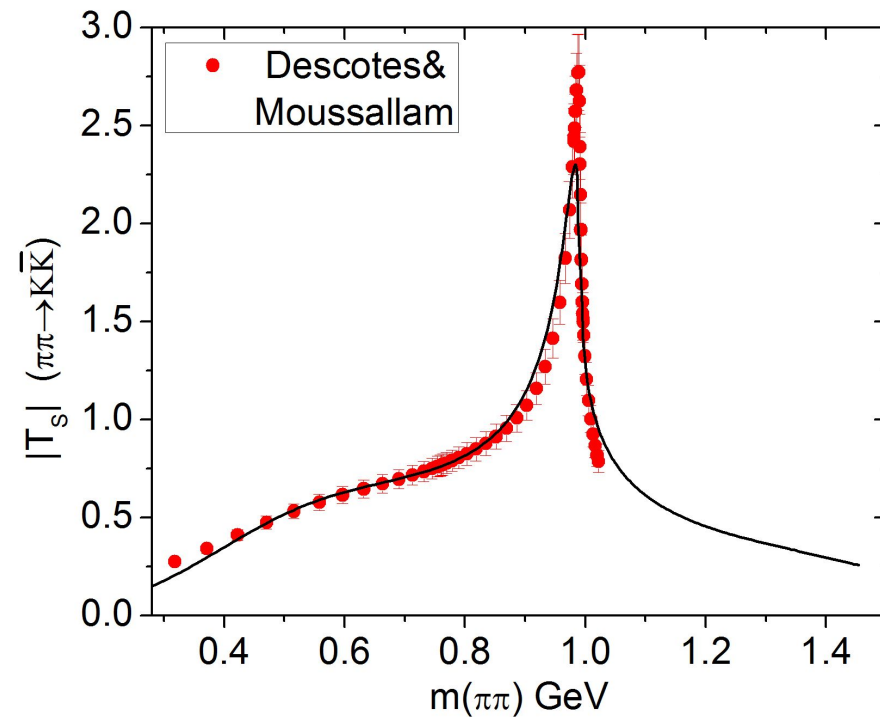
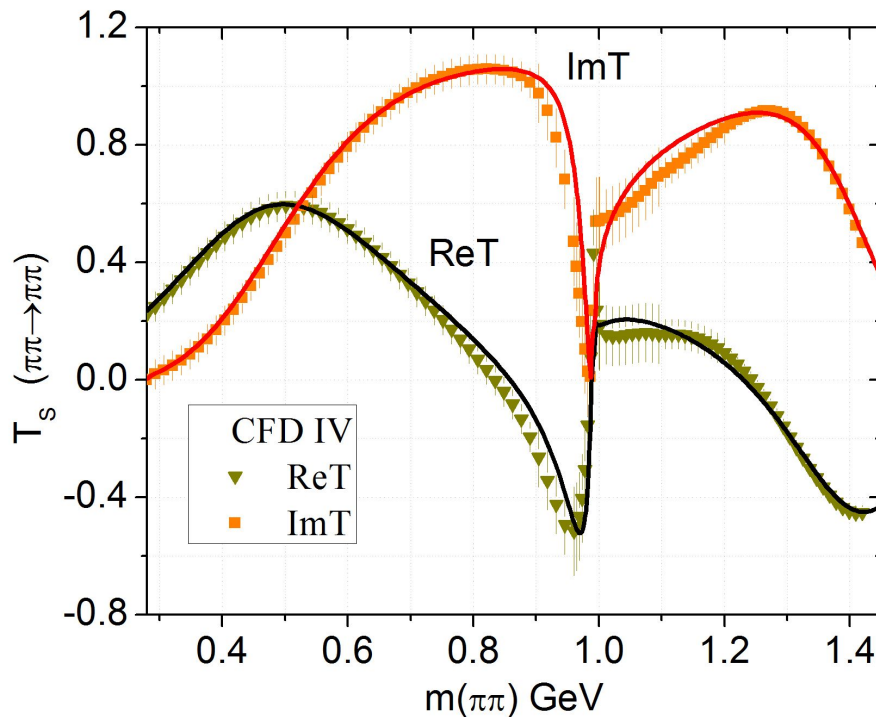
# Data: phase shift and inelasticity

- our fit



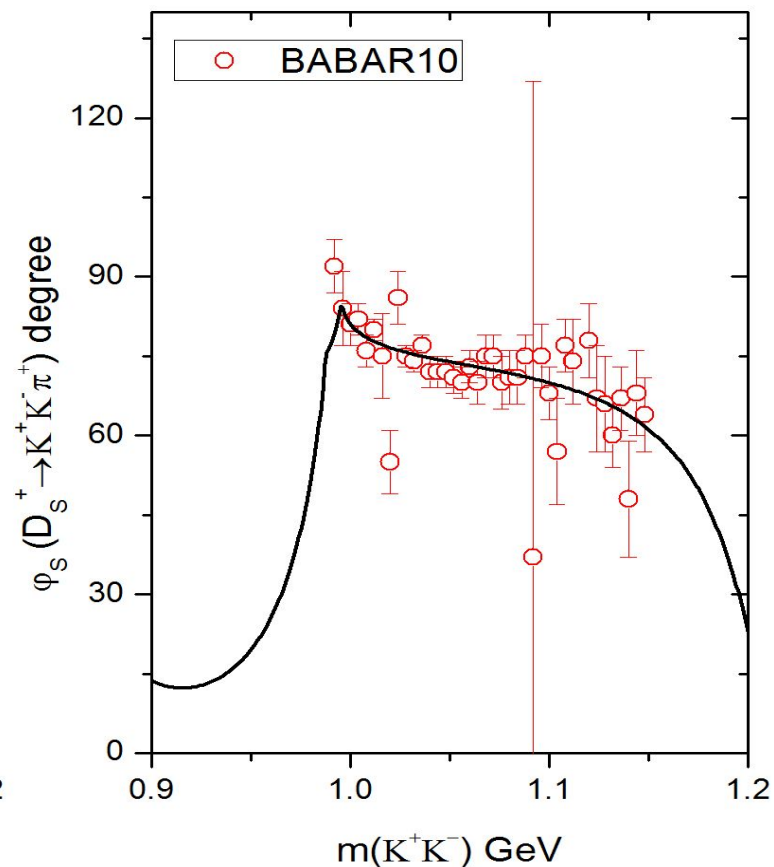
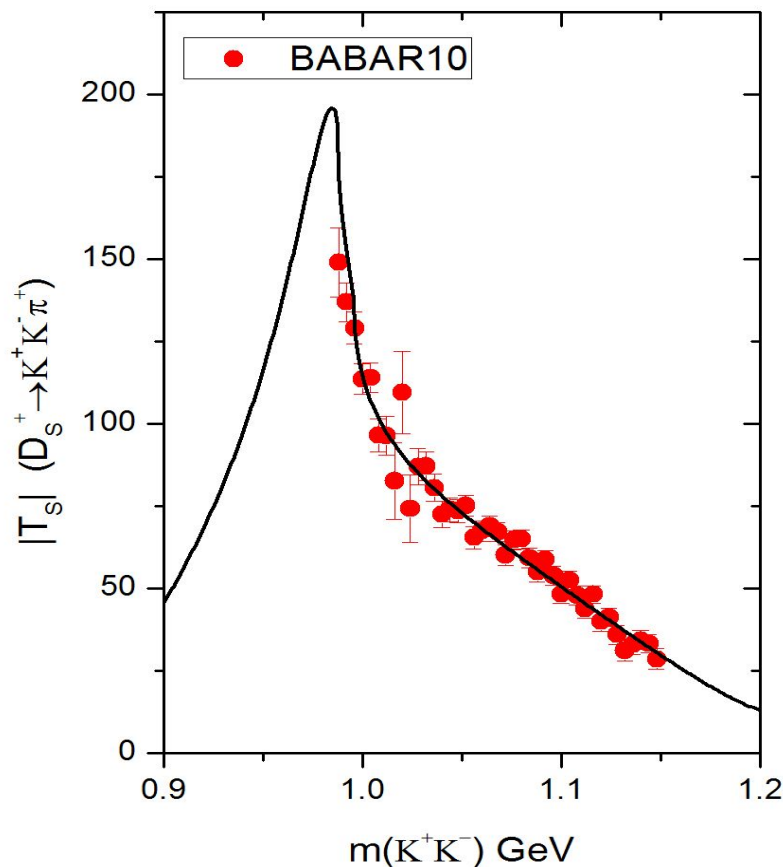
# Dispersion analysis constraints

- They use Roy like equation and take crossing symmetry, unitarity into account.



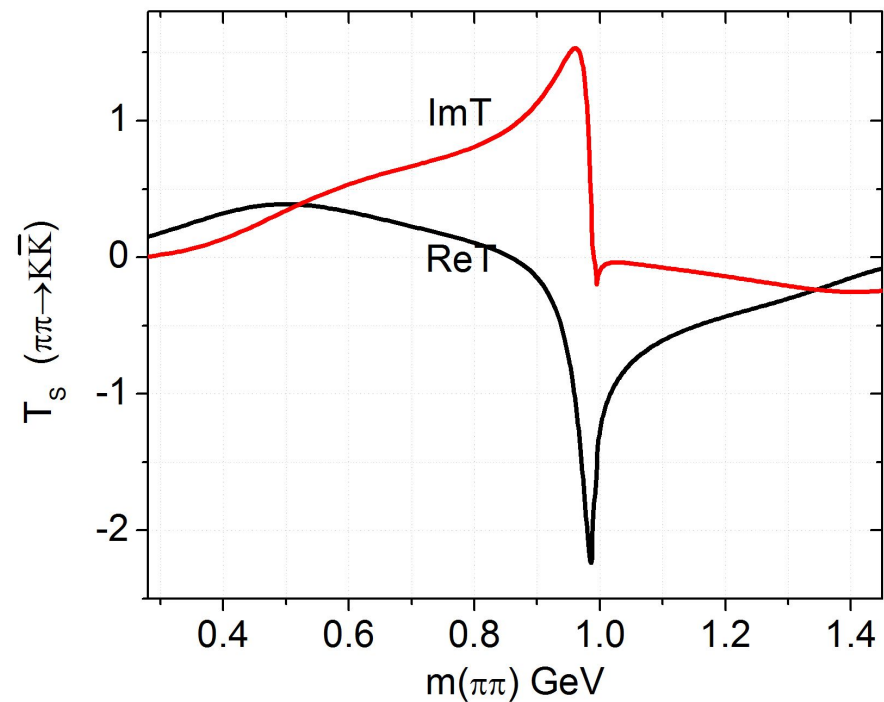
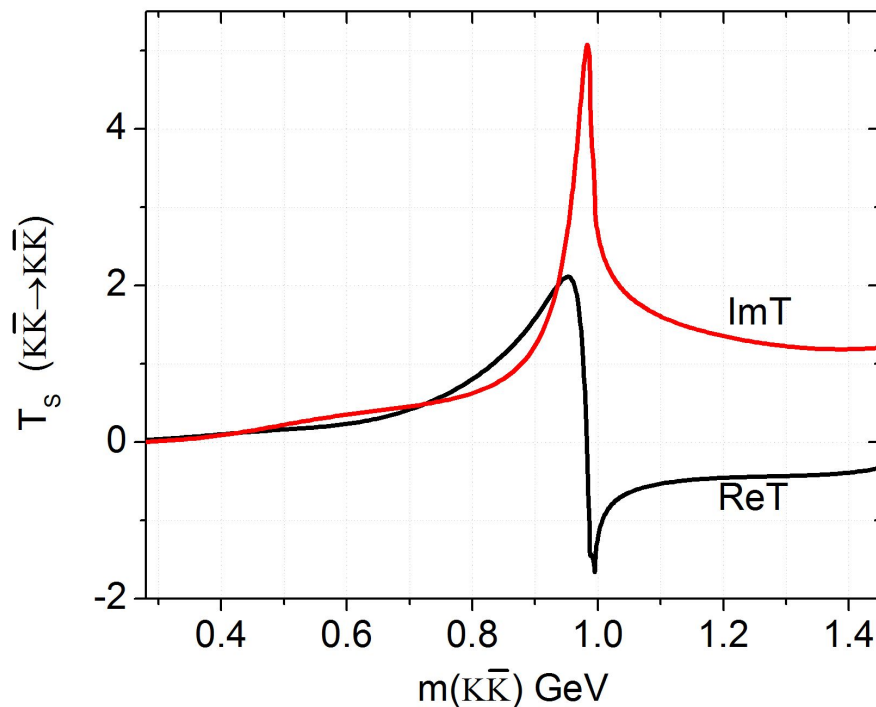
## BABAR's Dalitz plot analysis

- $K\bar{K}$  threshold region is quite important as it is around  $f_0(980)$ .



## Final T matrix

- We only list  $\pi\pi$ -KK coupled channel S-wave here, for Isospin 2 waves we use CFDIV.
- The  $\pi\pi - \pi\pi$  amplitude has been given in the past page.



# Poles from hadronic amplitudes

- pole locations in different Riemann sheets. It also gives hints for structure.

Morgan NPA543 (1992) 632.  
Pennington arXiv:1003.2549 [hep-ph]  
KK molecule?

Dai, Wang and Zheng  
CTP57 (2012) 841, CTP58  
(2012) 410  
Breit-Wigner origin?

| Resonance   | II               | III              |
|-------------|------------------|------------------|
| $\sigma$    | $0.441 - i0.272$ | $0.386 - i0.108$ |
| $f_0(980)$  | $0.995 - i0.042$ |                  |
| $f_0(1370)$ | $1.423 - i0.177$ | $1.406 - i0.344$ |
| $f_2(1270)$ | $1.270 - i0.081$ | $1.267 - i0.108$ |

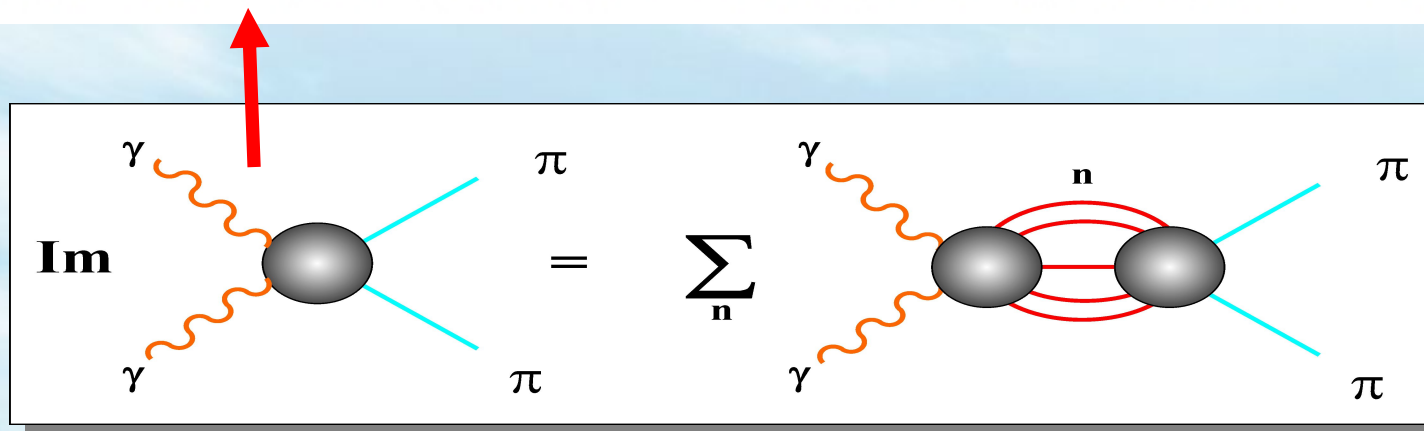
Table 4: Resonance pole positions on  $\sqrt{s}$  plane in unit of GeV.

### 3. Photon-photon collision

- With Final State Interaction Theorem (FSIT),  
We can construct the amplitudes of photon-photon scattering into meson pairs:

$$\mathcal{F}_{J\lambda}^I(\gamma\gamma \rightarrow \pi\pi; s) = \alpha_{1J\lambda}^I(s) \hat{T}_J^I(\pi\pi \rightarrow \pi\pi; s) + \alpha_{2J\lambda}^I(s) \hat{T}_J^I(\pi\pi \rightarrow \bar{K}K; s),$$

$$\mathcal{F}_{J\lambda}^I(\gamma\gamma \rightarrow \bar{K}K; s) = \alpha_{1J\lambda}^I(s) \hat{T}_J^I(\pi\pi \rightarrow \bar{K}K; s) + \alpha_{2J\lambda}^I(s) \hat{T}_J^I(\bar{K}K \rightarrow \bar{K}K; s).$$



# Photon-photon collision

- To constraint the di-photon amplitudes, we follow such steps:
  - We use dispersion relation to calculate the amplitudes below 0.6GeV, and give errors.
  - Fit the overall  $\gamma\gamma\rightarrow\pi\pi$  and  $\gamma\gamma\rightarrow K\bar{K}$  datasets, get a very narrowed patch of solutions.

