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Transplanckian masses in inflation

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Abstract

We explore the possibility that the transplanckian field values needed to accommodate the experimental results in minimally coupled single-field inflation models are only due to our insistence of imposing a minimal coupling of the inflaton field to gravity. A simple conformal transformation can bring the field values below the Planck mass without changing the physics at the expense of having a richer gravitational sector. Transplanckian field values may be the signal that we are (miss)interpreting phenomena due to gravity as being originated exclusively in the scalar sector.

Keywords:

1. Introduction

After the recent discovery of tensor modes at BICEP2 experiment [1][1], the theory of cosmological inflation [2] can claim to be the current (undisputed) paradigm of early universe cosmology. Inflation cannot only solve most of the problems of the Standard Big Bang Model but it offers the only available explanation for the origin of the large-scale structure of the universe based on causal physics. Even more, cosmological inflation is a predictive theory. It calls for an almost scale invariant spectrum of curvature perturbations which anticipates the characteristic oscillations in the angular power spectrum of cosmic microwave anisotropy maps, observed with high accuracy by WMAP [3] and Plank [4]. Unfortunately, inflation comes at a cost. Successful models of inflation, *i.e.* successful inflationary potentials require unusual features: the potentials have to be extremely flat so that enough inflation is produced to actually solve the above-mentioned issues, and, observations seem to require the inflaton field to travel over transplanckian distances in field space. In fact, following an argument due to Lyth [5] we have,

$$\frac{\Delta\theta}{M_{\rm Pl}} \gtrsim 5.8 \left(\frac{N_e}{50}\right)\left(\frac{r}{0.2}\right)^{1/2} \tag{1}$$

with $\Delta\theta$ the variation of the field during inflation, $r \simeq 13.8\ \epsilon$ the tensor-to-scalar ratio with ϵ the usual slow-roll parameter, N_e the number of e-folds of inflation since the relevant scales left the horizon till the end of inflation and $M_{\rm Pl} = (16\pi G)^{-1/2}$. Therefore, the value of $r = 0.2^{+0.07}_{-0.05}$ measured at BICEP2 implies transplanckian values for the inflaton field, $\Delta\theta/M_{\rm Pl} \gtrsim 5.8$. Fortunately, large (transplanckian) field values do not necessarily involve large (transplanckian) energies, which is the reason why transplanckian field values are not total anathema. In fact, transplanckian field values have been the norm rather than the exception in the inflationary game [10, 9, 8, 6]. There are (almost) no single field inflationary models which can be kept below Planck scale all the way. Yet another problem which has not been devoted enough attention to is the fact that the energy scale of inflation and the Planck scale are not that far from each other and therefore it is easy to imagine that corrections to Planck scale physics are bounded to play a role. Whether this role is significant or not is clearly a debatable issue. Going back to the transplanckian field values, one of the reasons why it is safe to entertain transplanckian field values (once checked that the ob-

[1]Throughout this work we will assume that although the exact numbers of BICEP2 may change, sizeable tensor modes, *i.e.* $r \gtrsim .1$ are an actual feature that will stay

servables are well behaved) is that a field is after all a "dummy" variable, *i.e.* it is "per-se" meaningless. Just a field redefinition will turn its value into the desired domain at no expense, all the observables will remain invariant. Nevertheless, field redefinitions may be gratis observable-wise, but they are not innocent. They will surface somewhere else: in a change of the kinetic terms, the couplings in the potential, etc. In [7], we conjecture about the possibility that the trasplanckian field values arising in single field inflationary models may be due to the fact that we are "forcing" our model to have Einstein gravity. We show that well-behaved and subplanckian modified gravity, as non-minimally coupled scalar fields and/or scalar tensor theories, can become transplanckian once we insist on interpreting them as minimally coupled scalar field theories in Einstein gravity.

