

Physical parameters of the electroweak crossover

Michela D’Onofrio

Department of Physics and Helsinki Institute of Physics, PL 64 (Gustaf Hällströmin katu 2), FI-00014 University of Helsinki, Finland

Kari Rummukainen

Department of Physics and Helsinki Institute of Physics, PL 64 (Gustaf Hällströmin katu 2), FI-00014 University of Helsinki, Finland

Anders Tranberg

Faculty of Science and Technology, University of Stavanger, N-4036 Stavanger, Norway

Abstract

We use large-scale lattice simulations to compute the rate of baryon-number violating processes, the *sphaleron rate*, the Higgs field expectation value, and the critical temperature of the electroweak phase transition in the Standard Model.

Keywords: baryogenesis, sphaleron rate, baryon number, electroweak crossover, lattice simulations, standard model

1. Introduction

Baryon and lepton numbers are classically conserved quantities, but the chiral nature of weak interactions gives rise to the anomalous violation of baryon and lepton number currents at the quantum level. In practice, however, the processes violating B- and L-numbers are suppressed below a temperature scale of $T_c \sim 100$ GeV, thus making B and L effectively conserved in the present Universe. The critical temperature corresponds to the electroweak scale, where it has been suggested [1] that baryogenesis might have taken place. In the Standard Model, baryon number is violated by the Adler-Bell-Jackiw anomaly

$$\partial_\mu j^{\mu 5} = -\frac{e^2}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}, \quad (1)$$

where $j^{\mu 5}$ is the axial vector current, e the gauge coupling, and F_{ij} the gauge field strength tensor. Eq. 1 expresses the fact that, in gauge theories, gauge invariance implies axial vector current non-conservation.

However, apart from B-violation, any successful model of baryogenesis has also to fulfill the other two

necessary Sakharov’s conditions of C and CP violation as well as departure from equilibrium. These are not naturally satisfied by the Standard Model as-is, but a minimal extension is required.

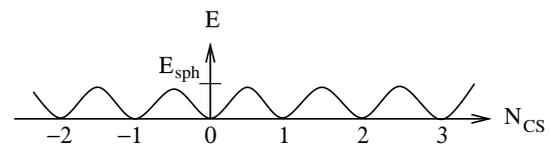


Figure 1: Vacuum structure of the pure-gauge electroweak theory. The energy of gauge field configurations as a function of Chern-Simons number [2].

At temperatures above the electroweak scale, the rate of the sphaleron transitions is unsuppressed and has been accurately measured using effective theories on the lattice. At temperatures substantially below the electroweak scale, the Higgs field expectation value is large and the sphaleron rate is strongly suppressed.

The work presented in these proceedings is based on our previous work [3], where we use an effective electroweak theory on the lattice with multicanonical and real-time simulation methods to calculate the sphaleron

rate through the electroweak crossover with Higgs mass of 125 GeV.

2. Theory

The electroweak theory possess a set of infinite non-trivial vacua (Fig. 1), each labeled by a *Chern-Simons number*

$$\begin{aligned} n_{CS} &\equiv \int d^3x J_{CS}^0 \\ &= -\frac{g^2}{64\pi} \int d^3x \epsilon^{ijk} \text{Tr} \left(A_i A_j A_k + i \frac{g}{3} A_i A_j A_k \right). \end{aligned}$$

The Chern-Simons current J_{CS}^μ is in turn related through the axial anomaly to the baryon- and lepton-number currents

$$\partial_\mu (J_B^\mu + J_L^\mu) = n_g \left(\frac{g^2}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} A_{\alpha\beta}^a A_{\mu\nu}^a \right), \quad (2)$$

$$\partial_\mu J_B^\mu = n_g \partial_\mu J_{CS}^\mu, \quad (3)$$

where the U(1) part of the theory is omitted. Transitions between vacua are possible by surmounting the potential barrier through *sphaleron transitions*. The sphaleron rate is strongly suppressed at low temperatures, where the potential barrier is high. At temperatures above the EWPT, though, transitions among vacua are made possible because of the availability of thermal energy.

