Uncertainties on the determination of the strong coupling α_s from the energy evolution of jet fragmentation functions at low z

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Abstract

The QCD coupling α_s is determined at NLO*+NMLLA accuracy from the comparison of experimental jet data to theoretical predictions of the energy-evolution of the parton-to-hadron fragmentation function moments (multiplicity, peak, width, skewness) at low fractional hadron momentum z. From the existing e^+e^- and $e^\pm p$ jet data, we obtain $\alpha_s(m_Z^2) = 0.1195\pm0.0021(\text{exp})^{+0.0015}_{-0.0}(\text{scale})$ at the Z mass. The uncertainties of the extracted α_s value are discussed.

Keywords: strong coupling, pQCD, jets, parton fragmentation functions, e^+e^- annihilation, deep-inelastic scattering

1. Introduction

In the chiral (massless quark) limit, the theory of the strong interaction –quantum chromodynamics (QCD)– has a single fundamental parameter: its coupling α_s which decreases logarithmically with increasing energy scale Q, i.e. $\alpha_s \propto \ln(Q^2/\Lambda_{\text{QCD}}^2)$, starting from a value $\Lambda_{\text{QCD}} \approx 0.2$ GeV where the perturbatively-defined coupling diverges. The running coupling α_s enters in the perturbative expansion of theoretical cross section for any hard process involving quarks and gluons, and the uncertainty in its value is a key (sometimes dominant) component of the theoretical error in all perturbative QCD (pQCD) predictions for any collision process involving hadrons. Many precision fits of the Standard Model as well as searches for new physics depend on α_s, whose value needs, thus, to be determined with good accuracy and precision.

The determination of α_s relies on the comparison of theoretical predictions for various observables, obtained perturbatively at a given level of accuracy (next-to-next-to-leading order, NNLO, in most cases), with the corresponding experimental measurements. The extracted α_s values at different energy scales are then compared to each other by translating the result in terms of the value at the Z mass pole: $\alpha_s(m_Z^2)$. The current value of $\alpha_s(m_Z^2) = 0.1185\pm0.0006$, has been obtained from a combination of measurements at e^+e^-, deep-inelastic scattering (DIS) e,ν-p, and hadron-hadron colliders [1]:

(i) hadronic τ decays (N^3LO), (ii) hadronic W,Z decays (N^3LO), (iii) radiative heavy-quarkonia decays (NLO), and (iv) event shapes and jet rates (NNLO), in e^+e^- collisions;

(v) scaling violations in parton distribution functions (NNLO), and jet cross sections (NLO), in e,ν-p DIS;

(vi) jet cross sections and angular correlations (NLO), and top-quark cross sections (NNLO), in p-p,p-$\bar{\text{p}}$ collisions.

In addition, comparisons of the predictions for different short-distance observables computed through lattice and pQCD methods provide extra "data points" for the determination of the $\alpha_s(m_Z^2)$ world-average, although the low systematic uncertainty assigned to such an approach has been questioned [2]. The current α_s uncertainty is of order ±0.5%, although a more conservative estimate sets it at the ±1% level [2], making

of α_s the least precisely known of all fundamental couplings in nature. In this context, having at hand extra independent approaches to determine α_s –with experimental and theoretical uncertainties different than those of the methods currently used– would be an obvious asset.

Attempts were carried out in the past (see e.g. [3]) to try to measure Λ_{QCD} (or, equivalently, α_s) using parton-to-hadron fragmentation functions (FFs) in the region of low fractional momenta $z = p_h/p_{parton}$ via the Modified Leading Logarithmic Approximation (MLLA) [4] which resums the soft and collinear singularities present in this region of phase-space. However, these older calculations were limited to LO approximations with a number of simplifying assumptions: (i) ad hoc cuts in the experimental distributions, (ii) simple fits in a restricted FF range, (iii) number of quark-flavours fixed to $N_f = 3$, and (iv) use of only one or two FF moments (Gaussian approximation). As a result, very imprecise (or even inconsistent) values of $\Lambda_{QCD} \approx$ 80–600 MeV were obtained. In Ref. [5] we have presented a novel extraction of α_s from the energy evolution of the first four moments of the parton-to-hadron FFs including, for the first time, resummations of higher-order (NNLL or NMLLA) logarithms and NLO running-coupling corrections. The approach has been successfully tested with the world experimental jet data from e^+e^- annihilation [5] and deep-inelastic $e^\pm p$ collisions [6]. We provide here a more detailed assessment of the systematic uncertainties involved in our α_s extraction.

