



# Hadronic total cross sections, Wilson loop correlators and the QCD spectrum

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## Abstract

We show how to obtain rising hadronic total cross sections in QCD, in the framework of the nonperturbative approach to soft high-energy scattering based on Wilson-loop correlators. Total cross sections turn out to be of “Froissart”-type, i.e., the leading energy dependence is of the form  $\sigma_{tot} \sim B \log^2 s$ , in agreement with experiments. The observed universality of the prefactor  $B$  is obtained rather naturally in this framework. In this case,  $B$  is entirely determined by the stable spectrum of QCD, and predicted to be  $B_{th} = 0.22$  mb, in fair agreement with experiments.

*Keywords:* total cross sections, QCD, nonperturbative approach

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## 1. Introduction

Explaining the behaviour of hadronic total cross sections at high energy is a very old problem, which is very rarely attacked within the framework of QCD. Most of the approaches to this problem are based on phenomenological models, which are sometimes QCD-inspired, but a full derivation from first principles of QCD is still lacking.

The observed rise of hadronic total cross sections at high energy is well described by a “Froissart-like” behaviour  $\sigma_{tot} \sim B \log^2 s$  with a universal prefactor  $B \simeq 0.27 \div 0.28$  mb [1]. This behaviour respects unitarity, as encoded in the Froissart-Lukaszuk-Martin bound [2, 3, 4],  $\sigma_{tot} \leq B_{FLM} \log^2 \frac{s}{s_0}$ , since  $B \ll B_{FLM} = \pi/m_\pi^2 \simeq 65$  mb.

Besides deriving  $\sigma_{tot}$  from the first principles of QCD, one should be able to explain the observed universality of  $B$ , and also the two orders of magnitude separating  $B$  and  $B_{FLM}$ . A step in this direction has been

made in Ref. [5], where we have derived the asymptotic behaviour of hadronic total cross sections in the framework of the nonperturbative approach to soft high-energy scattering [6, 7, 8], finding indeed a “Froissart-like” behaviour, and a theoretical prediction for  $B$  in fair agreement with experiments.

## 2. Nonperturbative approach to soft high-energy scattering

Understanding the rise of  $\sigma_{tot}$  is part of the problem of soft high-energy scattering, characterised by  $|t| \leq 1 \text{ GeV}^2 \ll s$ . In this regime perturbation theory is not fully reliable, and a nonperturbative approach is needed. In a nutshell, the nonperturbative approach to *elastic meson-meson scattering* is as follows [6, 7, 8]:

1. mesons are described as wave packets of transverse colourless dipoles;
2. in the soft high-energy regime, the dipoles travel essentially undisturbed on their classical, almost lightlike trajectories;
3. mesonic amplitudes are obtained from the dipole-dipole ( $dd$ ) amplitudes after folding with the appropriate squared wave functions,  $A(s, b) = \langle\langle A^{(dd)}(s, b; \nu) \rangle\rangle$ .

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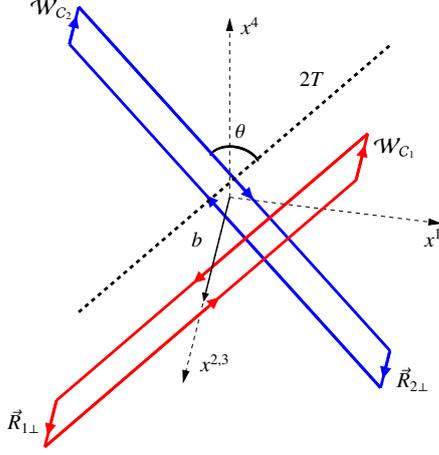


Figure 1: The relevant Euclidean Wilson loops.

Here  $b$  is the impact parameter<sup>2</sup>,  $\nu$  denotes collectively the dipole variables (longitudinal momentum fraction, transverse size and orientation), and  $\langle\langle \dots \rangle\rangle$  stands for integration over the dipole variables with the mesonic wave functions. This approach extends also to processes involving baryons, if one adopts for them a quark-diquark picture [9].