## Low energy di-photon amplitudes

- There are contribution of left hand cuts(LHCs), right hand cuts(RHCs) near real axis in  $F^I_{J\lambda}$ .
- According to FSIT, its phase should be the same as that of hadronic amplitudes. Thus we define

$$F^I_{J\lambda} = P^I_{J\lambda} \Omega^I_{J\lambda},$$

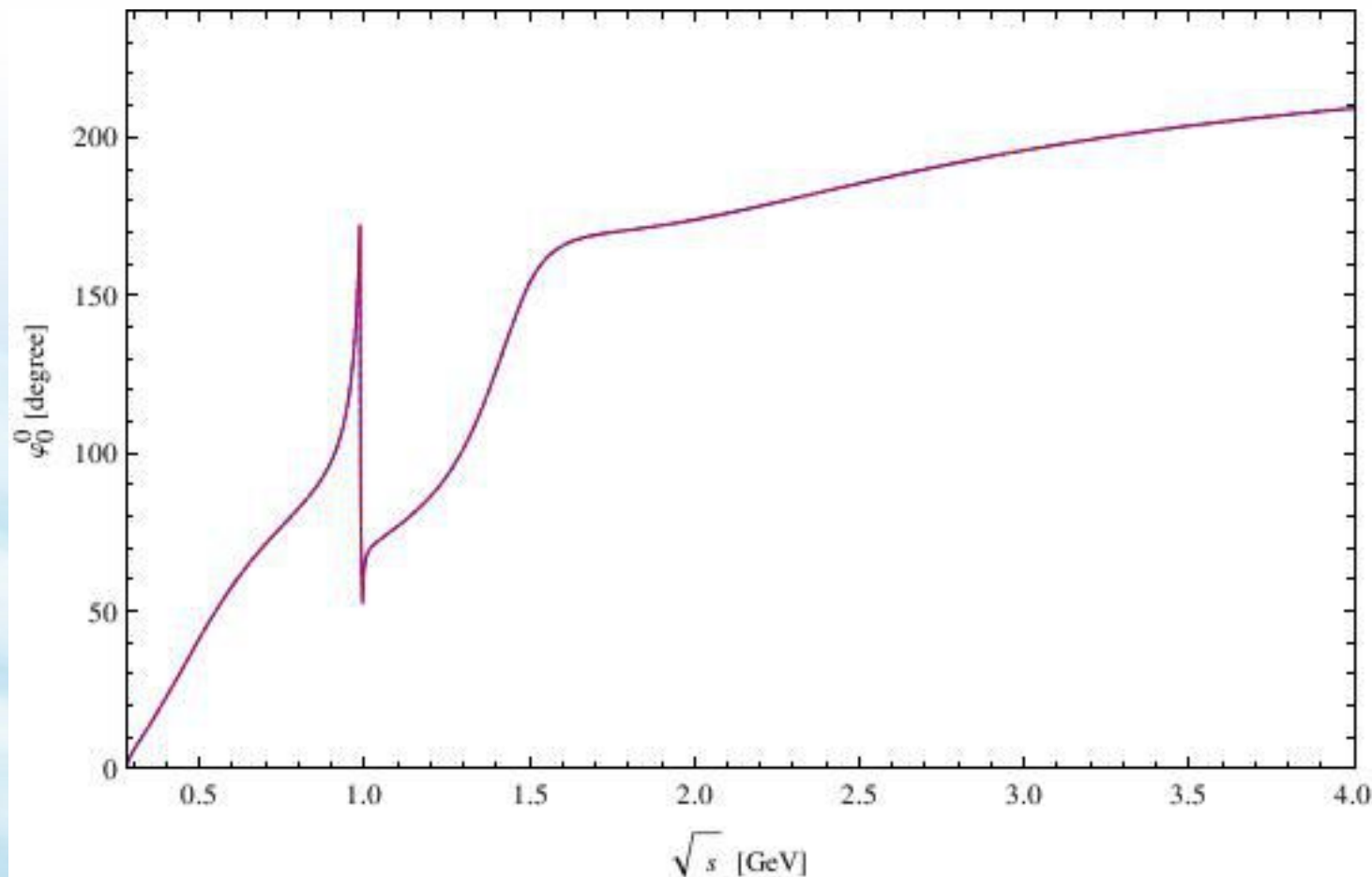
where  $P^I_{J\lambda}$  has only LHCs, and  $\Omega^I_{J\lambda}$ , contains RHCs.

$$\Omega^I_{J\lambda}(s) = \exp \left( \frac{s}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\varphi^I_{J\lambda}(s')}{s'(s' - s)} \right)$$

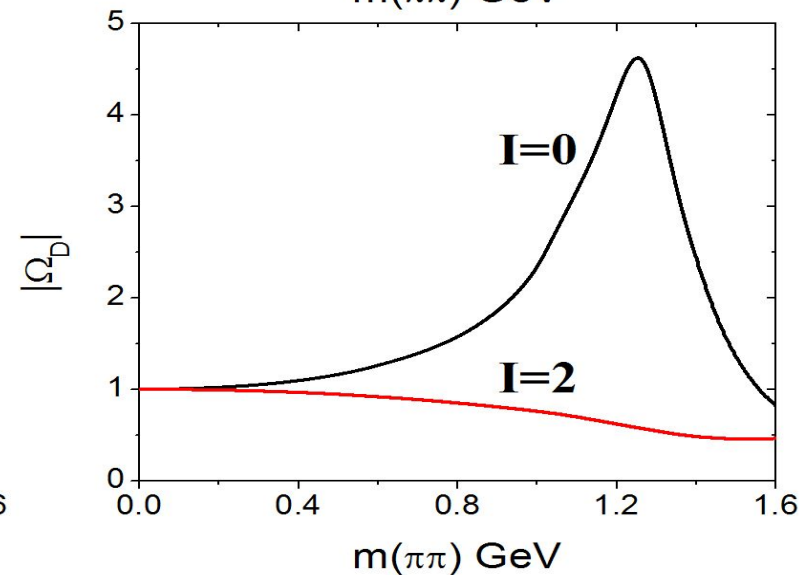
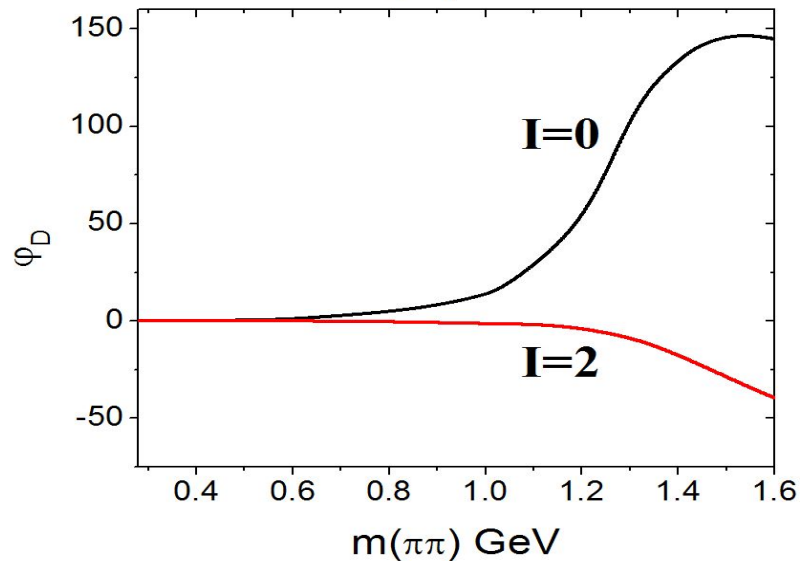
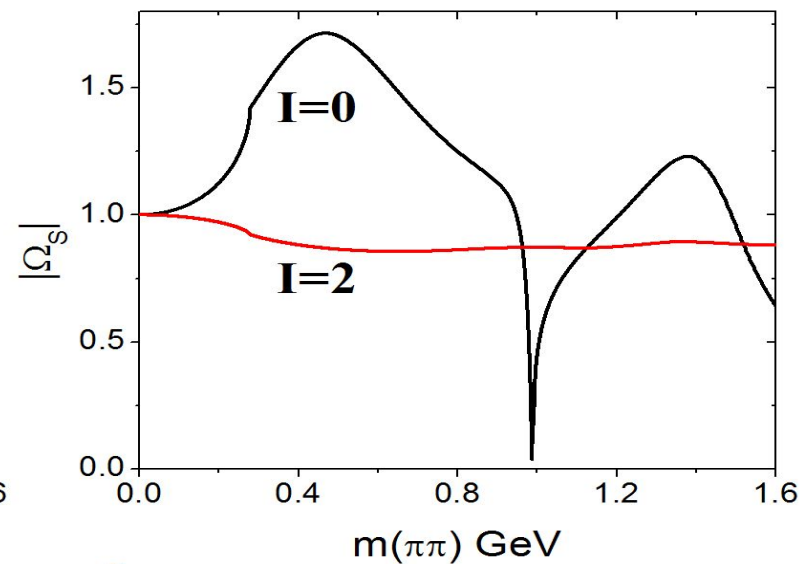
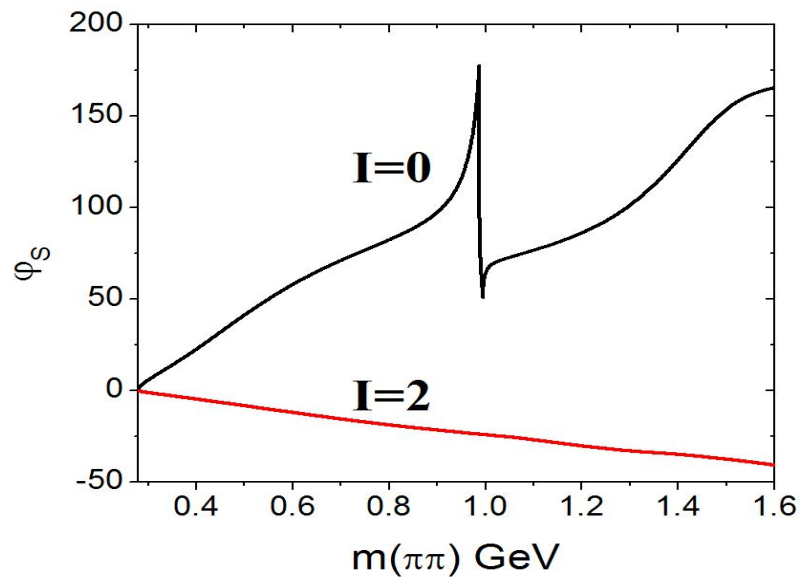
- Now  $\varphi^I_{J\lambda}$ , should be continued to high energy region, but it only give error bands for the low energy amplitudes.

## Low energy di-photon amplitudes

- Below 1.5 GeV it is from our fit, above 2 GeV from Regge behaviour.



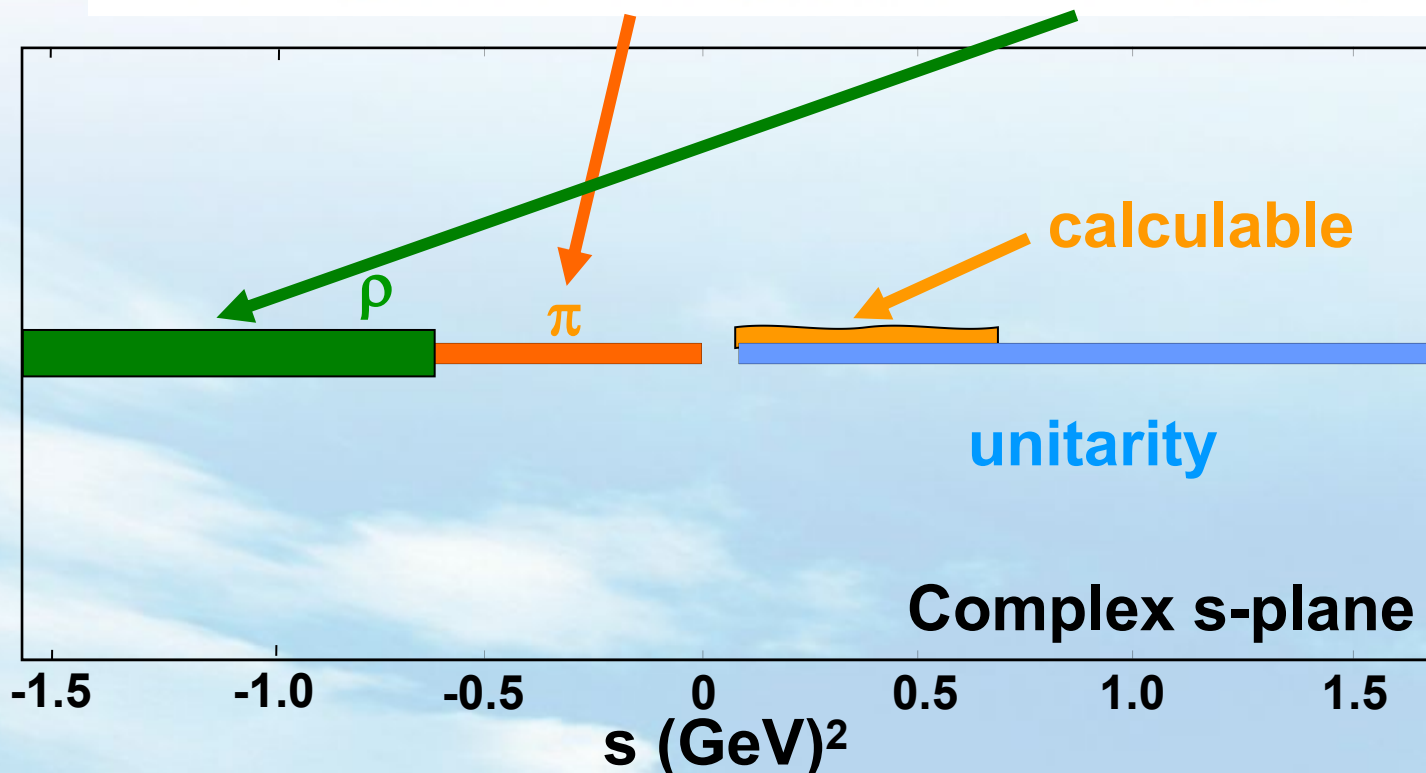
# Phases and Omnes function



## Dispersion relations

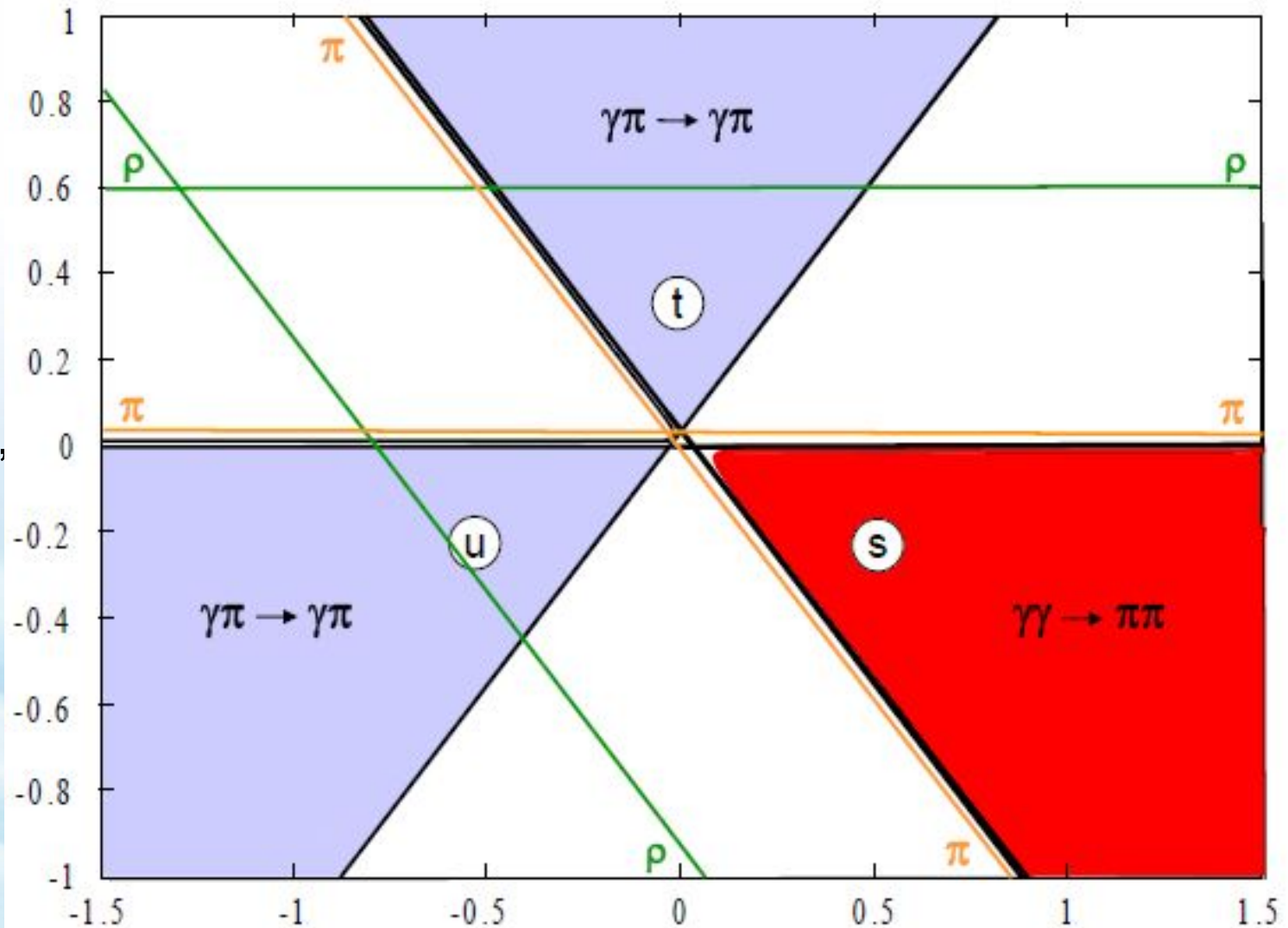
- Now what we need is the LHCs  $P_{J\lambda}^I$ , It is divided into two parts: Born term and other cross channel exchange terms. When  $s < 0$ :

$$\text{Im } \mathcal{F}_{J\lambda}^I = \text{Im } \mathcal{B}_{J\lambda}^I(s) + \text{Im } \mathcal{L}_{J\lambda}^I(s)$$



# Vector, Axial-Vector, Tensor contributions

- LHCs of  $\rho$ ,  $\omega$ ,  $a_1$ ,  $b_1$ ,  $h_1$  give an error band of low energy amplitudes.
- Remain parts are parametrized as an effective pole 'T'.