2. NLO*+NMLLA parton-to-hadron fragmentation functions

The parton-to-hadron FF, $D_{i \to h}(z, Q)$, encodes the probability that parton i fragments into a hadron h carrying a fraction z of the parent parton's momentum. Due to colour coherence and gluon-radiation interference inside a parton shower, not the softest partons but those with intermediate energies ($E_h \propto E_{jet}^{0.3}$) multiply most effectively in QCD cascades, leading to a single-particle spectrum with a typical "hump-backed plateau" (HBP) shape as a function of $\xi = \ln(1/z)$ (Fig. 1). Such an HBP shape can be parametrized, without any loss of generality, as a distorted Gaussian (DG) which depends on the original energy of the parton, $Y = \ln E\theta/Q_0$, evolved down to a shower cut-off scale $\lambda = \ln(Q_0/\Lambda_{QCD})$:

$$D(\xi, Y, \lambda) = \mathcal{N}/(\sigma\sqrt{2\pi}) \cdot e^{\left[\frac{1}{8}k - \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4\right]}, (1)$$

where $\delta = (\xi - \bar{\xi})/\sigma$, with moments: $\mathcal{N}$ (hadron multiplicity inside the jet), $\bar{\xi}$ (DG peak position), σ (DG

width), s (DG skewness), and k (DG kurtosis). Fig. 1 shows a typical example of FFs fitted to Eq. (1) with the corresponding extracted moments. Since the measured final-state hadrons are massive and the theoretical calculations assume massless partons/hadrons (i.e. $\xi_p = \xi_E$), the expression (1) for ξ needs to be modified by introducing an effective mass, via $E = \sqrt{p^2 + m_{eff}^2}$, with the corresponding Jacobian determinant correction. Such a change of variable only affects the DG fit for the softest hadrons (i.e. for the highest ξ values) [5].
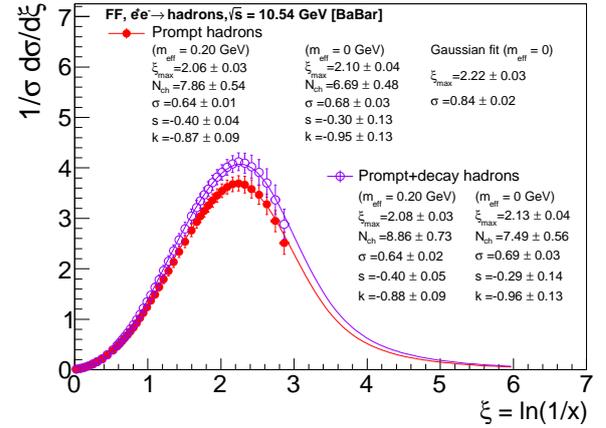

Figure 1: Jet fragmentation function measured by BaBar for prompt (closed dots) and inclusive (open dots) charged-hadrons in e^+e^- at $\sqrt{s} = 10.54$ GeV [7] fitted to the DG Eq. (1). The obtained moments are listed for two choices of the effective mass ($m_{eff} = 0, 0.2$ GeV).