At high energy the  $dd$  amplitude is given by the normalised connected correlator  $C_M$  of the Wilson loops (WL) running along the classical trajectories of the two dipoles,  $A^{(dd)}(s, b; \nu) = -C_M(\chi, b; \nu)$ , with  $\chi \simeq \log \frac{s}{m^2}$  the hyperbolic angle between the trajectories, and  $m$  the mass of the mesons (taken to be equal for simplicity).

The correlator  $C_M$  is obtained from the correlator  $C_E$  of two Euclidean WL at angle  $\theta$  (see Fig. 1),

$$C_E(\theta, b; \nu) \equiv \lim_{T \rightarrow \infty} \frac{\langle \mathcal{W}_{C_1} \mathcal{W}_{C_2} \rangle}{\langle \mathcal{W}_{C_1} \rangle \langle \mathcal{W}_{C_2} \rangle} - 1, \quad (1)$$

through the analytic continuation (AC) [10, 11]

$$C_M(\chi, b; \nu) = C_E(\theta \rightarrow -i\chi, b; \nu). \quad (2)$$

A more detailed discussion of the approach can be found in Ref. [5] and references therein. The Euclidean formulation has allowed the study of the relevant WL correlator by means of nonperturbative techniques, which include instantons [12, 13], the model of the stochastic vacuum [14], holography [15, 16, 17, 18, 19], and the lattice [13, 20, 21].

<sup>2</sup>We are interested in unpolarised scattering, so only the impact parameter modulus matters; for definiteness, we keep it oriented along direction 2 in the transverse plane.

### 3. Large- $s$ behaviour of $\sigma_{\text{tot}}$

The energy dependence of  $\sigma_{\text{tot}}$  is determined by the large- $s$ , large- $b$  behaviour of the amplitude through the “effective radius” of interaction  $b_c = b_c(s)$ , beyond which the amplitude is negligible, as  $\sigma_{\text{tot}} \propto b_c^2$ .

To determine  $b_c$ , in Ref. [5] we employed the following strategy. On the Euclidean side, we obtain information on the  $b$ - and  $\theta$ -dependencies of  $C_E$  by inserting between the WL operators (for large enough  $b$ ) a complete set of asymptotic states, characterised by their particle content and by the momenta and spin of each particle. After AC this gives us information on the  $s$ - and  $b$ -dependencies. This requires two crucial analyticity assumptions:

1. AC can be performed separately for each term in the sum;
2. WL matrix elements are analytic in  $\theta$ .

A few reasonable finiteness assumptions on the WL matrix elements are also made.

Under these assumptions, at large  $\chi, b$ , the relevant Minkowskian correlator reads [5]

$$C_M(\chi, b; \nu) \simeq \sum_{\alpha \neq 0} f_\alpha(\nu) \prod_a [w_a(\chi, b)]^{n_a(\alpha)}, \quad (3)$$

where the sum is over (non-vacuum) states  $\alpha$ , and  $n_a(\alpha)$  is the number of particles of type  $a$  in state  $\alpha$ . Here

$$w_a(\chi, b) = \frac{e^{[r^{(a)}\chi - b]m^{(a)}}}{\sqrt{2\pi b m^{(a)}}}, \quad r^{(a)} \equiv \frac{s^{(a)} - 1}{m^{(a)}}, \quad (4)$$

with  $(s^{(a)}, m^{(a)})$  spin and mass of particles of type  $a$ , and  $f_\alpha$  are functions of the dipole variables only. Particles of type  $a$  contribute only for  $b \lesssim r^{(a)}\chi$ , and so the effective radius of interaction is given by

$$b_c(s) = \left( \max_a r^{(a)} \right) \log \frac{s}{m^2} \equiv \frac{1}{\mu} \log \frac{s}{m^2}. \quad (5)$$

Here we assume the maximum to exist and to be positive. If it were zero or negative,  $\sigma_{\text{tot}}$  would be constant or vanishing at high energy. The spectrum is supposed to be free of massless states: in case they were present and with spin at most 1, the maximisation should be performed on the massive spectrum only [5].