2. Inflation in theories with non-minimal couplings to gravity.

We start from a general theory with gravity coupled to a single scalar field that will play the role of the inflaton. The action in the Jordan frame, with non-minimal coupling to gravity and assuming canonical kinetic terms, would be,

$$S = -\int d^4x\ \sqrt{-g}\left[\frac{k^2}{4}D(\theta)\,R - \frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\theta\partial_\nu\theta + V(\theta)\right]$$

where R is the scalar curvature, $k^2 = M_{\rm Pl}^2/(4\pi)$ and θ the scalar field. In the absence of any other sources of matter, and specialising for the case of a Friedmann-Robertson-Walker metric, $g_{\mu\nu} = {\rm diag}\{1, -a(t)^2, -a(t)^2, -a(t)^2\}$ the equations for the Hubble rate and the θ field become,

$$\begin{aligned} D(\theta)\,H^2 &= \tfrac{\dot\theta^2}{3k^2} + \tfrac{2V(\theta)}{3k^2} - \dot D(\theta)\,H \\ \ddot\theta + 3H\dot\theta &+ \tfrac{k^2}{4}\,D'(\theta)\,R + V'(\theta) = 0 \quad . \end{aligned} \tag{2}$$

Due to the addition of the extra source for perturbations we have introduced, $D(\theta)$, we need to include two more slow-roll parameters as compared to the standard case. The scalar-type perturbations will be affected by both of them, although only one (ϵ_3) will be relevant for the tensor perturbations [11, 12],

$$\begin{aligned} \epsilon_1 &= \frac{\dot H}{H^2} = \frac{H'\dot\theta}{H^2} \\ \epsilon_2 &= \frac{\ddot\theta}{H\dot\theta} \\ \epsilon_3 &= \frac{1}{2}\frac{\dot D}{HD} = \frac{1}{2}\frac{D'\dot\theta}{HD} \\ \epsilon_4 &= \frac{1}{2}\frac{\dot E}{HE} = \frac{1}{2}\frac{E'\dot\theta}{HE}\,, \end{aligned} \tag{3}$$

with $E = 3k^2(D')^2/2 + D$. Assuming $\dot\epsilon_i = 0$ and to linear order in the slow-roll parameters

$$\begin{aligned} n_s &= 1 + 2\,(2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) \\ n_T &= 2\,(\epsilon_1 - \epsilon_3) \\ r &= 13.8\mid \epsilon_1 - \epsilon_3 \mid \end{aligned} \tag{4}$$

As it is well-known, this model, as any non standard theory of gravity, can be mapped into a standard theory of gravity, at the expense of having a more complicated matter sector by a conformal transformation. Such a transformation is not just a coordinate redefinition (being general relativity a covariant theory, a coordinate redefinition would become trivial) rather, it is a transformation that mixes up the matter and gravitational degrees of freedom. The mapping we are alluding to, takes the original metric $g_{\mu\nu}$ into a new metric $\tilde g_{\mu\nu}$ according to $\tilde g_{\mu\nu} = e^{2\omega} g_{\mu\nu}$, with $e^{2\omega} = D(\theta)$.

The Hubble rate transforms as,

$$\tilde H = \frac{H + \dot D(\theta)/(2D(\theta))}{\sqrt{D(\theta)}}\,, \tag{5}$$

with $\dot D = \partial D/dt$, and the canonically normalised field in the Einstein frame is

$$\phi(\theta) = \pm\int\sqrt{\frac{3}{2}\left(\frac{D'(\theta)}{D(\theta)}\right)^2 + \frac{2}{k^2D(\theta)}}\;d\theta\,. \tag{6}$$

In terms of this rescaled field the action takes the form,

$$S = -\int d^4x\ \sqrt{-\tilde g}\left[\frac{k^2}{4}\tilde R - \frac{1}{2}\,\tilde g^{\mu\nu}\,\partial_\mu\phi\partial_\nu\phi + \tilde V(\phi)\right], \tag{7}$$

with $\tilde V(\phi) = V(\phi(\theta))/D(\theta)^2$.