## Dispersion relations

- Low's low energy theorem tells us that:

$$\mathcal{F}_{J\lambda}^I(s) \rightarrow \mathcal{B}_{J\lambda}^I(s) \quad \text{as } s \rightarrow 0$$

- Thus we can write a twice subtracted Dispersion relation on:  $(F-B)\Omega^{-1}$

$$\mathcal{F}_{00}^I(s) = \mathcal{B}_{00}^I(s) + \underbrace{b^I}_{\text{Solved by ChPT}} s \Omega_{00}^I(s) + \frac{s^2 \Omega_{00}^I(s)}{\pi} \int_L ds' \frac{\text{Im} [\mathcal{L}_{00}^I(s')] \Omega_{00}^I(s')^{-1}}{s'^2(s' - s)} - \frac{s^2 \Omega_{00}^I(s)}{\pi} \int_R ds' \frac{\mathcal{B}_{00}^I(s') \text{Im} [\Omega_{00}^I(s')^{-1}]}{s'^2(s' - s)}$$

Solved by ChPT

## Amplitudes for D-waves

- For D waves, we have

$$\mathcal{F}_{20}^I - B_{20}^I \sim s^2(s - 4m_\pi^2), \quad \mathcal{F}_{22}^I - B_{22}^I \sim s(s - 4m_\pi^2),$$

- Thus we write dispersion relations on

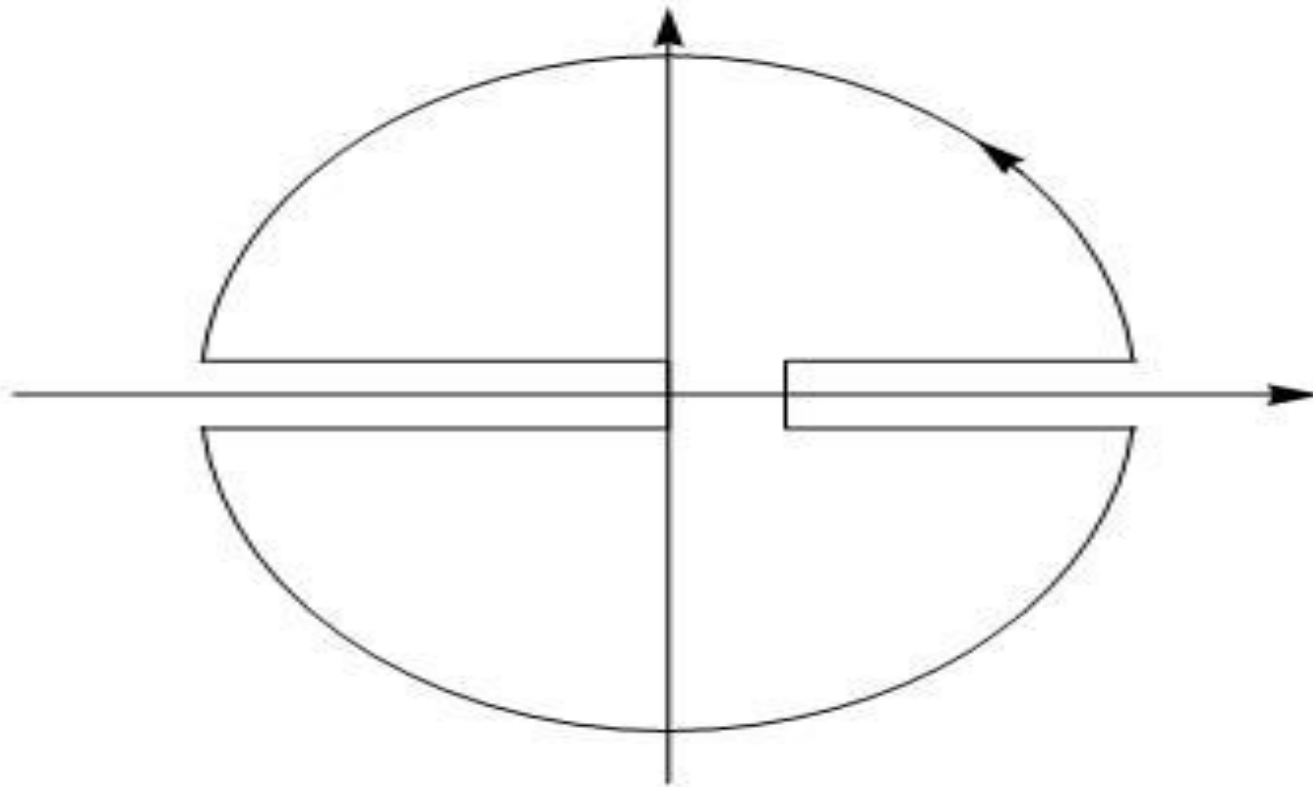
$$(\mathcal{F}(s) - \mathcal{B}(s))\Omega^{-1}(s)/s^n(s - 4m_\pi^2)^{J/2}$$

- and get

$$\begin{aligned} \mathcal{F}_{J\lambda}^I(s) = & \mathcal{B}_{J\lambda}^I(s) + \frac{s^n(s - 4m_\pi^2)^{J/2}}{\pi} \Omega_{J\lambda}^I(s) \int_L ds' \frac{\text{Im} [\mathcal{L}_{J\lambda}^I(s')] \Omega_{J\lambda}^I(s')^{-1}}{s'^n(s' - 4m_\pi^2)(s' - s)} \\ & - \frac{s^n(s - 4m_\pi^2)^{J/2}}{\pi} \Omega_{J\lambda}^I(s) \int_R ds' \frac{B_{J\lambda}^I(s') \text{Im} [\Omega_{J\lambda}^I(s')^{-1}]}{s'^n(s' - 4m_\pi^2)(s' - s)}. \end{aligned}$$

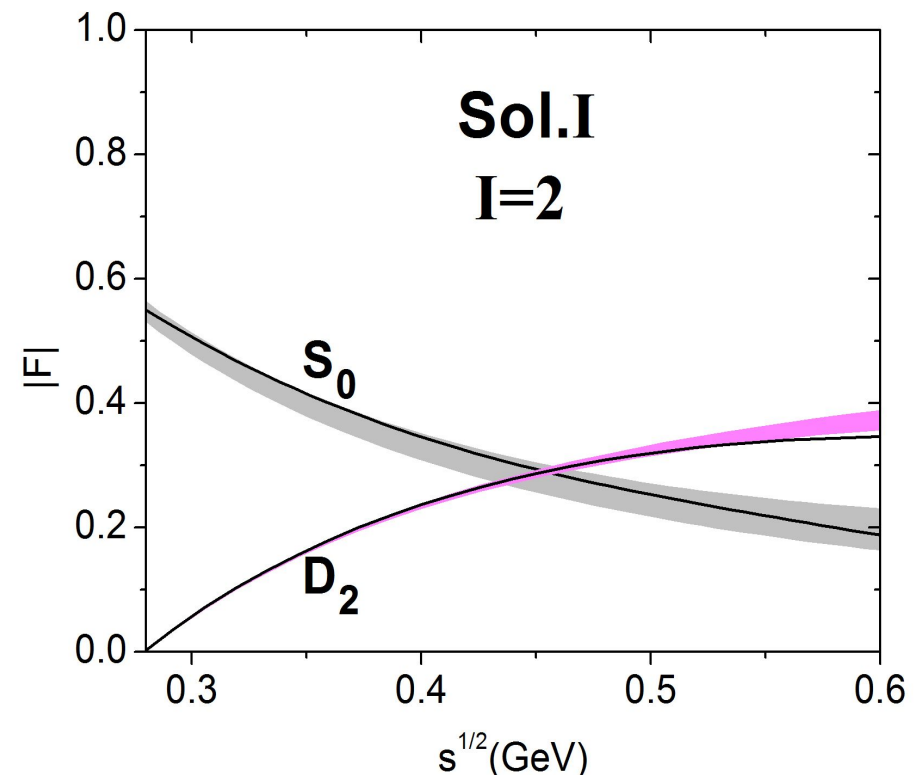
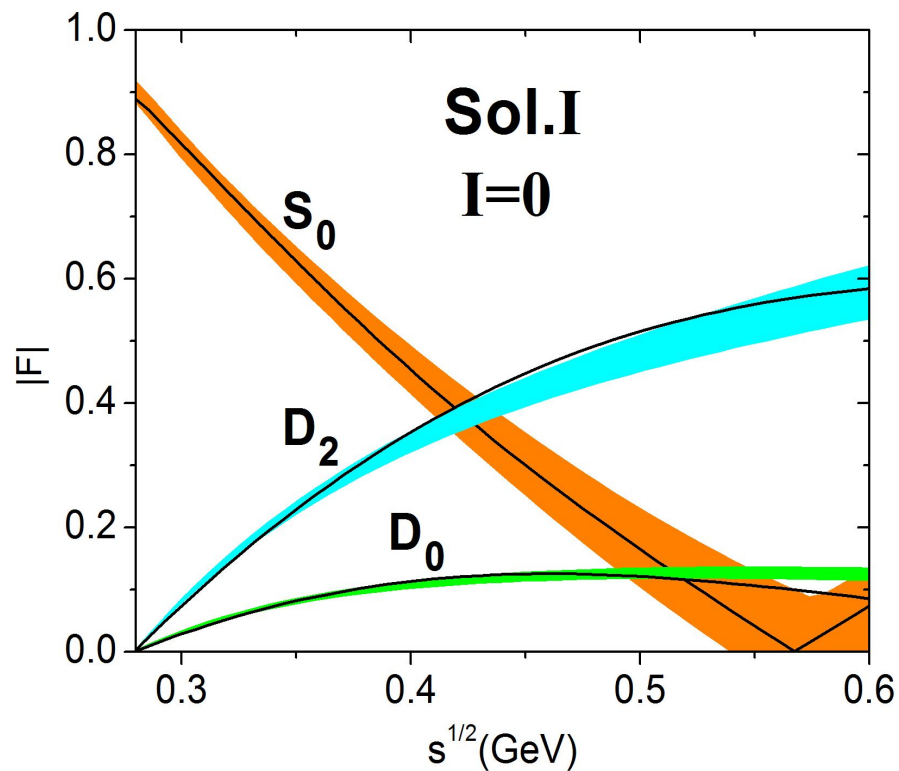
## Dispersion relation

- Now we get information in complex plane.



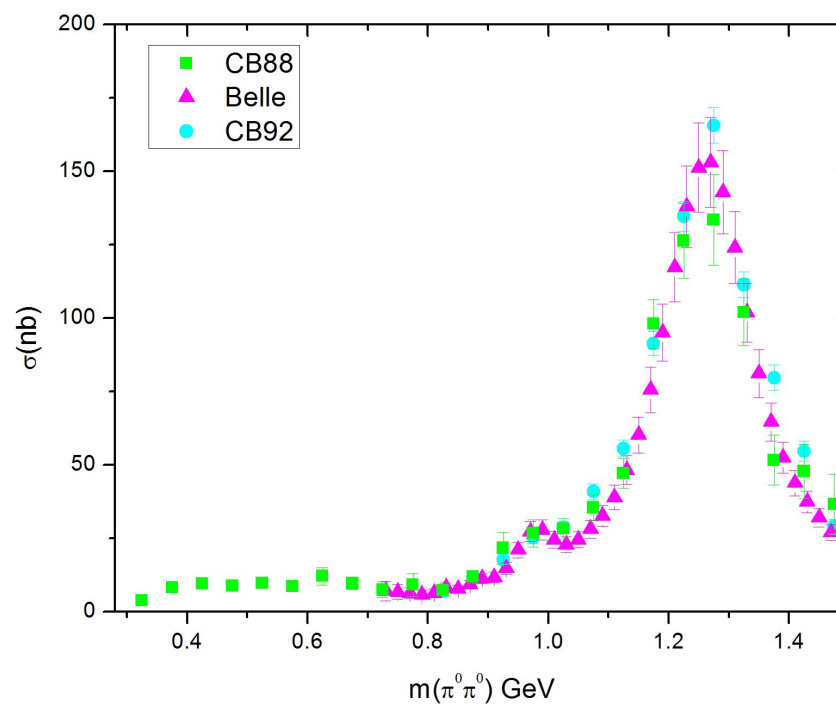
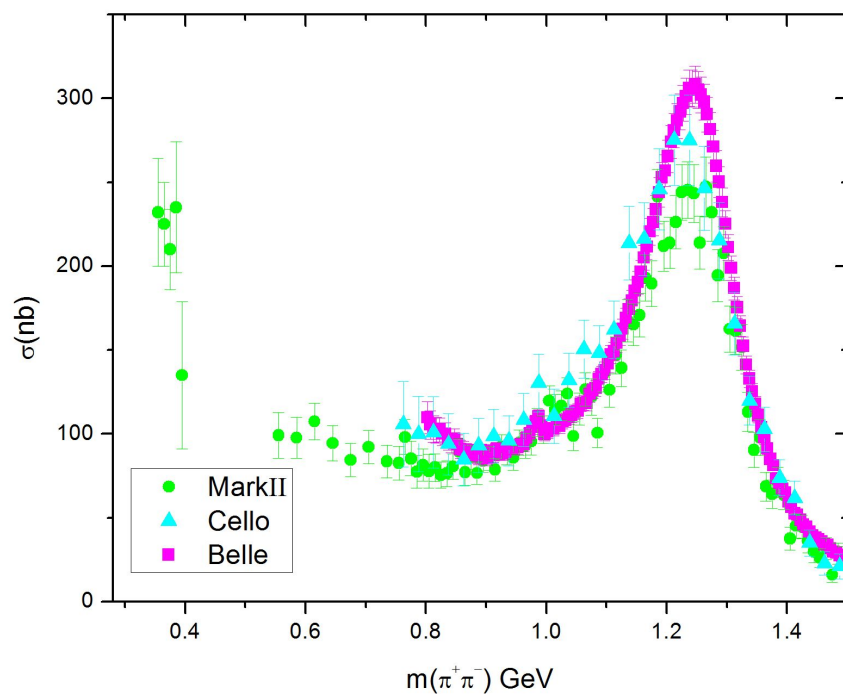
# Constraints on low energy amplitudes

- Finally we have the bands given by dispersion relations:

