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Event reconstruction in NEXT using the ML-EM algorithm

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Abstract

The NEXT collaboration aims to find the neutrinoless double beta decay in ^{136}Xe. The rareness of this decay demands an exceptional background rejection. This can be obtained with an excellent energy resolution, which has been already demonstrated in the NEXT prototypes. In addition to this, the $\beta\beta0\nu$ decay in gas produces a characteristic topological signal which could be an extremely useful extra handle to avoid background events.

The need for a satisfactory topology reconstruction has led the NEXT Collaboration to implement the Maximum Likelihood Expectation Maximization method (ML-EM) in the data processing scheme. ML-EM is a generic iterative algorithm for many kinds of inverse problems. Although this method is well known in medical imaging and has been used widely in Positron Emission Tomography, it has never been applied to a time projection chamber. First results and studies of the performance of the method will be presented in this poster.

Keywords: NEXT, ML-EM, neutrinoless double beta decay

1. NEXT (Neutrino Experiment with a Xenon TPC)

Neutrinoless double beta decay ($\beta\beta0\nu$) is one of the hottest topics in neutrino physics. A positive signal for this kind of events would clarify if neutrinos are Majorana particles. If that is the case, neutrino small masses could be explained via the see-saw mechanism. In addition, Majorana neutrinos also play an important role in several leptogenesis theories.

Latest results of current experiments indicate that the mean life of $\beta\beta0\nu$ is over 10^{25} years which means that background rejection is the main issue to address in order to find this extremely rare decay. With this in mind, energy resolution and radio purity are necessary for any experiment looking for this event.

The Neutrino Experiment with a Xenon TPC (NEXT)[1] will search for $\beta\beta0\nu$ in xenon using a high-pressure gaseous xenon (HPGXe) time projection chamber. To amplify the ionization signal the detector uses electroluminiscence, that is, the emission of scintillation light after atom excitation by a charge accelerated by a moderately large (no charge gain) electric field.

NEXT uses two detection planes, one composed of PMTs that will make energy measurements and another one composed of MPPCs that allow for pixel reconstruction. With this layout great energy resolution can be achieved (below 1% FWHM at $Q_{\beta\beta}$) while being able to do track reconstruction. Track reconstruction is a great handle to reject background because $\beta\beta0\nu$ leaves a unique trace in the detector consisting of a single erratic line with a blob in each of its ends. Furthermore, the design of NEXT is fully scalable to higher masses.

Currently the collaboration has been running in IFIC a small prototype of the final detector for 2 years. This prototype, NEXT-DEMO (DEMO), has served as a demonstrator and test bench of the NEXT design. Energy resolution proven at DEMO is 0.8% and first tracks have been obtained. NEXT-WHITE (NEW) will be the next step inside the project. It is a bigger prototype currently under construction in Laboratorio Subterráneo de Canfranc. It will be finished in the first half of 2015 and aims to prove the background model of the experiment as well as to reconstruct larger tracks that couldn't be fully contained in DEMO.

2. Maximum Likelihood Expectation Maximization

The Maximum Likelihood Estimation (MLE) is a method for solving many different kinds of inverse problems. When applied to a data set and given a statistical model that describes the forward problem, MLE provides estimates for the model's parameters. It basically consists in obtaining the parameters that maximize the likelihood of the statistical model given any outcome. These parameters are the most probable source of the outcome.

Since products are difficult to treat mathematically and because the logarithm is a monotone function, the log-likelihood function is generally used when using MLE. With this, the mathematical expression that defines MLE, considering x the input data (parameters we are looking for) and y the outcome, is:

$$x_{ML} = \arg\max \log \mathcal{L}(x|y) \tag{1}$$

For example, considering a Poisson process, one can get the following log likelihood function:

$$\begin{aligned} \log \mathcal{L}(\boldsymbol{x}|\boldsymbol{y}) &= \log \prod_l^m P(y_l|\boldsymbol{x}) \\ &= \log \prod_l^m \frac{e^{-\boldsymbol{Ax}|_l}(\boldsymbol{Ax}|_l)^{y_l}}{y_l!} \\ &= -\sum_l^m (\boldsymbol{Ax}|_l - y_l \log(\boldsymbol{Ax}|_l) + \log(y_l!)) \end{aligned} \tag{2}$$

Even with this consideration, problems that allow for analytical solutions are usually extremely hard to compute so numerical algorithms are used to calculate MLE. One of these algorithm is the Expectation Maximization.

This algorithm is especially suited for problems with incomplete data or if the maximization of the likelihood function is complex or even intractable. In this latter case, hidden variables are introduced purely as a mathematical gimmick in order to make the estimation tractable and in such a way that the knowledge of the supposed hidden variables considerably simplifies the maximization.

$$\mathcal{L}(\boldsymbol{x}|\boldsymbol{y}) = P(\boldsymbol{y}|\boldsymbol{x}) = \sum_z P(\boldsymbol{y}, z|\boldsymbol{x}) \tag{3}$$

The motivation is as follows. If we know the value of the parameters $\boldsymbol{x}$, we can find the value of the latent variables z by maximizing the log-likelihood over all possible values of z. Conversely, if we know the value of the latent variables z, we can find an estimate of the parameters $\boldsymbol{x}$ fairly easily by simply grouping the observed data points according to the value of the associated latent variable. This suggests an iterative algorithm, in the case where both $\boldsymbol{x}$ and z are unknown. The algorithm would work as follows:

1. Initialize the parameters $\boldsymbol{x}$ to some random values.
2. Expectation step (E-step): compute the best value for z given these parameter values.
3. Maximization step (M-step): Use the just-computed values of z to compute a better estimate for the parameters $\boldsymbol{x}$.
4. Iterate steps 2 and 3 until convergence.

