



# Study of Topological Distributions of Inclusive Three- and Four-jet Events at the LHC

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## Abstract

A study of inclusive topological distributions of three- and four-jet events has been conducted by the CMS Collaboration at the LHC with a data sample corresponding to an integrated luminosity of  $5.1 \text{ fb}^{-1}$  at a centre of mass energy of 7 TeV. Kinematic and angular distributions in inclusive multijet final states serve as a natural probe of quantum chromodynamics and can reveal its inner dynamics. Comparisons are carried out with the data and predictions of leading order calculations and parton shower generators. The compared data results are corrected for detector effects and can be directly compared with other models or next-to-leading order theoretical predictions.

*Keywords:* CMS, Multijet, QCD

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## 1. Introduction

The proton-proton collision events with high  $p_T$  jets serve as a direct test of perturbative Quantum Chromodynamics (pQCD). Here, tests are performed using inclusive three- and four-jet events. In order to study these final states, a class of observables are defined.

Many of the QCD event generators utilize the leading order matrix element in the primary  $2 \rightarrow 2$  process. These multijet variables are sensitive to the treatment of higher order processes and approximations used in these event generators. Agreement between the measured results and Monte Carlo predictions establishes the validity of the treatment of higher order effects. Any large deviation thereof may lead to large systematic uncertainties on searches of new physics.

## 2. Definition of variables

The topological variables used in this study are defined in the multijet centre of mass system (CM). The incoming partons are labelled with numbers 1 and 2 while the outgoing partons (measured as jets) are labelled with numbers 3, 4, ... ordered in descending energies in the multijet centre of mass frame.

The final state parton energy is an obvious choice for the topological variables for the three-jet final state. For simplicity,  $E_i$  ( $i = 3, 4, 5$ ) is often replaced by the scaled variable  $x_i$  ( $i = 3, 4, 5$ ), which is defined by  $x_i = 2E_i / \sqrt{\hat{s}_{345}}$ , where  $\sqrt{\hat{s}_{345}}$  is the centre of mass energy of the hard scattering process. It is also referred as the mass of the three-jet system and by definition,  $x_3 + x_4 + x_5 = 2$ .

The internal structure of the three-jet final state is determined by any two scaled parton energies. The third one is calculated using the above relation.

For the four-jet final state, the mass of the four-jet system ( $\sqrt{\hat{s}_{3456}}$ ) and two angular distributions characterizing the orientation of event planes are investigated. One of these is the Bengtsson-Zerwas angle ( $\chi_{BZ}$ ) [1] defined as the angle between the plane containing the two leading jets and the plane containing the two non-leading jets:

$$\cos \chi_{BZ} = \frac{(\vec{p}_3 \wedge \vec{p}_4) \cdot (\vec{p}_5 \wedge \vec{p}_6)}{|\vec{p}_3 \wedge \vec{p}_4| |\vec{p}_5 \wedge \vec{p}_6|}$$

The second variable is the cosine of the Nachtmann-Reiter angle ( $\cos \theta_{NR}$ ) [2] defined as the angle between the momentum vector differences of the two leading jets

and the two non-leading jets:

$$\cos \theta_{NR} = \frac{(\vec{p}_3 - \vec{p}_4) \cdot (\vec{p}_5 - \vec{p}_6)}{|\vec{p}_3 - \vec{p}_4| |\vec{p}_5 - \vec{p}_6|}$$

Historically,  $\chi_{BZ}$  and  $\theta_{NR}$  were proposed for  $e^+e^-$  collisions to study gluon self-coupling. Their interpretation in  $pp$  collisions is more complicated, but the variables can be used as a tool for studying the internal structure of the four-jet events.

### 3. Event Selection and Measurement

The CMS experiment [3] has collected an integrated luminosity of  $5.1 \text{ fb}^{-1}$  of  $pp$  collisions data at a centre of mass energy of 7 TeV from the Large Hadron Collider (LHC). The analysis is based on the dataset when all sub-detectors of the CMS experiment have been functioning properly.

Jets are reconstructed from particle flow objects [4] using the anti  $k_T$  clustering algorithm [5] with the size parameter  $R = 0.5$ . The jet energy corrections applied to jets used in this analysis are based on high  $p_T$  jet events generated by PYTHIA6 and then simulated using GEANT4 [6] and in situ measurements with dijet and photon+jet events [7]. In addition an offset correction is taken into account to subtract out the extra energy deposited in the jets due to pile up. Data events passing single-jet high level trigger (HLT) requirements with threshold at 110 and 370 GeV are used in this analysis.

Jets are selected with tight criteria on the neutral energy fractions (both electromagnetic and hadronic components) and all the jets are required to have  $|\eta| \leq 2.5$  and  $p_T$  above 50 GeV. The highest  $p_T$  jet is required to have  $p_T$  above a threshold as given by the requirement from the trigger turn on curve.

