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Lightest pseudoscalar exchange contribution to light-by-light scattering piece of the muon $g − 2$

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Abstract

Lightest pseudoscalar ($P = \pi^0$, η, η') exchange contribution to the light-by-light (LbL) scattering piece of the muon anomaly, $a_\mu = (g_\mu − 2)/2$, has been evaluated using a resonance chiral Lagrangian ($R\chi L$). Best description of pion transition form-factor (TFF) data is obtained with only tiny violations of one of the relations in the minimal consistent set of short-distance constraints on the anomalous $R\chi L$ couplings. $\eta^{(')}$ TFF, predicted in terms of the π TFF and the $\eta − \eta'$ mixing, are in good agreement with measurements. With this input, we obtain $a_\mu^{P,LbL} = (10.47 \pm 0.54) \cdot 10^{-10}$, consistent with the reference determinations in the literature, albeit with smaller error.

Keywords: Electromagnetic form-factors, Resonance Chiral Lagrangians, QCD, $1/N$ expansion, Muon anomalous magnetic moment.

The anomalous magnetic dipole moment of the muon, a_μ, is one the most precisely measured [1] and accurately predicted [2] observables. Since 2000, it exhibits a persistent discrepancy in the ballpark of three standard deviations, $a_\mu^{\exp} − a_\mu^{\text{th}} = (29 \pm 9) \cdot 10^{-10}$, ($3.3\sigma$) [2].

As an electromagnetic property a_μ is, first, a stringent test of QED. The one loop contribution computed by Schwinger [3] fixes its size universally to $\sim \alpha/(2\pi) \sim 10^{-3}$ for all charged leptons, which differs by six orders of magnitude with the current discrepancy $\sim 3 \cdot 10^{-9}$. A tremendous effort in computing higher-order terms lead to the complete five-loop QED result [4]. Its error is four orders of magnitude smaller than the previous value and the $O(\alpha^5)$ term is only $\sim 5 \cdot 10^{-11}$. Therefore, this anomaly cannot be attributed to uncalculated or imprecise QED contributions.

Conversely, hadronic effects are important in a_μ [1]: hadronic vacuum polarization ($LO + NLO$) contributes $(680.7 \pm 4.7) \cdot 10^{-10}$ to a_μ [6]. Although with much smaller central value, the Hadronic LbL contribution

($HLbL$) is crucial for the final theoretical uncertainty: $a_\mu^{HLbL} = (11.6 \pm 4.0) \cdot 10^{-10}$ [2], if we stick to the most conservative estimate.

The total error on the Standard Model prediction for a_μ, $6.3 \cdot 10^{-10}$, nearly equals the current experimental uncertainty, $6.4 \cdot 10^{-10}$. However, forthcoming experiments at FNAL and J-PARC [7] will soon reduce the latter to a fourth. This urges theoreticians to achieve a similar error reduction to benefit fully from the precision of these measurements. This is our main motivation to revisit the dominant contribution to a_μ^{HLbL} given by P exchange.

We have reconsidered [8] the lightest pseudoscalar ($P = \pi^0$, η, η') exchange contribution to a_μ^{HLbL}, which is one of the possible intermediate states in the four-point $VVVV$ Green function with one real and three virtual photons that enters a_μ^{HLbL}. This contribution turns out to basically saturate it, due to approximate cancellations between the remaining pieces [2]. Despite a simultaneous chiral and large-N_C expansion has been suggested [9] to tackle a_μ^{HLbL}, the relative size of the various terms is not fully understood yet. This raises reasonable doubts on the errors obtained studying invividual contributions isolately, which might translate into an increased overall error for a_μ^{HLbL} [10].

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[1]Electroweak effects are small and well under control at the required precision, $a_\mu^{EW} = (153.6 \pm 1.0) \cdot 10^{-11}$ [5].

The main difficulty for computing a_μ^{HLbL} has been that, contrary to the hadronic vacuum polarization, there was no way to relate it to measurements using dispersion relations and the optical theorem. However, very recently a formalism has been put forward [11], which would allow to extract the dominant one- and two-pion exchange contributions directly from data.

At the time being, nevertheless, the most precise experimental information that can be obtained on $a_\mu^{P,HLbL}$ comes from the corresponding TFF, where one photon is real to a very good approximation and the form-factor is measured as a function of the virtuality of the other photon up to roughly 6 GeV. It has been shown [2] that demanding an appropriate short-distance behaviour [12] to the P TFF and to the related VVP Green's function turns out to be crucial for the reliability of the $a_\mu^{P,HLbL}$ value. Dedicated studies of this question [13] have found that using $R\chi L$ [14] (rooted in the large-N_C limit of QCD [15] and chiral symmetry [16]) there exists a consistent set of short-distance constraints on the VVP Green function and related form factors [17]. For this, the antisymmetric tensor formalism needs to be employed [18] and pseudoscalar resonances be active degrees of freedom [19]. However, it is still an open question whether all short-distance constraints are already imposed working this way, or there are new genuine relations arising from the $VVVV$ Green function which have not been considered yet [20].