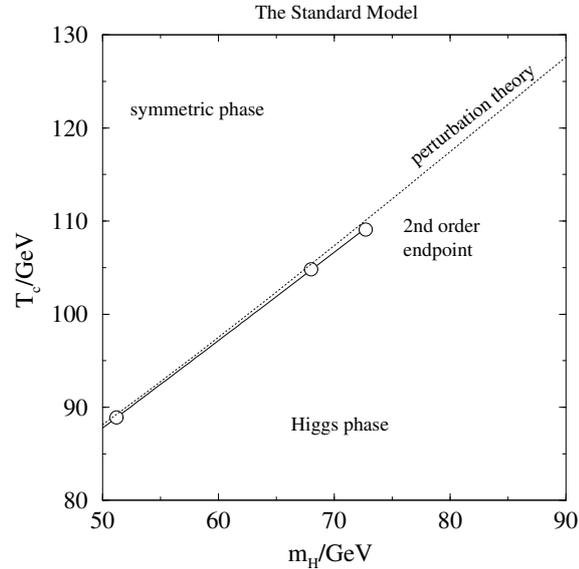


Figure 2: The order of the electroweak phase transition shown in terms of Higgs mass [4]. The phase transition is first order up to $m_H \sim 70$ GeV, where it becomes second order. For higher values of the Higgs mass, the transition is a crossover.

Each transition changes n_{CS} by one unit and the baryon number by $n_g = 3$

$$B(t_f) - B(t_i) = n_g [n_{CS}(t_f) - n_{CS}(t_i)].$$

3. Theory on the lattice

We use large-scale lattice simulations and compute the sphaleron rate, the Higgs field expectation value and the critical temperature of the electroweak phase transition in the Standard Model.

The thermodynamics of the 4-dimensional electroweak theory is studied in 3 dimensions through dimensional reduction [5], a perturbative technique giving the correspondence between 4D and 3D parameters. The result is a SU(2) effective theory with the Higgs field ϕ and gauge field A_μ (F_{ij})

$$L = \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \phi)^\dagger (D_i \phi) + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2, \quad (4)$$

and 3D effective parameters g_3^2 , λ_3 and m_3^2 . The time evolution of this effective SU(2) Higgs model is governed by Langevin dynamics [6]. The latter, however, is very slow on the lattice and can be substituted by any other dissipative procedure, heat bath in our case. One heat-bath sweep through the lattice corresponds to the real-time step $\Delta t = a^2 \sigma_{el}/4$ [7], where a is the lattice spacing and σ_{el} is the non-abelian color conductivity, the current response to infrared external fields.

4. Methods

In the symmetric phase we make use of canonical Monte Carlo simulations and approach the broken phase. At very low temperatures, the rate is highly suppressed and canonical methods do not work anymore.

Here, the computation is performed with *multicanonical methods* [8, 9], which make use of a weight function that compensates the low-temperature suppression in the baryon violation rate. The obtained sphaleron rate is

$$\Gamma \equiv \lim_{t \rightarrow \infty} \frac{\langle (n_{CS}(t) - n_{CS}(0))^2 \rangle}{V t}. \quad (5)$$

5. The sphaleron rate

The measured sphaleron rate is shown in Fig. 6 with a shaded error band. The freeze-out temperature T_* is solved from the crossing of Γ and the Hubble rate, shown with the almost horizontal

line. The sphaleron rate in the symmetric phase ($T > T_c$) is

$$\Gamma/T^4 = (18 \pm 3) \alpha_W^5,$$

and in the broken phase between $130 \text{ GeV} < T < T_c$ can be parametrized as

$$\log(\Gamma/T^4) = (0.83 \pm 0.01) T/\text{GeV} - (147.7 \pm 1.9).$$

The freeze-out temperature in the early Universe, where the Hubble rate wins over the baryon number violation rate, is $T_* = (131.7 \pm 2.3) \text{ GeV}$.

6. Conclusions

The discovery of the Higgs particle of mass 125–126 GeV enables us to fully determine the properties of the symmetry breaking at high temperatures. Using lattice simulations of a 3D effective theory, we have located the crossover range at $T_c = (159 \pm 1) \text{ GeV}$, determined the baryon number violation rate both above and well below the crossover point, and calculated the baryon freeze-out temperature in the early Universe, $T_* = (131.7 \pm 2.3) \text{ GeV}$.

It is the first time that this fundamental parameter of the Standard Model has been so extensively and precisely studied in the full temperature range of the electroweak crossover. The obtained accuracy is the greatest achievable in the circumstance of exponential suppression, which occurs in the broken phase. Significant improvement in this direction is not around the corner any time soon.