The process of parton radiation and splitting occurring in a jet shower, i.e. the evolution of the FF from a scale Q to Q' before the final parton-to-hadron transition, is governed by the DGLAP [8] equations (at large $z \gtrsim 0.1$, i.e. low $\xi \lesssim 2$) and by MLLA approaches (at small z, i.e. large $\xi \gtrsim 2$). In Ref. [5] we have solved the set of integro-differential equations for the FF evolution including[1] NLO* α_s and next-to-MLLA corrections, by expressing the Mellin-transformed hadron distribution in terms of the γ anomalous dimension: $D \simeq C(\alpha_s(t)) \exp\left[\int^t \gamma(\alpha_s(t'))dt'\right]$ for $t = \ln Q$, resulting in an expansion in half-powers of α_s: $\gamma \sim O_{DLA}(\sqrt{\alpha_s}) + O_{MLLA}(\alpha_s) + O_{NMLLA}(\alpha_s^{3/2}) + \cdots$, of which two new higher-order terms have been computed for the first time. The corresponding NLO*+NMLLA dependencies of the 5 moments of Eq. (1) as a function of Y and λ, have been derived. Evolving the fragmentation functions down to the lowest possible scale, $Q_0 \to \Lambda_{QCD}$ (i.e. $\lambda = 0$, or "limiting spectrum"), one obtains expressions for the

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[1] 'NLO*' indicates missing NLO terms in the splitting functions.
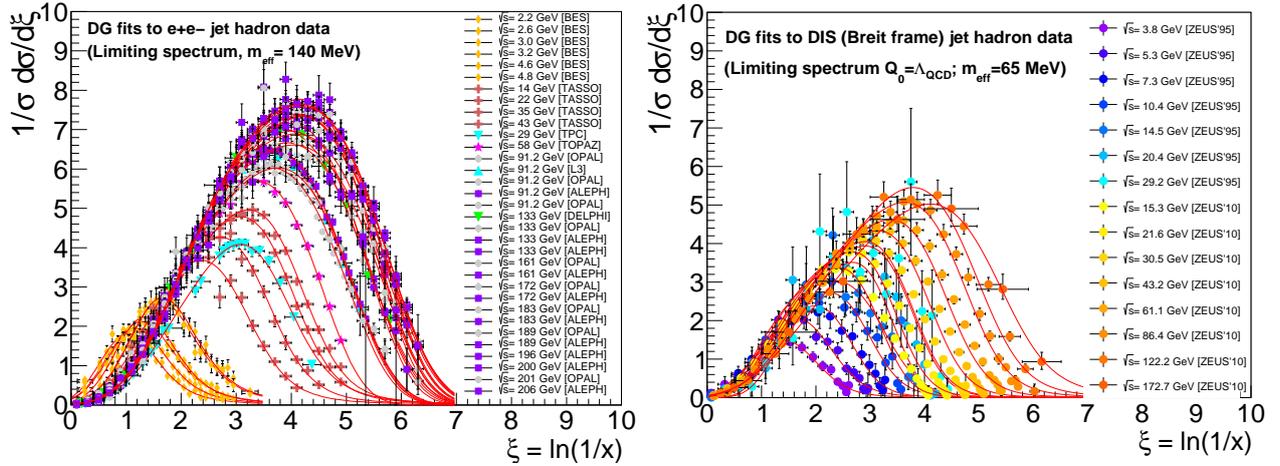
Figure 2: Charged-hadron distributions in jets as a function of $\xi = \ln(1/z)$ measured in e^+e^- at $\sqrt{s} \approx$ 2–200 GeV [5] (left) and $e^\pm$p (Breit frame) at $\sqrt{s} \approx$ 4–180 GeV [6] (right), individually fitted to the distorted Gaussian, Eq. (1), with the hadron mass corrections ($m_{\text{eff}} = 140, 65$ MeV) quoted.

HBP moments which depend *only* on Λ_{QCD}. Such an approach is justified for infrared-safe observables by the "local parton-hadron duality" hypothesis which states that the distribution of partons in jets is identical to that of the final hadrons (up to a constant $\mathcal{K}_{\text{ch}}$ which only affects the absolute normalization of the HBP distribution). Thus, by fitting the experimental hadron jet data at various energies to the DG parametrization (1), one can determine α_s from the corresponding energy-dependence of its fitted moments. Figure 2 shows the DG fits of the FFs measured in e^+e^- (32 datasets, left plot) [5] and DIS (15 datasets, right plot) [6]. We note that the DIS FFs need to be scaled up by a factor of 2 since only half of the total hadron production (in the current hemisphere of the Breit frame) is measured. For increasing jet energies the FF peak shifts to higher ξ, and its width broadens.

