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Natural quark mass structure in a U(1)' gauge extension

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Abstract

We propose a $U(1)'$ extension of the standard model with the addition of three quark singlets, two scalar singlets and one scalar doublet. By introducing mixing couplings between ordinary and exotic fermions ruled by global symmetries, we obtain mass relations among quarks with few parameters The model exhibits a "natural" scenery where a hierarchical mass spectrum can be obtained by providing Yukawa couplings with a nearly symmetric structure

Keywords: Quark masses, electroweak gauge sector, flavor symmetries

1. Introduction

The standard model (SM) is a theory based on the gauge group $G_{sm} = SU(3)_c \times SU(2)_L \times U(1)_Y$ that successfully explain most of the particle physics data [1, 2]. However, there are some questions that the SM does not explain in a satisfactory form. For example, in the quark flavor puzzle [3], all the quarks exhibit very different mass values, as shown in Fig. 1, even though they acquire masses at the same scale $v = 246$ GeV.

In the SM, the fermion masses are provided by Yukawa interactions consistent with the gauge symmetry and with the inclusion of only one scalar doublet. Calling q_L^i the usual left-handed quark doublet, U_R^i and D_R^i the right-handed up- and down-type singlets, and $i = 1, 2, 3$ the three phenomenological families, the Yukawa Lagrangian for the quark sector reads:

$$-\mathcal{L}_Q = \overline{q_L^i}(\phi h^U)_{ij}U_R^j + \overline{q_L^i}(\widetilde{\phi} h^D)_{ij}D_R^j + h.c., \quad (1)$$

where the scalar doublet ϕ acquire a vacuum expectation value (VEV) $v = 246$ GeV. After the symmetry breaking, the above Lagrangian leads to the mass matrices. Since the original symmetry G_{sm} does not impose any restriction in the family indices, there arise off-diagonal terms in the Yukawa matrices h^Q, leading to mass matrices of the form:

$$M_{U,D} = \frac{v}{\sqrt{2}}h^{U,D} = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}, \quad (2)$$

where $*$ indicates components proportional to v. However, the mass of the heaviest quark (top) is about 5 order of magnitude larger than the lightest quark (up). This require "unnatural" fine tuning of the Yukawa parameters without any apparent fundamental reason.

2. The Model

Let us assume the extension to the gauge symmetry $G' = G_{SM} \otimes U(1)_X$, with the following transformation rules for quarks and scalar doublet

$$q_L^{1(2,3)} = \begin{pmatrix} U^{1(2,3)} \\ D^{1(2,3)} \end{pmatrix}_L : (3, 2, \tfrac{1}{3}, \tfrac{1}{3}(0)),$$
$$U_R^i : (3^*, 1, \tfrac{4}{3}, \tfrac{2}{3}),$$
$$D_R^i : (3^*, 1, \tfrac{-2}{3}, \tfrac{-1}{3}),$$
$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \xi_1 + i\phi_1^0) \end{pmatrix} : (1, 2, 1, \tfrac{2}{3}),$$

where the values of the charges X are non-universal in the left-handed quark sector. However, this spectrum

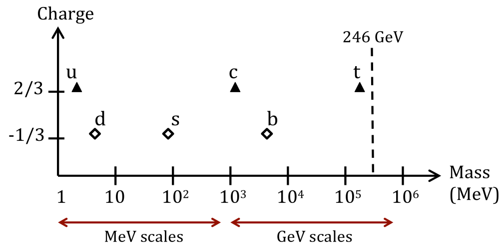

Figure 1: Mass scales for the up-type (charge 2/3) and down-type (charge −1/3) phenomenological quarks. The dashed line shows the electroweak breaking scale $v = 246$ GeV

exhibits the following problems: (a.) It predicts massless quarks, (b.) the local $U(1)_X$ predicts a second neutral and massless weak current, and (c.) the new neutral current induces anomalies which are not cancelled. To fixs these problems, we add the following particles:

$$\begin{aligned} T_{L(R)} &: (3, 1, \tfrac{4}{3}, \tfrac{1}{3}(\tfrac{2}{3})), \\ J^n_{L(R)} &: (3, 1, \tfrac{-2}{3}, 0(\tfrac{-1}{3})), \\ \phi_2 &= \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \xi_2 + i\phi_2^0) \end{pmatrix} : (1, 2, 1, \tfrac{1}{3}), \\ \chi_0 &= \tfrac{1}{\sqrt{2}}(v_\chi + \xi_\chi + i\zeta_\chi) : (1, 1, 0, \tfrac{1}{3}), \\ \sigma_0 &= \tfrac{1}{\sqrt{2}}(v_\sigma + \xi_\sigma + i\zeta_\sigma) : (1, 1, 0, \tfrac{1}{3}), \end{aligned} \tag{3}$$

where:

- The singlets T and J^n are up- and down-like quarks, respectively, with $n = 1, 2$.
- An additional scalar doublet ϕ_2 is introduced.
- An extra scalar singlet χ_0 with VEV v_χ is required to produce the breaking of the $U(1)_X$ symmetry at a large scale.
- Another scalar singlet σ_0 is introduced. Since it is not essential for the symmetry breaking mechanisms, we may choose a small VEV $\langle\sigma_0\rangle = v_\sigma \lesssim v$

We must assume the existence of additional global symmetries. They are:

- Z_2 *symmetries:* We restrict the couplings of scalar fields by requiring the discrete symmetries

$$\begin{aligned} &\phi_2 \to -\phi_2, \;\; \sigma_0 \to -\sigma_0, \\ &D^i_R \to -D^i_R \;\; T_{L,R} \to -T_{L,R}. \end{aligned} \tag{4}$$

- $U(1)_{T_3}$ *symmetry:* In the absence of the Yukawa couplings and after the gauge symmetry breaking, the model still has a global $SU(2)_L$ symmetry. Let us assume that the Yukawa Lagrangian break this global symmetry, but an $U(1)_{T_3}$ symmetry remains in the left-handed down sector, under which

$$D^2_L \to -D^2_L, \qquad D^3_L \to D^3_L. \tag{5}$$

Thus, by requiring the above global symmetries, we obtain extended mass matrices of the form:

$$\begin{aligned} M'_U &= \left(\begin{array}{c|c} M_U & k \\ \hline K & M_T \end{array}\right) \\ &= \left(\begin{array}{ccc|c} 0 & 0 & 0 & \bullet \\ * & * & * & 0 \\ * & * & * & 0 \\ \hline \bullet & \bullet & \bullet & \times \end{array}\right), \\ M'_D &= \left(\begin{array}{c|c} M_D & s \\ \hline S & M_J \end{array}\right) \\ &= \left(\begin{array}{ccc|c} 0 & 0 & 0 & \bullet \\ 0 & 0 & 0 & \bullet \\ * & * & * & 0 \\ \hline \bullet & \bullet & \bullet & \times \end{array}\right), \end{aligned} \tag{6}$$

where the symbols $*$ and $\bullet$ indicate terms at the electroweak scale v, and $\times$ terms at the large scale v_X. The above extended matrices have non-vanishing determinant, providing masses to all quarks. Specifically, the mass matrices $M'_{U,D}$ exhibit three eigenvalues at the scale $m_{u,d,s} \sim$ MeV, three at the scale $m_{c,b,t} \sim$ GeV and three at the scale $m_{T,J} \sim$ TeV, which is consistent with Fig. 1.

3. Numerical Results

Let us choose the following structures consistent with Equation (6):

$$M'_U = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & v_2 y_1 \\ v_1 Y_U & v_1 Y_U & v_1 Y_U & 0 \\ v_1 Y_U & v_1 Y_U & v_1 Y_t & 0 \\ v_\sigma c_1 & 0 & 0 & v_\chi h_\chi^T \end{pmatrix},$$

$$M'_D = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & h_D & 0 \\ 0 & 0 & 0 & 0 & h_D \\ v_2 Y_D & v_2 Y_D & v_2 Y_D & 0 & 0 \\ v_\sigma \Gamma_D & 0 & 0 & v_\chi k_{11} & 0 \\ 0 & v_\sigma \Gamma_D & 0 & 0 & v_\chi k_{22} \end{pmatrix}.$$