In this framework the π TFF can be written [17, 19]

$$\mathcal{F}_{\pi^0 \gamma\gamma}(Q^2) = -\frac{F}{3} \frac{Q^2 \left(1 + 32\sqrt{2}\frac{P_2 F_V}{F^2}\right) + \frac{N_C}{4\pi^2}\frac{M_V^4}{F^2}}{M_V^2(M_V^2 + Q^2)}, \quad (1)$$

and one of the high-energy constraints in the minimal consistent set demands that P_2 cancels the $O(Q^0)$ term for $Q^2 \to \infty$. We find that allowing a 4% violation of this condition yields the best fit to current data [2]. Fig. 1 compares our best fit result to all available data.

The fully off-shell form factor is also needed to evaluate $a_\mu^{P,HLbL}$ [2]. It depends only on one additional coupling unrestricted by high-energy behaviour, which can be fixed analysing the $\pi(1300) \to \gamma\gamma$ and $\pi(1300) \to \rho\gamma$ decays [19]. Upon the required integrations [2] one finds

$$a_\mu^{\pi^0,HLbL} = (6.66 \pm 0.21) \cdot 10^{-10}, \quad (2)$$

where the corresponding contributions to the error are discussed in detail in our paper [8].

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[2]This violation is not due to the difference between BaBar and Belle data points. Excluding BaBar data the violation is only reduced to 3%.
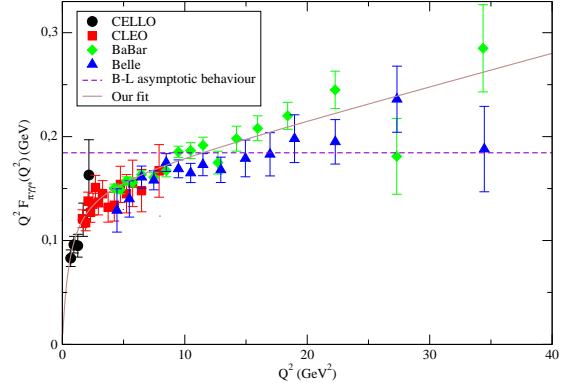

Figure 1: CELLO [21], CLEO [22], BaBar [23] and Belle [24] data for the πTFF are confronted to our best fit result using the form-factor in eq. (1).

Chiral dynamics and the η-η' mixing (whose uncertainty saturates the error on $a_\mu^{\eta^{(\prime)},HLbL}$) allow to relate the π TFF to the $\eta^{(\prime)}$ TFF [8]. Our predictions are compared to data in Figs. 2 and 3.
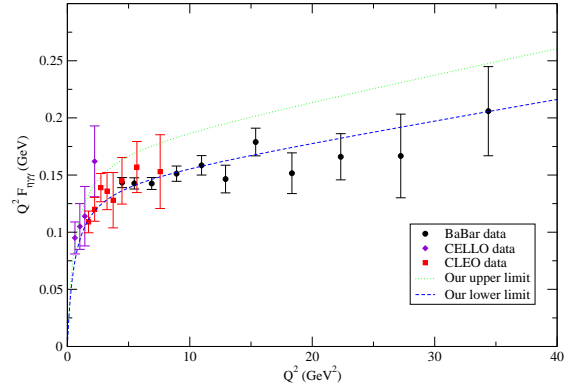

Figure 2: Our predictions for the ηTFF using the π TFF (1) and the η-η' mixing are confronted to BaBar [25], CELLO [21] and CLEO [22] data.

The corresponding contributions to $a_\mu^{P,HLbL}$ being

$$a_\mu^{\eta,HLbL} = (2.04 \pm 0.44) \cdot 10^{-10}, \quad (3)$$

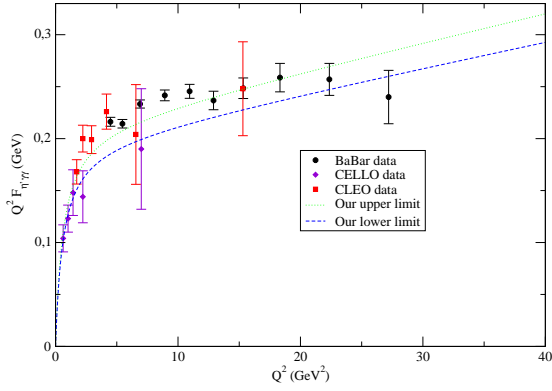$$a_\mu^{\eta',HLbL} = (1.77 \pm 0.23) \cdot 10^{-10}. \quad (4)$$

Figure 3: Our predictions for the η' TFF using the π TFF (1) and the η-η' mixing are confronted to BaBar [25], CELLO [21] and CLEO [22] data.