Events are selected with at least three jets passing the selection criteria as stated above. Additional selection requirements are also applied to reduce beam background and noise: (1) at least one good reconstructed primary vertex with number of degrees of freedom  $> 4$ ; (2) missing transverse energy to be small (the quantity  $\frac{E_T^{\text{miss}}}{\sum E_T}$  is required to be less than 0.3).

The jets are ordered in decreasing  $p_T$  and an event with at least three jets satisfying the jet selection criteria is classified as a three-jet event and likewise for four-jet events.

The leading experimental uncertainties are due to the jet energy scale, jet energy resolution and model dependence of corrections to the data. The contributions to the uncertainty from selection requirements and pileup effects are found to be negligible. These uncertainties

are calculated for each bin of the measured distributions and are added in quadrature. The overall systematic uncertainties are found to be smaller than the statistical uncertainty for most of the bins.

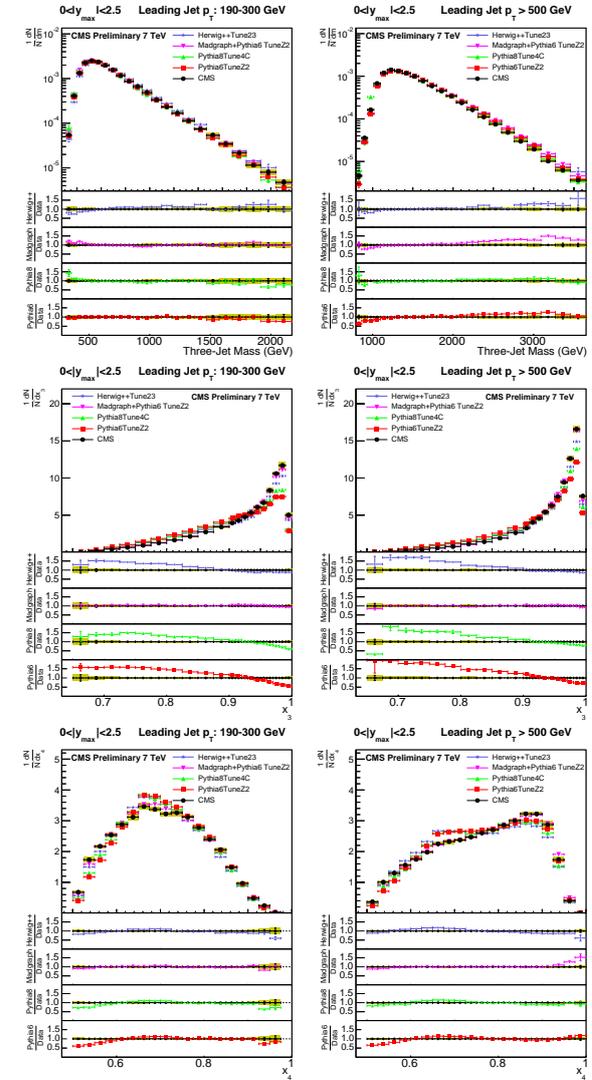


Figure 1: Distributions of three-jet variables being compared with predictions from the Monte Carlo models. The distributions are obtained from inclusive three-jet sample with the jets restricted in the region  $0.0 < |y| < 2.5$  for two ranges of leading jet  $p_T$ : between 190 and 300 GeV, and above 500 GeV.

### 4. Results

The normalized differential cross sections as a function of the three- and four-jet inclusive variables are compared with predictions from the four Monte Carlo

models: PYTHIA6, PYTHIA8, MADGRAPH + PYTHIA6 and HERWIG++. The variables considered for these comparisons are three-jet mass, scaled energies of the leading and next-to-leading jet in the three-jet sample in the three-jet center of mass frame, four-jet mass and the two angles  $\cos\theta_{NR}$  and  $\chi_{BZ}$ .