Using Eq. 3 we find for  $\sigma_{\text{tot}}$

$$\sigma_{\text{tot}} \underset{s \rightarrow \infty}{\simeq} 2\pi(1 - \kappa)[b_c(s)]^2 \simeq \frac{2\pi}{\mu^2}(1 - \kappa) \log^2 \frac{s}{m^2}, \quad (6)$$

with  $|\kappa| \leq 1$  due to unitarity [5]. In general  $\kappa$  depends on the colliding hadrons. Analyticity and crossing symmetry [22, 23] requirements show that universality is most naturally achieved if  $\kappa = 0$ , corresponding to a vanishing or oscillating correlator as  $\chi \rightarrow \infty$  at fixed  $b$ .

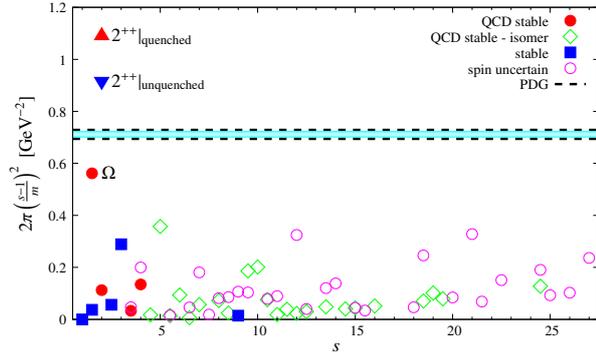


Figure 2: Plot of  $2\pi\left(\frac{s-1}{m}\right)^2$  for QCD-stable states. The  $2^{++}$  glueball state [24, 25] and the experimental value for  $B$  [1] are also shown. Nuclear data are taken from Ref. [26].

#### 4. Total cross sections from the hadronic spectrum

The total cross section satisfies the bound [5]

$$\sigma_{\text{tot}} \underset{s \rightarrow \infty}{\lesssim} \frac{4\pi}{\mu^2} \log^2 \frac{s}{m^2} = 2B_{\text{th}} \log^2 \frac{s}{m^2}, \quad (7)$$

with  $\mu^{-1} = \max_a r^{(a)}$  determined from the hadronic spectrum by maximising  $r^{(a)}$  over the stable states of QCD *in isolation*, as electroweak effects have been neglected from the onset. Only states with  $s^{(a)} \geq 1$  are considered, including nuclei (see Fig. 2).

Quite surprisingly, this singles out the  $\Omega^\pm$  baryon, which yields  $B_{\text{th}} \simeq 0.56 \text{ GeV}^{-2}$ . The resulting bound on  $\sigma_{\text{tot}}$  is much more stringent than the original bound, and the prefactor is of the same order of magnitude of the experimental value  $B_{\text{exp}} \simeq 0.69 \div 0.73 \text{ GeV}^{-2}$  [1]. Furthermore, our bound is not singular in the chiral limit [27].

If universality is achieved as discussed above in Section 3, then  $\sigma_{\text{tot}}$  is entirely determined by the hadronic spectrum, and reads [5]

$$\sigma_{\text{tot}} \underset{s \rightarrow \infty}{\simeq} \frac{2\pi}{\mu^2} \log^2 \frac{s}{m^2} = B_{\text{th}} \log^2 \frac{s}{m^2}. \quad (8)$$

This *prediction* for the prefactor is in fair agreement with  $B_{\text{exp}}$ , taking into account a systematic error of order 10% on  $B_{\text{exp}}$ , estimated by comparing the results of different fitting procedures [1, 28, 29, 30]. The same conditions leading to universal total cross sections also give universal, black-disk-like elastic scattering amplitudes [5].

Total cross sections are usually believed to be governed by the gluonic sector of QCD. However, in the *quenched* theory one finds from the glueball spectrum  $B_Q \gtrsim 1.6B_{\text{exp}}$ , suggesting large unquenching effects [5].

#### 5. Conclusions

In Ref. [5] we have derived the asymptotic, high-energy behaviour of hadronic total cross sections in the framework of the nonperturbative approach to soft high-energy scattering [6, 7, 8]. We find a ‘‘Froissart-like’’ behaviour  $\sigma_{\text{tot}} \sim B \log^2 s$ , with  $B$  (mainly) determined by the hadronic spectrum (see Eqs. 7 and 8), and in fair agreement with experiments.

Our main results do not depend on the detailed description of the hadrons in terms of partons: adding gluons and sea quarks to the wave functions would lead to more complicated WLs, but since our argument is independent of their detailed form, our conclusions remain unchanged.

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