It is trivial to show that the slow roll parameters in both frames are related as

$$\begin{aligned}
\tilde{\epsilon} &= \frac{k^2}{4}\left(\frac{\tilde{H}'}{\tilde{H}}\right)^2 = \epsilon_1 - \epsilon_3 \\
\tilde{\eta} &= \frac{k^2}{4}\frac{\tilde{H}''}{\tilde{H}} = \epsilon_2 - 3\epsilon_3 + \epsilon_4 \\
\tilde{n}_s &= 1 + 2\,(2\tilde{\epsilon} - \tilde{\eta}) \\
&= 1 + 2\,(2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) = n_s \\
\tilde{n}_T &= 2\tilde{\epsilon} = 2\,(\epsilon_1 - \epsilon_3) = n_T \\
\tilde{r} &= 13.8 \mid \tilde{\epsilon} \mid = 13.8 \mid \epsilon_1 - \epsilon_3 \mid = r
\end{aligned} \tag{8}$$

Leaving all the observables invariant, as it is obvious given the fact that changing from one frame to another one does not correspond to a change in the physics. However as the conformal transformation changes the space-time curvature (and also the scalar/matter field) phenomena that appear to be due to gravity in one frame may appear to be originated in the scalar sector in another. Besides, it is easy to see that as a result of the fact that the inflaton field in Einstein and Jordan frames are related in a highly non-trivial way, it can be expected that subplanckian values in a given frame, can correspond to transplanckian values in the second frame.

3. Single-field non-minimal models of inflation

As shown in the previous section, the field values in two different frames are correlated by an non-trivial function in a rather complicated way. Here, we will show that is possible and, in fact, quite natural and easy to find realistic examples of theories with non-minimal coupling to gravity, modified gravity or scalar tensor theories, that have transplanckian field values if we insist on imposing a minimal coupling to gravity, but are always subplanckian in their "natural" frame. This clearly does not imply that any conformal transformation will turn transplanckian minimally coupled scalar fields into the subplanckian regime once non-minimally coupled to gravity or once allowed to live in modified gravity schemes but, in our scheme, observations would select a subclass of conformal transformations.

3.1. Monomial Potentials

We start from the well known potential $V(\phi) = \lambda\phi^4$ in the Einstein frame. In the slow-roll regime $\dot{\phi}^2 \ll V(\phi)$, using the Eqs. of motion, we have,

$$H = \sqrt{\frac{2\lambda}{3}}\frac{\phi^2}{k} \quad , \quad \dot{\phi} = -2\sqrt{\frac{2\lambda k^2}{3}}\phi \tag{9}$$

and therefore, we have,

$$\epsilon = \frac{\dot{H}}{H^2} = \frac{H'\dot{\phi}}{H^2} = -\frac{4k^2}{\phi^2} \tag{10}$$

$$\eta = \frac{\ddot{\phi}}{H\dot{\phi}} = -\frac{2k^2}{\phi^2} \tag{11}$$

$$n_s = 1 + 4\epsilon - 2\eta = 1 - \frac{12k^2}{\phi^2} \tag{12}$$

Now, the number of e-foldings fixes the value of the field at which the scales of interest at present left the horizon,

$$N = \int H dt = -\left.\frac{\phi^2}{4k^2}\right|_{\phi_i}^{\phi_f} \simeq \frac{\phi_i^2}{4k^2} \tag{13}$$

Using $N \simeq 62$ we need $\phi_i \simeq 11 \times k \simeq 3.1 \times M_{\rm Pl}$, and we obtain $n_s = 1 - 3/N \simeq 0.95$ and $r \simeq 13\,\epsilon = 13/N \simeq 0.21$. Therefore, we see that this potential would be able to reproduce approximately the observed values for the spectral index and the tensor-to-scalar ratio, but only if the field values during inflation are well above the Planck mass.

