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τ hadronic spectral function moments in a nonpower QCD perturbation theory

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Abstract

The moments of the hadronic spectral functions are of interest for the extraction of the strong coupling and other QCD parameters from the hadronic decays of the τ lepton. We consider the perturbative behavior of these moments in the framework of a QCD nonpower perturbation theory, defined by the technique of series acceleration by conformal mappings, which simultaneously implements renormalization-group summation and has a tame large-order behavior. Two recently proposed models of the Adler function are employed to generate the higher order coefficients of the perturbation series and to predict the exact values of the moments, required for testing the properties of the perturbative expansions. We show that the contour-improved nonpower perturbation theories and the renormalization-group-summed nonpower perturbation theories have very good convergence properties for a large class of moments of the so-called "reference model", including moments that are poorly described by the standard expansions.

Keywords: strong coupling, tau decays

1. Introduction

The non-strange hadronic width of tau lepton is one of the best sources for the determination of strong coupling α_s [1]. The extraction of α_s from τ decays is plagued by the prescription chosen for implementing renormalization-group (RG) invariance [2, 3, 4]. Another source of ambiguity is the factorial growth of the coefficients of perturbative expansion of the Adler function. The additional sources of uncertainties are non-perturbative power corrections and the effects of quark-hadron duality violation (DV). Various moments have been used in the extraction of strong coupling. The non-perturbative condensates and terms involving DV may contribute to some moments depending on the structure of the relevant weight. Hence, it is important to study the properties of the perturbative expansions of the moments for an improved determination of the strong coupling. A large class of spectral function moments have been analyzed under different assumptions for the large-order behavior of the Adler function in two standard QCD perturbative expansions, the fixed-order and the contour-improved perturbation theories (FOPT and CIPT) [5]. It is shown that some moments that are commonly employed in α_s determinations from τ decays have unstable perturbative behavior.

In this work, we present the perturbative behavior of the moments in an alternative to FOPT and CIPT. This approached is referred as "renormalization-group-summed perturbation theory" (RGSPT) [6, 7, 8, 9, 10, 11, 12, 13]. Furthermore, we present our investigations in the framework of a novel formulation of QCD perturbation theory, defined by the accelerated series by means of a conformal mapping, applied to the Borel plane of QCD correlators[14, 15, 16]. This formulation is referred as "nonpower perturbation theory" (NPPT) [10, 11, 17]. In Refs. [8, 9], we have used a particular moment of the spectral function for the extraction of α_s from the total hadronic width. Now we present our results for several other moments, first considered in [5] and discussed in great detail in Refs. [10, 12], in the framework of CINPPT, RGSPT, and RGSNPPT.

2. Theory of RGSPT and RGSNPPT

The perturbative contribution to the τ hadronic spectral function moments can be written as

$$\delta^{(0)}_{w_i}(s_0) = \frac{1}{2\pi i}\oint_{|s|=s_0} \frac{ds}{s} W_i(s/s_0)\widehat{D}_{\rm pert}(s), \quad (1)$$

where $W_i(x)$ are weights and $\widehat{D}_{\rm pert}(s)$ is the perturbative part of the reduced Adler function given by

$$\widehat{D}(s) \equiv -s\,{\rm d}\Pi^{(1+0)}(s)/{\rm d}s - 1. \quad (2)$$

A natural expansion for the Adler function, that is referred as FOPT, is

$$\widehat{D}_{\rm FOPT}(s) = \sum_{n\geq 1}(a_s(\mu^2))^n\left[c_{n,1} + \sum_{k=2}^{n} k c_{n,k}\left(\ln\frac{-s}{\mu^2}\right)^{k-1}\right], \quad (3)$$

where $a_s(\mu^2) = \alpha_s(\mu^2)/\pi$. By setting $\mu^2 = -s$ in the expansion, one obtains the CIPT expansion

$$\widehat{D}_{\rm CIPT}(s) = \sum_{n\geq 1} c_{n,1}\,(a_s(-s))^n, \quad (4)$$

where the running coupling $a_s(-s)$ is determined by solving the RG equation

$$s\frac{da_s(-s)}{ds} = \beta(a_s). \quad (5)$$

In the $\overline{\rm MS}$ scheme for $n_f = 3$ flavors the coefficients $c_{n,1}$ calculated up to fourth order are [18]:

$$c_{1,1} = 1,\ c_{2,1} = 1.640,\ c_{3,1} = 6.371,\ c_{4,1} = 49.076. \quad (6)$$

The RGSPT expansion generalizes the summation of leading logarithms to non-leading logs, by summing all the terms available from RG invariance and can be written as [8, 9, 10, 11, 12, 13].

$$\widehat{D}_{\rm RGSPT}(s) = \sum_{n\geq 1}(\widetilde{a}_s(-s))^n\left[c_{n,1} + \sum_{j=1}^{n-1} c_{j,1} d_{n,j}(y)\right], \quad (7)$$

where

$$\widetilde{a}_s(-s) = \frac{a_s(\mu^2)}{1+\beta_0 a_s(\mu^2)\ln(-s/\mu^2)} \quad (8)$$

is the solution of the RG equation to one loop, $d_{n,j}(y)$ are calculable functions and $y \equiv 1 + \beta_0 a_s(\mu^2)\ln(-s/\mu^2)$.