3. Experimental uncertainties on α_s

The extraction of the FF moments in the DG fit has two possible sources of uncertainty: (i) the choice of the m_{eff} factor to account for finite hadron-mass corrections, and (ii) the use of experimental FFs with slightly different final charged-hadron definitions (the different experiments require tracks coming from within 1–10 cm from the primary interaction vertex, including or not a fraction of secondary hadrons from weak K_s^0 and Λ decays). To estimate both effects we have taken the recent high-quality prompt and inclusive pion/kaon/proton FFs measured by the BaBar experiment [7], and fitted their total (summed) distribution to DGs with varying values of m_{eff}. The resulting fitted moments for $m_{\text{eff}} = 0$–

0.2 GeV are listed in Fig. 1. In addition, since some older FF studies extracted the peak position and width of the single-hadron distributions assuming a pure Gaussian shape, we list also the extracted values of ξ_{max} and σ in the Gaussian approximation in the top-right legend of Fig. 1. The main findings are as follows:

(i) The fit uncertainties of all the extracted moments cover well the considered range of m_{eff}=0–0.2 GeV variations. The best fits (minimum χ^2) of the experimental FFs to the DG (Fig. 2) are obtained for $m_{\text{eff}} = 130\pm30$ MeV (e^+e^-) and 70 ± 20 MeV (DIS).

(ii) The DG moments obtained for the prompt and inclusive charged-hadrons FFs are all consistent within their associated uncertainties except, as expected, for the total multiplicity N_{ch} which is a factor of 8–10% smaller for the primary-hadrons FFs.

(iii) The obtained FF maximum ξ_{max} and width σ assuming a Gaussian distribution around the peak, overestimate their values by 6% and 20% respectively compared to the more realistic DG fits.

The conclusion is that our FF analysis is robust with respect to hadronization corrections, and that only the extracted value of N_{ch} needs to be reduced by a factor of (9 ± 1)% if the experimental distributions have not been corrected for secondary weak decays. In addition, the FF peaks and widths extracted in Gaussian fits in the oldest data analyses overestimate these parameters by 6% and 20% respectively compared to the DG fits.