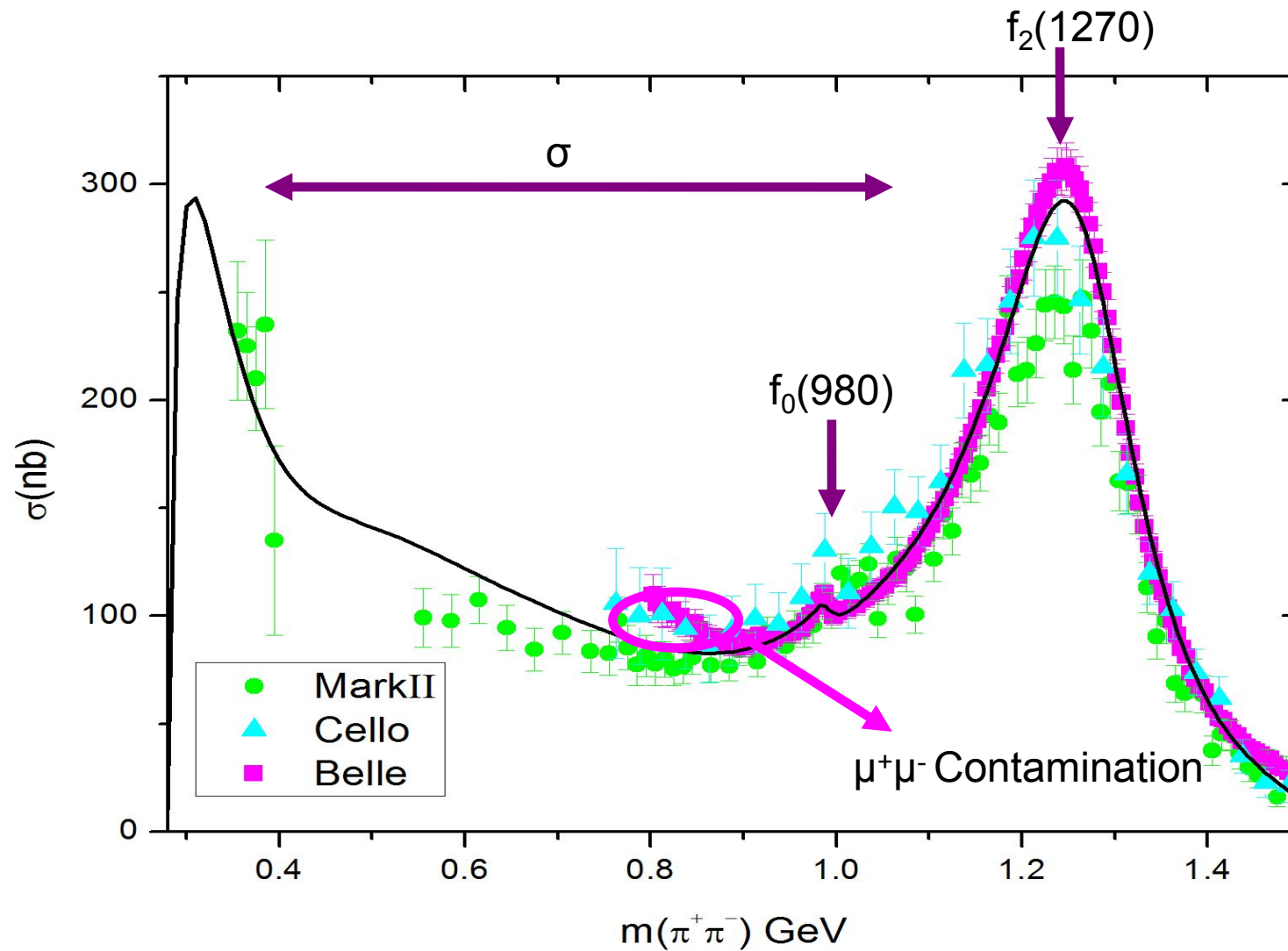


## $\gamma\gamma \rightarrow \pi\pi$ fit

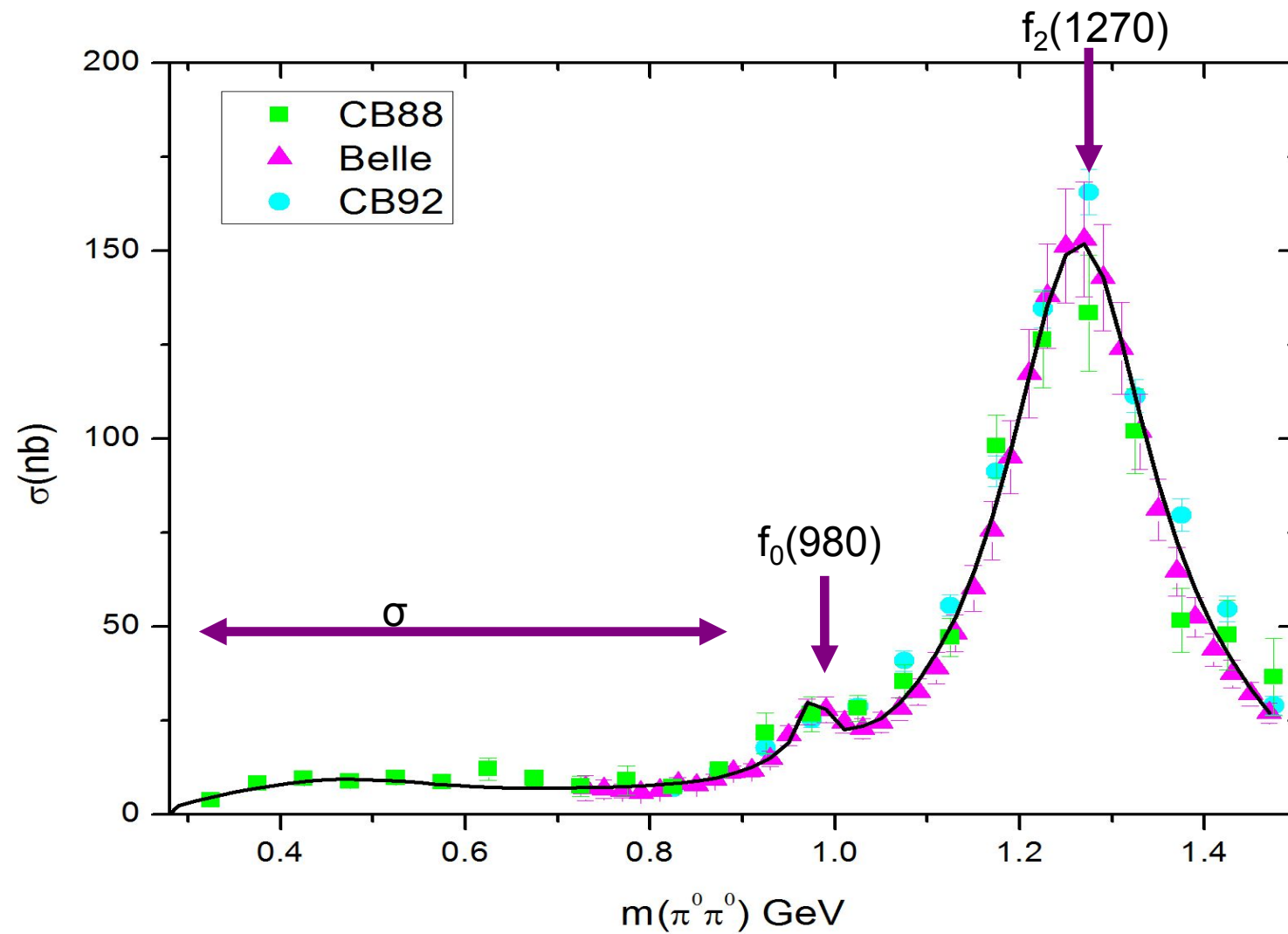
- With these constraints, we fit all datasets.
  - Here we present the integrated cross sections with limited angular coverage.



# $\gamma\gamma \rightarrow \pi^+\pi^-$ integrated cross section

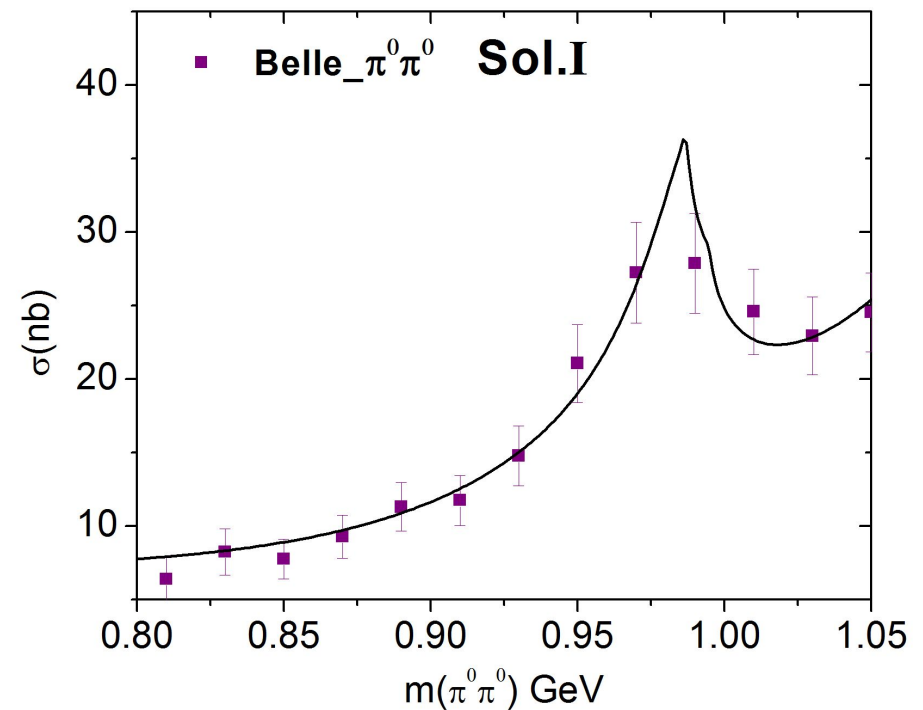
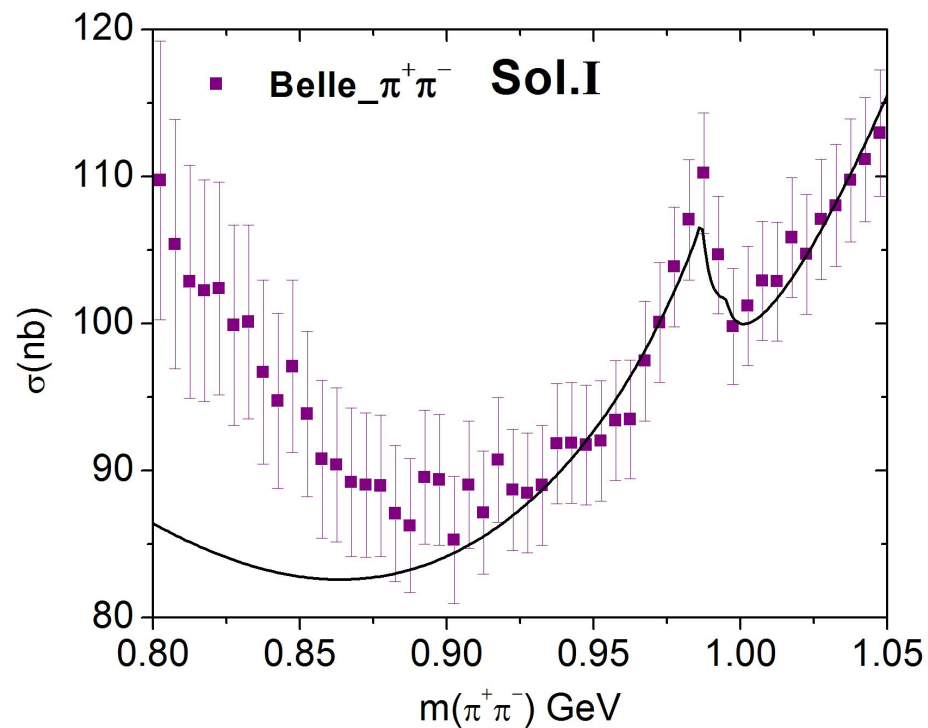


# $\gamma\gamma \rightarrow \pi^0\pi^0$ integrated cross section



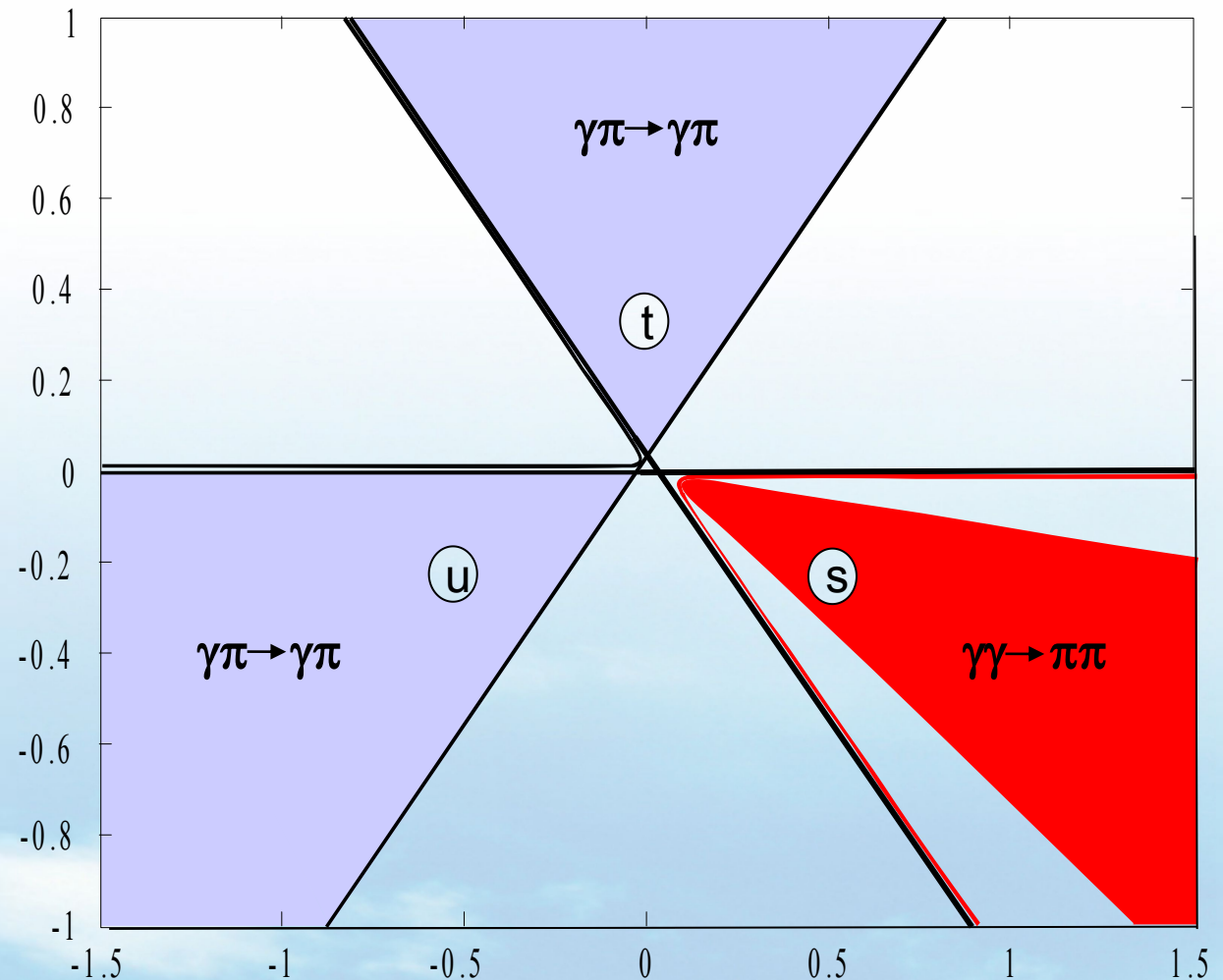
## integrated cross section around $f_0(980)$

- A small peak near  $f_0(980)$



## Why angular distribution?

- why angular distribution is so important when the integrated cross section has been fitted?
- We only have the limited angular distribution, is it possible to predict the full coverage ones?



## Differential cross section

- The differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\rho(s)}{128\pi^2 s} [ |M_{+-}|^2 + |M_{++}|^2 ]$$

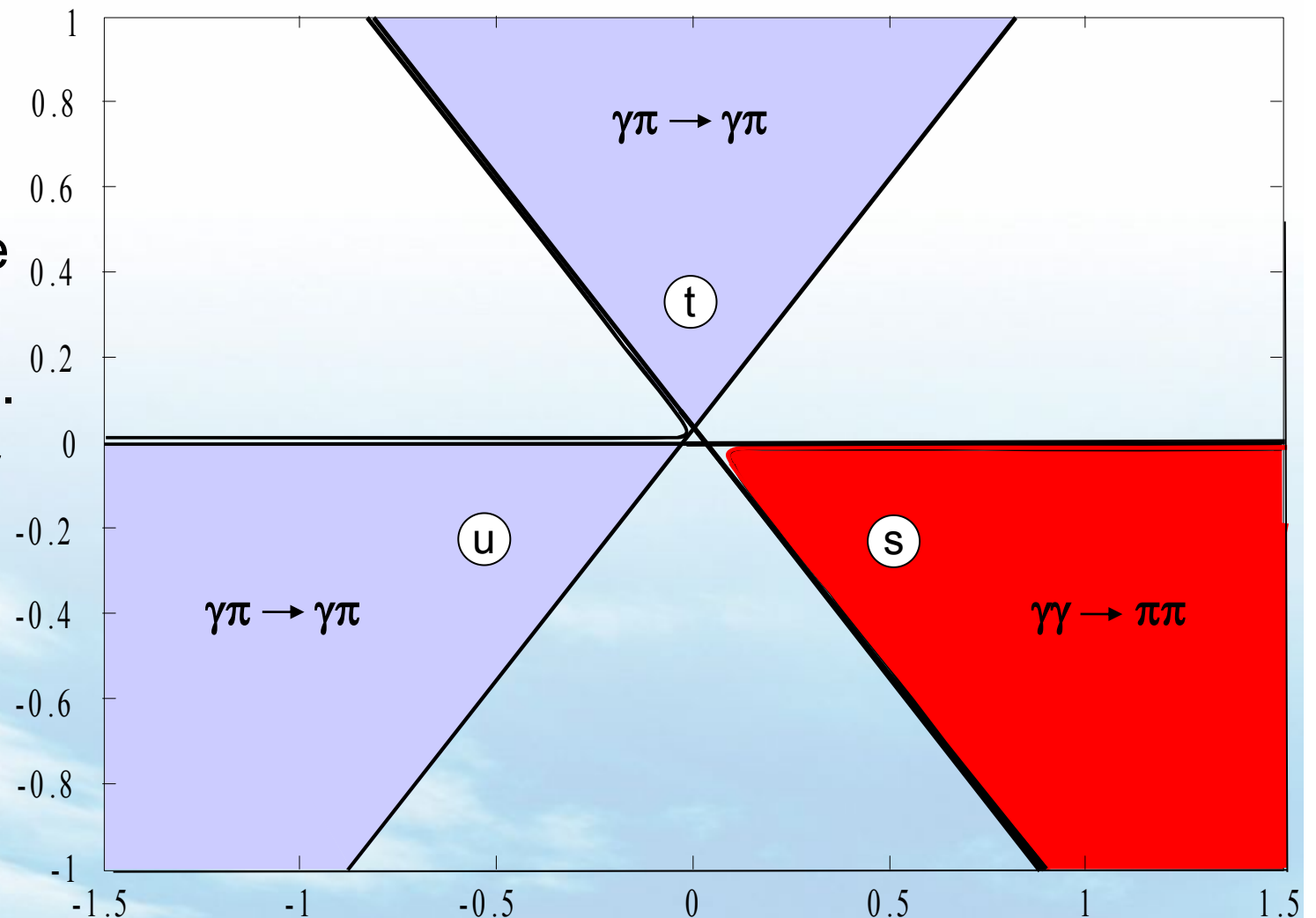
- And the helicity amplitudes are given by