However, if our field makes excursions well-beyond the Planck scale, or gets very close to it, we can expect gravitational corrections to come into play and to be very relevant. For example, higher-order curvature invariants should appear and it is then natural to consider also non-minimal couplings of the inflaton to gravity.

For instance, we could have a non-minimal coupling of the inflaton field, θ, to gravity, of the form $D(\theta) = (1 - \theta^2/(3k^2))$. The Einstein equations and the θ equations of motion in this frame, the Jordan frame, are,

$$\begin{aligned}
D(\theta)\,\tilde{H}^2 &= \tfrac{\dot{\theta}^2}{3k^2} + \tfrac{2\tilde{V}(\theta)}{3k^2} - \dot{D}(\theta)\,\tilde{H} \\
\ddot{\theta} + 3\tilde{H}\dot{\theta} &+ \tfrac{k^2}{4}\,D'(\theta)\,R + \tilde{V}'(\theta) = 0 \quad ,
\end{aligned} \tag{14}$$

with $\tilde{H}$ the Hubble rate in the Jordan frame related to the Hubble rate in the Einstein frame by Eq. (5). Then, the potential in the Jordan frame is,

$$\tilde{V}(\theta) = \frac{V\left(\phi(\theta)\right)}{\left(D(\theta)\right)^2} = \lambda\frac{\left(\phi(\theta)\right)^4}{\left(1 - \theta^2/(3k^2)\right)^2} \tag{15}$$

The fields in the Jordan and Einstein frame, are related by Eq. (6) that in this case can be integrated analytically to give

$$\theta(\phi) = \sqrt{3}\,k\tanh\left[\frac{\phi}{2\sqrt{6\pi}\,k}\right], \tag{16}$$

Therefore, we can see clearly that in the Jordan frame, the field θ is always subplanckian, $\theta \leq \sqrt{3/(4\pi)}\ M_{\rm Pl} \simeq 0.489\ M_{\rm Pl}$, and when $\phi \simeq 3.1\ M_{\rm Pl}$

the Jordan field $\theta \simeq 0.42\ M_{\rm Pl}$. It is a trivial exercise to show that using Eqs (3–8), and despite the fact the slow-roll parameters are different in both actions, two non-vanishing in one case corresponding to the usual slow-roll parameters, and four in the Jordan frame, all the observables are identical. Moreover, even in the Jordan frame, the potential is approximately quartic in θ at low field values, $\theta/(\sqrt{3}k) \ll 1$, as can be seen from Eq. (15). Therefore, already by the end of inflation, both theories are indistinguishable.

3.2. Generic scalar-tensor theories

In the previous model, we have specified the potential in the Einstein frame and the transformation to the Jordan and we have seen that the transplanckian values of the field may be simply due to our attempt to write in Einstein form a theory that has a non-minimal coupling to gravity.

Here we will use a different strategy, we will start from an ansatz that guarantees inflation in the Jordan frame and obtain the potential and the non-minimal coupling to gravity from there.

We start from the requirement that space inflates exponentially with the inflaton field being responsible for it, $a = \exp(-\theta/b)$ and therefore $H = \dot{a}/a = -\dot{\theta}/b$ [13].

This ansatz establishes also the number of e-foldings in this scenario, which is given by

$$N_e = \int H dt = -\int \frac{\dot{\theta}}{b} dt = \frac{-1}{b}\int d\theta = \frac{1}{b}\left(\theta_i - \theta_f\right) \simeq \frac{\theta_i}{b}\,, \tag{17}$$

where the fact that the scalar field is rolling down (θ is decreasing) becomes transparent and we have chosen $\theta_f = 0$ at the end of inflation for simplicity.