From the parton-to-hadron FF moments obtained for jets of various energies, one can extract α_s by fitting
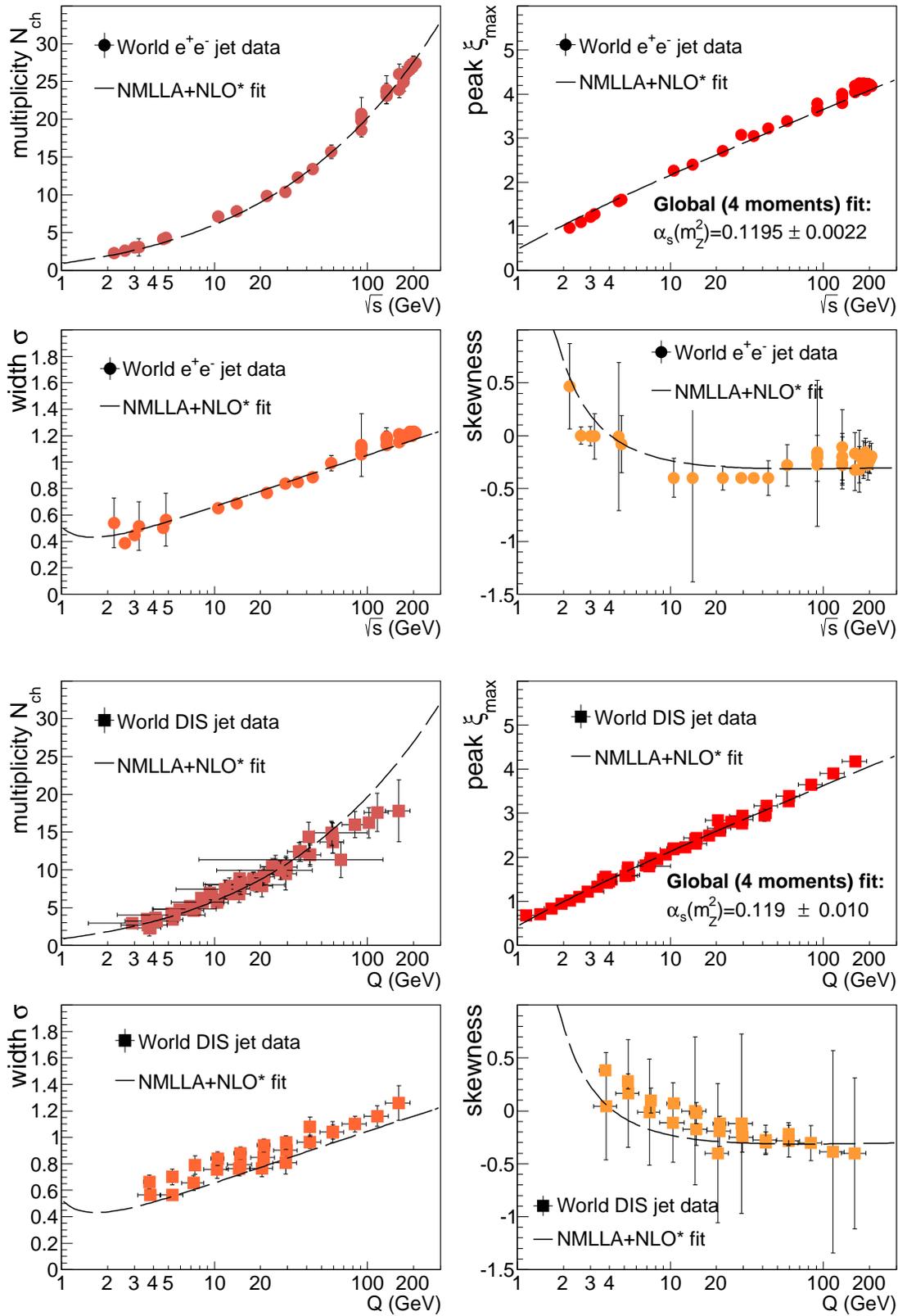
Figure 3: Energy evolution of the moments (charged hadron multiplicity, peak, width and skewness) of the jet FFs measured in e^+e^- collisions at $\sqrt{s} \approx$ 2–200 GeV [5] (top panels) and $e^\pm p$ collisions (Breit frame) at $\sqrt{s} \approx$ 4–180 GeV [6] (bottom panels). The extracted values of the NLO $\alpha_s(m_Z^2)$ are quoted for the the two independent data-sets fits.

their combined evolutions to our NLO*+NMLLA predictions, which include also corrections to account for the increasing number of flavours, N_f = 3,4,5, at the corresponding heavy-quark production thresholds: $E_{jet} > m_{charm,bottom} \approx 1.3,4.2$ GeV. The energy dependence of the multiplicity, peak, width and skewness of the jet FFs are shown in Fig. 3 for e^+e^- annihilation at $\sqrt{s} \approx 2$–200 GeV [5] (top panels), and for DIS $e^{\pm}$p and νp collisions in the range of four-momentum transfers $Q \approx 4$–180 GeV [6] (lower panels). We note that the DIS plot includes more moments (published in previous studies) than those obtained directly fitting the data of Fig. 2, right. The hadron multiplicities measured in DIS jets (scaled by a factor of two to cover both hemispheres) are smaller (especially at high energy) than those measured in e^+e^- collisions [5], a fact pointing likely to limitations in the FF measurement only in half (current Breit) $e^{\pm}$p hemisphere and/or in the determination of the relevant Q kinematic variable. The DIS HBP widths show also larger scatter than observed in e^+e^-, which is not unexpected as we are including moments fitted in the Gaussian approximation which overestimates their value by about 20% as discussed in the previous section. The skewness has the largest uncertainties of all moments, and the kurtosis (not shown) is very close to zero and not properly reproduced by the calculations [5]. The most robust FF moment for the determination of Λ_{QCD} is the peak position ξ_{max} which has the simplest theoretical expression for its NLO*+NMLLA energy evolution [5], and it is largely insensitive to the uncertainties (finite-mass corrections, secondary decays, DG fit) associated with our method.