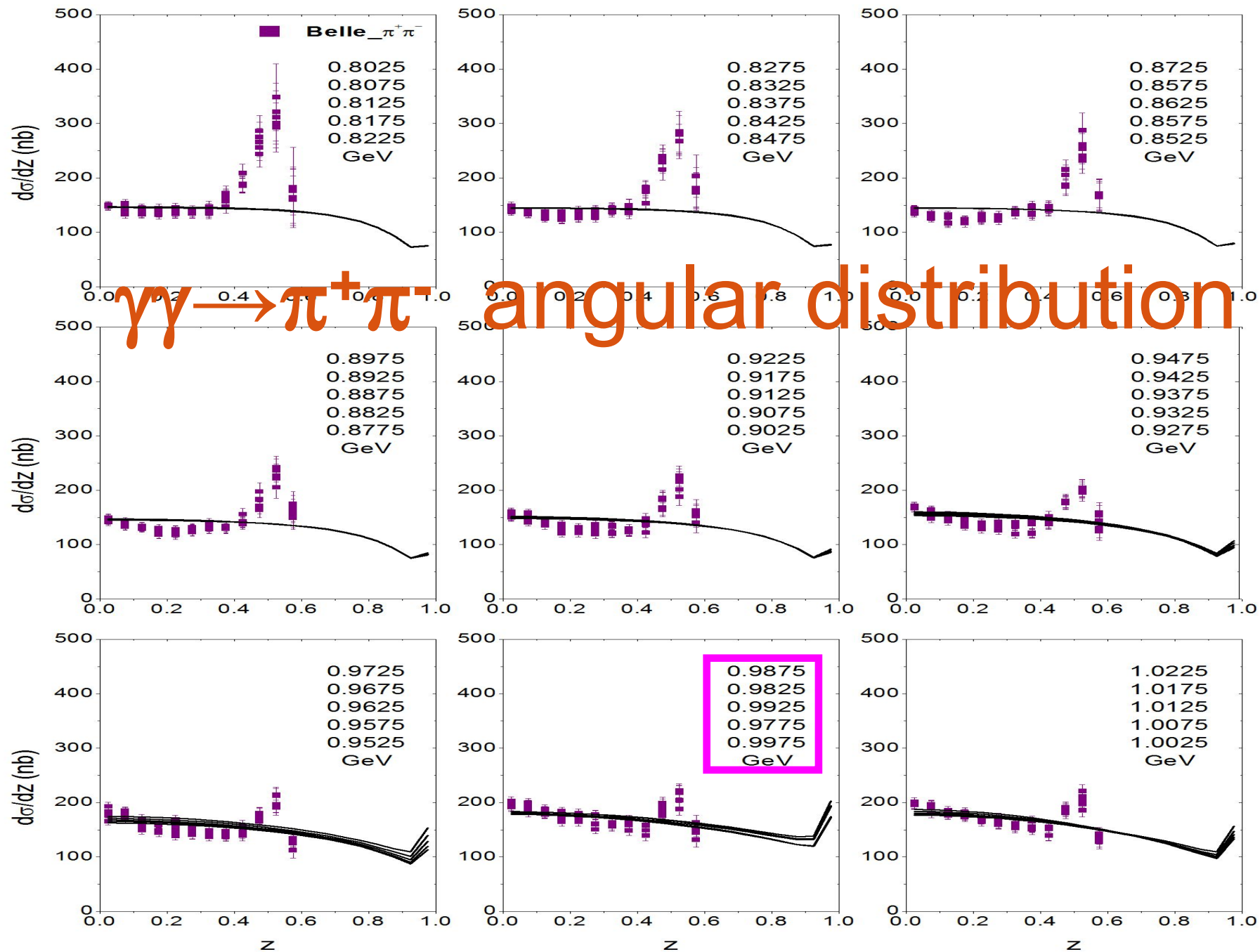
$$M_{++}(s, \theta, \phi) = e^2 \sqrt{16\pi} \sum_{J \geq 0} F_{J0}(s) Y_{J0}(\theta, \phi) ,$$
$$M_{+-}(s, \theta, \phi) = e^2 \sqrt{16\pi} \sum_{J \geq 2} F_{J2}(s) Y_{J2}(\theta, \phi) .$$

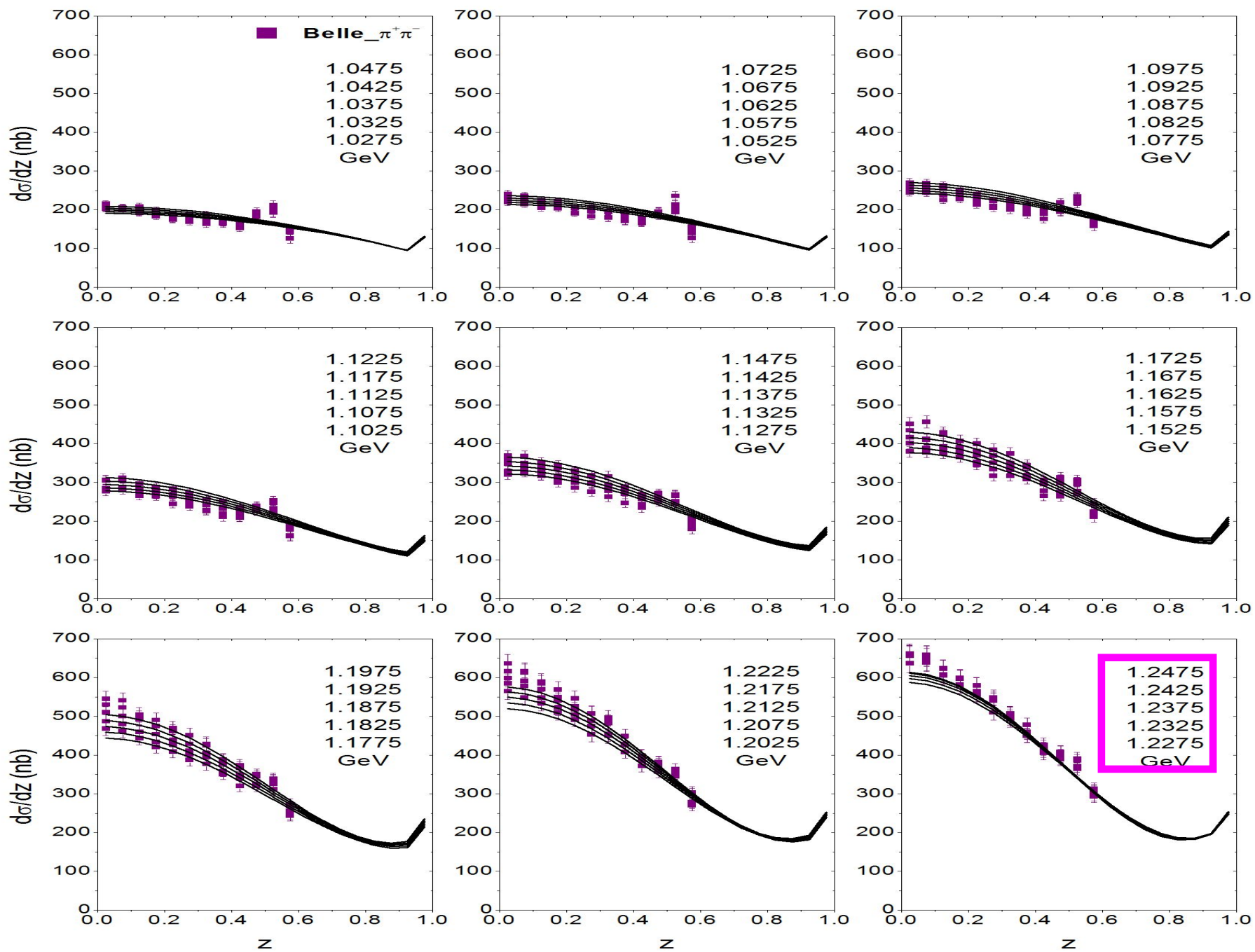
- For the integrated cross section, S-D<sub>0</sub> waves will interfere strongly through 1-1.4GeV.
- Angular distribution it is more efficient to distinguish each ipartial wave.

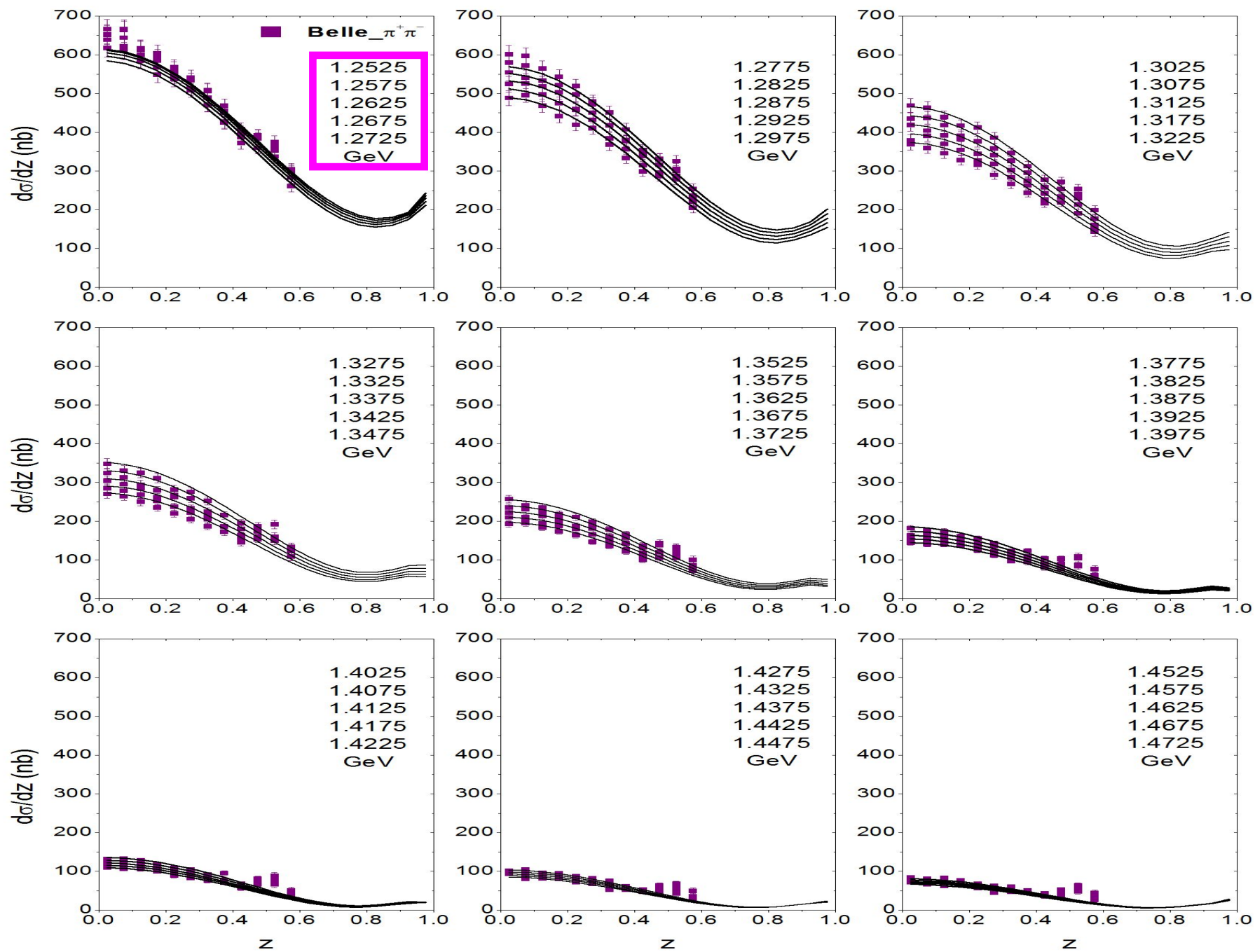
## $\gamma\gamma \rightarrow \pi\pi$ angular distribution

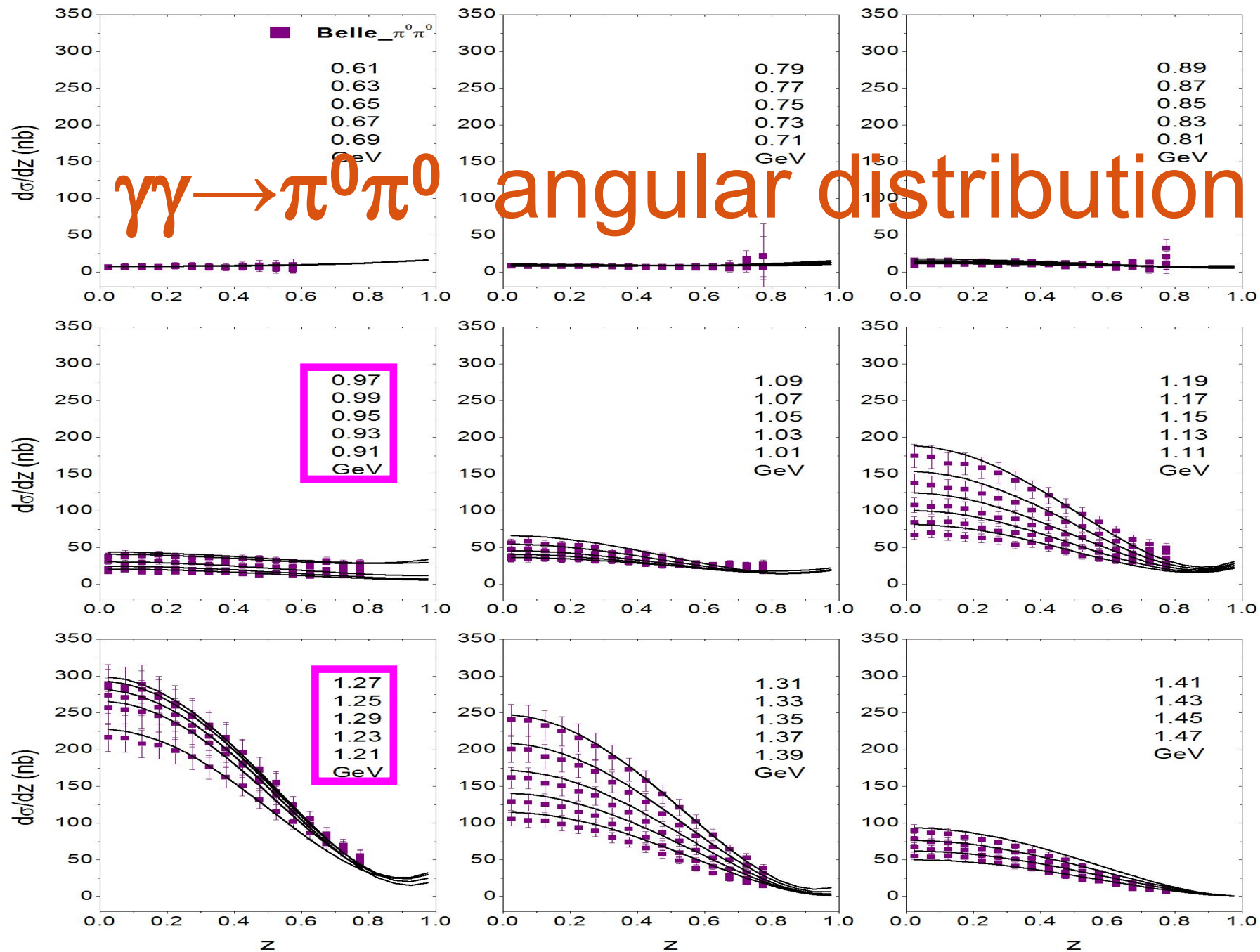
- we can predict the full cross section if we know each partial wave.
- The angular distribution is helpful to separate each partial wave.