Using Eqs. (2), with $H = -\dot{\theta}/b$ and $\dot{f} = f'\dot{\theta}$, we can now obtain the relation between the Hubble rate and the coupling to gravity that will sustain the exponential period of expansion we are longing to have,

$$\frac{H'}{H} = \frac{2b/k^2 + bD'' + D'}{2D - bD'} \tag{18}$$

The following step is clear, we need to choose either a coupling to gravity (as we did in the previous section) or a Hubble rate and then obtain the other one via this second order differential equation. In this section, we are going to choose the form of the Hubble rate and, from there, obtain the non-minimal coupling to gravity. For simplicity, we want to obtain analytic expressions for this coupling, and then not many choices for H'/H are possible. One of the simplest choices is $H'/H = 1/M$. In this case we can solve exactly the

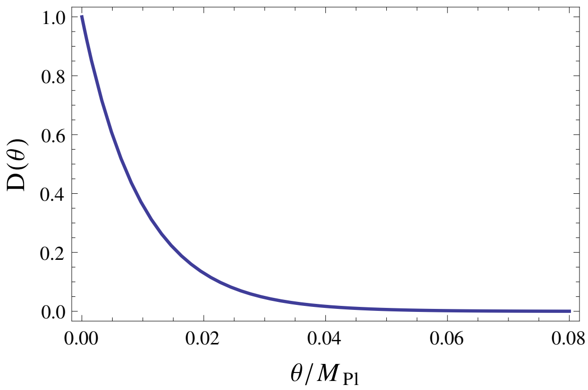

Figure 1: Coupling of the scalar field to the Ricci scalar curvature R. The figure is produced for the slow-roll regime, $\alpha \ll 1$

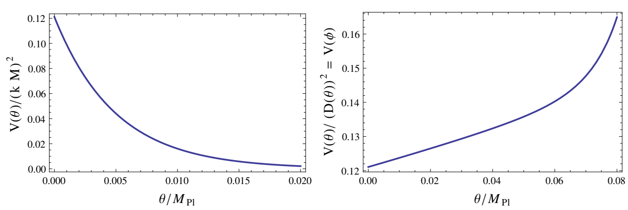

Figure 2: Potential of the scalar field in the Jordan frame in terms of the non-minimally coupled field (left). The potential in the Einstein frame in terms of the non-minimally coupled variable is shown in the right panel. The figures are produced for the slow roll regime, $\alpha \ll 1$.

equation for the coupling to curvature although the expression for it is not particularly enlightening. However, by looking at Figure 1 we can see that, for the set of parameters needed to produce the correct inflation phenomenology, the behaviour of $D(\theta)$ is in fact quite simple. The number of e-foldings in this scenario can be written in terms of the new mass scale M and the parameter α as $N_e \simeq \theta_i/b = \theta_i/(\alpha M)$.

The slow-roll parameters defined in Eqs. (3), in the interesting region $\frac{M^2}{k^2} < e^{-2N_e\alpha}$ (*i.e.* basically 120 $\alpha \sim O(1)$), become,

$$\begin{aligned}
\tilde{\epsilon} &= \epsilon_1 - \epsilon_3 \simeq -2\alpha^2 \\
\tilde{\eta} &= \epsilon_2 + \epsilon_4 - 3\epsilon_3 \simeq -2\alpha^2 \\
n_s &= 1 + 4\tilde{\epsilon} - 2\tilde{\eta} \simeq 1 - 4\alpha^2 \\
r &= 13.8\,|\tilde{\epsilon}| \simeq 27.6\alpha\,.
\end{aligned} \tag{19}$$

As before, we can get the potential, which has a rather baroque expression in full form, although it is basically an exponential potential $e^{\frac{2\theta}{M}}$, as can be seen in the left-side plot of Figure 2. Here, we see that the potential in the θ field is decreasing and seems not able to produce inflation. However, in the Jordan frame, we must take also into account the effects of the non-minimal cou-