The fits of the energy-dependencies of the HBP moments allow one to extract QCD coupling values: $\alpha_s(m_Z^2) = 0.1195 \pm 0.0022$ (from the e^+e^- data) [5], and $\alpha_s(m_Z^2) = 0.119 \pm 0.010$ (from DIS jets alone) [6], in perfect agreement with the current world average. The quoted propagated $\alpha_s(m_Z^2)$ uncertainties have been obtained following the "χ^2 averaging" method [1]. Namely, we fit first the energy-dependence for each individual moment to its corresponding theoretical prediction and if the goodness-of-fit χ^2 is larger than the number of degrees of freedom (ndof), all the data-points of the moment are enlarged by a common factor such that χ^2/ndof equals unity. We then perform the combined fit of *all* the energy-evolutions of the four moments letting Λ_{QCD} as the *single* free parameter. Such an approach takes into account in a well defined manner any correlations among the four extractions of α_s, as well as any missing extra systematic uncertainties. Our discussion of the experimental uncertainties indicates that such an

error assignment covers perfectly well the range of α_s variations induced by finite hadron-mass corrections, experimental final-hadron definitions, and DG fits.

4. Theoretical uncertainties on α_s

The advantages of our α_s determination compared to all other methods used so far are two-fold:

(i) Our approach relies on a theoretically convergent resummation of soft and collinear logs down to $Q_0 \rightarrow \Lambda_{QCD}$ and, thus, it is much more robust with respect to corrections due to the transition from partons to hadrons. Hadronization uncertainties are (much) smaller and limited to the choice of the m_{eff} factor accounting for finite-hadron mass effects as discussed in the previous Section.

(ii) Our expansion in half-powers of α_s includes almost the full set of α_s NLO corrections plus NNLL terms. The latter log corrections partially account for contributions from missing (NNLO) terms and, in this sense, our approach is more complete than existing α_s extractions at NLO accuracy.
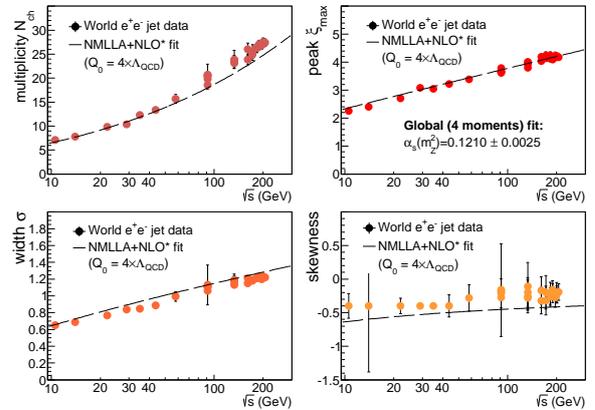
Figure 4: Energy evolution of the moments (charged hadron multiplicity, peak, width and skewness) of the jet FFs measured in e^+e^- collisions at $\sqrt{s} \approx 10$–200 GeV, fitted to the NLO*+NMLLA predictions evaluated at a scale $\lambda = 1.4$ (i.e. $Q_0 = 4\Lambda_{QCD} \approx 1$ GeV).

Notwithstanding this last item, the state-of-the-art α_s determinations are at NNLO accuracy [1] whereas our method is NLO*+NNLL. In order to estimate the size of theoretical uncertainties associated with missing fixed-order terms in our truncation of the α_s expansion, we have redone the fit of the moments versus jet energy for a different value of the scale $\lambda = \ln(Q_0/\Lambda_{QCD})$ at which the parton evolution is stopped. We vary λ