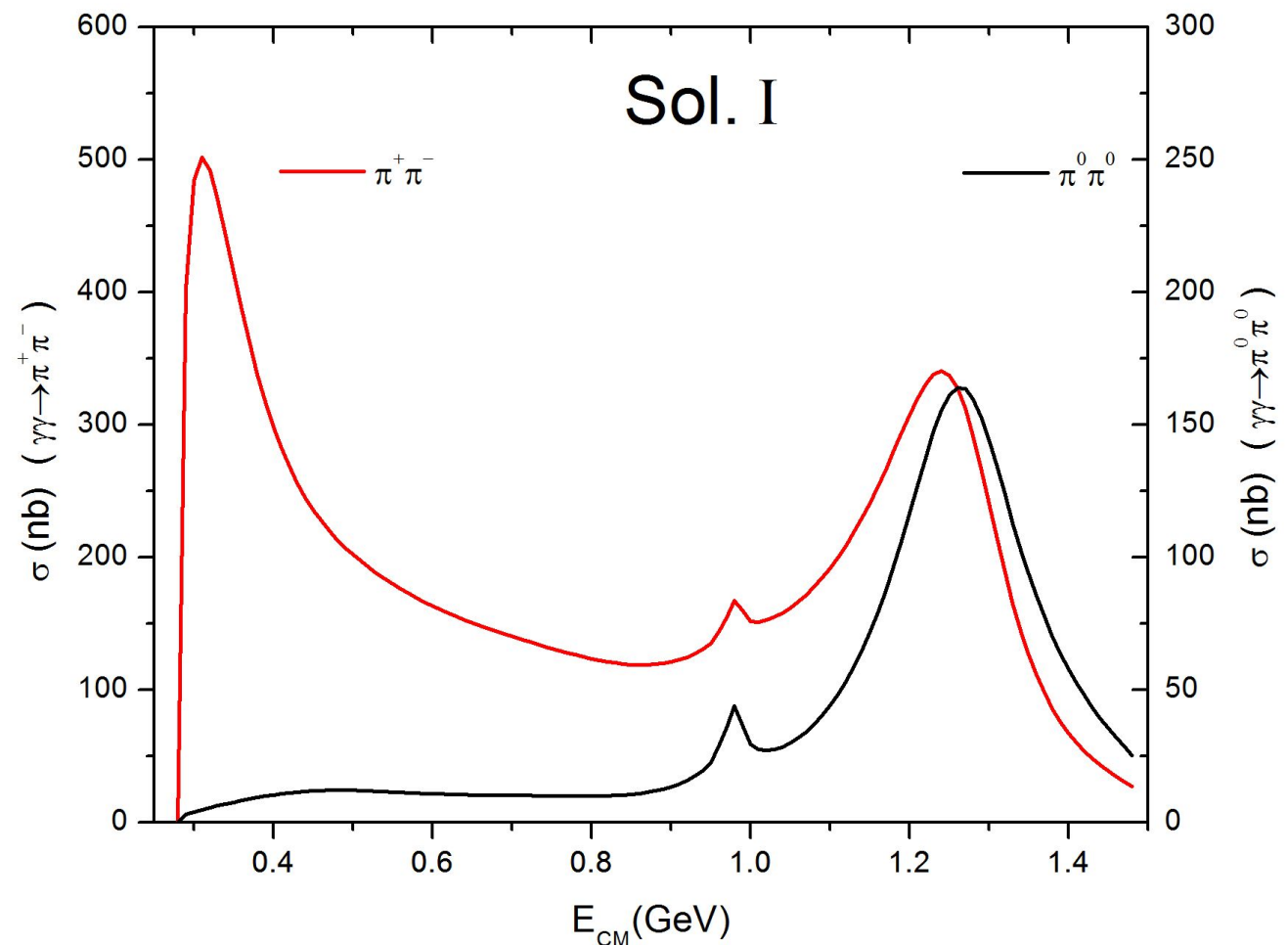






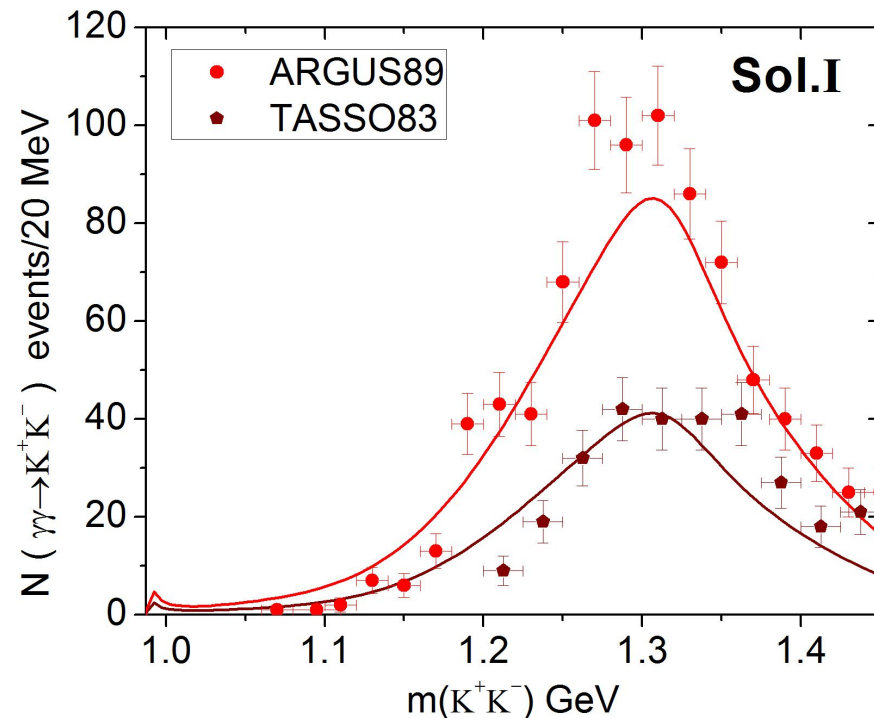
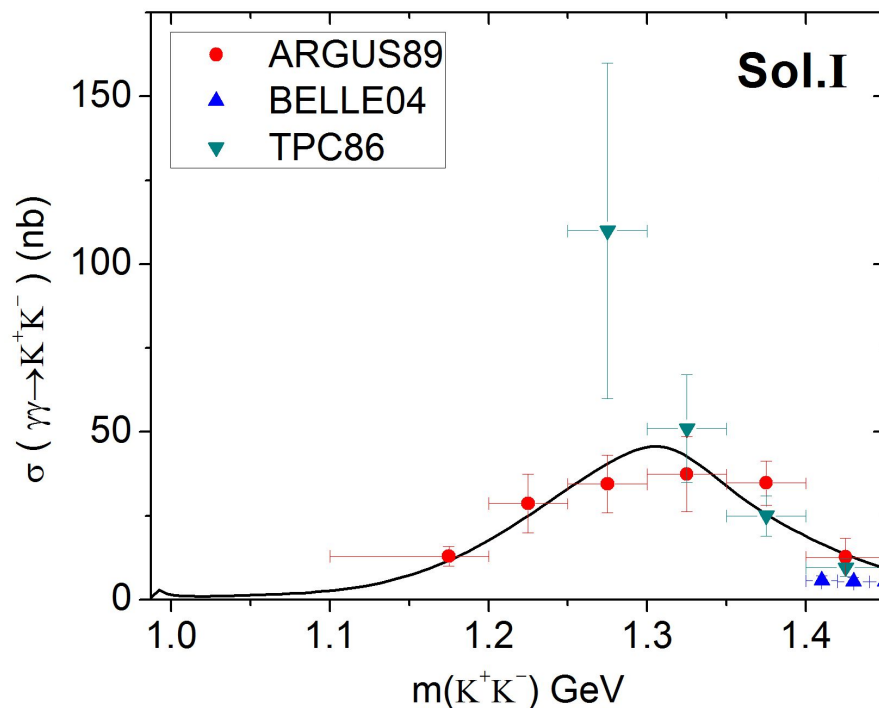
## Prediction of the full angle coverage

- Only with the full coverage amplitudes can we predict the couplings.



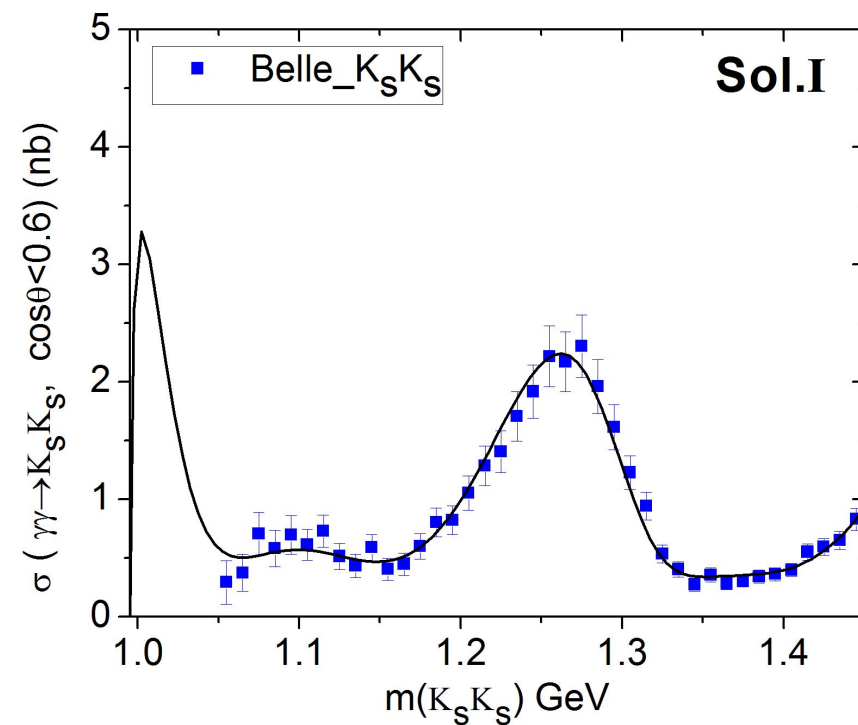
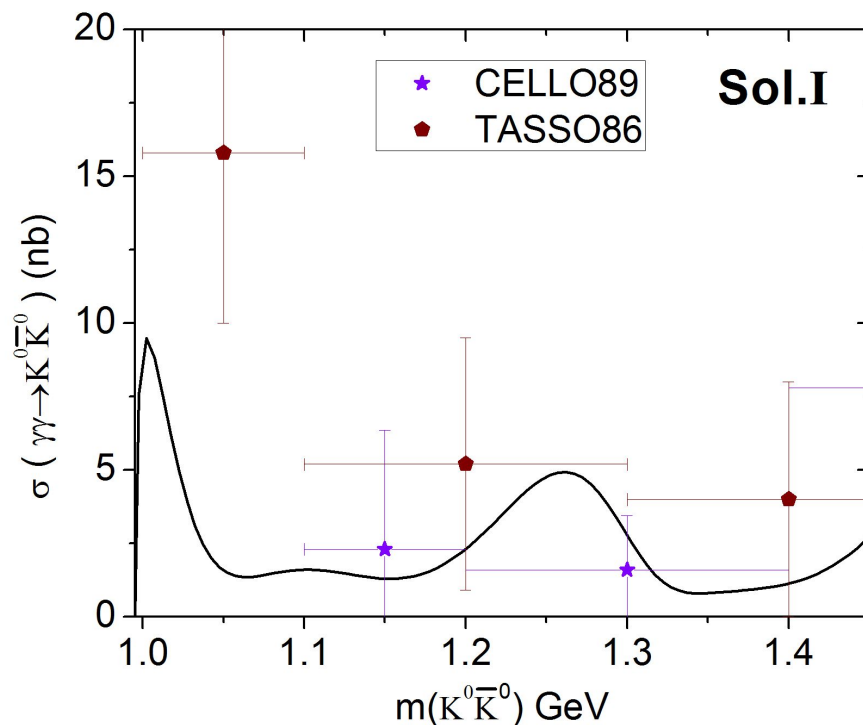
## $\gamma\gamma \rightarrow K^+K^-$ integrated cross section

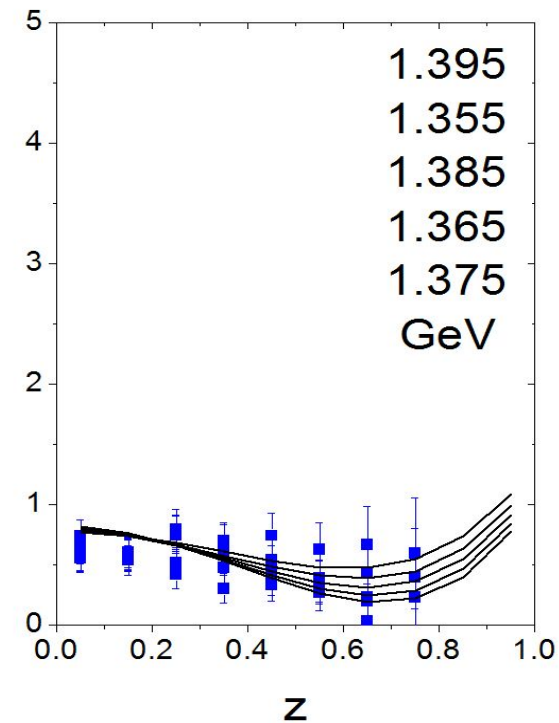
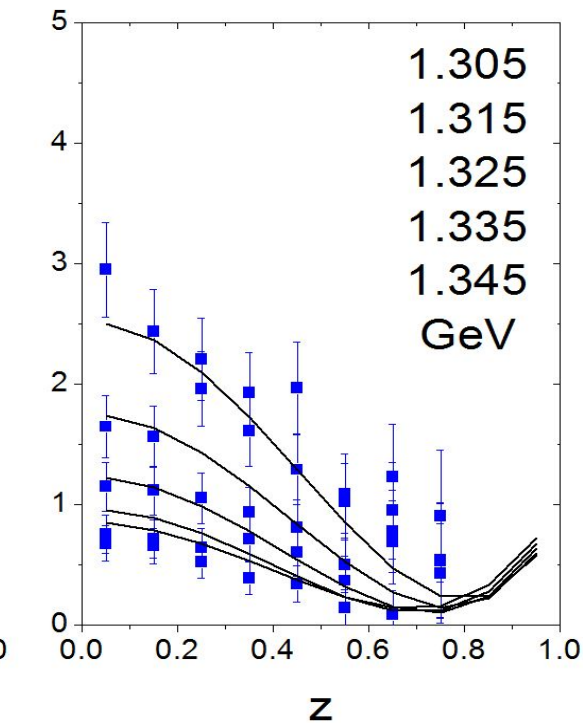
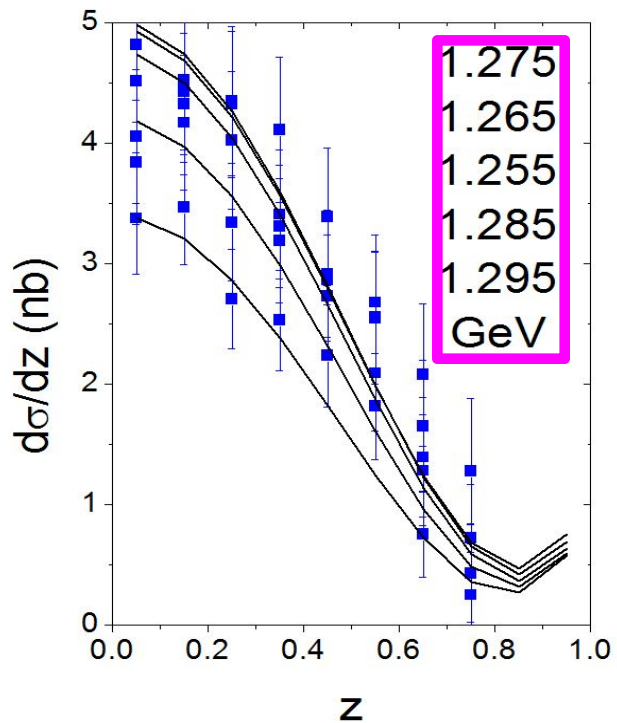
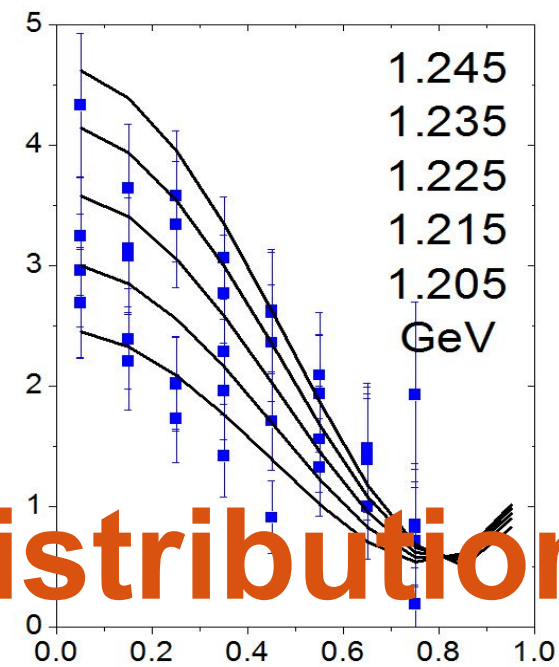
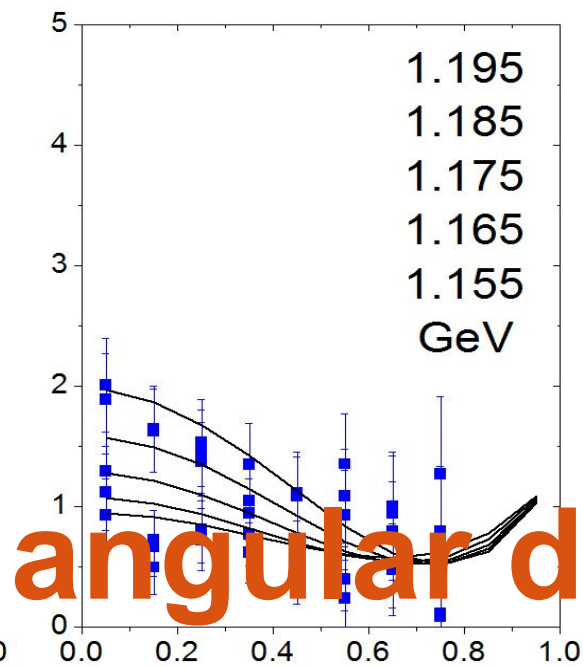
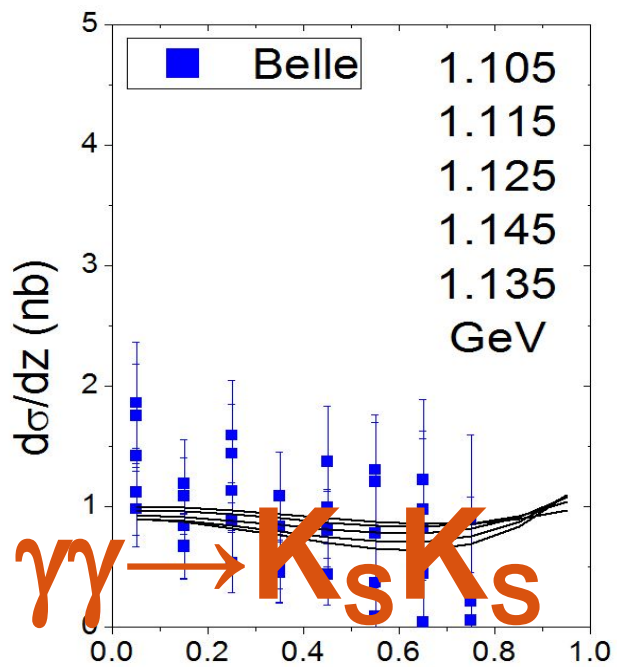
- If only fit  $\gamma\gamma \rightarrow \pi\pi$ , we will get a region of solutions.  $\gamma\gamma \rightarrow KK$  is helpful to select solutions.
- The integrated cross section data has large error bars and the events shape data are not coincide with each other.



