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Phenomenology and formal studies on small-x physics by using Monte Carlo techniques

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Abstract

We review our recent studies on a number of BFKL related projects in QCD and N = 4 SYM which were done with the use of advanced Monte Carlo techniques. We discuss briefly the new setup of our code as a Monte Carlo tool that produces theoretical predictions ready to be directly compared against experimental data in order to perform a number of important phenomenological studies at the LHC.

Keywords:
QCD, Small-x Physics, BFKL, Monte Carlo

1. Introduction

The term "small-x physics" in the title refers to the study of scattering amplitudes when the colliding center-of-mass energy squared, s, is very large. Logarithms in energy are then enhanced and need to be resummed to all orders in the strong coupling α_s. This is done by solving the Balitsky-Fadin-Kuraev-Lipatov equation at leading (LO) [1] and next-to-leading logarithmic (NLO) accuracy [2].

Indeed, the BFKL resummation framework has been used to study the properties of scattering amplitudes in Quantum Chromodynamics (QCD) and $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory in certain kinematic regions, namely, in multi-Regge (MRK) and quasi-multi-Regge kinematics. Physical observables such as hadron structure functions at small values of Bjorken x in deep inelastic scattering or inclusive di-jet production with a significant rapidity separation at the Large Hadron Collider (LHC) are characteristic cases [3, 4, 5, 6, 3, 7, 8, 9] where the BFKL approach is suitable.

Apart from a rich phenomenology list, the BFKL framework presents an interesting connection to more formal theoretical works. Actually, it was in a generalized leading logarithmic approximation, where the Bartels-Kwiecinski-Praszalowicz (BKP) equation was proposed [10, 11] and found to have a hidden integrability [12, 13, 14, 15, 16, 17, 18, 19, 20]. Moreover, corrections to the Bern-Dixon-Smirnov (BDS) iterative ansatz [21] were found in MRK and within the BFKL formalism in [22, 23]. These corrections have been understood as part of the finite remainder to the amplitude which corresponds to the anomalous contribution of a conformal Ward identity [24, 25, 26, 27, 28, 29, 30].

In the next Section, we present the iterative solution of the BFKL equation and some of our previous results. We spare all technical details for which we refer the reader to the original publications [31, 32]. One should mention that a key idea to work within the BFKL framework is the k_T factorization scheme [33, 34, 35].

2. Discussion

The BFKL equation is an elegant mathematical construction, it is not easy to solve though. At present, analytic solutions are known for the LO kernel in QCD and the NLO kernel in $\mathcal{N} = 4$ SYM.

Nevertheless, it is possible to solve the BFKL equation iteratively, in a numerical fashion, by applying

Monte Carlo integration techniques. The physics encoded in the BFKL equation can be “visualized” in a graphic way by using reggeized gluons, ordinary gluons and effective vertices that involve the former two (Fig. 1). This is a picture that also makes clearer the numerical iterative solution.

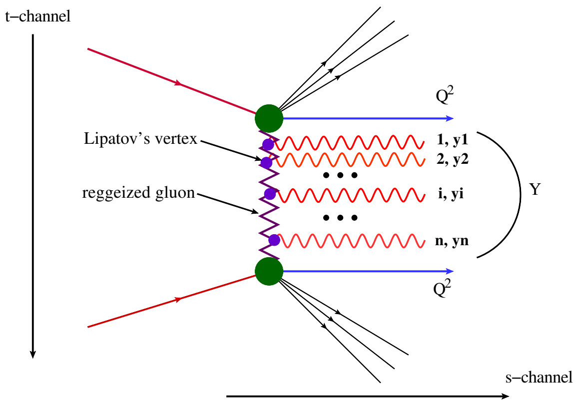

Figure 1: Two projectiles collide and produce two hard jets (Q^2 is a hard scale so that we can use perturbation theory) and n gluons flying in the s-channel: gluon-1, gluon-2,...,gluon-n, with corresponding rapidities y_1, y_2,...,y_n. In the t-channel, a reggeized gluon is exchanged. The cyan-blue blobs are the so-called Lipatov effective vertices.

What Fig. 1 implies is that to calculate the process depicted, one has to take into account all possible diagrams which differ in the number of the t-channel emitted gluons. In other words, one must solve the BFKL equation and obtain the gluon Green’s function (GGF) $\mathcal{H}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y})$. $\mathbf{q}_1$ and $\mathbf{q}_2$ in Fig. 1 are the momenta of the reggeized gluons above gluon-1 and below gluon-n respectively, whereas $\mathbf{q}$ is the momentum transfer. Y is the rapidity span from gluon-1 to gluon-n. In principle, one needs to consider an infinite sum of terms: the 1st one with no gluon emission ($n = 0$), the 2nd one with one gluon emission ($n = 1$), the 3rd one with two gluon emission, etc. Every term is an integral over the emitted gluon momenta and their individual rapidities. Depending on Y, one can truncate the sum to a finite N (max n=N) in order to have a “numerically acceptable” result.