Assuming that $v_\chi \gg v_{1,2,\sigma}$, we find for the up sector that:

$$\begin{aligned} m_u &\approx y_1 c_1 \left(\frac{v_2 v_\sigma}{2 m_T}\right) \\ m_{c,t} &= \frac{v_1}{2\sqrt{2}}(Y_U + Y_t)\left[1 \mp \sqrt{1 + 4 y_{Ut}\epsilon_{Ut}}\right] \\ m_T &\approx \frac{1}{\sqrt{2}} h_\chi^T v_\chi, \end{aligned} \tag{7}$$

where we define the asymmetry parameters

$$y_{Ut} = \frac{Y_U}{Y_U + Y_t}, \quad \epsilon_{Ut} = \frac{(Y_U - Y_t)}{(Y_U + Y_t)}. \tag{8}$$

For the down sector we find:

$$\begin{aligned} m_{d,s} &\approx \Gamma_D \left(\frac{h_D v_\sigma}{2 m_{J^{1,2}}}\right) \\ m_b &= \frac{1}{\sqrt{2}} Y_D v_2 \\ m_{J^{1,2}} &\approx \frac{1}{\sqrt{2}} k_{11,22} v_\chi. \end{aligned} \tag{9}$$

Using the second equation from (7) and the definitions (8), we find the ratio

$$\frac{m_c}{m_t} \approx \frac{-y_{Ut}\epsilon_{Ut}}{1 + y_{Ut}\epsilon_{Ut}}, \tag{10}$$

while from the first equation in (9) for the down sector we find

$$\frac{m_d}{m_s} = \frac{m_{J^2}}{m_{J^1}}. \tag{11}$$

Regarding the up and bottom quarks, we find that

$$\frac{m_u}{m_b} = \left(\frac{y_1 c_1}{\sqrt{2} Y_D}\right) \frac{v_\sigma}{m_T}. \tag{12}$$

Taking only the central values, we consider the following experimental masses [2]:

$$\begin{aligned} & m_u = 2.3 \text{ MeV}, m_d = 4.8 \text{ MeV}, m_s = 95 \text{ MeV}, \\ & m_c = 1.275 \text{ GeV}, m_b = 4.65 \text{ GeV}, m_t = 173.5 \text{ GeV}. \end{aligned}$$

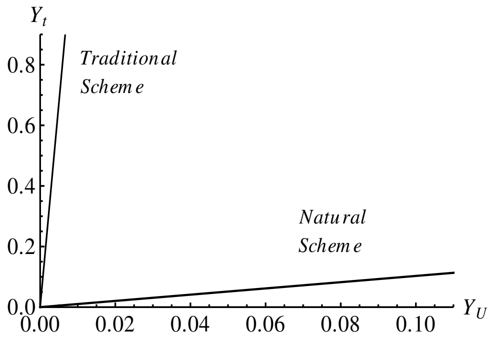

Figure 2: Ratio between the top (Y_t) and light (Y_U) Yukawa couplings. Both lines show two schemes: the *traditional scheme* with $Y_t/Y_U \approx 135.05$ and the *natural scheme* with $Y_t/Y_U \approx 1.03$.

Using the above values and the ratios (10), (11) and (12), we obtain the following relations:

$$y_{Ut}\epsilon_{Ut} = \frac{-m_c/m_t}{1 + m_c/m_t} \approx -7,3 \times 10^{-3}, \tag{13}$$

$$m_{J^1} \approx 20\, m_{J^2}, \tag{14}$$

$$\frac{v_\sigma}{m_T} \approx \left(5 \times 10^{-4}\right) \frac{\sqrt{2} Y_D}{y_1 c_1}. \tag{15}$$

Fig. 2 shows the top quark coupling Y_t as function of the light-quark coupling Y_U according to (13), which exhibits two possible solutions which lead to two different mass schemes. First, the *traditional scheme*, where $Y_t/Y_U \approx 135.05$ is required to fit the experimental masses, and where small variations of Y_U imply large variations of Y_t. This ratio implies an asymmetry factor $\epsilon_{Ut} \approx -0.985$. Second, we obtain a "*natural scheme*" where $Y_t/Y_U \approx 1.03$, which is consistent with a symmetry where degenerated up-type Yukawa couplings is favored, with $\epsilon_{Ut} \approx -0.015$.

acknowledgment

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References

[1] S.L. Glashow, Nucl. Phys. 22, 579 (1961); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N.Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[2] J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012).

[3] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B147, 277 (1979).