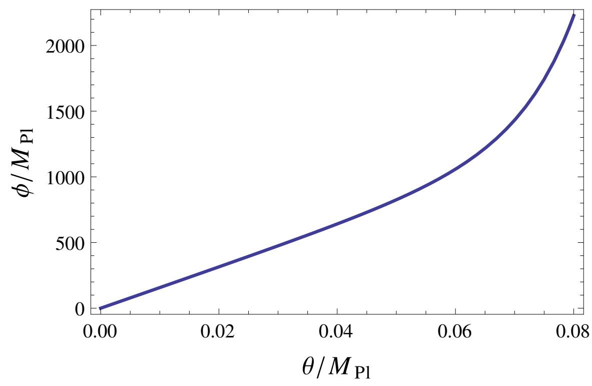

Figure 3: Minimally coupled scalar field in terms of the non-minimally coupled field. The figure is produced for the slow roll regime, $\alpha \ll 1$.

pling to gravity, and then the corresponding potential in the Einstein frame becomes much more attractive. This is shown in the right-side plot in Fig. 2, where we plot the potential in the Einstein frame as a function of the θ field.

In figure 3, we show the relation between both fields, where as before our point becomes transparent: the non-minimally coupled field is subplanckian (if $\alpha M < 1/N_e$), while the minimally coupled one is not. At this point, it is important to stress that, in this case, we have not engineered the coupling to gravity in order to support our point. Instead, we have only asked our scale factor to sustain an inflationary period and looked for the simplest possible choice allowing to solve analytically the second order differential equation which relates the Hubble rate to the curvature coupling. In this context, the emergence of a subplanckian field value in the Jordan frame cannot be considered the result of a fine-tuning.

3.3. f(R) gravity models

To finish we will consider the case where inflation is entirely nourished by gravity [14],

$$L = \frac{k^2}{4} f(R)\,. \tag{20}$$

In this case, equations for the background become

$$\begin{aligned} H^2 &= \frac{1}{3F(R)}\left(\frac{RF(R)-f(R)}{2} - 3H\dot{F}(R)\right) \\ \dot{H} &= -\frac{1}{2F(R)}\left(\ddot{F}(R) - H\dot{F}(R)\right) \end{aligned} \tag{21}$$

where $F(R) = \partial f(R)/\partial R$ and $R = -6(2H^2 + \dot{H})$. Face value, this case is quite distant from the previous ones, as now there is no conformal transformation capable of driving us to the Einstein frame. However, once we depart in a non-trivial way from the standard gravity, the field equations for R become higher-order effectively, signalling the presence of additional degrees of freedom. This feature can be taken care by the introduction of an auxiliary scalar field and going, as an intermediate step, through a Brans-Dicke form of our model [14]. Then, in a similar way as in the previous cases, a conformal transformation will take us away from our $f(R)$ gravity to the kingdom of Einstein gravity plus a minimally coupled scalar field with a specific potential. In the case we are studying, under a conformal transformation, the metric is redefined as $\hat{g}_{ab} = \Omega^2 g_{ab}$ where Ω is a spacetime position dependent factor and is defined to be

$$\Omega^2 = F(R) = \exp\left(\sqrt{\frac{2}{3k^2}}\phi\right) \tag{22}$$

and ϕ is the new dynamical variable we have obtained after conformal transformation of the Brans-Dicke auxiliary field,

$$\phi = \sqrt{\frac{3k^2}{2}}\ln F(R) \tag{23}$$

whose Lagrangian is given by

$$L = -\left(\frac{k^2}{4}\hat{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right) \tag{24}$$

where $\hat{R}$ is the conformally transformed Ricci scalar and the potential has the form

$$V(\phi) = \frac{k^2}{4}\frac{f(R) - RF(R)}{F^2(R)} \tag{25}$$

For the sake of concreteness and to make our point even more transparent, we will consider the following gravity during inflation

$$f(R) = R\left(1 + (R/M^2)^{5/4}\right) \tag{26}$$

where M is an arbitrary mass scale and we assume that during inflation, the second term dominates over the first one, *i.e.* during inflation the second term is driving the Einstein action, implying that during inflation $H^2 \gg M^2$. Clearly, once the inflationary phase is over, we smoothly approach Einstein gravity. In this case

$$F(R) = \exp\left(\sqrt{\frac{2}{3}}\phi\right) \tag{27}$$

and

$$V(\phi) = \frac{-5k^2}{54}\left(\frac{2}{3}\right)^{3/5} M^2 e^{-2\sqrt{\frac{2}{3}}\phi}\left(e^{\sqrt{\frac{2}{3}}\phi} - 1\right)^{9/5} \tag{28}$$