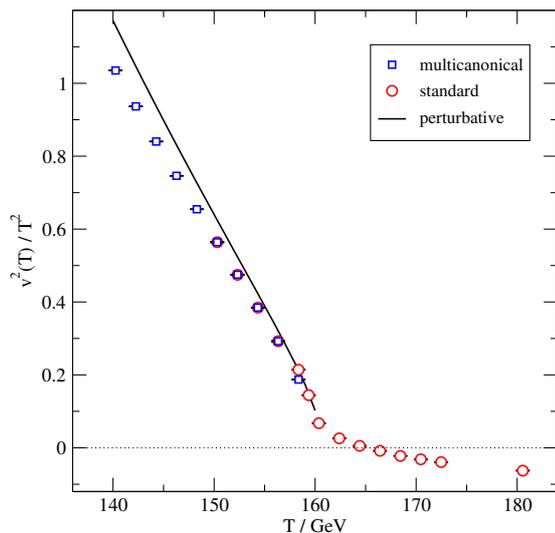


Figure 3: Higgs field expectation value $\langle \phi^2 \rangle$ as a function of temperature. and compared with the perturbative result in [12]. We notice a perfect match between the canonical and multicanonical results and a smooth transition from the symmetric to the broken phase.

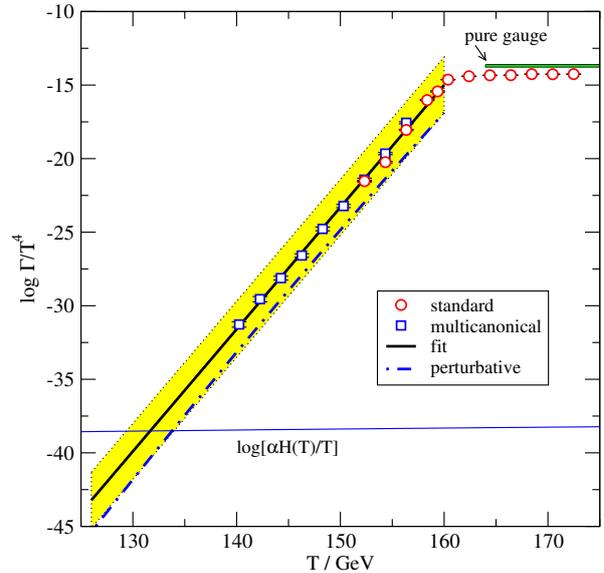


Figure 4: The sphaleron rate as a function of temperature at the crossover. Both canonical and multicanonical results are shown and are in good agreement. The perturbative result is from [10]. Pure gauge refers to the rate in hot SU(2) gauge theory [11].

These results represent intrinsic properties of the Minimal Standard Model, as well as provide input for leptogenesis calculations, in particular for models with electroweak-scale leptons. The sphaleron rate obtained here also provides a benchmark for future computations of the sphaleron rate in extensions of the Standard Model.

References

- [1] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36
- [2] J. M. Cline, hep-ph/0609145.
- [3] M. D’Onofrio, K. Rummukainen and A. Tranberg, Phys. Rev. Lett. **113** (2014) 141602 [arXiv:1404.3565 [hep-ph]].
- [4] M. Laine and K. Rummukainen, Nucl. Phys. Proc. Suppl. **73** (1999) 180 [hep-lat/9809045].
- [5] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379].
- [6] D. Bodeker, Phys. Lett. B **426** (1998) 351 [hep-ph/9801430].
- [7] G. D. Moore and K. Rummukainen, Phys. Rev. D **63** (2001) 045002 [hep-ph/0009132].
- [8] G. D. Moore, Phys. Rev. D **59** (1999) 014503 [hep-ph/9805264].
- [9] M. D’Onofrio, K. Rummukainen and A. Tranberg, JHEP **1208** (2012) 123 [arXiv:1207.0685 [hep-ph]].
- [10] Y. Burnier, M. Laine and M. Shaposhnikov, JCAP **0602** (2006) 007 [hep-ph/0511246].
- [11] G. D. Moore and K. Rummukainen, Phys. Rev. D **61** (2000) 105008 [hep-ph/9906259].
- [12] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020].