## $\gamma\gamma \rightarrow K^0 \bar{K}^0$ integrated cross section

- The latest  $K_S K_S$  data of Belle make the accurate couple channel analysis possible. Especially the angular distribution.



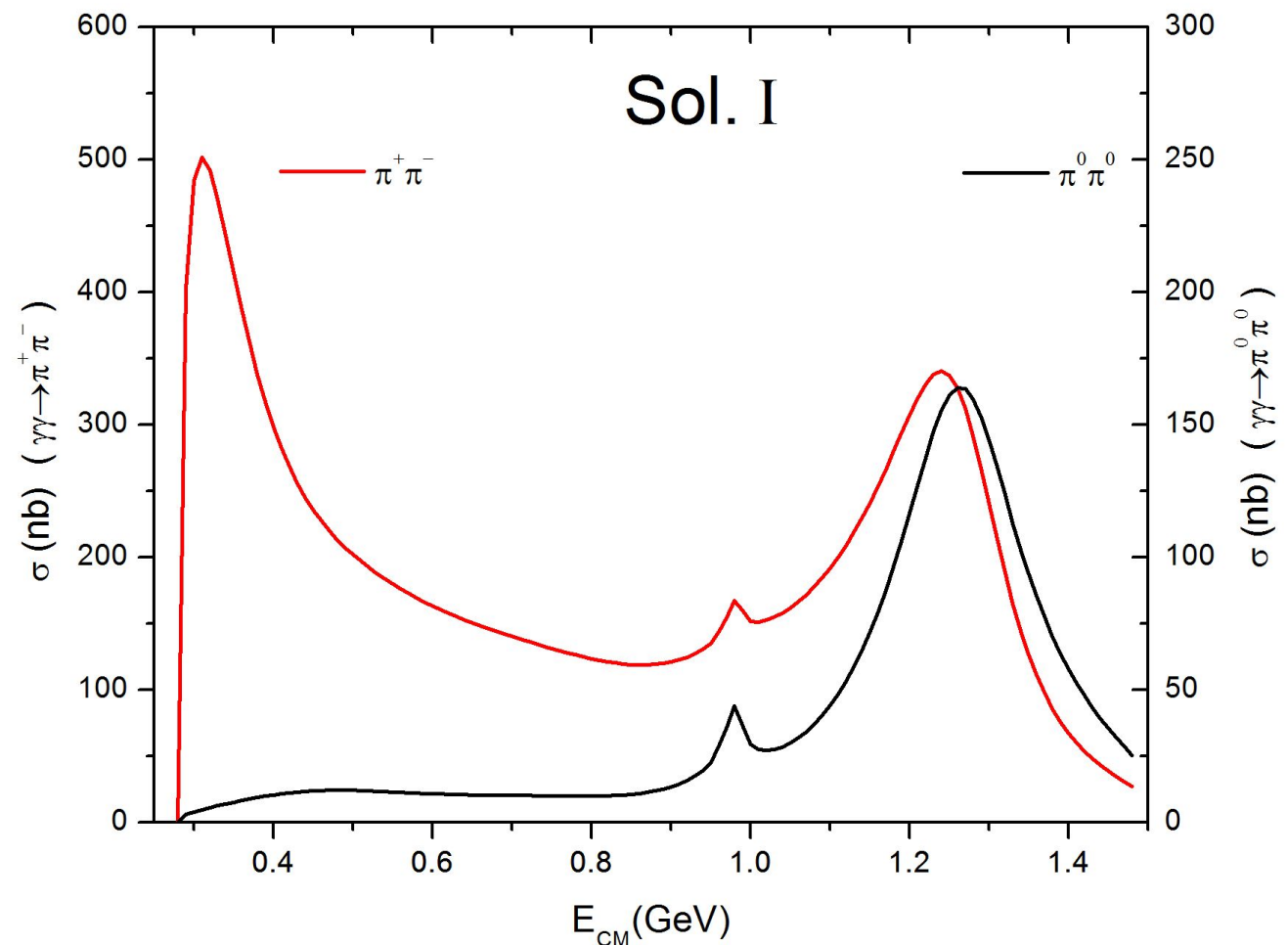


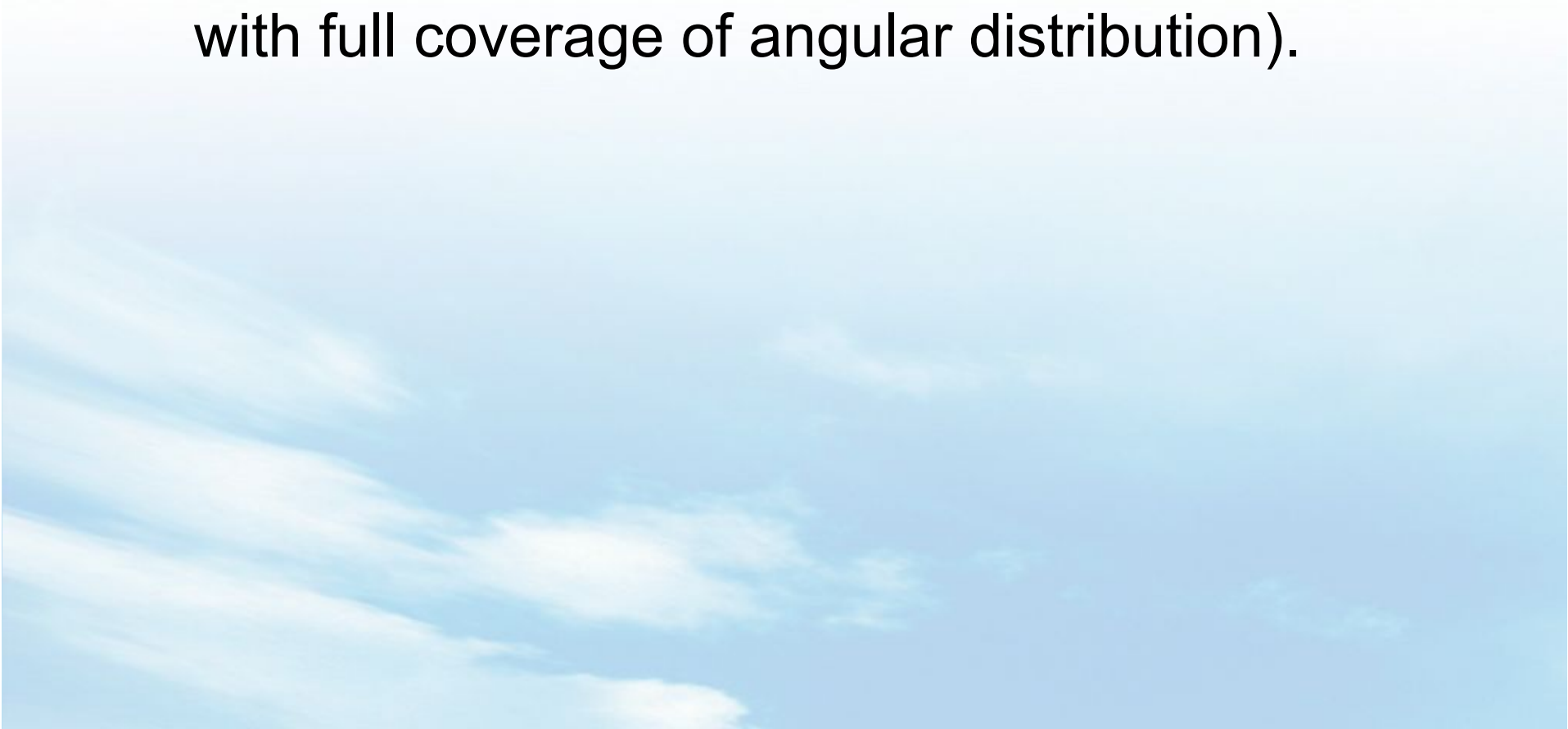
$\gamma\gamma \rightarrow K_s K_s$

angular distribution

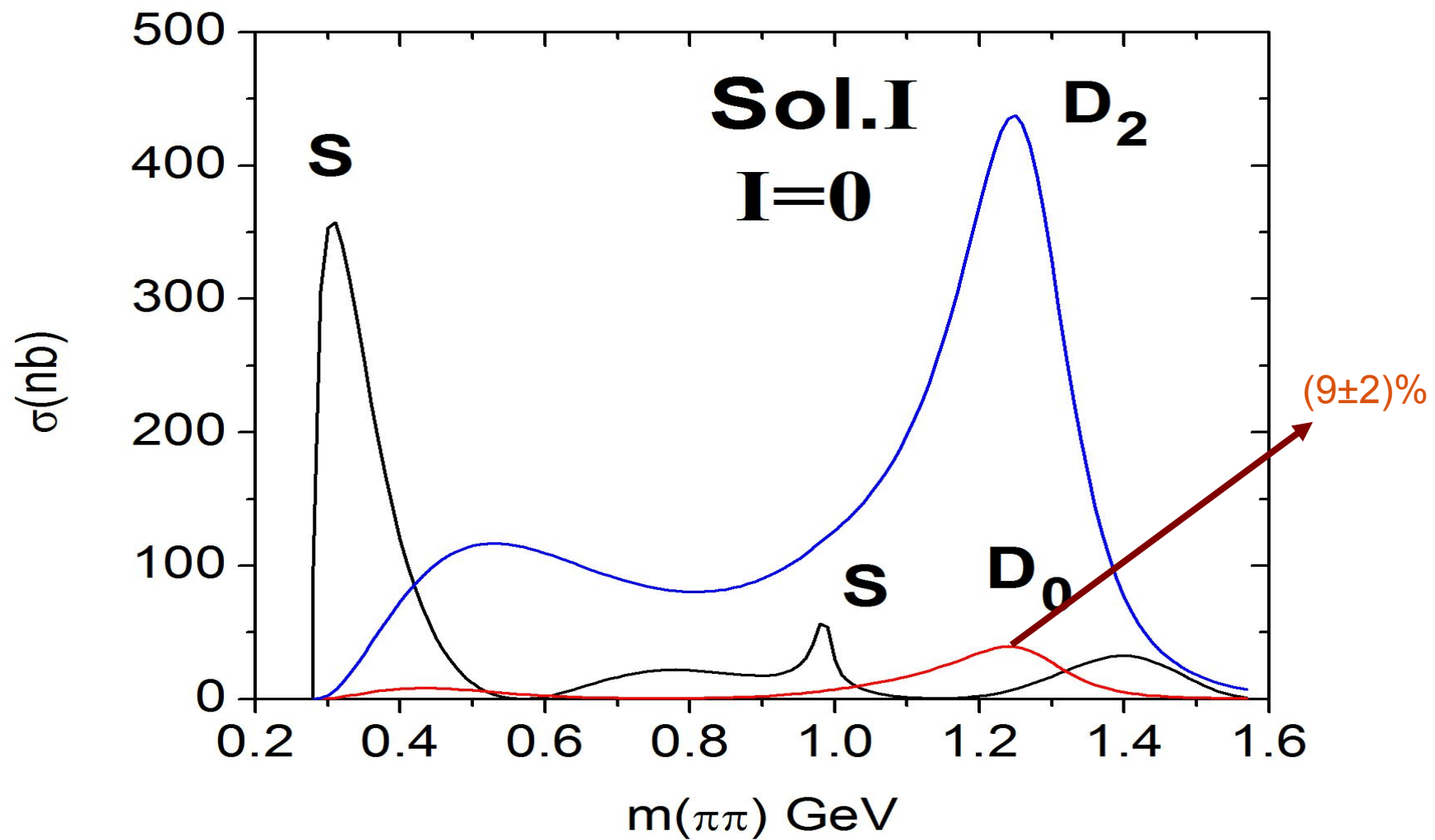
## 4. Photon couplings

- Only with the full coverage amplitudes can we predict the couplings.

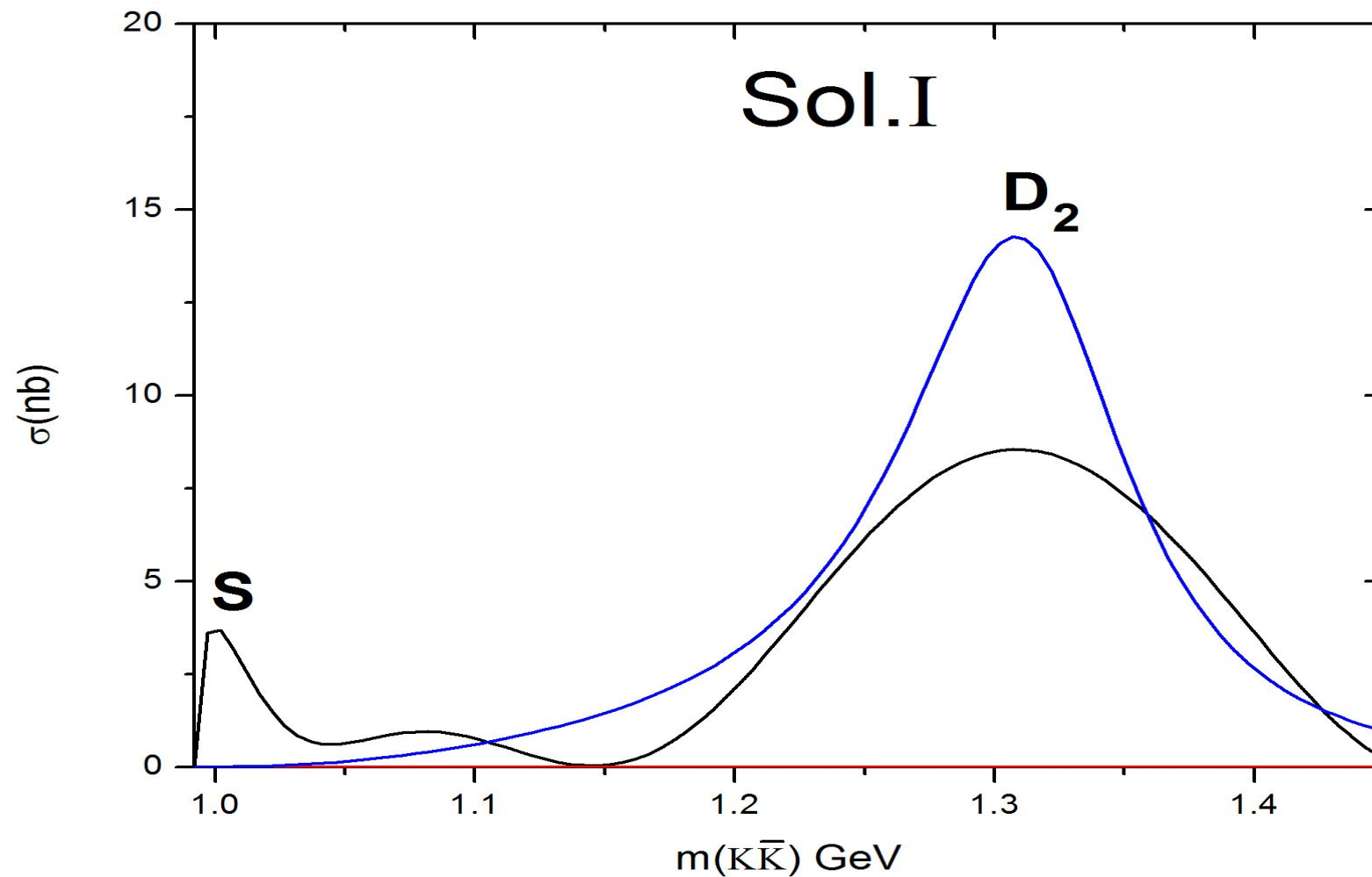


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- To get the information of scalars, tensors, we need the individual partial waves(of course, with full coverage of angular distribution).
- 