The qualitative description above can be cast in formal mathematical language thus yielding the iterative BFKL equation:

$$\mathcal{H}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}; \mathrm{Y}) = \left(\frac{\lambda^2}{\sqrt{\mathbf{q}_1^2 \mathbf{q}_1'^2}}\right)^{c_{\mathcal{R}} \bar{\alpha}_s \mathrm{Y}}$$
$$\times \Bigg\{ \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2) + \sum_{n=1}^{\infty} \prod_{i=1}^{n} c_{\mathcal{R}} \int \frac{d^2\mathbf{k}_i}{\pi \mathbf{k}_i^2} \theta(\mathbf{k}_i^2 - \lambda^2) \frac{\bar{\alpha}_s}{2}$$
$$\times \left(1 + \frac{\left(\mathbf{q}_1' + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2 (\mathbf{q}_1 + \sum_{l=1}^{i} \mathbf{k}_l)^2 - \mathbf{q}^2 \mathbf{k}_i^2}{(\mathbf{q}_1' + \sum_{l=1}^{i} \mathbf{k}_l)^2 \left(\mathbf{q}_1 + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2}\right)$$
$$\times \int_0^{y_{i-1}} dy_i \left(\frac{\left(\mathbf{q}_1 + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2 \left(\mathbf{q}_1' + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2}{\left(\mathbf{q}_1 + \sum_{l=1}^{i} \mathbf{k}_l\right)^2 \left(\mathbf{q}_1' + \sum_{l=1}^{i} \mathbf{k}_l\right)^2}\right)^{\frac{\bar{\alpha}_s}{2} y_i}$$
$$\times \delta^{(2)}\left(\mathbf{q}_1 + \sum_{l=1}^{n} \mathbf{k}_l - \mathbf{q}_2\right) \Bigg\}. \tag{1}$$

We have chosen to present here the (LO) BFKL equation in a form that is valid for any color representation, setting $c_{\mathcal{R}} = 1$ gives the usual color singlet case. A mass parameter λ is also used to regularize the phase space integral of the “real emission” sector. The dependence on λ cancels out.

As indicated previously, an interesting question is to study the convergence of the sum defining the function $\mathcal{H}$ in Eq. (1). For a fixed value of Y and the coupling $\bar{\alpha}_s$, we expect that only a finite number of terms in the sum is needed to reach a good accuracy for the gluon Green’s function. This is shown in Fig. 2 (forward case) and Fig. 3 (non-zero momentum transfer).

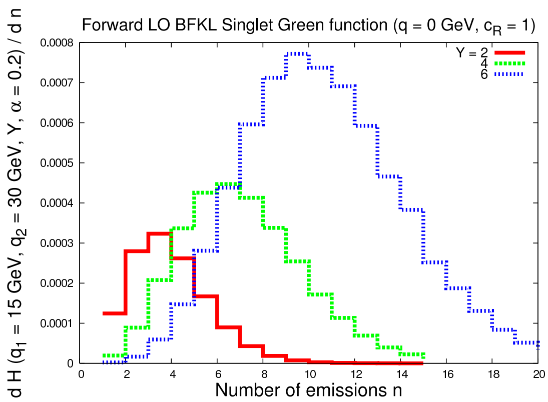

Figure 2: Distribution in the contributions to the LO BFKL gluon Green function with a fixed number of iterations of the kernel, plotted for different values of the center-of-mass energy, a fixed $\bar{\alpha}_s = 0.2$ and momentum transfer $\mathbf{q} = 0$.

In Figs. 2 and 3, one can see the contribution to the gluon Greens function by each term of the sum in Eq. 1. It is evident that the larger the Y, the more emissions need to be considered. Roughly speaking, the area below the red (Y=2), green (Y=4) and blue (Y=6) lines

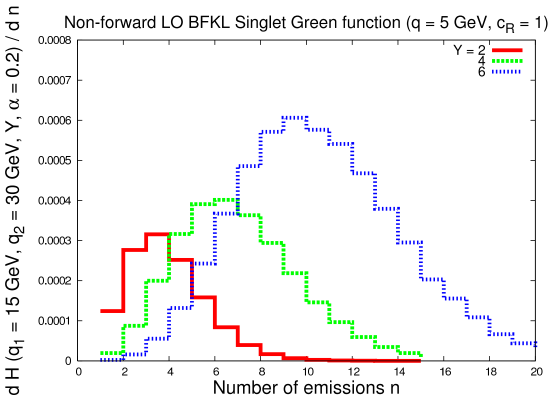

Figure 3: Distribution in the contributions to the LO BFKL gluon Green function with a fixed number of iterations of the kernel, plotted for different values of the center-of-mass energy, a fixed $\bar{\alpha}_s = 0.2$ and momentum transfer $\mathbf{q} = 5$.

gives the value of the gluon Greens function. In Fig. 2, for zero momentum transfer, q=0, and in Fig. 3, for momentum transfer q=5 GeV.

At present, our C++ Monte Carlo implementation [36] covers the LO and NLO kernels in N=4 SYM and in QCD, for zero and non-zero momentum transfer in both the color singlet and color octet representations. Up till now, we have mainly used our code to perform studies on the properties of the gluon Green's function. This first stage, allowed us to experiment with different code optimization techniques. We are now ready to proceed to a number of phenomenological studies for the LHC. We expect to release our first results in the near future.

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