We consider two models where the Adler function is defined as [4]

$$\widehat{D}(s) = \frac{1}{\beta_0}\,{\rm PV}\int_0^\infty \exp\left(\frac{-u}{\beta_0\tilde{a}_s(-s)}\right)B(u)\,{\rm d}u, \quad (9)$$

and

$$\frac{B(u)}{\pi} = B_1^{\rm UV}(u) + B_2^{\rm IR}(u) + B_3^{\rm IR}(u) + d_0^{\rm PO} + d_1^{\rm PO}u. \quad (10)$$

The free parameters are fixed by imposing the known information about the series. Then all higher order coefficients can be predicted. The reference model (RM) is parameterized by the UV and first IR renormalons[4]. In the alternative model (AM), the first IR renormalon is removed by hand [5]. We employ series acceleration by conformal mapping for moments of the Adler function. The technique was first proposed by Ciulli and Fischer[19]. The large-order behaviour of the expansion is improved by analytic continuation in the Borel plane. We consider the general class of conformal mappings [14, 17]

$$\widetilde{w}_{lm}(u) = \frac{\sqrt{1+u/l} - \sqrt{1-u/m}}{\sqrt{1+u/l} + \sqrt{1-u/m}}, \quad (11)$$

where j, k are positive integers satisfying $j \geq 1$ and $k \geq 2$. The function $\widetilde{w}_{jk}(u)$ maps the u plane cut along $u \leq -j$ and $u \geq k$ onto the disk $|w_{jk}| < 1$ in the plane $w_{jk} \equiv \widetilde{w}_{jk}(u)$, such that $\widetilde{w}_{jk}(0) = 0$, $\widetilde{w}_{jk}(-j) = -1$ and $\widetilde{w}_{jk}(k) = 1$. Then, we consider the functions

$$S_{lm}(u) = \left(1 - \frac{\widetilde{w}_{lm}(u)}{\widetilde{w}_{lm}(-1)}\right)^{\gamma_1}\left(1 - \frac{\widetilde{w}_{lm}(u)}{\widetilde{w}_{lm}(2)}\right)^{\gamma_2}, \quad (12)$$

where l, m are positive integers satisfying $l \geq 1$ and $m \geq 2$. γ_j, $j = 1, 2$, are suitable exponents, defined such as to preserve the behavior of $B(u)$. Thus, we obtain a new class of RGSNP expansion

$$\widehat{D}_{\rm RGSNPPT}(s) = \sum_{n\geq 0} c^{(lm)}_{n,\rm RGSPT}(y)\,\mathcal{W}^{(lm)}_{n,\rm RGSPT}(s), \quad (13)$$

$$\mathcal{W}^{(lm)}_{n,\rm RGSPT}(s) = \frac{1}{\beta_0}{\rm PV}\int_0^\infty \exp\left(\frac{-u}{\beta_0\tilde{a}_s(-s)}\right)\frac{(\widetilde{w}_{lm}(u))^n}{S_{lm}(u)}\,{\rm d}u. \quad (14)$$

Similar expansions are also obtained for CIPT and FOPT[17].

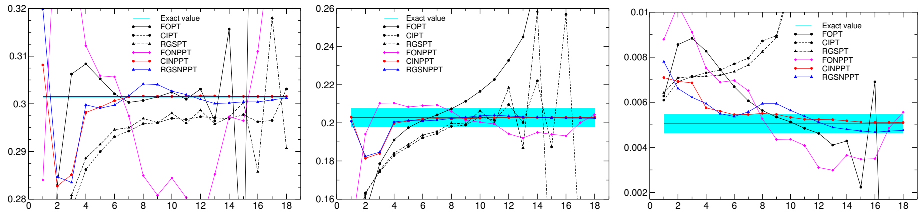

Figure 1: $\delta^{(0)}_{w_i}$ for the weights W_1, W_6, and W_{16} calculated for the RM, as functions of the perturbative order up to which the series was summed. The horizontal bands give the uncertainties of the exact values. We use $\alpha_s(M_\tau^2) = 0.3186$.

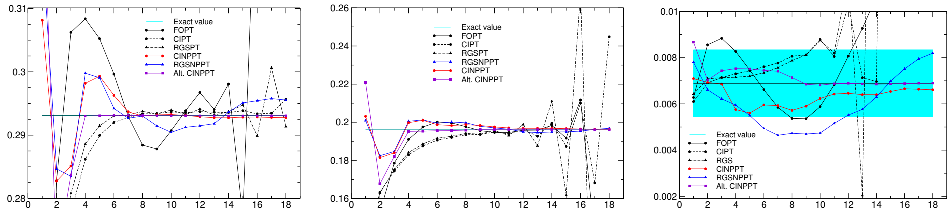

Figure 2: $\delta^{(0)}_{w_i}$ for the weights W_1, W_6, and W_{16} calculated for the AM, as functions of the perturbative order up to which the series was summed.

3. Results

In Fig. 1, we show the perturbative behavior of a few moments in the case of RM for the standard expansions FOPT, CIPT, and RGSPT with their optimal non-power expansions. The weights shown in the figures are $W_1(x) = 2(1-x)$, $W_6(x) = (1-x)^2$, $W_{16}(x) = \frac{1}{210}(1-x)^4(13 + 52x + 130x^2 + 120x^3)$. The horizontal bands give the uncertainties of the exact values calculated in RM. We observe that RGSPT provides results close to CIPT. The RGSNPPT and CINPPT expansions provide impressive approximation to exact value. In Fig. 2, we show the perturbative behavior of the same moments in the case of AM. In this case, RGSNPPT and CINPPT approach the exact value at higher orders.

4. Conclusion

We have investigated several spectral function moments of the massless Adler function in the frame of a new class of "non-power" perturbative expansions in QCD, where the powers of the coupling are replaced by more adequate functions [9, 14, 15, 16, 17]. The CINPPT and RGSNPPT expansions provide a good perturbative description of a large class of τ hadronic spectral function moments, including some for which all the standard expansions fail. A program that employs these expansions for the simultaneous determination of the strong coupling and other parameters of QCD from hadronic τ decays is of interest for future investigations.

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