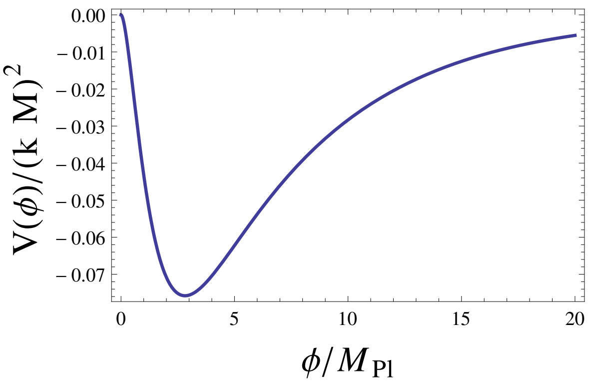

Figure 4: Potential for the scalar field introduced in the conformal transformation. From the figure it is obvious that the potential does have the right shape to inflate.

as before, the form of the potential looks rather complicated but its shape is pretty simple, as can be seen from figure 4. In fact, already by eye, we can guess that this kind of potential should be able to accommodate a decent period of inflation.

But we can do way better that guessing; the analysis of our potential is straightforward. The modes we are interested in studying are those that left the horizon, 60 efoldings before the end of inflation, where $\phi_{\rm end}$ the field value at the end of inflation, is calculated by asking $\epsilon = 1$ obtaining a field value that is well beyond $M_{\rm Pl}$, $\phi_{\rm hor} \approx 15\ M_{\rm Pl}$ giving, $n_s = 0.97$ and $r = 0.16$. Once again, we see that an innocent modification of gravity when casted as a minimally coupled scalar field rolling down a potential ends up giving transplanckian field values. Once more, we like to stress that, as in the previous example, we have not designed a modification of gravity able to accommodate transplanckian field values. We have just chosen an $f(R)$ capable of producing sizeable tensor modes and found that this corresponds to transplanckian field values once analysed as a minimally coupled scalar field.

4. Conclusions

In this work, we have explored the possibility that the transplanckian field values needed to accommodate the experimental results in minimally coupled single-field inflation models are only due to our insistence of imposing a minimal coupling of the inflaton field to gravity. A simple conformal transformation can bring the field values below the Planck mass, although the inflaton coupling to gravity becomes non-minimal and the potential is changed.

We are perfectly aware that the field value by itself carries no information, it is after all a "dummy" variable, but the fact that its vacuum expectation value turns out to be well above the Planck mass may be telling us that it is gravity (or its couplings to gravity), and not only the inflaton potential couplings, the true drivers of inflation.

We have shown that not only it is possible to turn the most popular inflationary potentials into the desired regime by choosing an appropriate coupling to gravity, but also that scalar tensor theories, designed exclusively to sustain inflation by asking the scale factor to grow exponentially, also turn subplanckian even in the simplest cases. We have also presented a case where gravity itself is solely responsible for inflation, and again results in transplanckian field values once interpreted as single field inflation.

In summary, we have seen that is possible to find realistic examples of theories with non-minimal coupling to gravity that have transplanckian field values if we insist on imposing a minimal coupling to gravity, but are always subplanckian in their "natural" frame. Thus, we have proven that single-field inflation models can still accommodate a large tensor-to-scalar ratio with subplankian field values in the presence of non-minimal coupling to gravity.

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