# $\gamma\gamma \rightarrow \pi\pi$ individual partial waves, $I=0$



# $\gamma\gamma \rightarrow K\bar{K}$ individual partial waves, I=1



## Breit-Wigner photon widths

- From a Breit-wigner description

$$\Gamma(R \rightarrow \gamma\gamma) = \frac{\sigma_{\gamma\gamma}(\text{res.peak}) M_R^2 \Gamma_{tot}}{8\pi(\hbar c)^2(2J+1) \text{BR}}$$

- we get:
  - $\Gamma(f_2 \rightarrow \gamma\gamma) = 3.28 \pm 0.37 \text{ keV}$
  - $\Gamma(a_2 \rightarrow \gamma\gamma) = 0.94 \pm 0.10 \text{ keV}$

## Extract photon couplings

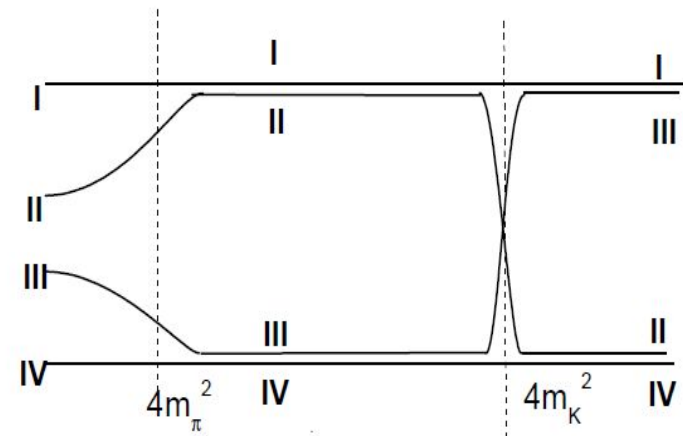
- A more efficient way is to extract the photon couplings from different Riemann sheets.

$$T^{II} = \mathcal{B}_{II} T, \quad T^{III} = \mathcal{B}_{III} T, \quad T^{IV} = \mathcal{B}_{IV} T$$

$$F^{II} = B_{II} F \equiv \begin{pmatrix} \frac{1}{1+2i\rho_1 T_{11}^I} & 0 \\ \frac{-2i\rho_1 T_{12}^I}{1+2i\rho_1 T_{11}^I} & 1 \end{pmatrix} F,$$

$$F^{III} = B_{III} F \equiv \begin{pmatrix} \frac{1+2i\rho_2 T_{22}^I}{\det S} & \frac{-2i\rho_2 T_{21}^I}{\det S} \\ \frac{-2i\rho_1 T_{12}^I}{\det S} & \frac{1+2i\rho_1 T_{11}^I}{\det S} \end{pmatrix} F$$

$$F^{IV} = B_{IV} F \equiv \begin{pmatrix} 1 & \frac{-2i\rho_2 T_{21}^I}{1+2i\rho_2 T_{22}^I} \\ 0 & \frac{1}{1+2i\rho_2 T_{22}^I} \end{pmatrix} F.$$



Xiao&Zheng  
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## Extract photon couplings

- In the second sheet:

$$g_\pi^2 = -\frac{T(z_{II})}{S'(z_{II})}, \quad g_\gamma g_\pi = -\frac{F(z_{II})}{S'(z_{II})}$$

- In the third sheet:

$$\begin{aligned} g_\gamma g_\pi &= -\frac{F_1^I(z_{III})S_{22}(z_{III})}{(\det S)'(z_{III})} + \frac{F_2^I(z_{III})2i\rho_2(z_{III})T_{21}^I(z_{III})}{(\det S)'(z_{III})}, \\ g_\gamma g_K &= \frac{F_1^I(z_{III})2i\rho_1(z_{III})T_{12}^I(z_{III})}{(\det S)'(z_{III})} - \frac{F_2^I(z_{III})S_{11}(z_{III})}{(\det S)'(z_{III})}. \end{aligned}$$

$$g_\pi^2 = \frac{S_{22}(z_{III})}{2i\rho_1 \det S'(z_{III})}, \quad g_K^2 = \frac{S_{11}(z_{III})}{2i\rho_2 \det S'(z_{III})}$$

## Extract photon couplings

- With  $F_1 = \alpha_1(s)T_{11}(s) + \alpha_2(s)T_{12}(s)$  we simply have

$$g_{\gamma\gamma}^{Sh} = \alpha_1(s_R) g_{\pi\pi}^{Sh} + \alpha_2(s_R) g_{KK}^{Sh}$$

- Sh represents for Riemann sheets II and III,  
Our translation of couplings into

$$\Gamma(R \rightarrow \gamma\gamma) = \frac{\alpha^2}{4(2J+1)m_R} |g_{\gamma\gamma}|^2$$

| State       | Sh  | pole locations<br>(GeV) | $g_{\gamma\gamma} =  g e^{i\varphi}$ |                 |               | $\Gamma(f_J \rightarrow \gamma\gamma)$<br>(keV) | $\lambda = 0$<br>fraction % | C.L. |
|-------------|-----|-------------------------|--------------------------------------|-----------------|---------------|-------------------------------------------------|-----------------------------|------|
|             |     |                         | $J_\lambda$                          | $ g $ (GeV)     | $\varphi$ (°) |                                                 |                             |      |
| $f_2(1270)$ | II  | $1.270 - i0.081$        | $D_0$                                | $0.37 \pm 0.03$ | $172 \pm 6$   | $3.50 \pm 0.41$                                 | $8.3 \pm 1.4$               | **** |
|             |     |                         | $D_2$                                | $1.24 \pm 0.08$ | $176 \pm 5$   |                                                 |                             |      |
|             | III | $1.267 - i0.108$        | $D_0$                                | $0.35 \pm 0.03$ | $168 \pm 6$   | $2.95 \pm 0.43$                                 | $8.6 \pm 1.7$               | **** |
|             |     |                         | $D_2$                                | $1.13 \pm 0.08$ | $173 \pm 6$   |                                                 |                             |      |
| $a_2(1370)$ | IV  | $1.313 - i0.053$        | $D_2$                                | $0.75 \pm 0.04$ | $-139 \pm 2$  | $1.14 \pm 0.11$                                 | $0^\dagger$                 | **   |
| $f_0(500)$  | II  | $0.441 - i0.272$        | S                                    | $0.26 \pm 0.01$ | $107 \pm 3$   | $2.06 \pm 0.22$                                 | 100                         | **** |
| $f_0(980)$  | II  | $0.995 - i0.042$        | S                                    | $0.16 \pm 0.01$ | $185 \pm 3$   | $0.32 \pm 0.03$                                 | 100                         | **** |
| $f_0(1370)$ | II  | $1.423 - i0.177$        | S                                    | $1.00 \pm 0.10$ | $5 \pm 1$     | $9.4 \pm 1.9$                                   | 100                         | *    |
|             | III | $1.406 - i0.344$        | S                                    | $0.70 \pm 0.12$ | $-149 \pm 7$  | $4.7 \pm 1.5$                                   | 100                         | *    |

# Comparing with some models

$f_2 : 2.95 \pm 0.43 \text{ keV}$

$\sigma : 2.06 \pm 0.22 \text{ keV}$

?

$f_0 : 0.32 \pm 0.03 \text{ keV}$

| composition                      | prediction (keV) | author(s)             |
|----------------------------------|------------------|-----------------------|
| $(\bar{u}u + \bar{d}d)/\sqrt{2}$ | 4.0              | Babcock & Rosner      |
|                                  | $< 1^\dagger$    | Giacosa <i>et al.</i> |
| $\bar{s}s$                       | 0.2              | Barnes                |
|                                  | 0.062            | Giacosa <i>et al.</i> |
| $[\bar{n}s][ns]$                 | 0.27             | Achasov <i>et al.</i> |
| $\bar{K}K$                       | 0.6              | Barnes                |
|                                  | 0.22             | Hanhart <i>et al.</i> |
| $gg$                             | 0.2–0.6          | Narison <i>et al.</i> |

# Tensors&Scalars

- Our widths give  $n \sim 0.7-1.3$

$$\frac{\Gamma(\sigma \rightarrow \gamma\gamma)}{\Gamma(f_2 \rightarrow \gamma\gamma)} = \frac{15}{4} \left( \frac{m_\sigma}{m_{f_2}} \right)^n$$

'2' by F.E. Close *et. al*,  
PRD43 (1991) 2161

$n=3$ , Coulomb potential

$n=0.7-1.3$

$n=0$ , linear confinement

## 5. Summary

### Amplitudes

Including all new datasets and analyticity, unitarity, crossing symmetry, we perform an amplitude analysis on photon photon collision.

### poles

With these amplitudes we extract the accurate pole locations and photon couplings.

### Structure

$f_0(980)$  is most likely to be  $K\bar{K}$  molecule, though a combination with  $q\bar{q}$  is still possible.

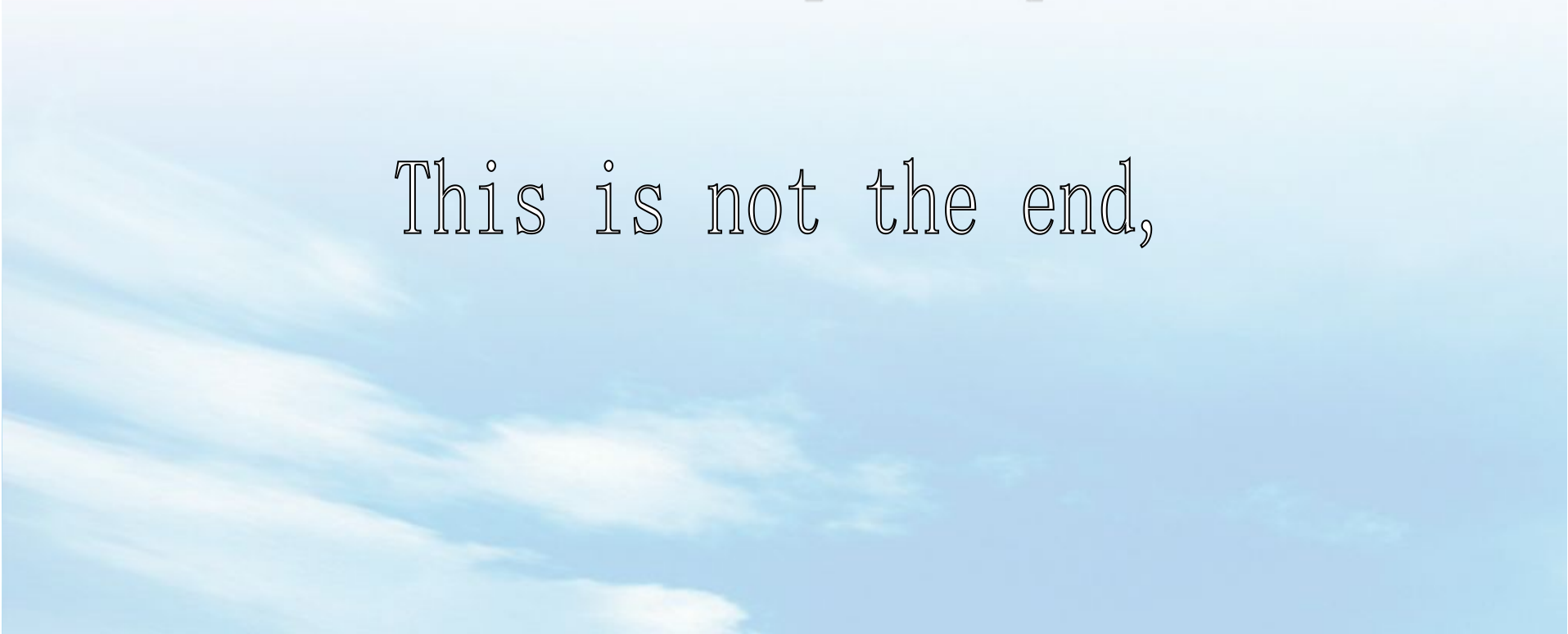
### Future?

Both theory and experiment effort are needed for deeper understanding.

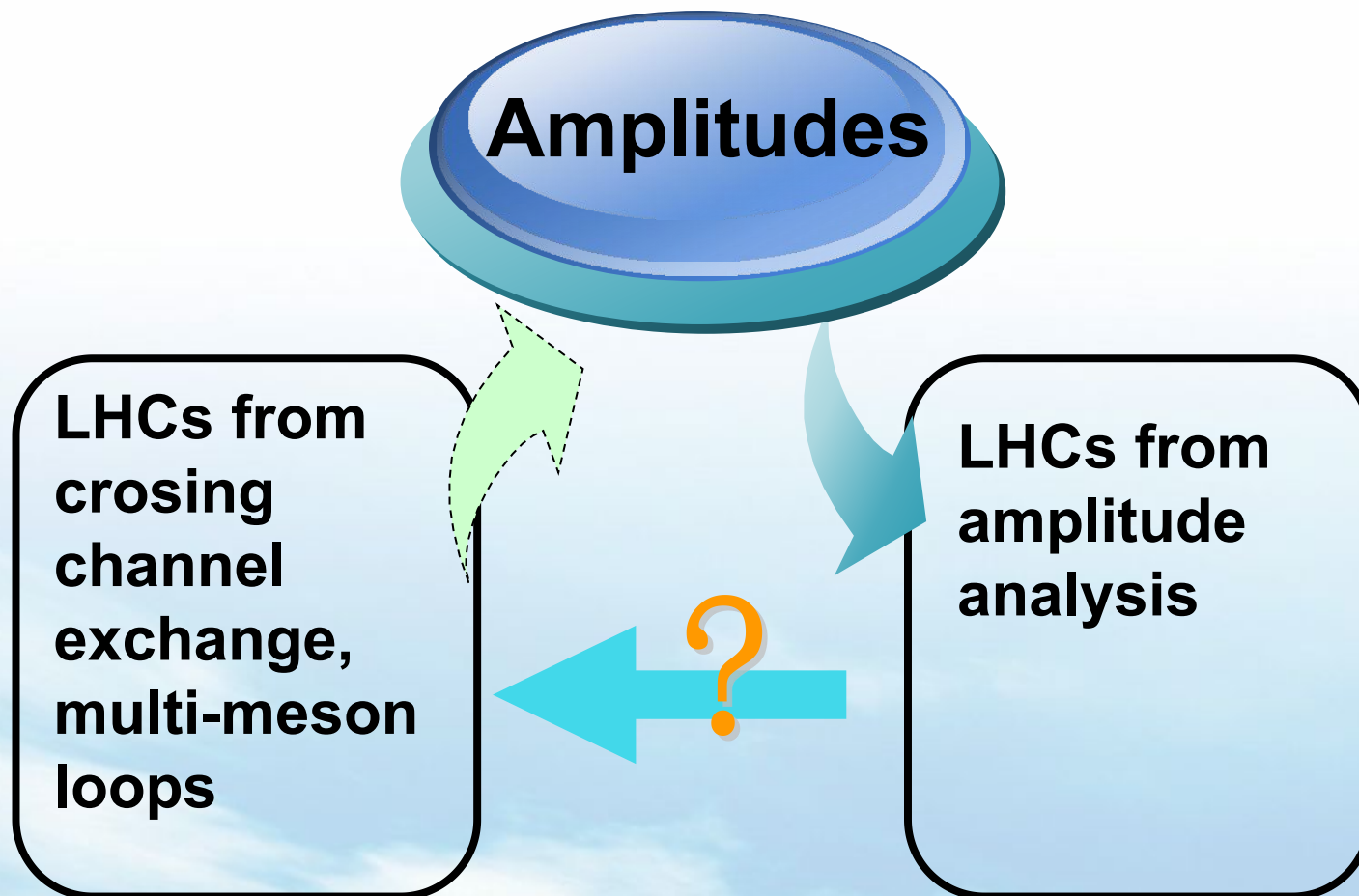


# Thank You For your patience !

This is not the end,



# Prospects



# Preliminary results