from the default limiting-spectrum value ($\lambda = 0$, i.e. $Q_0 = \Lambda_{QCD}$) to $\lambda = 1.4$ (i.e. $Q_0 = 4\,\Lambda_{QCD} \approx 1$ GeV) and fit the experimental data for jets energies in the range $Q = 10$–200 GeV, so as to leave some room for parton QCD evolution, i.e. avoiding jet data with energies too close to the shower cutoff at 1 GeV. In the relevant range of experimental jet energies $E_{jet\,max} \approx 2, 10$–$200$ GeV, the average scale at which α_s is effectively evaluated in our approach is given by the geometric mean: $Q = \sqrt{E_{jet\,max} \times Q_0} \approx 0.6$–$2$ GeV (for the limiting spectrum) and $Q = 3$–14 GeV (for the $\lambda = 1.4$ case). The results of the fits for the e^+e^- data are shown in Fig. 4. We obtain $\alpha_s(m_Z^2) = 0.1210 \pm 0.0025$, which is consistent with the value determined in the limiting spectrum case, $\alpha_s(m_Z^2) = 0.1195 \pm 0.0022$, although larger by $+0.0015$. We assign this difference as a (positive) source of systematic error associated with the scale uncertainty of our calculations.

5. Conclusions

The QCD coupling has been determined at NLO*+NMLLA accuracy from the energy evolution of the moments of the parton-to-hadron fragmentation functions measured in the $E_{jet} = 2$–200 GeV range in e^+e^- and DIS collisions: $\alpha_s(m_Z^2) = 0.1195 \pm 0.0022$(fit), and $\alpha_s(m_Z^2) = 0.119 \pm 0.010$(fit), respectively. We have studied in detail the experimental and systematic uncertainties associated with our α_s extraction, finding that the fit uncertainties obtained through the "χ^2 averaging" method fully cover the range of $\alpha_s(m_Z^2)$ variations driven by hadronization (finite hadron-mass) corrections, different experimental final-hadron definitions, as well as overall fit procedure. An additional theoretical-scale uncertainty of $+0.0015$ has been calculated by redoing the fit of the moments using a different shower cut-off value (i.e. relaxing the limiting-spectrum criterion). The final value of $\alpha_s(m_Z^2)$ obtained from a simple weighted-average of the couplings extracted independently from the e^+e^- and DIS jet data, yields: $\alpha_s(m_Z^2) = 0.1195 \pm 0.0021\,(\text{exp})\,^{+0.0015}_{-0.0}\,(\text{scale})$, which is in perfect accord with the current world-average of $\alpha_s(m_Z^2) = 0.1185 \pm 0.0006$ (obtained at NNLO accuracy). Work is in progress to combine the e^+e^- and DIS fragmentation-function moments into a *single* global fit [9], which will better account for any residual correlated uncertainty. In Fig. 5 we compare our $\alpha_s(m_Z^2)$ value to all other existing results at NLO accuracy. All the strong coupling values plotted are from the latest PDG compilation [1] except for the most recent jet results obtained from the CMS [10] and ATLAS [11] data. Our result is the most precise of all

approaches with a totally different set of experimental and theoretical uncertainties. The weighted average of all the NLO values yields: $\alpha_s(m_Z^2) = 0.1188 \pm 0.0012$. The methodology presented here provides a new promising approach for the determination of the QCD coupling constant complementary to other existing jet-based methods –such as jet shapes, and inclusive jet production cross sections in e^+e^-, DIS and p-p collisions– and can be used to reduce the overall final uncertainty of the least well-known interaction coupling in nature.
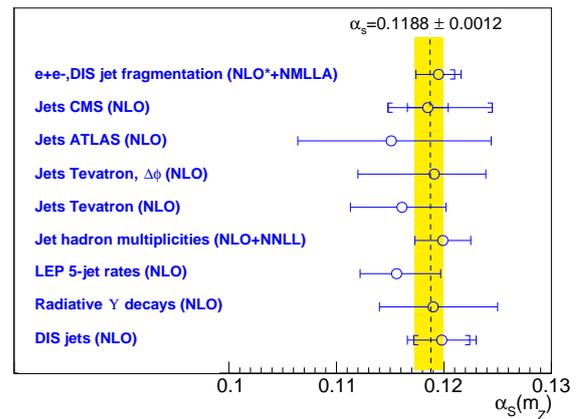

Figure 5: Summary of NLO α_s determinations from different methods. The error "brackets" (if present) indicate the theoretical uncertainties of each extraction. The dashed line and shaded (yellow) band indicate their weighted average (listed also on the top).

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