Our main result is the value

$$a_\mu^{P,HLbL} = (10.47 \pm 0.54) \cdot 10^{-10} , \qquad (5)$$

for the contribution of the three lightest pseudoscalar mesons (π^0, η and η') to the muon anomaly, which is in good agreement with the two reference values: $(9.9 \pm 1.6) \cdot 10^{-10}$ (Jegerlehner and Nyffeler) and $(11.4 \pm 1.3) \cdot 10^{-10}$ (Prades, de Rafael and Vainshtein) [2] but has a reduced error, mainly thanks to the new BaBar and Belle data on the P TFF extending to larger energies.

Using the values in the literature for the remaining contributions to a_μ^{HLbL} [2] yields

$$a_\mu^{HLbL} = (11.8 \pm 2.0) \cdot 10^{-10} , \qquad (6)$$

which would translate into a theoretical uncertainty on a_μ of $\pm 5.1 \cdot 10^{-10}$, $\sim 20\%$ smaller than the current estimate.

Finally, we also propose [8] the measurement of $\sigma(e^+e^- \to \pi^0 \mu^+ \mu^-)$ and the corresponding differential distribution as a function of the di-muon invariant mass to better characterize the P TFF and hopefully further reduce the error of $a_\mu^{P,HLbL}$.

Acknowledgements

References

[1] G. W. Bennett et al. [Muon G-2 Collaboration], Phys. Rev. D **73** (2006) 072003; Phys. Rev. Lett. **92** (2004) 161802.

[2] F. Jegerlehner and A. Nyffeler, Phys. Rept. **477** (2009) 1. J. Prades, E. de Rafael and A. Vainshtein, Advanced series on directions in high energy physics. Vol. 20.

[3] J. S. Schwinger, Phys. Rev. **73** (1948) 416.

[4] T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. **109** (2012) 111808.

[5] A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. **76** (1996) 3267. M. Knecht, S. Peris, M. Perrottet and E. De Rafael, JHEP **0211** (2002) 003. A. Czarnecki, W. J. Marciano and A. Vainshtein, Phys. Rev. D **67** (2003) 073006 [Erratum-ibid. D **73** (2006) 119901].

[6] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C **71** (2011) 1515 [Erratum-ibid. C **72** (2012) 1874]. B. Krause, Phys. Lett. B **390** (1997) 392.

[7] T. Mibe contribution to arXiv:1407.4021 [hep-ph]. G. Venanzoni for the new MUON g-2 Coll., these proceedings.

[8] P. Roig, A. Guevara and G. López Castro, Phys. Rev. D **89** (2014) 073016.

[9] E. de Rafael, Phys. Lett. B **322** (1994) 239.

[10] A. Nyffeler, Nuovo Cim. C **037** (2014) 02, 173.

[11] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, arXiv:1402.7081 [hep-ph].

[12] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B**147** (1979) 385; 448.

[13] B. Moussallam, Phys. Rev. D **51** (1995) 4939; Nucl. Phys. B **504** (1997) 381. J. Bijnens, E. Gamiz, E. Lipartia and J. Prades, JHEP **0304** (2003) 055. M. Knecht and A. Nyffeler, Eur. Phys. J. C **21** (2001) 659.

[14] G. Ecker et. al., Nucl. Phys. B **321** (1989) 311, Phys. Lett. B **223** (1989) 425.

[15] G. 't Hooft, Nucl. Phys. B **72** (1974) 461; **75** (1974) 461. E. Witten, Nucl. Phys. B **160** (1979) 57.

[16] S. Weinberg, Physica A **96** (1979) 327. J. Gasser and H. Leutwyler, Annals Phys. **158** (1984) 142; Nucl. Phys. B **250** (1985) 465. J. Bijnens et. al., JHEP **9902** (1999) 020, Eur. Phys. J. C **23** (2002) 539.

[17] P. Roig and J. J. Sanz-Cillero, Phys. Lett. B **733** (2014) 158.

[18] P. D. Ruiz-Femenía, A. Pich and J. Portolés, JHEP **0307** (2003) 003.

[19] K. Kampf and J. Novotny, Phys. Rev. D **84** (2011) 014036.

[20] J. Bijnens, E. Gamiz, E. Lipartia and J. Prades, JHEP **0304** (2003) 055. K. Melnikov and A. Vainshtein, Phys. Rev. D **70** (2004) 113006. B. Ananthanarayan and B. Moussallam, JHEP **0406** (2004) 047. A. Nyffeler, Phys. Rev. D **79** (2009) 073012. M. Knecht and A. Nyffeler, work in progress.

[21] H. J. Behrend et al. [CELLO Collaboration], Z. Phys. C **49** (1991) 401.

[22] J. Gronberg et al. [CLEO Collaboration], Phys. Rev. D **57** (1998) 33.

[23] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D **80** (2009) 052002.

[24] S. Uehara et al. [Belle Collaboration], Phys. Rev. D **86** (2012) 092007.

[25] P. del Amo Sanchez et al. [BaBar Collaboration], Phys. Rev. D **84** (2011) 052001.