3. Event reconstruction using ML-EM in NEXT

The ML-EM algorithm can be applied to several problems and, specifically, it's been broadly used in medical imaging. The problem of reconstructing events in NEXT is very similar to the one in PET (Positron Emission Tomography) where extremely complex images have to be reconstructed. In addition, in both NEXT and PET cases the emitted light is detected by photomultipliers and the detection process can be described by Poisson statistics. This makes both cases equal from a mathematical point of view.

Taking that into account, the log likelihood function for the NEXT case is given by eq. (2) which, after rewriting, suppressing constant terms and working with it, results in:

$$log\mathcal{L}(\boldsymbol{x}|\boldsymbol{y}) = \sum_l^m (y_l \log(\boldsymbol{Ax}|_l) - \boldsymbol{Ax}|_l) \tag{4}$$

This expression can be solved using the Expectation Maximization [2, chap. 5.3] algorithm resulting, writing the sums and probabilities explicitly, in:

$$\lambda_m(v) = \frac{\lambda_{m-1}(v)}{\sum_d P(v,d)} \sum_d \frac{n(d)P(v,d)}{\sum'_v \lambda_{m-1}(v')P(v',d)} \tag{5}$$

where $\lambda(v)$ is the charge of the voxel v, $P(v,d)$ is the probability of detection by the detector d when having a photon emitted from voxel v and $n(d)$ is the number of photoelectrons produced in the detector d.

With this, the first step in order to be able to use ML-EM in NEXT is to voxelize the active volume of the detector. Then one has to compute the probabilities associated to each voxel and detector. There are several sources that may affect the signal so a MC simulation is used to convolute all the effects into a probability matrix. This matrix is calculated by simulating the response of a charge produced at each voxel and checking the fraction of light detected at each sensor.

The input for the method is the signal detected in both the tracking and energy plane of the TPC. Using that information events can be reconstructed with both a great energy resolution and precise tracking simultaneosly. This signal can also be used as the seed for the first iteration of ML-EM, this is $\lambda_0(v)$. The seed could be random but in order not to bias the result in any way a uniform distribution of the total number of photoelectrons produced at the sensors is used.

The output will be a collection of voxels, each one with a reconstructed charge. While an estimation of the energy of the event can be made by simply adding the charge of each voxel (fig. 1), this collection of voxels is directly the charge distribution of the event inside the detector (fig. 2).

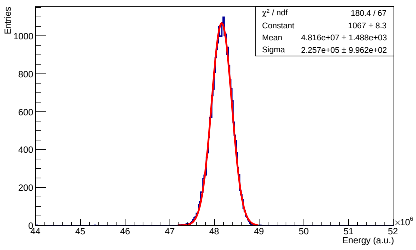

Figure 1: Energy resolution of electrons of the energy of the ^{137}Cs decay (662 keV). The resolution extrapolated to $Q_{\beta\beta}$ (2458 keV) is 0.57% at FWHM.

Due to computational limitations only a bidimensional probability matrix has been tested. With this consideration, two reconstructions modes have been developed and are being studied. One is a bidimensional reconstruction of the event. This is made by integrating all the charge measured by each sensor over time of the signal. With this as an input the ML-EM can be applied inmediately and the resultant collection of voxels directly is the transversal projection of the event. This mode of reconstruction uses a low computational charge making it really fast and, therefore, suitable for all kind of quick analysis.

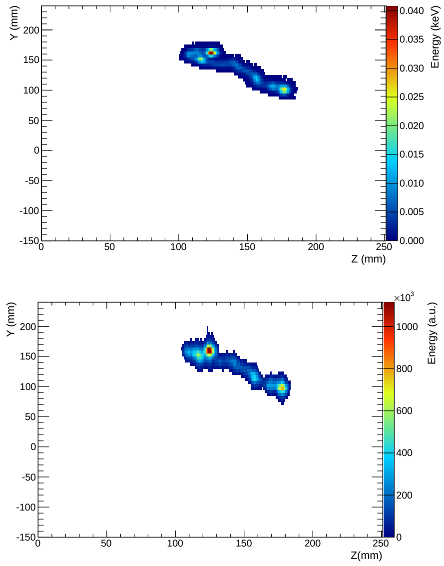

Figure 2: True $\beta\beta0\nu$ event with diffusion (top) and its ML-EM reconstruction (bottom) in NEW. The 2 blobs can be clearly seen in both cases.

The other approach of reconstruction allows for a pseudotridimensional reconstruction of the track. This is made by slicing the signal in time divisions instead of integrating it. The ML-EM algorithm can then be applied to each division obtaining a transversal reconstruction of that segment. Combining all the time slices gives then a tridimensional reconstruction of the event. This method implies applying ML-EM several times, one per each division, making it slower, but is optimal for well defined tracking.

Using a bidimensional matrix implies that all the longitudinal position dependent effects like diffusion or attachment are not considered in the matrix thus the method will produce tracks smeared by these effects. Still, results obtained are satisfactory for both methods. Future plans include a more detailed study of the topology response of the method and the implementation of a full 3D probability matrix.

References

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