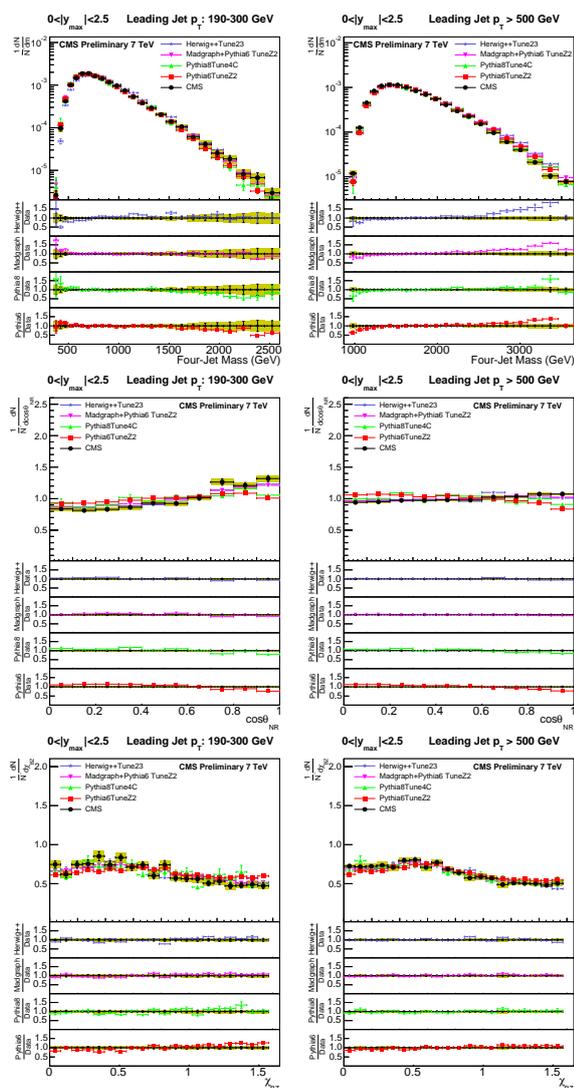


Figure 2: Distributions of four-jet variables being compared with predictions from the Monte Carlo models. The distributions are obtained from inclusive four-jet sample with the jets restricted in the region  $0.0 < |\eta| < 2.5$  for two ranges of leading jet  $p_T$ : between 190 and 300 GeV, and above 500 GeV.

For all the comparison plots, the top part shows the data and the model predictions with the corresponding statistical uncertainty. For the data, the shaded area shows the statistical and systematic uncertainty added

in quadrature. The bottom part from each plot shows the ratio of Monte Carlo prediction for each model and the data. Comparisons are made for two different ranges of the leading jet transverse momentum,  $190 < p_T < 300$  GeV and  $p_T > 500$  GeV.

Monte Carlo models have different ways of modeling the underlying events or hadronization of the partons into hadrons. The effect of different hadronization models has been investigated by comparing the ratio of the parton-level distribution to the particle-level distribution for two different Monte Carlo models: PYTHIA6 and HERWIG++. The difference in the two Monte Carlos is typically below 5%. Distributions for six different underlying event tunes of PYTHIA6 are compared. The resulting distributions differ typically within 5% and again the disagreement with the data cannot be fully explained by this effect. The Monte Carlo models use CTEQ6 as the default PDF parametrization. The effect of PDF set choice on the multijet variables is calculated according to the recommendation of PF4LHC group [8]. The uncertainties are found to be typically at the level of 1-2% depending on the variable type and  $p_T$  range considered.

The distributions studied in this analysis get different level of agreement from the Monte Carlo models and many of these disagreements cannot be explained in terms of the choice of PDF, hadronization models or underlying event modelling. The predictions from MADGRAPH followed by hadronization due to PYTHIA6 give a more consistent description of all six multijet distributions described in this analysis. The mean deviations for the jet mass distributions are below 4.6% with the exception of four-jet mass distribution for leading jet  $p_T$  below 190 GeV (where the mean deviation is 8.6%). The agreement for scaled jet energy is better than 3.0% for the second most energetic jet and better than 5.4% for the most energetic jet. The agreements for the angular variables are at the level of 1.6% (3.7%) for  $\cos\theta_{NR}$  ( $\chi_{BZ}$ ) when the leading jet energy is above 500 GeV. This becomes somewhat worse for low energy jets - the worst agreement is for events with leading jet  $p_T$  between 110 and 190 GeV with mean values around 7.0% (10.4%) for  $\cos\theta_{NR}$  ( $\chi_{BZ}$ ). This is the only model which provides multijet final states at the tree level and this may be the key reason why predictions from this model are in agreement with the data at an acceptable level for all the variables.

The two PYTHIA Monte Carlo models give a fairly good description of the three- and four-jet mass distributions. The level of agreement with the data is similar to that for MADGRAPH + PYTHIA6 Monte Carlo and sometimes it is even better (PYTHIA6 provides better description of three-jet mass distribution). However, both these

models give poor descriptions of the angular variables for the four-jet final state and the agreement is even worse in the case of scaled energies of the leading and next-to-leading jets for inclusive three-jet events. This behaviour could be due to the choice of the ordering method used in the parton shower. The predictions of PYTHIA8 seem to be closer to the data for these variables than those from PYTHIA6 Monte Carlo.

Out of the four Monte Carlo models, HERWIG++ provides the worst description for the jet mass distributions (between 4.0% and 14.3% for three-jet mass and between 6.9% and 14.7% for four-jet mass). However, this model gives reasonable description (almost as good as MADGRAPH + PYTHIA6) for the four-jet angular variables. The agreement for scaled energies of the next-to-leading jet in the three-jet sample is within 8.0% while that for the leading jet is somewhat worse. HERWIG++ is also a leading order Monte Carlo model like PYTHIA but it uses angular ordering in the parton shower. This could be one source of improvement in this Monte Carlo model.

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